Pension reform, capital markets, and the rate of return

Axel Börsch-Supan, Florian Heiss,
Alexander Ludwig and Joachim Winter

May 2002
Pension reform, capital markets, and the rate of return

Axel Börsch-Supan**, Florian Heiss*, Alexander Ludwig*, and Joachim Winter*

* Mannheim Research Institute for the Economics of Aging (MEA), University of Mannheim
** National Bureau for Economic Research (NBER), Washington, DC

First version: May 2000
This revision: May 2002

Abstract: This paper discusses the consequences of population aging and a fundamental pension reform – that is, a shift towards more pre-funding – for capital markets in Germany. We use a stylized closed-economy, overlapping-generations model to compare the effects of the recent German pension reform with those of a more decisive reform that would freeze the current pay-as-you-go contribution rate and thus result in a larger funded component of the pension system. We predict rates of return to capital under both reform scenarios over a long horizon, taking demographic projections as given. Our main finding is that the future decrease in the rate of return is much smaller than often claimed in the public debate. Our simulations show that the capital stock will decrease once the baby boom generations enter retirement even if there were no fundamental pension reform. The corresponding decrease in the rate of return, the direct effect of population aging, is around 0.7 percentage points. While the capital market effects of the recent German pension reform are marginal, the rate of return to capital would decrease by an additional 0.5 percentage points under the more decisive reform proposal.

Keywords: aging; pension reform; rates of return

JEL classification: E27; H55; J11

This paper is a substantial revision of a paper presented at the workshop “Reforming Old-Age Pension Systems” of the Herbert-Giersch-Stiftung at Schloß Wendgräben, May 25-26, 2000. We thank Alan Auerbach, Melanie Lührmann, Reinhold Schnabel, Gerhard Schwödiauer, Jan Walliser, and two anonymous referees for helpful comments and discussions, and Hendrik Engelmann-Pilgers for excellent research assistance. This research was supported by the Volkswagenstiftung, the Deutsche Forschungsgemeinschaft (DFG), and the Gesamtverband der Deutschen Versicherungswirtschaft (GDV).

Corresponding author: Dr. Joachim Winter
Mannheim Research Institute for the Economics of Aging (MEA)
University of Mannheim
D-68131 Mannheim
E-mail: winter@uni-mannheim.de
1. Introduction

Germany is moving towards a multi-pillar pension system. Most economists and politicians have accepted, some reluctantly, that the pressures on the pure pay-as-you-go system caused by population aging must be alleviated by introducing a pre-funded component. The pension reform enacted in 2001, sometimes referred to as the “Riester reform” in the public debate, is an important step in that direction. Until the year 2000, pension reforms in Germany have been of the parametric type, changing some features of the pay-as-you-go pension system, but avoiding more radical measures. Germany’s 2001 pension reform is more fundamental because for the first time after the pay-as-you-go-system was introduced in 1957\(^1\), a funded component has officially been added to the public pension system.\(^2\)

There is no need to repeat the arguments for (and against) fundamental pension reform. The interested reader is referred to Raffelhüschen (1993), Buslei and Kraus (1996), Börsch-Supan (1998, 2000, 2002), Wissenschaftlicher Beirat (1998), Sinn (1999) and Rürup (2000) for different perspectives on the pension crisis in Germany and alternative reform proposals. This paper concentrates on an important aspect of pension reform that has gained attention relatively late in the pension reform debate in Germany, namely, the capital market effects of population aging and of a pension reform that involves a shift towards more pre-funding.\(^3\)

Population aging has incisive macroeconomic effects which in turn determine capital market outcomes. Labor supply will be relatively scarce, whereas capital will be relatively abundant. This will drive up wages relative to the rate of return to capital, reducing households’ incentive to save if the interest elasticity of saving is positive. In addition, some fraction of the capital stock may become obsolete due to the shrinking labor force and diminishing returns to scale, making the accumulation of capital even less attractive. In general, these mechanisms should eventually result in a declining rate of return to capital. An alternative interpretation is that once the baby-boom generations retire around the year 2030, they start consuming out of their retirement savings; this will result in capital market outflows, and via declining prices for financial assets the rate of return will decrease. This is the so-called “asset market melt-

\(^1\) When the public pension system was enacted by Bismarck in 1889, it was fully funded. A partial pay-as-you-go system was introduced in 1957, and once the system’s funds were depleted in 1969, the system became purely pay-as-you-go.

\(^2\) Schnabel (2001) contains a brief description of the main features of the reform.

\(^3\) This paper is a substantially revised version of Börsch-Supan et al. (2000).
down hypothesis”, see Poterba (2001) for a review. Worries about declining rates of return are also expressed in the public debate about fundamental pension reform.

To address these concerns, we use a stylized overlapping generations model of pension reform to calculate the rate of return to capital over a long horizon, using demographic projections by Birg and Börsch-Supan (1999) and alternative scenarios. Based on these demographic projections, we compute the size of the aggregate labor force and analyze the economic consequences of two pension reform schemes. As a benchmark, we model the pure PAYG pension system as it was in place before the 2001 reform and compare the results under this counterfactual system to both the “Riester” reform of 2001 and a more decisive reform proposal, the “freezing reform” as suggested in Börsch-Supan (2002). In our benchmark population and labor force scenario, the comprehensive (or equilibrium) contribution rate would increase from currently 28 percent to about 42 percent without a fundamental pension reform.4 In contrast, the 2001 reform is projected to result in a reduction of the contribution rate relative to the pure PAYG system of about 2.7 percentage points in 2050. The Riester reform will therefore alleviate, but not solve the financing problems of the German pension system.

The more radical freezing reform scheme assumes that the contribution rate to the public pension system remains fixed (“frozen”) at its current level in order to insure the long-term viability of a reduced pay-as-you-go system. Under such a scheme, the replacement rate has to drop by more than projected by the government for its current reform. This implies also that households will need to save more for retirement than anticipated by government projections. Once the large baby-boom generations retire, they start to consume out of their accumulated retirement savings. Our simulations show quite substantial decreases in the aggregate savings rate due to population aging (about 9.2 percentage points) and corresponding decreases in the rate of return to capital of about 0.7 percentage points in 2050. While this is non-negligible, the word “melt-down” appears exaggerated as a description of this magnitude.

We compare this direct effect of population aging on German capital markets with the effects of fundamental pension reforms. According to our simulations, the (additional) capital market effects of the 2001 reform will be only marginal. The more radical “freezing” reform proposal would have significantly larger capital market effects: In our benchmark scenario, the rate of

4 This comprehensive rate includes the general tax revenues used to finance the various federal subsidies to the public pay-as-you-go pension system, see below.

