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Banks without Parachutes – Competitive Effects of Government Bail-out Policies

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Banks without Parachutes —
Competitive Effects of
Government Bail-out Policies†

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Abstract: The explicit or implicit protection of banks through government bail-out policies is a universal phenomenon. We analyze the competitive effects of such policies in two models with different degrees of transparency in the banking sector. Our main result is that the bail-out policy unambiguously leads to higher risk-taking at those banks that do not enjoy a bail-out guarantee. The reason is that the prospect of a bail-out induces the protected bank to expand, thereby intensifying competition in the deposit market and depressing other banks’ margins. In contrast, the effects on the protected bank’s risk-taking and on welfare depend on the transparency of the banking sector.

Keywords: Government bail-out, banking competition, transparency, “too big to fail”, financial stability.

JEL-Classification: G21, G28, L11.

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1 Introduction

In most countries, part of the banking sector is protected through implicit or explicit government guarantees. Some of these guarantees, such as deposit insurance, affect all banks more or less in the same way; others privilege a subset of banks, such as public banks, or large banks that are “too big to fail”. Such asymmetric bail-out policies are the subject of our study.

Political and academic discussions have focused on the detrimental effects of such guarantees on the risk-taking behavior of the protected banks. In contrast, the reactions of the remaining banks in the banking system have not been dealt with in the literature. We close this gap by analyzing the competitive effects of government bail-out policies on those banks that do not enjoy a public guarantee. An understanding of other banks’ reactions is indispensable for the judgment of the overall welfare effects of public bail-out policies.

The relevance of such competitive effects can be illustrated with an example from Japan. Since the 1990s the profitability of Japanese private banks has been compromised by thin interest margins. These have been attributed to the competition from government financial institutions as well as from (mostly large) banks receiving disguised subsidies.\(^1\) In particular, private banks face strong competition from Japan’s postal savings system, the biggest deposit taker in the world, which benefits from an explicit government guarantee and tax exemptions and is subject to limited prudential supervision. The extent of welfare losses arising from this type of “unfair competition” (see Fukao, p. 25, 2003b) depends essentially on how smaller private banks adjust their risk-taking in reaction to shrinking profitability due to the subsidization of public and larger banks.

The relationship between banks’ profit margins and their risk-taking is one of the central themes in the literature on competition and stability in the banking sector. The basic idea is that competition tends to reduce rents in the banking sector. In reaction, banks increase their asset risk because of the well-known risk-shifting problem described by Jensen and Meckling (1976). Similarly, public bail-out guarantees to a subset of banks lead to a reduction of rents at the competitor banks. Hence there will be a risk-shifting problem at those banks that are not expected to be bailed out. This effect will be the driving force in our model.\(^2\) It should be noted that the effect of competition on banking stability is sensitive to the way that banking competition is modeled. As shown by Boyd and De Nicoló (2005), there may be a countervailing effect stemming from loan market competition, as will be discussed in the robustness section to our model.

\(^1\)See Fukao (2003a,b) and Kashyap (2002) for an extensive overview of these problems. See also the diagnosis in the Annual Report of the Bank for International Settlements (2002, pp. 133).

\(^2\)A similar mechanism has been used by Acharya (2003) to model international spill-overs between banking systems governed by different regulations. This paper will be discussed in more detail in the literature review.
Our starting point is a paper by Allen and Gale (2000, chapter 8.3), in which the tradeoff between competition and stability is analyzed in a static agency model (see also Allen and Gale (2004)). Because of its clarity and simplicity, the model is well-suited to capture the effect of the size of rents on banks’ risk-taking behavior. Like Allen and Gale, we model competition on the liabilities side of banks’ balance sheets in a Cournot fashion. However, we modify their model by introducing an asymmetric government bail-out policy: some banks are bailed out with higher probabilities than others. In contrast to Allen and Gale, who assume full deposit insurance for all banks, depositors care about the risk of banks’ assets and demand default premia in order to be compensated for expected losses from bank insolvencies.

Moreover, we consider two time structures with different patterns of information revelation: In the first model, banks are opaque in the sense that depositors cannot observe risk before setting deposit rates. Hence, as in Allen and Gale (2000, chapter 8.3), default premia are set before deposit volumes and risk choices are determined. In the second model, we reverse the timing. Depositors can observe their bank’s risk choice and the level of deposits before setting default premia. We call banks transparent in this case.

Our main result is that the government bail-out policy unambiguously leads to higher risk-taking at banks that do not enjoy a government guarantee. The reason is that the subsidization induces the protected bank to expand its deposit volume, no matter whether banks are opaque or transparent. Since deposit volumes are strategic substitutes in our model, the competitor banks react by decreasing their deposit volumes. However, the overall effect on aggregate deposits is positive so that there is an increase in the deposit rate, depressing the competitor banks’ margins and inducing them to take higher risks.

Another important result concerns the protected bank’s risk-taking. Contrary to conventional wisdom, the effect of the guarantee on the protected bank appears to depend on the transparency in the banking sector. In the model with opaque banks, the protected bank may have lower incentives to take risks, because – as in the work of Keeley (1990) and Allen and Gale (2000, chapter 8) – the subsidy increases the bank’s rents. With transparent banks, risk-taking unambiguously increases in the bail-out probability. Here, the argument is similar to that in the literature discussing excessive risk-taking in the context of unfairly priced deposit insurance.

The implications for welfare are ambiguous. Within the setup of our model, a government bail-out policy may increase or decrease welfare ex ante, depending, among other things, on the information structure in the banking sector. Hence welfare effects are much more complicated than suggested by public discussions of government bail-out policies.

\(^3\)Matutes and Vives (2000) use similar time structures in their analysis of the tradeoff between competition and stability.
The paper proceeds as follows: Section 2 contains a brief review of the related literature. In section 3, we derive the competitive effects of an asymmetric bail-out policy for the cases of opaque and transparent banks. In both models, we first analyze the monopoly case with a single bank; we then consider the oligopoly case where banks have different bail-out probabilities. Welfare implications are discussed for each model, and a number of checks for robustness are carried out. Section 4 summarizes our major findings and discusses empirical and normative implications. Proofs are given in the appendix.

2 Related literature

Our paper is related to two strands of literature: first, to the extensive literature on competition and stability in the banking sector; and second, to the literature on the effects of public bail-out guarantees.

A paper by Keeley (1990) was the first of a large number of papers to establish the trade-off between competition and stability in the banking sector. In a simple model, Keeley shows that the reduction of rents through competition exacerbates the risk-shifting problem at banks caused by limited liability or unfairly priced deposit insurance.\(^4\) Hence, the creation of “charter value” (i.e. the present discounted value of future rents) through restrictions on competition can induce banks to refrain from overly risky behavior if the expected loss of the charter value is larger than the expected gains from increased risk-taking. Hellmann, Murdoch, and Stiglitz (2000) and Repullo (2004) discuss regulatory responses.

The work by Keeley has been extended in a number of ways, with differing conclusions about the existence of the presumed tradeoff. Allen and Gale (2000, chapter 8.3) generalize Keeley’s results in a static agency model, confirming the negative relationship between competition and stability. While the tradeoff appears to be robust to the introduction of product differentiation (see Matutes and Vives (2000) and Cordella and Yeyati (2002)), it may break down in the presence of competition in loan markets (and not just deposit markets) (see Koskela and Stenbacka (2000), Caminal and Matutes (2002), and Boyd and De Nicoló (2005)). Dynamic models yield contradictory results (see Hellmann, Murdoch, and Stiglitz (2000), Perotti and Suarez (2002), Repullo (2004), and Allen and Gale (2000, chapter 8.4)).

Similar to the theoretical literature, the empirical literature yields ambiguous results about the trade-off between competition and stability. Keeley (1990) presents some

\(^4\)This literature review is restricted to the papers most closely related to ours. For more detailed surveys on the relationship between competition and stability in banking, see Canoy, van Dijk, Lemmen, de Mooij, and Weigand (2001) and Carletti and Hartmann (2003). Allen and Gale (2004) provide a useful overview of what type of models tends to yield what type of results regarding the sign of the relationship between competition and stability.
evidence for the view that the surge in bank failures in the 1980s in the United States may be explained by the disappearance of monopoly rents in banking due to financial deregulation. Similarly, the accumulation of systemic banking crises in developed and developing countries in the past two decades has been attributed to financial liberalization, which has also been shown to be accompanied by declining charter values in banking (see Demirgüç-Kunt and Detragiache (1999)). In a similar vein, Demirgüç-Kunt, Laeven, and Levine (2004) find that banks’ interest margins are higher in countries with tighter restrictions on competition in banking. In contrast, a recent cross-country study by Beck, Demirgüç-Kunt, and Levine (2003) shows that systemic banking crises are less likely in countries with more concentrated banking sectors, but more likely in countries with tighter restrictions on entry and banking activities. These findings are inconsistent with the “charter value hypothesis,” according to which crises should be less likely in the latter case as well. De Nicoló, Bartholomew, Zaham, and Zephirin (2003) find that the probability of the failure of the five largest banks is positively correlated with bank concentration. This also contradicts the charter value hypothesis.