3
return to capital would decrease by about 0.5 percentage points. Again, while being non-negligible, this effect is much less than often claimed in the public debate. Moreover, these estimates are an upper bound on the rate-of-return effects since our closed-economy model abstracts from all diversification effects in a world with capital mobility. This is shown by Börsch-Supan, Ludwig, and Winter (2002) in a more stylized overlapping generations model that allows for capital mobility.

The remainder of this paper is structured as follows. Section 2 presents the demographic and labor force projections and the pension reform schemes that form the background of our analysis. In section 3, we present a stylized overlapping generations model that evaluates the effects of population aging and pension reform. Section 4 contains benchmark simulations on household savings, the capital stock, and the rate of return. In Section 5, we carry out an extensive sensitivity analysis with respect to our demographic assumptions and the calibration parameters of the model; this allows to trace out the range of potential capital market effects of population aging and pension reform. Section 6 concludes.

2. Demographic projections and the implementation of pension reform

It is helpful to begin with some notation that will link demographics, labor force participation, pension systems and the overlapping generations model. In some instances, noted below, this notation is more stylized than our actual simulation model.

Demography enters our simulation model via time-specific sizes of the living cohorts in year \( t \), denoted by \( N_{t,a} \), where \( a \) is age. We include three population groups among \( N_{t,a} \), workers, unemployed and pensioners. The economic life of a cohort begins at the age of twenty years, for which we set \( a = 1 \) and ends with certain death at \( a = \Omega_\kappa \), where \( \Omega_\kappa \) denotes life-expectancy of cohort \( \kappa = t-a \). Life expectancy corresponds to the underlying demographic projections. For ease of presentation, we take \( N_{t,a} \) to be total population for \( a = 1, ..., 80 \). In our simulations, we include age and time-specific weights that represent the fraction of the population that is currently working, unemployed or retired. Age specific labor force participation and unemployment rates are derived from our labor market scenarios to be described in more detail below. The fraction of pensioners increases from 0 to 1 over an extended retirement window from age 48 through 80. The time paths of these weights are cohort-specific, reflecting shifts in labor supply and retirement behavior. To summarize, \( \nu_{t,a}N_{t,a} \) therefore reflects the number of workers, \( \mu_{t,a}N_{t,a} \) the number of unemployed individuals and \( \pi_{t,a}N_{t,a} \) the number of pensioners of age \( a \) in time period \( t \).
To fill this abstract framework with life, we employ long-term projections of the demographic structure of the German population computed by Birg and Börsch-Supan (1999).\(^5\) We distinguish several scenarios for demographic change and stratify these projections along three dimensions which together determine population aging: life expectancy, fertility and migration. Figure 1 summarizes these projections by showing the old-age dependency ratio (i.e., the ratio of the number of persons aged 65 and older to the number of persons aged 15 to 64) for three demographic scenarios: (1) weak aging, increasing fertility and strong gains through immigration, (2) modest aging, constant fertility and modest gains through immigration, and (3) strong aging, constant fertility and weak gains through immigration. The underlying assumptions of these scenarios are concisely summarized in tables 1 to 3.

**Insert Tables 1 to 3**

The demographic projections are augmented by labor force projections based on age and gender specific employment rates. Rather than allowing for endogenous labor supply decisions, we feed our simulation model with various paths of aggregate employment, again following Birg and Börsch-Supan (1999). While our scenario-based approach to modeling labor supply does not allow us to analyze the effects of endogenous labor supply in an aging society, it allows us to analyze labor-supply variations along two dimensions that are only partially modeled in many other models, female labor supply and unemployment. Overall, our labor market projections comprise the following dimensions: female labor force participation, retirement age, and unemployment. We combine variations along these dimensions to obtain three labor force scenarios which can be summarized in the overall labor market participation rate. We consider (1) almost no increase, (2) a modest increase, and (3) a strong increase in labor force participation rates. Tables 4 to 6 summarize the assumptions on labor force participation in these scenarios.

**Insert Tables 4 to 6**

The declining labor force participation rates after the peak participation age enable us to calculate the number of pensioners in each year.\(^6\) Dividing the number of pensioners by the number of workers results in an economic dependency ratio of 59.9 percent in 2000 which is very close to the official figure of 59.7 percent.

---

\(^5\) In our simulation model we assume that a final steady state is reached in 2200. To achieve this, we make the simplifying assumption that the population size is constant after 2100, the end date of the demographic projections by Birg and Börsch-Supan (1999).

\(^6\) The details of this procedure are described in Ludwig (2002).
Figure 2 shows the future evolution of this economic dependency ratio for the entire range of projections derived from our demographic and labor force scenarios. It combines the most optimistic population and labor force scenarios (lower curve) with the most pessimistic scenarios (upper curve). The figure also shows the combination of the medium population and the medium labor force scenario. We consider this combination as the most likely development and take this as our benchmark scenario for all simulations that follow. The capital market consequences of the alternative labor force scenarios will be analyzed in the sensitivity analysis of section 5. In the benchmark scenario, the old-age dependency ratio flattens after the year 2035. We therefore take 2035 to evaluate the macroeconomic effects of aging while we evaluate the effects of pension reform in 2050.

Our next step is to model the pension system. This is complicated by the fact that the German pension system is linked with the other social protection systems in Germany as well as the general tax system. The pure pay-as-you-go (PAYG) pillar of the German pension system is financed by contributions (19.1 percent of gross earnings in 2002) and general tax revenues (about 9 percent of gross earnings), resulting in a comprehensive contribution rate of about 28 percent of gross earnings. It provides a pension that is approximately proportional to life-time earnings. The system is much less redistributive than the U.S. social security system and most other European pension systems. The current net replacement rate, the average net pension income as a fraction of current net labor income, is about 70.4 percent (U.S. social security provides only about 50 percent). The average retirement age is just 59.5 years, mainly due to an actuarially unfair pension formula that favors early retirement. In addition to old-age pensions, the system provides a generous disability pension and survivor benefits. Public pensions account for almost 12 percent of GDP, a share two-and-a-half times larger than in the United States. Börsch-Supan (2000) provides more details on the German pension system and its current problems.

In addition to the direct contribution rate to the public PAYG pension system, denoted as $\tau_t^{PS}$, we model contributions to unemployment insurance, $\tau_t^{UI}$, and contributions to health and long-term care insurance summarized in $\tau_t^H$. We denote the overall social security contribution rate, the sum of $\tau_t^{UI}$, $\tau_t^{H}$ and $\tau_t^{PS}$, as $\tau_t^{SS}$. Moreover, taxes on wage income enter our model via the income tax rate $\tau_t^{INC}$ and are used to finance government grants to the pension

---

7 Net pension income is gross pension income net of contributions to health and long-term care insurance. Income taxes paid by pensioners are not included in this definition because they are usually quite small.
system and general government consumption. Net wages, indexed by $n$, are therefore gross wages, indexed by $g$, net of the employee’s social security contributions and income taxes.

Since the pension-related government sector is not allowed to issue debt, the budget constraint of the pension system is always balanced such that

$$\tau_{i}^{PS} \sum_{a=1}^{80} w_{i,a}^{g} \nu_{i,a} N_{t,a} + Z_{i} + \tau_{i}^{PS} 0.8 \sum_{a=1}^{80} w_{i-a}^{n} \mu_{i,a} N_{t,a} =$$

$$\phi_{i} \sum_{a=1}^{80} \pi_{i,a} \lambda_{t,a} N_{t,a} + G_{i}^{PS} + 0.5 \cdot \tau_{i}^{H} \phi_{i} \sum_{a=1}^{80} \pi_{i,a} \lambda_{t,a} N_{t,a} +$$

The left hand side of equation (1) describes contributions to the pension system which consists of three elements. The first term captures contributions of the current work force as the product of the direct contribution rate to the PAYG pension system and aggregate gross wage income, where the only difference between individual wages is due to age (see section 3.1). The second term, $Z_{i}$, are government grants to the social security system which are assumed to be exclusively financed by income taxes and thus implicitly account for indirect contributions of the current work force. The third term describes contributions of the unemployment insurance system as the product of the direct contribution rate $\tau_{i}^{PS}$ and 80 percent of the aggregate base for unemployment benefits. We assume that each individual is unemployed for only one year. Therefore unemployment benefits of each unemployed individual are based on net wage income during the last working period.

The right hand side of equation (1) describes expenditures of the public pension system, again consisting of three elements. The first term describes aggregate pension payments which are the product of the current pension value (the gross pension an average pensioner receives per month, or per year in our model), denoted by $\phi_{i}$, times the number of effective pensioners, $\sum_{a=1}^{80} \lambda_{t,a} \pi_{t,a} N_{t,a}$. Here, $\lambda_{t,a}$ is the weight attached to each pensioner of age $a$ in time period $t$ which adjusts the pension entitlement of each pensioner relative to the pension entitlement of an average pensioner who worked for 45 years and earned average income (“Eckrentner”). The entitlement to pensions of each individual pensioner are determined by her earning points, which depend on the relative amount of insured income – the sum of gross wage income relative to average wage income over all working periods – and the number of months (years in our model) contributions were made. The second term, $G_{i}^{PS}$, captures administrative government expenditures within the pension system. The third term describes aggregate contributions of the social security administration to health and long-term care insurance. The
base for these contributions is gross pension income of each pensioner. Half of the contributions are paid by the social security administration and half by the individual pensioner.

We employ three pension scenarios. The first scenario ("pure PAYG") is a counterfactual benchmark: In this scenario, we assume that the pre-2001 pay-as-you-go pension system is maintained without any reform that introduces pre-funding. Accordingly, future benefits are computed by an adjustment formula referred to as net wage adjustment, and contribution rates follow from the budget equation (1). The current pension value in this first scenario evolved according to the recursion

\[
\phi_t = \phi_{t-1} \cdot \frac{AGI_{t-1}}{AGI_{t-2}} \cdot \frac{NIR_{t-1}}{NIR_{t-2}} \cdot \frac{NPBR_{t-2}}{NPBR_{t-1}},
\]

where \(AGI_t\) is average gross wage income, \(NIR_t\) is the net income ratio (the ratio of net to gross income), and \(NPBR_t\) is the net pension benefit ratio (the ratio of net to gross pension benefits). The change in the net income ratio captures the effect of increases in income taxes and social security contributions. If the aggregate tax burden is higher in year \(t-1\) than in year \(t-2\), this dampens the increase in the current pension value. The change in the net pension benefit ratio works in the opposite direction. It amplifies the increase in the current pension value if the tax burden of pensioners (half of the contributions to health and long-term care insurance) increases relative to the tax burden of workers.

The second scenario models the pension system how it is currently projected after the 2001 reform (the so-called “Riester reform”). The objective of the Riester reform is to keep the net contribution rate below 20 percent until 2020 and below 22 percent until 2030. In order to achieve this, the recursive equation for the current pension value is replaced by a new adjustment formula that will be implemented in 2003:

\[
\phi_t = \phi_{t-1} \cdot \frac{AGI_{t-1}}{AGI_{t-2}} \cdot \frac{\theta_t - \tau_{t-1}^{PS} - \psi_{t-1}}{\theta_t - \tau_{t-2}^{PS} - \psi_{t-2}},
\]

where \(\psi_t\) denotes the share of private retirement provision relative to household income that is subsidized by the German government. \(\psi_t\) increases linearly from 0.5 percentage points in 2002 to 4 percentage points in 2009. Since the numerator of the third term in equation (2) will always be lower than the denominator during this adjustment period, this effectively dampens the increase of the current pension value in addition to the dampening effect by in-

---

8 While the increase in \(\psi_t\) is 0.5 percentage points each year, the fraction of private retirement provision relative to income subsidized by the German government increases by 1 percentage point every two years.
creases in the contribution rate, \( \tau_t^{PS} \). After 2009, \( \psi_t \) remains at the level of 4 percentage points. Since the factor enters equation (3) with lagged values, its dampening effect will disappear in 2011. Therefore, \( \theta_t \) will be reduced from currently 1 to 0.9 in 2011 which increases the sensitivity of the current pension value to increases in the contribution rate. Note that in contrast to equation (2), the reformed system described by equation (3) contains only the contribution rate to the pension system and no other social security contributions or taxes in the new adjustment formula. Equation (3) is therefore referred to as modified gross (or net) wage adjustment.

Equations (1) and either (2) or (3) determine the evolution of the current pension value over time. We convert this current pension value into the more prevalent concept of the net replacement rate. Note that \((1-0.5\tau_t^H)\phi_t\) is the net pension benefit of an average pensioner which is just equal to the product of the net replacement rate and average net income of the current work force. By reducing the current pension value, the Riester reform will therefore reduce the net replacement rate provided by the pay-as-you-go pillar of the pension system.\(^9\)

We do not explicitly model the funded component of the pension system and therefore also do not account for any savings incentives due to subsidies granted in the Riester reform scheme.\(^10\) In our model, the funded component consists entirely of voluntary, private savings, as given by households’ optimal life-cycle decisions.