The second strand of literature related to our paper concerns the effects of public bail-out guarantees. With respect to public banks, the literature is scarce. The most important empirical findings are that government ownership of banks is pervasive all over the world and that it tends to be associated with poorly operating financial systems and slower growth performance (see Barth, Caprio, and Levine (2001) and La Porta, de Silanes, and Shleifer (2002)). The evidence on the relationship between the governmental ownership of banks and banking stability is mixed. Caprio and Martínez Peria (2000) find that government ownership tends to increase bank fragility. In contrast, Barth, Caprio, and Levine (2004) show that government ownership has no robust impact on bank fragility, once one controls for banking regulation and supervisory practices.

There exist a fairly large number of papers on the so-called “too-big-to-fail” (hereafter TBTF) problem. Large banks may be subject to an incentive problem because the public authorities cannot credibly commit to not supporting these banks when failure is impending. The effects on risk-taking are similar to the ones discussed in the deposit insurance literature. In fact, a TBTF policy can be described as a complete insurance of all deposits and other liabilities at zero costs. Since Merton (1977), it has been well-known that unfair deposit insurance entails a risk-shifting problem, similar to the problem arising from limited liability. Hence one may expect that a more concentrated banking sector with TBTF banks entails higher risk-taking at the largest banks, and thus higher fragility. Since a higher concentration implies

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5See Matutes and Vives (1996, 2000) for theoretical analyses of the relationship between deposit insurance, competition and bank stability. Empirical evidence for the adverse effects of deposit insurance on banking stability has been presented by Demirgüç-Kunt and Detragiache (2002).

6Erlenmaier and Gersbach (2001) show that, in the absence of risk-shifting problems, a TBTF policy dominates random bail-out schemes in terms of welfare, whereas the random scheme leads to higher stability.
less competition, this result is just the opposite of what would be predicted by the 
“charter value literature” described above. Our paper aims to resolve this apparent 
contradiction.

The TBTF problem also seems to be an empirically relevant phenomenon. Boyd and 
Gertler (1994) document a TBTF problem at the largest commercial banks in the 
United States in the 1980s. Schnabel (2004, 2005) describes a similar phenomenon 
at the so-called “great banks” in Germany at the time of the Great Depression. 
The episode studied most intensively is the near-failure of Continental Illinois in 
1984 and the consequent public announcement by regulators that the 11 largest US 
banks were too big to be allowed to fail. In an event study, O’Hara and Shaw (1990) 
find significant positive abnormal returns for TBTF banks after the announcement. 
This is consistent with the existence of a positive subsidy to TBTF banks. Studies 
using bond market data tend to confirm the existence of conjectural government 
guarantees (see Flannery and Sorescu (1996) and Morgan and Stiroh (2002)).

Another strand of the empirical literature looks at the question whether the prospect 
of becoming TBTF is a motivation for bank mergers. While Benston, Hunter, and 
Wall (1995) reject this hypothesis for the years 1981 to 1986, the evidence for the 
1990s in Kane (2000) and Penas and Unal (2001) is consistent with the hypothesis.

The main difference between bail-out policies and deposit insurance is that the for-
mer affect different banks asymmetrically if bailout probabilities differ across banks. 
To our knowledge, there exists no paper that explicitly models this asymmetry in the 
context of a domestic banking system with heterogeneous banks. The only related 
empirical finding is by O’Hara and Shaw (1990) who find negative effects on banks 
not included in the list of banks deemed to be “too big to fail;” they attribute this 
finding to the self-financing character of the deposit insurance system.

The paper most closely related to ours is the one by Acharya (2003) on the desirabil-
ity of an international convergence of capital adequacy regulation. Acharya models 
the asymmetric treatment of banks across banking systems governed by different 
regulations. Similar to our paper, the differences in bail-out probabilities across 
countries may lead to competitive spill-overs. However, Acharya does not model 
competition explicitly, but he assumes spill-overs through the banks’ (nonmonetary) 
cost functions, which he interprets as competitive effects. In contrast, our model 
explicitly incorporates an industrial organization model of banking. Interestingly, 
the two models yield quite different results, as will be pointed out below.

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7 The possibility of risk reduction through the commitment to a bail-out policy has also been 
noted by Cordella and Yeyati (2003).

8 This argument is similar to the dynamic arguments in the literature on competition and sta-
bility. It was first stated by Hunter and Wall (1989) and Boyd and Graham (1991).
The basic setup of our model is similar to that in Allen and Gale (2000, chapter 8.3). We consider an economy with \(n\) chartered banks, indexed by \(i = 1, \ldots, n\). Bank \(i\) collects deposits \(d_i\) and invests them in a risky project. Projects yield a return \(y_i\) per invested unit with probability \(p(y_i)\), otherwise they return zero. The success probability is a decreasing and concave function of the target return, i.e. \(p'(y_i) < 0\), and \(p''(y_i) \leq 0\). Each bank can choose the “risk level” of its investment by fixing \(y_i\). The aggregate amount of deposits in the economy is \(D = \sum_{i=1}^{n} d_i\). Depositors demand an expected return \(R(D)\), with \(R'(D) > 0\) and \(R''(D) \geq 0\). Banks and depositors are assumed to be risk neutral.

So far the model is identical to that used by Allen and Gale (2000, chapter 8.3). However, Allen and Gale assume that deposits are fully insured, so that depositors do not need to be concerned about default probabilities. In contrast, we assume that bank \(i\) is bailed out by the government with probability \(\beta_i \in [0; 1]\) in the case of failure. The government can commit itself to this (exogenous) bail-out probability. Given this, depositors are repaid with probability \(p(y_i) + \beta_i (1 - p(y_i))\), while with probability \((1 - \beta_i) (1 - p(y_i))\), they receive nothing. In order to obtain an expected return of \(R(D)\), they demand a nominal return of \(\rho_i R(D)\); the “default premium” \(\rho_i\) will depend on \(\beta_i\) and \(y_i\).

The expected profit of bank \(i\) consists of three factors: the probability of success \(p(y_i)\), the deposit volume \(d_i\), and some “margin” given by the difference between \(y_i\) and the nominal repayment \(\rho_i R(D)\). It hence is a function of four endogenous variables, namely its risk level \(y_i\), its default premium \(\rho_i\), its deposit volume \(d_i\), and the competitors’ deposit volume \(D_{-i} = \sum_{k \neq i} d_k = D - d_i\).

\[
\Pi_i(y_i, \rho_i, d_i, D_{-i}) = p(y_i) [y_i d_i - \rho_i R(d_i + D_{-i}) d_i] = p(y_i) d_i [y_i - \rho_i R(D)].
\] (1)

Within this setting, we define two games characterized by the degrees of transparency in the banking sector, modeled through different time patterns of actions and information revelation (similar to Matutes and Vives (2000)). In each game, we first discuss the monopoly case with \(n = 1\). This yields insights into the banks’ incentives to take risks and expand volume, abstracting from competitive effects. These insights will be useful in the subsequent analysis of the oligopoly case. Finally, we will briefly discuss welfare implications and carry out some robustness checks.
Figure 1: Time structure when banks are opaque

• Depositors (anticipating \(d_i\) and \(y_i\)) set the default premia \(\rho_i\).
• Banks (knowing \(\rho_i\)) choose \(d_i\) and \(y_i\); \(R(D)\) is determined in the deposit market.
• Projects mature and return \(y_i\) with probability \(p(y_i)\). Banks pay \(\rho_i R(D)\) to their depositors if possible. Otherwise, the government pays \(\rho_i R(D)\) with probability \(\beta_i\).

3.1 Opaque banks

In the first model, depositors set the default premia \(\rho_i\) before banks choose their deposit volumes \(d_i\) and their target returns \(y_i\) (see figure 1). This means that depositors cannot exert any market discipline because they cannot react to the actual risk-taking of banks, which is revealed only after depositors have set the default premia. Therefore, we call banks opaque in this case. The model structure is equivalent to a commitment problem, in which banks cannot commit to a particular risk level. If there were a possibility for commitment or if the risk-taking of banks were contractible, depositors could discipline banks by demanding default premia that increase with risk-taking. This time structure generates a moral hazard problem between depositors and banks, known as risk-shifting or asset substitution.

3.1.1 Monopoly

To abstract from competitive effects, we first look at the case with only one bank \((n = 1)\), so that \(D = d_1\). For readability, we omit all indices. As usual, we analyze the problem backwards. First, we determine \(d\) and \(y\) for given \(\rho\). The implicit equations for optimal \(d\) and \(y\) are

\[
\frac{\partial \Pi}{\partial d} = p(y) \left[ y - \rho [R(d) + d R'(d)] \right] = 0 \quad \text{and} \quad \frac{\partial \Pi}{\partial y} = d \left[ p(y) + [y - \rho R(d)] p'(y) \right] = 0.
\]

If a solution to (2) and (3) exists, it is unique given our assumptions on \(R(d)\) and \(p(y)\), yielding the implicit functions \(d(\rho)\) and \(y(\rho)\). Then we can derive the following lemma.

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9Depositors in our model should be thought of as investors who are not (fully) insured through a deposit insurance scheme. Because of the risk neutrality of depositors, \(\beta_i\) can also be interpreted as the fraction of deposits that the government refunds in the case of bank failure.
**Lemma 1 (Optimal \(d\) and \(y\) for given \(\rho\))** The optimal deposit volume \(d\) decreases in the default premium \(\rho\), i.e. \(d'(\rho) < 0\). The optimal risk level \(y\) rises in the default premium \((y'(\rho) > 0)\) if and only if

\[
\frac{R'(d)}{R(d)} < 1 + d \frac{R''(d)}{R'(d)}.
\]

(4)

Intuitively, a rise in \(\rho\) results in a decline in the bank’s profit margin \(y - \rho R(d)\). The bank can react in two ways to compensate for this decline: It can either decrease its volume \(d\), implying a fall in the deposit rate \(R(d)\) and an overall contraction; or it can raise \(y\), at the cost of a falling success probability \(p(y)\). Lemma 1 tells us that the bank will always decide to shrink. In contrast, the bank’s risk-taking behavior depends on the inverse elasticity of the deposit supply (see the left hand side of (4)). Risk-taking increases in the default premium, if the inverse elasticity is sufficiently small, or if the elasticity of the deposit supply is sufficiently large. Hence, if the deposit rate reacts only weakly to a decline in deposits, the bank will increase risk. However, it will decrease risk if the inverse elasticity is large.

Now we turn to the determination of \(\rho\). Anticipating \(d\) and \(y\), depositors set a default premium \(\rho\) to obtain an expected return of \(R(d)\), yielding

\[
\rho = \frac{1}{p(y) + \beta (1 - p(y))}.
\]

(5)

We can now characterize the effects of bail-out policies. An increase in \(\beta\) (i.e. the bank is bailed out with a higher probability) leads to a decrease in \(\rho\). As shown by lemma 1, this induces the bank to expand \((d\) rises), whereas the effect on risk-taking \(y\) depends on the elasticity of deposit supply. In turn, the reaction of \(y\) feeds back into \(\rho\). The following proposition describes the overall reaction of \(\rho\), \(d\), and \(y\).

**Proposition 1 (Effects of bail-out policy in monopoly)** In an opaque monopolistic banking system, an increase in the bail-out probability induces depositors to demand a lower default premium, \(\partial \rho / \partial \beta < 0\). The bank reacts by choosing a higher deposit volume, \(\partial d / \partial \beta > 0\). It chooses a higher risk level \((\partial y / \partial \beta > 0)\) if and only if \(y'(\rho)\) is negative, i.e. if the supply of deposits is sufficiently inelastic.

An increase in \(\beta\) implies that depositors are compensated with a higher probability (or to a higher degree) when there is a bank failure. According to the proposition, this reduces the default premium even after taking into account feedback effects. The results on \(d\) and \(y\) follow directly from lemma 1. As before, the effect on risk-taking is ambiguous and depends on the elasticity of the deposit supply. In particular, risk-taking increases only if the supply of deposits is sufficiently inelastic. This ambiguous result contradicts the conventional wisdom according to which a higher bail-out probability always leads to an increase in risk-taking.\footnote{This result also contradicts proposition 1 in Acharya’s (2003) paper. In fact, the deposit volumes and risk-taking of banks always move in the same directions in that paper. This is driven...}
3.1.2 Oligopoly

Assume now that \( n \) banks have been chartered instead of just one. We are interested in how the market as a whole reacts when the government changes the bail-out policy for one bank. Without a loss of generality, assume that the government raises the bail-out probability \( \beta_1 \) of bank 1.

Assume for the moment that the deposit volume \( D_{-1} \) of competitor banks is given. Then proposition 1 implies that, just as in the monopoly case, the increase in \( \beta_1 \) leads to a fall in the default premium \( \rho_1 \). This induces bank 1 to increase its volume \( d_1 \). The question then is how bank 1's behavior affects the remaining banking sector.

In our model, banks interact only in the deposit market, namely through the deposit rate \( R(D) \). In equilibrium, the deposit volume of each bank must be an optimal reaction to the volume choices of all competitors. Lemma 2 summarizes the strategic interactions in the deposit market.

**Lemma 2 (Strategic interactions in the deposit market)** The reaction function \( d_j(D_{-j}) \) of any bank \( j \) is a strictly decreasing function. Starting from an equilibrium with positive deposit volumes at all banks, an outward shift of one bank’s reaction function leads to an increase in that bank’s deposit volume and a decrease in competitor banks' deposit volumes. The former effect dominates the latter, hence aggregate deposits \( D \) increase.

The first part of lemma 2 implies that deposit volumes are strategic substitutes in our model. Figure 2 plots the reaction functions for a numerical example.\(^{11}\) From proposition 1, we know that the reaction function of bank 1 shifts outward as \( \beta_1 \) increases, while the reaction functions of the competitors remain unchanged.

The second part of lemma 2 implies that an increase in \( \beta_1 \) leads to an expansion of deposits at the subsidized bank and a contraction of deposits at the remaining banks. Finally, the overall effect is an expansion of the aggregate deposit volume. This last point is crucial: It means that the market rate \( R(D) \) increases, implying higher risk-taking by the competitor banks due to shrinking margins. The following proposition sums up actions and reactions of bank 1 and its competitors.

**Proposition 2 (Competitive effects of bail-out policy)** In an opaque banking system, an increase in the bail-out probability \( \beta_1 \) leads to

1. an expansion of deposits at bank 1 and a contraction of deposits at its competitor banks \( j \neq 1 \), \( \partial d_1 / \partial \beta_1 > 0 \) and \( \partial d_j / \partial \beta_1 < 0 \);

by his assumption that there is only one risky investment opportunity, and not a continuous risk choice as in our model.

\(^{11}\)In the example used in the figures, condition (4) is satisfied with equality, so there is no effect on risk-taking in the monopoly case. The effects on risk-taking observed in the oligopoly case can hence be attributed to competitive effects alone.
Figure 2: Reaction functions in the deposit market for varying $\beta_1$

The graph is based on the functions $p(y) = 1 - y$ and $R(d) = d$. For comparability, the same functions are used throughout the paper. Here and in the following graphs, we consider an oligopoly with two banks. Black lines stand for $\beta_1 = \beta_2 = 0.1$, the gray line for $\beta_1 = 0.13$. The equilibria are indicated by dotted lines. Notice that the increase in $d_1$ is larger than the decrease in $d_2$, so that aggregate deposits increase.

2. an increase or decrease in risk at bank 1, depending on the elasticity of the deposit supply; in all cases where risk decreases in the monopoly, it decreases also in the oligopoly, while the opposite is not true; in either case, the default premium falls, $\partial \rho_1 / \partial \beta_1 < 0$;

3. an increase in risk at the competitor banks $j$, accompanied by higher default premia, $\partial y_j / \partial \beta_1 > 0$ and $\partial \rho_j / \partial \beta_1 > 0$.

The most notable results are the ones on risk-taking: There is an unambiguous increase in the risk-taking of the competitor banks in response to a higher bail-out probability of one bank, whereas the risk at the protected bank may increase or decrease. In fact, there is a lower tendency for the protected bank to take risks than when there is a monopoly.

Proposition 2 is illustrated in figure 3 for a numerical example with two banks. As the bail-out probability $\beta_1$ rises, bank 1 raises its deposit volume. At the same time, bank 2 is crowded out. Due to a lower nominal deposit rate $\rho_1 R(D)$, bank 1 reduces its riskiness. For bank 2, the nominal rate rises, leading to higher risk-taking. If bail-out policies become too asymmetric, the less protected bank’s incentives to take risks become overwhelming, inducing depositors to demand ever higher default premia.