The third pension scenario (“freezing reform”) fixes the contribution rate to the PAYG pension system from 2003 onwards at its level in the year 2000 (19.3 percent of gross earnings). It also freezes the state subsidy \( Z_t \) in the budget equation (1). Hence, the comprehensive contribution rate remains constant at about 28 percent. As we show in section 4, such a freezing reform results in much lower replacement rates than the Riester reform scheme and therefore induces much higher private savings to prepare for retirement.

Both reform schemes result in lower public pensions for future retirees and higher net wages due to the reduced contribution rates and a higher degree of capital deepening. We assume that households chose their consumption path in order to maximize their discounted life-time

---

\(^9\) The official definition by the German government departs from this definition of the replacement rate by also accounting for the maximum share of private savings. This accounting procedure effectively raises the nominal replacement rate to higher figures (see Schnabel, 2001). In order to maintain comparability of replacement rates across the three pension schemes, we apply the replacement rate concept as defined above.

\(^10\) While such subsidies would not be neutral in our model, we assume that the savings incentives generated by these subsidies do not lead the households to “oversave” in the sense that they accumulate more savings than
utility, given their income path. When the government announces a pension reform scheme, households anticipate the resulting changes in the income path over the life cycle, re-optimize, and increase their savings. For both reform schemes, we assume that they are announced in 2001 such that households start to re-optimize in 2002.

The accumulated savings in the funded pillar of both reform scenarios depend on the rate of return while the pension benefits derived from the PAYG pillar depend on the wage level. The overlapping generations model described in the following section endogenizes both rate of return and wage level in a dynamic general equilibrium, taking the dynamics of the population age structure and the projections of the labor force as given.

3. Aging and pension reform in a stylized overlapping generations model

Our dynamic macroeconomic model is based on a version of the familiar overlapping generations model (Samuelson, 1958; Diamond, 1965) introduced by Auerbach and Kotlikoff (1987, chapter 3). Overlapping generations models have been used extensively to study the effects of population aging on social security systems. Recent examples include Kotlikoff, Smetters and Walliser (1999), De Nardi et al. (1999) and Altig et al. (2001) for the United States, Miles (1999) for Great Britain, and Fehr (1999) and Hirte (2002) for Germany. Miles and Iben (2000) present a comparative analysis of pension reform schemes for the United Kingdom and Germany. Kotlikoff (1998) provides an overview of earlier applications of overlapping generations models. We present the features of the simulation model in section 3.1, followed by a description of the solution method and the calibration of the model in section 3.2.

3.1 The simulation model

Since the purpose of this paper is to study the effects of a fundamental pension reform on the rate of return to capital, we focus our analysis on capital market effects. To this end, we take great care to get the first-order effects of demographic change right by using annual demographic projections. On the other hand, we do not model the households’ labor supply decision, since feedback effects created by an endogenous labor supply is of second order to the capital market consequences of pension reform. While feedback effects of labor supply in the

projected by the OLG model. In fact, proponents of the Riester reform argue that one needs the subsidies to exactly reach the equilibrium savings level.

11 There is a recent literature that argues that boundedly rational individuals might not be able to make savings decisions that allow them to attain their desired levels of retirement wealth (O’Donoghue and Rabin, 1999). We take up this issue in the conclusions.
presence of distorting taxes and social security contributions certainly also affect the capital market, we restrict our attention to households’ life-cycle savings decisions as the primary response on the capital market to demographic changes and decreasing public pensions. We may reiterate, however, that we account for potential unemployment and labor force participation reactions in our scenario framework described in section 2.

The first cornerstone of our general equilibrium model is the production sector where, given factor inputs (capital and labor), output and factor prices are determined. The production sector consists of a representative firm that uses a constant-returns-to-scale CES production function given by

\[
Y_t = \left( \alpha K_t^{1-\beta} + (1-\alpha) \left( A_t \sum_{a=1}^{80} e_{t,a} \nu_{t,a} N_{t,a} \right)^{1-\beta} \right)^{1/(1-\beta)},
\]

where \( \alpha \) and \( \beta \) are the factor share and the elasticity of substitution, \( e_{t,a} \) is age-specific productivity and \( A_t \) is the aggregate level of labor productivity that grows with a constant rate \( g \). For ease of presentation we divide equation (3) by \( A_t \sum_{a=1}^{80} e_{t,a} \nu_{t,a} N_{t,a} \) to obtain the representation of the production function in terms of efficiency units,

\[
y_t = f(k_t) = \left( \alpha k_t^{1-\beta} + (1-\alpha) \right)^{1/(1-\beta)}
\]

From static profit maximization, we obtain the interest rate as

\[
r_t = f''(k_t) - \delta,
\]

(where \( \delta \) is the rate of depreciation) and gross wages as

\[
w_t = A_t \frac{(f(k_t) - k_t f''(k_t))}{1 + 0.5 \cdot \tau_{ss}^t},
\]

where the term in the denominator denotes the employer’s contributions to the social security system.

In order to determine aggregate consumption, we next consider optimal household behavior derived from intertemporal utility maximization. By choosing an optimal consumption path, each generation \( a \) maximizes, at any point in time \( t \), the sum of discounted future utility. We assume that the within-period utility function can be characterized by constant relative risk aversion, and that preferences are additively separable over time. The objective function of generation \( a \)'s maximization problem at time \( t \) is then given by
where $\sigma$ denotes the coefficient of relative risk aversion and $\rho$ is the discount rate. The planning horizon of each household in the model is constrained by life expectancy. Maximization is subject to a dynamic budget constraint which for generation $a$ at time $t$ is given by

$\sum_{j=1}^{\infty} \frac{1}{(1+\rho)^j} \left( C_{t+j-a, j} \right)^{1-\sigma}$,

which reflects households’ trade-off between current and future utility. As in any life-cycle model, this trade-off is determined by the ratio of the interest rate and the time preference rate, and by the degree of risk aversion. We determine the life-time consumption paths of all generations backwards, starting with zero wealth in the final period of life, and then iterating using the Euler equation. The resulting time paths of consumption determine aggregate saving and wealth in the household sector. Note that we do not model any form of indirect consumption taxes and capital income taxes. All government consumption in the model is financed by income taxation.