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12The result on the risk-taking of competitor banks here again differs from the effect discussed in Acharya (2003). There, the risk-taking of the banks in one country may also decrease in reaction to higher forbearance in another country.
which in turn fuel risk-taking, so that the process reaches no new equilibrium. The less protected bank closes. If there are only two banks initially, the other bank is left with a monopoly as described in section 3.1.1. At that point, deposits at the remaining bank jump up and risk-taking drops.

### 3.1.3 Welfare considerations

We now briefly discuss some welfare implications of our model. A fully-fledged welfare analysis is beyond the scope of this paper because we have not modeled the benefits of bail-outs, such as the avoidance of contagion and systemic crises. Also, distortions from taxation to finance bail-outs would have to be taken into account.

Three factors in our model are relevant for welfare assessments: risk-taking, the level of deposits, and the entrance or exit of banks. Banks’ repayments to depositors and the bail-out payments from the government are welfare-neutral in our model.

Let us consider the increase in the bail-out probability of one bank. In the case of a relatively inelastic deposit supply, welfare is reduced by the overall increase in risk-taking, which is always excessive in this model. If the deposit supply is sufficiently elastic, the effect of risk-taking on welfare is ambiguous. The aggregate increase in deposits tends to increase welfare as long as expected returns $y_i p(y_i)$ outweigh $R(D)$. However, banks may grow excessively due to subsidization through bail-outs; then an expansion of deposits reduces welfare. Finally, an asymmetric bail-out policy may crowd out the least protected banks completely, leading to a drop in the risk-taking of the remaining banks and in aggregate deposits. The effect of banks’ exit on welfare depends on the relative impact of the deposit contraction and risk reduction. Given the multitude of effects going in different directions, the overall effect of bail-out policies is ambiguous in the opaque model.
Banks (anticipating $\rho_i$) choose $d_i$ and $y_i$.
Depositors (knowing $d_i$ and $y_i$) set the default premia $\rho_i$; $R(D)$ is determined in the deposit market.
Projects mature and return $y_i$ with probability $p(y_i)$. Banks pay $\rho_i R(D)$ to their depositors if possible. Otherwise, the government pays $\rho_i R(D)$ with probability $\beta_i$.

3.2 Transparent banks

In our second model, the time structure is reversed (see figure 4), so that depositors observe banks’ risk choices before setting default premia. Here depositors can (and do) exert market discipline. We call banks transparent in this case. In making their risk choices, banks take into account that they will be punished for excessive risk-taking. Again the model could alternatively be phrased in terms of a commitment problem. If banks could commit to a certain risk level or if risk-taking were contractible, default premia would depend directly on the level of risk-taking, exerting discipline on banks.

We assume that the government does not condition its policy on banks’ risk levels $y_i$. This assumption may be motivated by the time inconsistency problem that the government faces. Banks are bailed out because this is ex post optimal for the economy, even if they have taken excessive risks before they failed. An exogenous, unconditional bail-out probability may be a good approximation of such a policy.

The disciplining effect of this time structure can be seen most clearly in the extreme case of $\beta_i = 0$, i.e. the bank is never bailed out. In this case, depositors demand a default premium $\rho_i = 1/p(y_i)$, and expected profits of the bank are given by $\Pi_i = d_i (y_i p(y_i) - R(D))$. Then the bank’s optimal $y_i$ is equal to the first-best solution because the bank itself must bear the entire cost from excessive risk-taking. If $\beta_i > 0$, there is again a risk-shifting problem because the costs of the implicit government guarantee are not completely borne by the bank. To keep our proofs tractable, we assume that $R(D)$ and $p(y_i)$ are linear. Furthermore, we assume that $\beta_i > 0$ is not too large, i.e., $\beta_i \approx 0$. In section 3.3, we will discuss the robustness of our results to these assumptions.

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13 This assumption is stronger than actually needed. A sufficient assumption would be that $R(D)$ and $p(y_i)$ do not bend too much in the neighborhood of the equilibrium.
3.2.1 Monopoly

Again, we start with the case of a monopolistic bank to abstract from competitive effects. First, we determine the default premium $\rho$, taking the level of deposits $d$ and the bank’s risk choice $y$ as given. The expression for $\rho$ looks exactly the same as in (5),

$$\rho = \frac{1}{p(y) + \beta \left(1 - p(y)\right)}.$$  \hspace{1cm} (6)

If $\beta$ is smaller than one, depositors charge a higher default premium for higher $y$. In the extreme case, with $\beta = 1$, the default premium is independent of the chosen risk level, with $\rho = 1$. In contrast, the default premium does not directly depend on the deposit volume. Furthermore, $\rho$ depends negatively on the bail-out probability $\beta$. We can directly incorporate the default premium into the profit function (1),

$$\Pi = p(y) d \left( y - \frac{R(d)}{p(y) + \beta \left(1 - p(y)\right)} \right).$$ \hspace{1cm} (7)

Now we determine the optimal deposit volume $d$ and risk level $y$ by maximizing (7),

$$\frac{\partial \Pi}{\partial d} = p(y) \left[ y - \rho(y) \left[R(d) + d R'(d)\right] \right] \equiv 0,$$ \hspace{1cm} (8)

$$\frac{\partial \Pi}{\partial y} = d \left[ p'(y) \left[ y - \rho(y) R(d) \right] + p(y) \left[ 1 - \rho'(y) R(d) \right] \right] \equiv 0.$$ \hspace{1cm} (9)

If a solution to (8) and (9) exists, it is unique given our assumptions on $R(d)$ and $p(y)$. The two first-order conditions yield the implicit functions $d(y)$ and $y(d)$. The equilibrium is given by the intersection of these two functions. We are interested in the reactions of the optimal $d$ and $y$ to a change in $\beta$.

For given $d$ and $y$, the default premium $\rho$ will decrease if $\beta$ increases. This means that risk-taking becomes less costly for the bank. Therefore, it will increase risk in anticipation of the less severe punishment for risk-taking. The decrease in $\rho$ and the increase in $y$ increase the bank’s margin, which induces the bank to increase its deposit volume. At the same time, larger margins tend to lower risk, leading to a countervailing effect on $y$. Moreover, the changes in $y$ feed back into the default premium $\rho$. The following proposition characterizes the overall effects of an increase in the bail-out probability on the equilibrium choices of $\rho$, $d$, and $y$.

**Proposition 3 (Effects of bail-out policy in monopoly)** In a transparent monopolistic banking system, an increase in the bail-out probability induces the bank to raise its deposit volume $d$ and its risk level $y$, i.e. $\partial d/\partial \beta > 0$ and $\partial y/\partial \beta > 0$. The default premium $\rho$ decreases ($\partial \rho/\partial \beta < 0$) if $R(d)$ is small enough (i.e. if $R(d) < 2 y p(y) \left(1 - p(y)\right)$).
The proposition shows that the direct effect on risk always dominates the indirect effects. The monopolistic bank increases its risk in reaction to the subsidization by the government because part of the potential losses can be shifted to the government. The effect on $\rho$ is ambiguous: The rise in $\beta$ tends to lower $\rho$, whereas the rise in $y$ tends to raise $\rho$. Proposition 3 states that the effect of $\beta$ dominates for small $R(d)$. Finally, the overall effect on the bank’s margin is positive, inducing the bank to expand its deposit volume.

3.2.2 Oligopoly

Now we turn to the oligopoly case with $n$ banks. Analytically, we simultaneously solve the system of equations $(\partial \Pi_i / \partial d_i = 0)_i$ and $(\partial \Pi_i / \partial y_i = 0)_i$, $i = 1, \ldots, n$. Again we assume without loss of generality that the government raises $\beta_1$.

The chain of reactions is almost identical to the one in the opaque model in section 3.1.2. There is a direct effect on bank 1, as described in proposition 3. Thereby, the rise in $\beta_1$ leads to an increase in bank 1’s risk and deposit volume, taking the deposit volumes of competitors as given. As before, the behavior of bank 1 spills over to the other banks through the deposit market. Lemma 3 describes the strategic interactions in the deposit market.

**Lemma 3 (Strategic interactions in the deposit market)** The reaction function $d_j(D_{-j})$ of any bank $j$ is a strictly decreasing function. Starting from an equilibrium with positive deposit volumes at all banks, an outward shift of one bank’s reaction function leads to an increase in that bank’s deposit volume and a decrease in competitor banks’ deposit volumes. The former effect dominates the latter, hence aggregate deposits $D$ increase.

This lemma is identical to lemma 2, and so are the mechanisms at work. As before, deposit volumes are strategic substitutes. In fact, banks’ reactions functions look like the ones in figure 2, with slightly different slopes. Hence an increase in $d_1$ is again accompanied by a decrease in $d_j$ for $j \neq 1$. The overall effect on total deposits is positive. As a result, the market rate rises, and competitors’ risk levels increase accordingly. Proposition 4 summarizes the reactions of bank 1 and its competitors to an increasing bail-out probability of bank 1.

**Proposition 4 (Competitive effects of bail-out policy)** In a transparent banking system, an increase in the bail-out probability $\beta_1$ leads to

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14Note that the condition given in the proposition applies to the case where $\beta$ is small and $y$ is close to the first-best level of risk. For larger $\beta$, the condition may be overly restrictive. In our numerical simulations, $\rho$ decreases in $\beta$ globally, even when the condition is violated.
1. an expansion of deposits at bank 1, and a contraction of deposits at its competitor banks $j \neq 1$, i.e. $\partial d_1/\partial \beta_1 > 0$ and $\partial d_j/\partial \beta_1 < 0$;

2. an increase in risk at bank 1, i.e. $\partial y_1/\partial \beta_1 > 0$; the default premium decreases ($\partial \rho_1/\partial \beta_1 < 0$) if $R(D)$ is small enough;

3. an increase in risk at the competitor banks $j$, accompanied by higher default premia if $\beta_j > 0$, i.e. $\partial y_j/\partial \beta_1 > 0$ and $\partial \rho_j/\partial \beta_1 > 0$; for $\beta_j = 0$, $y_j$ and $\rho_j$ are constant.

The proposition is illustrated in figure 5 for the case of two banks. As the bail-out probability $\beta_1$ rises, bank 1 expands, while bank $j$ is crowded out. Since the increase in $d_1$ is larger than the decrease in $d_j$, the aggregate deposit volume increases, leading to a higher market rate $R(D)$. As a result, the risk-shifting problem at bank $j$ is exacerbated. In the case of transparent banks, $y_1$ also increases unambiguously. Interestingly, the effect of competition on risk-taking may be even stronger than the direct effect: For large $\beta_1$, $y_2$ exceeds $y_1$ in the numerical example.$^{15}$

The most important result is that the competitive effects of the bail-out policy on the remaining banking sector are independent of the time and information structure of the model: In both of our models, the subsidized bank expands, causing a rise in the market rate, which aggravates the risk-shifting problems at competitor banks.

### 3.2.3 Welfare considerations

The factors affecting welfare are the same as in the opaque banking system. Again we consider an increase in the bail-out probability of one bank. In the transparent

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$^{15}$To demonstrate this possibility, we chose a different $\beta_2$ in the graphs than in the model with opaque banks. For $\beta_2 = 0.25$, the two curves just touch at $\beta_1 = 1$. 

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Figure 5: Effects of an asymmetric bail-out policy

![Diagram](image-url)
model, the increase in risk-taking at all banks clearly decreases welfare. The effect of an expansion of deposits is again ambiguous. However, the welfare gain due to the aggregate deposit expansion can outweigh the welfare losses from excessive risk-taking only if the number of banks is small. If the economy is sufficiently competitive (i.e. \( n \) is sufficiently large), welfare will decrease even for small \( \beta \). Finally, less protected banks may leave the market if asymmetries become too large. Summing up, bail-out policies tend to decrease welfare in the transparent model if the number of banks in the system is sufficiently large.

3.3 Robustness

In this section, we discuss various extensions of our model to check whether our results are robust when the main assumptions are relaxed.

**Loan market competition** Our model assumes that banking competition is taking place in the market for deposits only. However, Boyd and De Nicoló (2005) have shown that the impact of banking competition on stability may be reversed if one allows for loan market competition.

Boyd and De Nicoló point out that the relationship between banking competition and stability depends on whether banks are facing a *portfolio problem* or an *optimal contracting problem*. Most papers on the trade-off between competition and stability have used the first approach, where banks directly choose the riskiness of their portfolios. In contrast, the second approach assumes that the banks’ borrowers, not the banks themselves, choose among projects, and face a moral hazard problem. Boyd and De Nicoló show that the relationship between competition and stability may be positive in that case.

Let us discuss both approaches in the context of our model. For the sake of simplicity, we abstract from deposit market competition. If banks face a *portfolio problem*, the introduction of loan market competition does not alter our main results. The protection of one bank lowers its refinancing costs, and causes it to become more aggressive in the loan market. The protected bank grows, whereas the remaining banks shrink. Overall, the loan volume will rise, and loan rates will drop. The risk-taking behavior of the protected bank will depend on the transparency of the system, just as in our model. The competitor banks get lower profit margins, which induces them to take more risk, independent of transparency.

If banks face an *optimal contracting problem*, our results are overturned. For the same reasons as above, the protection of one bank will induce a drop in loan rates. However, this drop induces the banks’ borrowers to take fewer risks. This reduces the portfolio risk at all banks, independent of the transparency of the system.
Summing up, our model is robust to the introduction of loan market competition as long as banks face a portfolio problem. However, the results are overturned if banks face an optimal contracting problem. In reality, both types of problems may be relevant. In such a setting, the aggregate result will depend on the relative strength of the two effects.

Price competition Another extension is to model the strategic interactions among banks as price competition with product differentiation or transportation costs.\textsuperscript{16} We believe that, in such a model, the protected bank’s risk-taking behavior would still depend on the system’s transparency, and that competitor banks would again be pushed towards higher risk-taking.

Suppose, for example, that banks are located on a Salop circle and that the bail-out probability of one bank increases. Then for given deposit rates of the competitors, the protected bank is in a local monopoly, hence it should react as discussed in the monopoly sections 3.1.1 and 3.2.1: It becomes more aggressive, and risk-taking should depend on transparency. As a result, the neighboring banks find themselves threatened by more competition from the protected bank. They may react by increasing their deposit rates to regain some “territory.” This effect spills over to the neighbors of the neighbors, and so forth. In equilibrium, all competitor banks lose some territory and increase their deposit rates; this is accompanied by higher risk-taking.

Continuous variation of transparency So far, we have looked at two extreme cases of bank transparency. In section 3.1, banks were perfectly opaque, so that risk choices were not observable at all; in section 3.2, banks were perfectly transparent, and risk was observable without any distortion. We now discuss what happens in the range between the two extremes.

Assume the following time structure: Banks fix their risk levels \( y_i \) before depositors set their default premia \( \rho_i \). However, depositors learn \( y_i \) only with probability \( \eta \). Then for \( \eta = 0 \), we are back in the perfectly opaque case, whereas for \( \eta = 1 \), banks are perfectly transparent. Instead of solving this model formally, we have run a large number of simulations. All these simulations confirm our main results: The competitor banks increase their risk, independently of \( \eta \). In addition, higher transparency amplifies the risk-taking of protected banks in reaction to a higher bail-out probability. Formally, the simulations suggest that the cross-derivative \( \frac{\partial^2 y_1}{\partial \beta_1 \partial \eta} \) is positive.

\textsuperscript{16}This has been done by Matutes and Vives (2000) and Cordella and Yeyati (2002), however without considering government bail-outs.
Tractability assumptions In the section on transparent banks, we have proven the main results only under the assumptions of linear $R(D)$ and $p(y)$, and small $\beta$. Therefore, we ran a large number of simulations to support the claim that our main results still hold in the case of strictly convex $R(D)$, strictly concave $p(y)$, and large $\beta$. Again the simulations confirmed our main results: If the protection of one bank increases, all competitors shrink and take more risks; the risk-taking behavior of the protected bank depends on the transparency of the system.

Dynamic model Our static model neglects the incentive effects arising from the banks’ charter values in a dynamic setting. A simple iteration yields a dynamic version of our model. Then, the expectation of future profits may induce banks to refrain from risk-taking. An increase in the bail-out probability of one bank increases the protected bank’s current and future profits, and hence its “charter value.” As a result, this bank will show an even lower tendency for risk-taking than in the one-shot game. This implies that, even in a transparent setting, in response to more protection the protected bank may take fewer risks. In contrast, the current and future profits of the unprotected banks drop, implying a decline in charter values. Therefore, the incentives of the unprotected banks for higher risk-taking are still reinforced. Even in a multi-period setting, the protection of one bank will increase the risk-taking of the competitor banks.