Finally, two aggregation conditions ensure market clearing and general equilibrium. In all periods $t$, we aggregate over the living generations ($a = 1, \ldots, 80$). The aggregation equations for capital and consumption goods, respectively, are given by

$K_t = \sum_{a=1}^{80} W_t^a \cdot N_t^a$ and

$B_{t,a} = \sum_{j=1}^{\infty} \frac{1}{(1+\rho)^j} \left( C_{t+j-a, j} \right)^{1-\sigma}$,
3.2 Solution method and calibration

We determine the equilibrium path of this overlapping generations model by using the recursive Gauss-Seidel algorithm. It starts with picking an arbitrary initial path for the capital stock. Since labor supply is exogenous in our model, we solve the static optimization problem of the representative firm for a given trial value of the capital stock and the labor inputs implied by the demographic projections. We then compute time paths of the factor prices (i.e., the wage and interest rates). Given factor prices, we solve the age and time-specific intertemporal optimization problems of all cohorts at all points in time, which yields, after aggregating, time paths of aggregate consumption and savings. Since we operate under the assumption of a closed economy, aggregate household saving determines the economy’s aggregate capital stock. This completes the solution for all endogenous variables, given the arbitrary initial capital stock. This initial capital stock will in general not coincide with the size of the capital stock that is consistent with household optimization (conditional on factor prices). Hence, we adjust the initial capital stock and repeat the entire computation recursively until convergence between the two capital stock paths is achieved. This defines the intertemporal equilibrium of our dynamic economy.

We start calculations in 1960 and end in 2200 assuming a final steady state. We calibrate the model with actual demographic data from 1960 to 2000. As an initial condition to determine the assets of households who already live in 1960, we assume that they behave in 1960 as if the economy was initially in a steady state.

Since agents in the model are forward looking and since we solve for the saddle path of all relevant variables from 1960 to 2000 we assume that agents have anticipated all past changes to the environment, especially those that alter the structure of the public pension system. While this is a plausible first order approximation for general political reforms and demographic developments, it is certainly not a good approximation to the German reunification. We therefore model the reunification as a shock that increases the number of agents but only marginally increases the capital stock. This results in a decrease in the capital stock per effi-

\[ C_t = \sum_{a=1}^{8t} C_t^a \cdot N_t^a. \]

12 German general unemployment benefits (Arbeitslosengeld) amount to 60 percent of the last (in our model last period’s) net wage income for households without children and to 67 percent for households with children. We simplify by assuming only 60 percent for all households.
ciency unit – even though aggregate efficiency $A$ also decreases, see below – and a corresponding increase in the rate of return to capital of about 0.2 percentage points. Since the Vertrag zur deutschen Wirtschafts- und Währungsunion was signed in October 1990, we assume that agents re-optimize at the beginning of 1991, taking the new conditions as given.

Since we treat all agents alike, we make an implicit assumption about immigrants by assuming first, that all net migrants of age $a$ in time $t$ have the same amount of assets as natives of the same age and second, that there are no skill differences between immigrants and natives. Given that we are not interested in the effects of different immigration policies, we consider these assumptions as good first order approximations. The interested reader it referred to Bonin et al. (2000), Razin and Sadka (1999), and Storesletten (2000) as examples of studies that analyze the interactions between migration and fiscal policy.

For the simulations, we set $\tau_{t}^{\text{PS}}$ to the actual value of 0.193 in 2000, the beginning year of our calculations. The contribution rate to the unemployment insurance system is assumed to evolve proportional to the unemployment rate underlying our labor market projections. The respective rate decreases from currently 6.5 percent to 2.3 percent in 2050 in our benchmark scenario – an order of magnitude last reached in 1977. Contribution rates to health and long-term are assumed to remain constant at their current levels of 13.8 and 1.7 percent. The tax rate on wage income is initially set to 16 percent but increases proportional to government grants to the pension system.  

We initially normalize the current pension value, $\varphi_{t}$, such that the replacement rate is equal to the current value of 70.4 percent (year 2000). Furthermore, we set the ratio of administrative expenditures to total pension payments to 1.8 percent (as it was in 2000). Solving out equation (1) for the ratio of government grants to overall contributions yields a ratio of 35.7 percent which implies an implicit tax rate on income of 6.9 percent – a bit less than the earlier noted 9 percent. Our model thus slightly underestimates overall expenditures of the German pension system. This minor error, however, does not much affect our predictions. For the simulations we assume that the ratio of government grants to total contributions remains constant such that the amount of government grants relative to work income increases over time.

---

13 However, we assume that mortality and fertility of immigrants differ from natives in the demographic model (see section 2).

14 We do not carry out a sensitivity analysis with regard to these assumptions. See e.g. Sinn and Thum (1999) for a detailed analysis on how assumptions on other taxes affect predictions of the PAYG system contribution rate in a partial equilibrium framework.
if the direct contribution rate to the PAYG pension system increases. As noted before, we do not model any form of indirect taxation. Therefore, such a relative increase of government grants yields to an increase in the tax rate on wage income. For the backward solution of the model until 1960, we use the social security contribution rates observed in the past and solve out equation (1) for the replacement rate. This results in an average replacement rate for Germany of about 72 percent which is only slightly higher than the observed value.

Further parameters of the model are the household’s preference parameters, the parameters of the production function and values for the age-specific productivity profile. For the latter, we apply estimates for Germany presented in Fitzenberger et al. (2001). These authors use data from the West German Beschäftigungsstatistik of the Federal Employment Service to separate the evolution of wages into a life-cycle wage profile independently of the calendar year and a macroeconomic time trend for four different education groups. We employ their estimated age-wage-profiles and weight them with the respective shares of each education group taken from the German Income and Expenditure Survey 1993 (Einkommens- und Verbrauchsstichprobe). This provides us with a representative age-wage profile that peaks at the age of 52 and then decreases slightly.