Endogenous bail-out probabilities Finally, we have assumed that bail-out probabilities are exogenous. In the case of public banks, this assumption seems to be reasonable. However, in the case of a “too-big-to-fail” policy, the bail-out probability should depend on the size of banks. We believe that our results are reinforced if bail-out probabilities depend on size. In that case, we would get an additional strategic effect. Since high bail-out probabilities are beneficial for banks, a strategic tendency towards increased volume would develop. This raises the deposit rate, exacerbating the risk-shifting problem.

4 Conclusion

We have started from the question of how government bail-out policies affect competition in the banking sector. While the existing literature has focused on the effects of bail-out policies on the bank that enjoys the public guarantee, we are mainly interested in the competitive effects of such policies on the remaining banking sector.

We have presented two models, differing with respect to their time and information structures. In the first model, with opaque banks, risk-taking is unobservable by depositors. Therefore, a bank’s risk choice does not directly affect its refinancing costs. In the second model, with transparent banks, investments are perfectly observable. As a consequence, deposit rates react promptly to a bank’s risk choice.
Our main contribution has been in showing that an increase in the bail-out probability of one bank unambiguously leads to an increase in the risk-taking of the competitor banks. At the same time, competitor banks are crowded out. The competitive effects are particularly strong in an opaque setting. This is due to a multiplier effect, which reinforces the original effect of risk-taking on the default premium through the feedback from the default premium on risk-taking.

The effect on the protected bank’s risk-taking depends, among other things, on the degree of transparency in the banking system. If banks are opaque, the protected bank may take less risk, while it always assumes more risk in a transparent environment (at least in a static setting). This adds a qualification to the existing literature, which suggests that an increase in the bail-out probability always leads to higher risk-taking at the protected bank. The welfare effects of bail-out policies are ambiguous. Welfare may increase or decrease, depending on the transparency of the banking system, the degree of competition within the system, the degree of protection, and the asymmetry of banks.

Our results have proven to be robust to a number of modifications. The only caveat concerns the modeling of banking competition (see Boyd and De Nicoló (2005)): If there is loan market competition and banks’ behavior is best described by an optimal contracting problem, an increase in the bail-out probability of one bank may make all banks less risky.

Our paper raises interesting empirical issues, concerning e.g. the role of transparency for banking stability. Most empirical studies on the risk-taking of large banks find that these banks tend to be riskier than smaller banks (e.g. De Nicoló (2000) and Boyd and Graham (1998)). However, most of these studies have used data on developed countries, which may be expected to have a higher degree of transparency. One interesting question for future research is whether the risk behavior of protected banks depends on the transparency of the system. Moreover, our results may help to interpret some of the findings of Beck, Demirgüç-Kunt, and Levine (2003). If the indicators of the institutional quality in banking used in that study measure transparency (or market discipline), rather than competition, it is not surprising that these indicators reduce fragility. The positive relationship between concentration and stability may indicate that the banking systems are, on average, quite opaque. Hence, the results do not necessarily contradict the charter value hypothesis. An interesting way to extend this research would be to interact the concentration measure with an indicator of bank transparency. The empirical exploration of the effects of transparency on the trade-off between banking competition and stability seems to be one fruitful area for future research.

Another interesting question is whether the competitive effects of public bail-out policies can be confirmed empirically. Anecdotal evidence on Japan, mentioned in the introduction, and on France, cited by Acharya (2003), seems to support our main results. Similarly, the negative stock price reactions of smaller banks to the announcement of larger banks being “too big to fail” is consistent with our
model (see O’Hara and Shaw (1990)). In future research, this question is still to be systematically analyzed.

The normative implication of our model is that governments should refrain from bail-out policies, especially in transparent banking markets. The overall welfare effects of such policies are highly ambiguous, and the effects on the competitor banks are always detrimental. Only the subsidized bank stands to profit, at the cost of increased instability in the remaining banking sector. Regulatory initiatives aiming at greater transparency should be accompanied by a commitment not to bail out banks. Since our results are driven solely by market expectations of bail-outs (and not by actual bail-outs), the government should try to build up a reputation of being committed to a “zero bail-out policy.” Market transparency and government intervention are substitutes for one another; they should not prevail at the same time.

A Appendix

A.1 Proofs for opaque banks

Proof of lemma 1: Treating $d$ and $y$ as functions of $\rho$, we take the derivatives of (2) and (3) with respect to $\rho$, and get

$$y'(\rho) - R(d) - \left[ d + 2 \rho d'(\rho) \right] R'(d) - d \rho d''(\rho) R''(d) = 0,$$

$$\left[ y - \rho R(d) \right] y'(\rho) p''(y) + \left[ 2 y'(\rho) - R(d) - \rho d'(\rho) R'(d) \right] p'(y) = 0.$$

Solving for $d'(\rho)$ and $y'(\rho)$ yields

$$d'(\rho) = \frac{1}{-\rho p''(y) \left[ y - \rho R(d) \right] [2 R'(d) + d R''(d)] + p'(y) [3 R'(d) + 2 d R''(d)]} < 0,$$

$$y'(\rho) = \frac{p'(y) R(d) [R'(d) + d R''(d)] - d [R'(d)]^2}{\text{same denominator as above}}.$$

All terms in square brackets are positive. Because $p'(y) < 0$ and $p''(y) \leq 0$, $d'(\rho)$ must be negative. This proves the first part of the lemma. The denominator of $y'(\rho)$ is negative and $p'(y)$ is negative; hence $y'(\rho)$ is positive if $R(d) [R'(d) + d R''(d)] - d [R'(d)]^2$ is positive. (4) is obtained by a simple transformation.

Proof of proposition 1: Again we make use of the implicit function theorem. (5) can be rewritten as

$$1 = \rho [p(y) + \beta (1 - p(y))]. \quad \text{(A1)}$$

Treating both $y$ and $\rho$ as functions of $\beta$ and taking derivatives yields

$$0 = \rho'(\beta) [p(y) + \beta (1 - p(y))] + \rho [1 - p(y) + (1 - \beta) p'(y) y'(\rho) \rho'(\beta)].$$
Now we solve for $\rho'(\beta)$,

$$
\rho'(\beta) = -\frac{1 - p(y)}{(1 - \beta) p'(y) y'(\rho) + \rho^{-2}}.
$$

If $y'(\rho)$ is negative, we see immediately that the whole expression is negative. If $y'(\rho)$ is positive, the denominator is positive for $\beta = 1$, hence the whole expression is again negative. For smaller $\beta$, the denominator shrinks, and it may eventually become zero. This is the point at which the moral hazard problem becomes so severe that the bank can no longer collect deposits: The nominal deposit rate would induce risk-shifting, which in turn would raise deposit rates, and this vicious circle would not come to an end. Here, $\rho'(\beta)$ becomes minus infinity. We conclude that in the region where an inner solution exists, $\rho'(\beta)$ is negative even if $y'(\rho)$ is positive. This proves the first part of the proposition.

Since $\beta$ affects $d$ and $y$ only through $\rho$, $d'(\beta)$ can be written as $d'(\beta) = d'(\rho) \rho'(\beta)$, and $y'(\beta)$ as $y'(\beta) = y'(\rho) \rho'(\beta)$. Given our results from lemma 1 and the negative sign of $\rho'(\beta)$, we directly get the results from the second part of the proposition. ■

**Proof of lemma 2:** We first show that a bank’s deposit volume shrinks as competitors’ deposit volumes expand. Consider the profits of bank $i$ ($i = 1, \ldots, n$),

$$
\Pi_i = p(y_i) d_i \left[ y_i - \rho_i R(d_i + D_{-i}) \right],
$$

where $D_{-i}$ is the aggregate deposit volume of bank $i$’s competitors. For given $\rho_i$, the first-order conditions for bank $i$’s behavior are

$$
y_i - \rho_i \left[ R(d_i + D_{-i}) + d_i R'(d_i + D_{-i}) \right] = 0 \quad \text{and} \quad p(y_i) + [y_i - \rho_i R(d_i + D_{-i})] p'(y_i) = 0.
$$

Treating $d_i$ and $y_i$ as functions of $D_{-i}$ and taking the derivatives with respect to $D_{-i}$ yields

$$
y_i'(D_{-i}) - \rho_i \left[ (1 + 2 d_i'(D_{-i})) R' + (1 + d_i'(D_{-i})) d_2 R'' \right] = 0,
y_i'(D_{-i}) p'(y_i) + p'(y_i) [y_i'(D_{-i}) - \rho_i (1 + d_i'(D_{-i})) R']
+ [y_i - \rho_i R] y_i'(D_{-i}) p''(y_i) = 0,
$$

where $R$ always stands for $R(d_i + D_{-i})$. Now we solve for $d_i'(D_{-i})$, yielding

$$
d_i'(D_{-i}) = -\frac{(y_i - \rho_i R) [R' + d_i R''] p''(y_i) + [R' + 2 d_i R''] p'(y_i)}{(y_i - \rho_i R) [2 R' + d_i R'''] p''(y_i) + [3 R' + 2 d_i R'''] p'(y_i)},
y_i'(D_{-i}) = \frac{\rho_i p'(y_i) (R')^2}{\text{same denominator}}.
$$

All the terms in square brackets are positive, so that $d_i'(D_{-i}) < 0$ and $y_i'(D_{-i}) > 0$. Because the terms in the square brackets in the numerator of $d_i'(D_{-i})$ are smaller
than those in the denominator, we have $|d_i'(D_{-i})| < 1$. The (marginal) contraction in deposits is smaller than the expansion of $D_{-i}$.