Our choice of benchmark calibration parameters for the production function are concordant with parameter estimates for Germany by Börsch-Supan (1995). The estimate of the share of capital used in production is 0.4099 and the estimate of the substitution elasticity between capital and labor is 0.999. We therefore take 0.4 and 1 as benchmark values respectively, i.e. we initially assume a Cobb-Douglas production function. The capital share is relatively high – given a narrow concept of capital without human capital – and usual values for the elasticity of substitution range from 0.8 to 1 (compare e.g. Altig et al., 2001). We therefore consider a range from 0.3 to 0.4 for $\alpha$ and a range from 0.8 to 1 for $\beta$ in the sensitivity analysis, both producing lower estimates of the return to capital than our benchmark values. For $g$, the growth rate of labor augmenting technical change, we use 1 percent per year as in other OLG models such as Altig et al., 2001). This is a conservative estimate of future growth and at the lower bound of parameter values employed in the literature: Cutler et al. (1990) suggest 1.4 percent whereas Pemberton (1999) uses 2 percent. We therefore apply a range from 1 to 2 percent in the sensitivity analysis. The average depreciation rate of capital was 0.0528 in Germany from 1960 to 1990. We therefore use 0.05 throughout all the scenarios. The remaining parameter is $A$, the aggregate efficiency parameter in the production function. We adjust $A$ twice, first in 1990 when we solve the model prior to the German reunification employing equation (4) and GDP data for Germany. We then solve $A_t = A_{t-1}(1+g)$ backward until
1960 and forward until 2200 which is consistent with our deterministic model. We readjust $A$ after the reunification in 2000 and again solve $A_t = A_{t-1}(1+g)$ backward until 1991 and forward until 2200.

An often applied value for the discount rate of households is 0.011 based on an estimate by Hurd (1989). We use 0.01 as a benchmark value and apply a range from 0.01 to 0.03 in the sensitivity analysis. The coefficient of relative risk aversion is usually chosen to lie between 2 and 4. We consider this range in the sensitivity analysis and use 2.7 as a benchmark value – less than the mean of the range we consider in the sensitivity analysis in order to achieve reasonable values for the capital to output ratio. The parameter values used in the calibration of our model are summarized in table 7.

Given the benchmark calibration parameters, the model performs well in matching the empirical counterparts of the capital to output ratio over the period 1960-1990. The empirical average of the capital to output ratio is 2.604, our model predicts 2.609. We somewhat overestimate the return to capital and underestimate the savings rate of households. The savings rate of households predicted in our model is around 4.5 percent while the empirical value increased from 4.35 percent in 1960 to about 12 percent in 2000. In our model of perfect foresight, there is no role for uncertainty and thus the predicted rate of return to capital should be scaled to the rate of return to risk-free assets. We use 4.5 percent in 2000 as a reasonable estimate of a risk-free rate of return.

4. Simulation results

We begin our presentation of simulation results with the contribution and replacement rates under the three pension system scenarios described in section 2. The comprehensive contribution rate – a more meaningful concept than the direct contribution rate – is initially at 28 percent. It is predicted to increase to about 42.1 percent by 2050 if the current generous replacement rates were maintained, see figure 3. This implies an increase of the tax rate on wage income from currently 16 percent to 19.7 percent and a direct contribution rate of about 29.7 percent in 2050.

Figure 3 also shows the evolution of the contribution rate to the PAYG pillar under the Riester reform scheme. We arrive at contribution rates that are somewhat less optimistic than those calculated by the German government which uses more optimistic population and labor force scenarios. The difference between the comprehensive contribution rate under the pure PAYG system and the Riester reform scheme is 2.7 percentage points in 2050. Both results
are in line with the findings by Schnabel (2001). By definition, the contribution rate in the “freezing reform” scenario is constant at 28 percent after the implementation of the reform scheme.

Figure 4 plots the net replacement rates associated with the different pension reform scenarios. As the figure indicates, the replacement rate under the pure PAYG system is predicted to remain almost constant at the current level. The freezing reform however would lead to a significant reduction of pensions provided by the public pension system – the replacement rate will be reduced by about 30 percentage points relative to maintaining a pure PAYG pension system. The replacement rate under the Riester reform lies in between and arrives at a still rather generous level: the net replacement rate decreases by about 6.7 percentage points relative to the pure PAYG pension system.\footnote{15}

We now present the results of our macroeconomic simulation model. To separate the direct effect of population aging on capital markets from potential feedback effects through pension reform, we first present projections under the counterfactual assumption of an unchanged pension system (isolating the population aging effect) and then add the two pension reform scenarios described in section 2.

We begin by looking at the aggregate savings rate, defined as household savings as a percentage of disposable income. Figure 5 shows that the projected aggregate savings rates decline substantially due to aging. For example, in the year 2038, when the peak of the aging problem occurs, the savings rate is projected to be substantially negative at -2 percent under the pure PAYG system. The difference between the projected peak of the savings rate that will be reached in year 2015 and the trough in 2035 is about 9.2 percentage points. Savings under a very fundamental pension reform – the freezing reform – would be substantially higher than under the present system. The overall difference in the aggregate savings rate between the freezing reform system and the present system would be 1.4 percentage points in 2050. This shows that optimal life-cycle behavior generates additional savings under a fundamental pension reform. Our model therefore shows that additional retirement saving induced by a pension reform does not crowd out other saving totally, as is sometimes claimed. The Riester reform will only marginally change household’s optimal savings behavior. This result is due to the small impact of the Riester reform on contributions to the public pension system and the pension level (see figures 3 and 4).
Next, we aggregate savings to obtain the economy’s capital stock. Figure 6 shows projections of the total capital stock for alternative pension schemes. Since crowding out is only partial, the extra savings induced by both reform schemes result in a higher long-term capital stock. Figure 6 also shows how misplaced the notion of an “asset meltdown” actually is. While saving rates decline sharply during the peak of the aging process and the capital stock does decrease between the years 2030 and 2045 under all pension systems, the size of this decline is small. Moreover, as soon as the projected old age dependency ratio becomes roughly stable, the effect of technological progress dominates the demographic effects and the capital stock increases again. The projected maximum decrease of the capital stock – between 2030 and 2043 under the pure PAYG pension system – is about 5.5 percent.

The projected time path of the capital stock shown in figure 6 has several implications. First, the magnitude of accumulated new savings under the freezing reform scheme is manageable, relative to the current capital stock. The long run value (year 2050) represents about 16 percent of the current (year 2000) capital stock under the freezing reform and 4 percent under the Riester reform. It is about equal to today’s value of life insurance savings and occupational pensions. Second, compared to the pure PAYG system scenario, there is less decline in the capital stock between the years 2030 to 2045 when the baby boomers retire because the baby boom retirement entry stretches about 10 years during which the pre-funded component has not yet matured. The continued increase in new funded-pillar pension accounts compensates dissaving among the retired baby boomers.

Population aging leads to a dramatic increase of the capital-labor ratio. It is exacerbated by the two pension reforms. This increase translates into a reduced return to capital. This is depicted in figure 7. The decrease in the return to capital due to population aging is projected to be around 0.7 percentage points from year 2000 to year 2028. Relative to this population aging effect, the additional decrease in the rate of return due to the Riester pension reform is only marginal. Under the more radical freezing reform this additional decrease would amount to some 0.5 percentage points. These effects are much smaller than often claimed in the public debate. They reflect the small decrease in the capital stock when the baby boomers retire. The main reason for the difference to the “meltdown” projections are the equilibrium effects modeled in our overlapping generation model. Saving rates adjust to demographic changes, and they rebound after the trough around 2040, when the demographic changes slow down.

15 The reader is reminded of our definition of the replacement rate, which – unlike the current government’s definition – preserves comparability under the various pension reform schemes. See section 2.
Before turning to the sensitivity of our results, it is worth noting that these estimates are based on a closed-economy model. International diversification will further reduce the decrease in the rate of return since international capital mobility will diversify the demographic shock. We will discuss this briefly in the concluding section.

5. Sensitivity analysis

How do the core results of our analysis – a rather muted response of the aggregate saving rate and the rate of return to population aging, and an even smaller additional effect of pension reform on these two variables – stand up when key parameters and assumptions of our simulation model are changed?

Our sensitivity analysis is carried out along two dimensions: varying the underlying demographic and labor force projections, and varying the key parameters of the OLG model. Regarding the first dimension, we project savings and returns under all possible combinations of labor force and population scenarios, keeping all other parameters of the model fixed. Since we consider three of each of these scenarios, this provides us with a total of nine combinations.

In terms of the second dimension, we combine three values – the mean, the maximum and the minimum (see table 7) – of five key model parameters (the share of capital, the rate of substitution between capital and labor, the rate of technological progress, the coefficient of relative risk aversion, and the discount rate). In this exercise, we keep the population and labor force scenario at its benchmark values. This exercise provides us with 243 combinations.

We then compute the mean, median, minimum and maximum of the impacts of population aging and pension reform on the aggregate savings rate and the rate of return. The results are presented in tables 8 and 9. Note that treating the 9 and 243 combinations, respectively, with equal weight when computing means and medians deviates from the conventional approach where the benchmark scenarios and parameters are given a higher weight.16

Some summary statistics of the first exercise – the sensitivity analysis regarding alternative population and labor force scenarios – are given in table 8. Since the benchmark scenario is the median scenario of the combination of nine scenarios, the results of our benchmark calculations are close to the mean and median of the combination of all scenarios. Note that the minimum and maximum of the respective values are reached under the extreme scenarios. For

16 See e.g. Altig et al., 2001 for what is referred to as “the conventional approach” in the text.
example, the projected maximum decrease in the rate of return to capital of 0.7 percentage points between the freezing reform and the pure PAYG system occurs in the combination of the strong aging population scenario with a weak increase in the labor force – a fairly unrealistic scenario. And even under this assumption, the effect remains small.

Next, we turn to the sensitivity analysis with regard to the calibration parameters. As table 9 indicates, the projected decrease in the savings rate due to population aging is higher in the benchmark scenario relative to the mean (median) of the sensitivity analysis. This is due to the fact that our benchmark values for the share of capital used in the production function and the elasticity of substitution between capital and labor are at the upper bound and the discount rate of households is at the lower bound of the range of calibration parameters used in the sensitivity analysis. All three changes of these parameters in the sensitivity analysis relative to the benchmark – a decrease of $\alpha$ and $\beta$ and an increase in $\rho$ – lead to a decrease in savings. The positive effect on savings caused by an increase in $g$, the growth rate of the productivity parameter, is not strong enough to offset this decrease on average. It follows that the average capital stock obtained in the sensitivity analysis is lower and that therefore the return to capital is higher. Table 9 also shows that the increase in savings due to a fundamental pension reform is on average lower than the increase in our benchmark scenario. Due to decreasing returns of scale, this smaller mean increase in savings relative to the benchmark leads to a higher reduction in the rate of return to capital. However, the maximum projected decrease in the rate of return to capital under the freezing reform scenario is still rather low at about 0.9 percentage points, while the mean is about 0.6 percentage points.

Along both dimensions – demographic and labor force scenarios on the one hand, and key model parameters on the other hand – the sensitivity analysis has therefore provided substantial confidence in the stability of the conclusions which we have drawn at the end of the preceding section.

6. Conclusions

In this paper, we have analyzed the consequences of population aging and fundamental pension reform – that is, a shift towards more pre-funding of pension benefits – for capital markets in Germany, focusing on savings and the rate of return. A transition to a partially funded system results initially in a higher capital stock. When the baby boom generations begin to consume their retirement savings, the capital stock decreases. Our simulations suggest that the
resulting decrease in the rate of return is not negligible, but it is much smaller than often claimed in the public debate.

Moreover, there are two mechanisms that suggest that the estimates of the decrease in the rate of return due to population aging and fundamental pension reform derived from our model represent an upper bound. First, the model presented in this paper assumes that Germany is a closed economy. This is a counterfactual assumption, and the return to capital can be improved by international diversification. While the patterns of population aging are similar in most countries, the timing differs substantially, in particular between industrialized and less developed countries. Among others, Pemberton (1999), Reisen (2000), and Börsch-Supan et al. (2002) show that in a more realistic open-economy scenario, these differences in population aging generate international capital flows that moderate our rate-of-return predictions even further. Closed-economy models of pension reform exaggerate both the effects of population aging and the feedback effects of pension reform on capital markets.

Second, there may be additional feedback effects associated with strengthened capital markets through pension reforms that our fixed production structure given by equation (4) misses. It has been argued that as pension funds become more important in households’ portfolios, equity culture in Germany will improve, both deepening and broadening capital markets and possibly strengthening corporate governance (see Deutsche Bank Research, 1996). The underlying mechanisms are discussed in detail by Börsch-Supan and Winter (2001). They start from the observation that few households in Germany hold financial assets with at least some minimal ownership rights. These assets are highly concentrated among few households, in stark contrast to countries in which a substantial share of retirement income is financed through pension funds. A lack of relatively actively managed pension funds contributes to a financial system with diffuse control structures and weak corporate governance, which in turn results in low capital productivity relative to other countries. Pension reform towards a higher degree of pre-funding can therefore strengthen corporate governance, increase capital productivity at constant or even increasing levels of labor productivity, and increase total factor productivity. Even if such productivity effects of pension reform are small, Börsch-Supan and Winter argue that they change the growth path of an economy and therefore have large effects in the long run, strengthening savings and further reducing the “meltdown” effect.