So far, we have only considered the direct effects of a change in the competitors’ volumes, holding $\rho_i$ constant. However, a rise in $y_i$ will translate into a rise in $\rho_i$, which in turn affects $d_i$ and $y_i$. According to lemma 1, $d_i$ will decrease, so the direct effect is reinforced by the indirect effect, and the overall effect on $d_i$ is negative, which proves the first part of lemma 2.

So far, we have shown that an increase in the deposit volume of one bank is accompanied by a decrease in deposits at the competitor banks. We now show that the aggregate effect on deposits is positive. The initial equilibrium levels of deposits are denoted by $d^*_1, \ldots, d^*_n$. When $\beta_1$ rises marginally, equilibrium levels adjust to $d^{**}_1, \ldots, d^{**}_n$. We have already shown that the direct effect of the rise of $\beta_1$ is an expansion of $d_1$, and that this leads to a contraction of $d_2, \ldots, d_n$. However, this contraction will never overcompensate the increase in $d_1$: When $d_2, \ldots, d_n$ contract so much that the original $D^*$ is reached again, for each bank $j \neq 1$, the choice $d^*_j$ would again be optimal. This contradicts the fact that deposit volumes have decreased. Since bank 1’s deposit volume rises in any case, $D$ has to expand.

As shown above, the contraction of $d_2, \ldots, d_n$ leads to a further expansion of $d_1$, which again entails a further contraction of $d_2, \ldots, d_n$. Eventually, this convergence process comes to an end. Possibly, $d_j = 0$ for some $j \in \{2, \ldots, n\}$. The contraction of $d_2, \ldots, d_n$ will never overcompensate the expansion of $d_1$, hence in the new equilibrium

$$d^{**}_1 > d^*_1, \quad d^{**}_j < d^*_j \quad \text{for all } j \in \{2, \ldots, n\}, \quad \text{and} \quad D^{**} > D^*.$$ 

This proves our second assertion. $\blacksquare$

**Proof of proposition 2:** The direct effects of an increase in $\beta_1$ on bank 1 are the same as in a monopoly: $\rho_1$ declines, $d_1$ rises, and the effect on $y_1$ is ambiguous.

An application of lemma 2 proves that $d_j$ (for $j \neq 1$) decreases in reaction to the increase in $d_1$. Then, using the same procedure as in the proof of proposition 1, we can show that $\rho_j$ increases. Writing $y_j$ as a function of $\rho_j$ and $D_{-j}$, and $\rho_j$ as a function of $D_{-j}$, we take the derivative of (A1) to get

$$\frac{\rho_j'(D_{-j})}{\rho_j} + \rho_j (1 - \beta_j) p'(y_j) \frac{\partial y_j}{\partial \rho_j} \rho_j'(D_{-j}) + \frac{\partial y_j}{\partial D_{-j}} = 0. \quad (A2)$$

Solving for $\rho_j'(D_{-j})$ yields

$$\rho_j'(D_{-j}) = \frac{\rho_j^2 (1 - \beta_j) p'(y_j) \partial y_j / \partial D_{-j}}{1 + \rho_j^2 (1 - \beta_j) p'(y_j) \partial y_j / \partial \rho_j}. \quad (A3)$$

The numerator of this expression is negative. Regarding the denominator, we can make the same argument as above to prove that it is positive for both negative and
positive $y'_j(\rho_j)$. This proves that $\rho'_j(D_{-j})$ is positive. Since $\beta_j$ is constant, it follows directly that $y_j$ must also increase when the competitors’ volume increases (this can also be seen from (A2)). Note that this is also true when $y_j$ decreases in response to the increase in $\rho_j$. The feedback from $\rho_j$ into $y_j$ never overcompensates the direct effect that was shown to be positive in the proof of lemma 2.

The decrease in the competitor banks’ deposit volumes induces bank 1 to increase its deposit volume even further, and so on. This reinforces the effect on bank 1’s deposit volume as well as all the effects on the competitor banks. Similarly, the initial effect on $\rho_1$ is reinforced by the decrease in $D_{-1}$: (A3) shows that $\rho_1$ decreases even for constant $\beta_1$, so the decrease will be even stronger for rising $\beta$. The initial effect on risk-taking $y_1$ depends again on the inverse elasticity of the deposit supply. However, the inverse elasticity of individual supply is smaller than that of aggregate supply,

$$\frac{\partial R(D)}{\partial d_1} \frac{d_1}{R(D)} = \left(1 + \frac{\partial D_{-1}}{\partial d_1}\right) \frac{R'(D)}{R} < \frac{R'_1(D)}{R_1} = \frac{\partial R(D)}{\partial D} \frac{D}{R(D)}.$$  

This means that the condition in (4) is more likely to be satisfied. So there is a stronger tendency for the protected bank to take risks in response to an increase in $\rho_1$ than in a monopoly. In turn, this implies that there is a lower tendency for the protected bank to take risks in response to an increase in $\beta_1$ because this induces $\rho_1$ to fall. Again the indirect effects reinforce the initial effect. This completes the proof of proposition 2. ■

A.2 Proofs for transparent banks

Proof of proposition 3: The proof of proposition 3 falls into three steps. In a first step, we characterize the function $d(y)$, in a second step, the function $y(d)$. We then show how these curves shift in reaction to a change in $\beta$.

Step 1. We first show that, for $\beta > 0$, $d(y)$ is strictly increasing in the neighborhood of the optimal $y$, i.e. $d_y > 0$. If $\beta = 0$, then $d_y = 0$.

From the first-order condition in (8), we can derive the slope of the function $d(y)$,

$$d_y = \frac{\partial d}{\partial y} = \frac{1 - \rho'(y) [R(d) + d R'(d)]}{\rho(y) [2 R'(d) + d R''(d)]}. \tag{A4}$$

Substituting for $\rho(y)$ in (A4), we get

$$d_y = \frac{\beta + (1 - \beta) [p(y) + y p'(y)]}{2 R'(d) + d R''(d)} = \frac{\beta + (1 - \beta) [p(y) + y p'(y)]}{2 R'(d)}$$

if $R(d)$ is linear, as was assumed for the transparent model. If $\beta$ is small, then $p(y) + y p'(y) \approx 0$. Hence, for $\beta > 0$, we get

$$d_y \approx \frac{\beta}{2 R'(d)} > 0,$$  

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Figure A1: Risk levels and deposit volumes for varying $\beta$ in the monopoly

![Figure A1: Risk levels and deposit volumes for varying $\beta$ in the monopoly](image)

Equilibria (i.e. intersections of curves pertaining to the same $\beta$) are denoted by the solid black curve.

and $d_y = 0$ for $\beta = 0$, which was to be shown.

Figure A1 displays $d(y)$ for varying $\beta$. As predicted, $d(y)$ increases in the neighborhood of the first-best solution; it is locally constant only for $\beta = 0$.

**Step 2.** We now show that, for $\beta > 0$, $y(d)$ is strictly increasing, i.e. $y_d > 0$. If $\beta = 0$, then $y_d = 0$.

Plugging $\rho(y)$ into (9), we get

$$0 = [p(y) + y p'(y)] [p(y) + \beta (1 - p(y))]^2 - \beta p'(y) R(d). \quad (A5)$$

Taking the derivative with respect to $d$ and assuming linear $p(y)$ yields

$$0 = y_d [2 p'(y) + y p''(y)] [p(y) + \beta (1 - p(y))]^2 + [p(y) + y p'(y)] 2 (1 - \beta) y_d p'(y) [p(y) + \beta (1 - p(y))] - \beta y_d p''(y) R(d) + p'(y) (R'(d)) = y_d [2 p'(y)] [p(y) + \beta (1 - p(y))]^2 + [p(y) + y p'(y)] 2 (1 - \beta) y_d p'(y) [p(y) + \beta (1 - p(y))] - \beta (p'(y) R'(d)). \quad (A6)$$

Solving for $y_d$ and assuming that $\beta$ is small, and hence $p(y) + y p'(y) \approx 0$, we get

$$y_d = \frac{\beta R'(d)}{2 [p(y) + \beta (1 - p(y))]^2}.$$ 

If $\beta > 0$, $y_d > 0$; if $\beta = 0$, $y_d = 0$, which was to be shown.

Figure A1 displays the function $y(d)$ for different choices of $\beta$. $y(d)$ increases in $d$ if $\beta > 0$; it is globally constant at the first-best $y$ if $\beta = 0$. 

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Step 3. In the final step, we show how the curves \( d(y) \) and \( y(d) \) move in the \((y,d)\)-space when \( \beta \) increases. Then we analyze the corresponding changes in the equilibrium values of \( y \) and \( d \).

We first show that, ceteris paribus, the deposit volume \( d \) increases as \( \beta \) increases. Substituting \( \rho \) in (8) and taking the derivative with respect to \( \beta \) yields

\[
0 = y (p(y) + \beta (1 - p(y))) - R(d) - d R'(d),
0 = y (1 - p(y)) - d_\beta (2 R'(d) + d R''(d)).
\]

For linear \( R(d) \), we get

\[
d_\beta = \frac{y (1 - p(y))}{2 R'(d)} > 0.
\]

Hence the curve \( d(y) \) shifts upwards in the \((y,d)\)-space (see figure A1). Next, we show that the curve \( y(d) \) shifts to the right as \( \beta \) rises. Applying the same procedure as above to (9), we get

\[
0 = [p(y) + y p'(y)] [p(y) + \beta (1 - p(y))]^2 - \beta p'(y) R(d),
0 = y_\beta [2 p'(y) + y p''(y)] [p(y) + \beta (1 - p(y))]^2
+ [p(y) + y p'(y)] 2 [p(y) + \beta (1 - p(y))] [1 - p(y) + y_\beta (1 - \beta) p'(y)]
- R(d) [p'(y) + \beta p''(y) y_\beta].
\]

(A7)

If \( p(y) \) is linear and if \( \beta \) is not too large, so that \( p(y) + y p'(y) \approx 0 \), (A7) becomes

\[
0 = y_\beta [-2 p(y)] [p(y) + \beta (1 - p(y))]^2 + p(y) R(d),
y_\beta = \frac{R(d)}{2 [p(y) + \beta (1 - p(y))]^2} > 0.
\]

The changes in equilibrium values follow directly. Our findings imply that the curves \( y(d) \) and \( d(y) \) look as depicted in figure A1: Both curves are increasing, and \( y(d) \) is steeper than \( d(y) \) for small \( \beta \) (in fact, the \( y(d) \)-curve is nearly vertical, whereas the \( d(y) \)-curve is nearly horizontal). Shifting the two curves in the directions described above yields the desired result: both \( y \) and \( d \) must increase in equilibrium when \( \beta \) goes up.

The effect on \( \rho \) remains to be shown. \( \rho \) falls if \([p(y) + \beta (1 - p(y))] \) rises. Therefore, we examine

\[
\frac{\partial [p(y) + \beta (1 - p(y))]}{\partial \beta} = y_\beta p'(y) + (1 - p(y)) - \beta y_\beta p'(y)
= 1 - p(y) + (1 - \beta) y_\beta p'(y)
= 1 - p(y) + (1 - \beta) \frac{p'(y) R(d)}{2 [p(y) + \beta (1 - p(y))]^2}
\]

\[\approx 1 - p(y) - \frac{R(d)}{2 y p(y)}.\]
This is positive whenever $R(d)$ is small enough,

$$R(d) < (1 - p(y)) \times (2y p(y)). \quad (A8)$$

This completes the proof of the proposition. ■

**Proof of lemma 3:** In order to derive the slope of the banks’ reaction functions, we must consider the simultaneous reactions of the optimal $y_j$ and $d_j$ to an increase in the competitors’ deposit volume $D_{-j}$. It can be shown that both curves move downwards in the $(y_j,d_j)$-space as $D_{-j}$ rises. This implies that in principle $y_j$ and $d_j$ could go up or down when competitors’ deposits increase. However, as the following calculations will show, $y_j$ unambiguously increases, whereas $d_j$ decreases.

$y_j$ and $d_j$ are determined simultaneously by the following implicit equations:

$$\frac{\partial \Pi_j}{\partial d_j} = 0 = y_j (p(y_j) + y_j (1 - p(y_j)) + R(d_j + D_{-j}) + d_j R'(d_j + D_{-j}), \quad (A9)$$

$$\frac{\partial \Pi_j}{\partial y_j} = 0 = (p(y_j) + y_j p'(y_j)) (p(y_j) + y_j (1 - p(y_j))) - \beta_j p'(y_j) R(d_j + D_{-j}). \quad (A10)$$

Generally, if we have two equations, $F(y,d,\epsilon) = 0$ and $\tilde{F}(y,d,\epsilon) = 0$, that implicitly define a dependence $d(\epsilon)$, then the implicit function theorem implies that

$$\frac{\partial d}{\partial \epsilon} = \frac{F_y \tilde{F}_\epsilon - F_\epsilon \tilde{F}_y}{F_d F_y - F_y F_d}.$$

Applied to equations (A9) and (A10), we receive

$$\frac{\partial d_j}{\partial D_{-j}} = -\frac{(1 - \beta_j)([5 \beta_j + 4 (1 - \beta_j) p(y_j)] p(y_j) + [\beta_j + 2 (1 - \beta_j) p(y_j) - y_j p'(y_j)]) + \beta_j^2}{(1 - \beta_j)([11 \beta_j + 8 (1 - \beta_j) p(y_j)] p(y_j) + [3 \beta_j + 4 (1 - \beta_j) p(y_j) - y_j p'(y_j)]) + 3 \beta_j^2}.$$

If $\beta_j$ is small, we get $\partial d_j/\partial D_{-j} \approx -1/2 < 0$, proving the first part of the lemma.

We still have to show that the aggregate effect on deposits is positive. The argument here is completely analogous to the one used in the proof of lemma 2, and is therefore skipped. This completes the proof. ■

**Proof of proposition 4:** Lemma 3 states that competitors react to an expansion of $d_1$ by contracting $d_j$. From the point of view of bank 1, the interest rate on the deposit market can be described by a function $\tilde{R}(d_1)$, which incorporates the other banks’ reactions to bank 1’s behavior. Since $D$ is larger in the new equilibrium (cf. lemma 3), $\tilde{R}(d_1)$ has a positive slope, just as $R(D)$. Therefore, the results for bank 1 from proposition 4 follow directly from proposition 3 if one replaces $R(d)$ with $\tilde{R}(d)$. Hence $y_1$ and $d_1$ increase, whereas $\rho_1$ decreases (for $\tilde{R}(d_1)$ small enough).

From the point of view of the competitor banks, an increase in $\beta_1$ is equivalent to a rise in $D_{-j}$. Therefore, the reaction of $d_j$ is already described by lemma 3. Using a
procedure similar to the one above, one can also derive the reaction of $y_j$:

$$\frac{\partial y}{\partial \epsilon} = \frac{F_d \tilde{F}_c - F_c \tilde{F}_d}{F_y \tilde{F}_c - F_c \tilde{F}_y},$$

$$\frac{\partial y_j}{\partial D_{-j}} = \frac{\beta_j R'(D)}{(1-\beta_j)(11\beta_j+8(1-\beta_j)p(y_j)+[3\beta_j+4(1-\beta_j)p(y_j)]y_jp'(y_j))+3\beta^2_j}.$$ 

If $\beta_j$ is small (and hence $y_j$ is close to the first-best), we get

$$\frac{\partial y_j}{\partial D_{-j}} \approx \beta_j \frac{R'(D)}{(11\beta_j+8(1-\beta_j)p(y_j)+[3\beta_j+4(1-\beta_j)p(y_j)]y_jp'(y_j))+3\beta^2_j} > 0$$

as long as $\beta > 0$. Hence $y_j$ increases in reaction to the increase in $d_1$. If $\beta = 0$, risk-taking does not change at all; in that case, competitor banks always choose the first-best risk level, independent of other banks’ behavior. The reaction of $\rho_j$ is obvious. Because the risk level $y_j$ rises and $\beta_j$ remains constant, the default premium $\rho_j$ must also rise. This completes the proof of the proposition. ■

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