To conclude this paper, a few remarks on the economic model we used to simulate the macro-economic effects of a fundamental pension reform are in order. We have already mentioned that our overlapping-generations model is stylized. Some important economic mechanisms are
not taken into account, most importantly, endogenous labor supply decisions and taxation. While it would be interesting to explore these issues in our model, we do not anticipate that they would change the basic message of our analysis. In addition, our sensitivity analysis with respect to labor market scenarios covers a wide range of potential endogenous labor market reactions.

At a more fundamental level, while the framework of rational, forward-looking life-cycle decisions is a convenient and broadly accepted tool for the analysis of the effects of pension reforms and other public policy measures, the underlying assumption of rational behavior has been criticized as being unrealistic. In many applications, one might ignore this problem by following an as if argument. For example, Rodepeter and Winter (1999) argue that simple rules of thumb might help individuals to achieve savings outcomes that are quite similar to the solution of the intertemporal optimization problem. Another problem might arise when individuals suffer from a lack of self-control: They might not be able to make time-consistent savings decisions that allow them to attain their desired levels of retirement wealth (O’Donoghue and Rabin, 1999; Diamond and Köszegi, 1999).

It is difficult to quantify the importance of deviations from the rationality assumption, in particular in an OLG model. If anything, we would argue that in Germany, in contrast to the United States, people have saved too much for retirement rather than too little (see Bösches-Supan et al., 2001), and undersaving due to time-inconsistent preferences has not been a problem in the past. Also, recent polls by Boeri et al. (2001) suggest that people are well aware of the need to save for their retirement. In any case, it should be clear that a fundamental pension reform needs to provide appropriate incentives that ensure that individuals save enough to close the future pensions gap, and that individuals invest their retirement savings wisely. Providing retirement savings products that facilitate such decisions and allow for self-commitment is an important task for insurance companies and banks, but there is no reason to believe that this cannot be achieved by the market sector. The 2001 pension reform in Germany provides such products, but it remains to be seen whether the incentives are strong enough to generate sufficient new saving; Börsch-Supan (2002) discusses this issue further.

Another aspect which is not reflected in our overlapping generations model is financial markets risk. Our analysis concentrated on the long-term path of the rate of return to capital in a model with no stochastic aggregate fluctuations, so there was no role for risk. Real-world investments are risky, and in their savings and portfolio decisions, households are concerned not only about the (expected) rate of return, but also about its variance, that is, about portfolio
risk. However, it is important to note that the average rates of return to capital projected in this paper are much lower than the rates of return to equity experienced on stock markets in the last few years. It is certainly not necessary that stock market valuations of productive capital grow at such rates to sustain a funded pension system. Even a considerable drop in the stock market relative to the record levels seen in the spring of 2000 would not jeopardize long-run average rates of return. Diamond (1999) and Poterba (2001) provide overviews of this issue and discuss what average rates of return might be expected in the future; these are in line with the results we obtained from our simulations.

Future research should incorporate risk aspects into OLG models such as the one we presented in this paper. Brooks (2000) provides an example. While his results, based on stylized demographic assumptions of an aging economy, are difficult to compare with our results which are based on realistic demographic projections and detailed reform proposals, the interest rate effects he obtains are in the same order of magnitude as the ones we report in this paper. Brooks reports that during the initial phase of demographic transition, when the baby boomers save more and drive the capital stock up, the expected rate of return to capital rises above its steady-state level by ten base points, while the risk-free rate is 23 base points higher. When the baby boomers get old and dissave, the rate of return to capital falls by ten base points, and the risk-free rate by 24 base points, below their steady-state levels.

The risk aspect of pension reform has generated strong statements. Burtless (2000) concludes his study of social security privatization and financial markets risks by taking an extreme position: “Because public social security is backed by taxing and borrowing authority of the state, it can spread risks over a much larger population ..., including contributors and beneficiaries in several generations.” We do not think that this statement can be backed by a formal analysis. Exact welfare comparisons in stochastic overlapping generations models that allow for (idiosyncratic or aggregate) risk are conceptually quite difficult (see Sargent, 1999), and they need to take account of the nature of risks involved, including the various risks inherent in pensions based on policy decisions. Moreover, one should not forget what is at stake. Given the severity of the population aging problem that lies ahead, the “tax and borrow” approach to reforming social security is just not feasible for a country aging as deeply as Germany. The political strain caused by the degree of intergenerational redistribution (and the associated tax rates) required for sustaining the existing pay-as-you-go system is likely to out-
weigh the benefits of mandated intergenerational risk sharing by far. While more research is needed on the risk aspects of a fundamental pension reform, our analysis suggests that the rate of return to capital can sustain such reforms.

17 Miles and Timmermann (1999), Bohn (1999), Shiller (1999), Storesletten et al. (1999), and Rangel and Zeckhauser (1999) provide formal analyses of institutional arrangements for intragenerational, intergenerational, and international risk sharing.
References


**Table 1:** Assumptions on mortality in the demographic model

<table>
<thead>
<tr>
<th>Year</th>
<th>Mortality Scenario</th>
<th>Male – West</th>
<th>Female – West</th>
<th>Male – East</th>
<th>Female – East</th>
<th>Male – Migrants</th>
<th>Female – Migrants</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994/1996</td>
<td>Scenario 1</td>
<td>73</td>
<td>79.5</td>
<td>70.3</td>
<td>77.9</td>
<td>80.3</td>
<td>85.2</td>
</tr>
<tr>
<td>2080</td>
<td></td>
<td>79</td>
<td>85</td>
<td>79</td>
<td>85</td>
<td>81</td>
<td>87</td>
</tr>
<tr>
<td>1994/1996</td>
<td>Scenario 2</td>
<td>73</td>
<td>79.5</td>
<td>70.3</td>
<td>77.9</td>
<td>80.3</td>
<td>85.2</td>
</tr>
<tr>
<td>2080</td>
<td></td>
<td>81</td>
<td>87</td>
<td>81</td>
<td>87</td>
<td>81</td>
<td>87</td>
</tr>
<tr>
<td>1994/1996</td>
<td>Scenario 3</td>
<td>73</td>
<td>79.5</td>
<td>70.3</td>
<td>77.9</td>
<td>80.3</td>
<td>85.2</td>
</tr>
<tr>
<td>2080</td>
<td></td>
<td>83</td>
<td>89</td>
<td>83</td>
<td>89</td>
<td>81</td>
<td>87</td>
</tr>
</tbody>
</table>

**Note:** A more detailed description of the transition paths is given in Birg and Börsch-Supan (1999).

**Source:** Birg and Börsch-Supan (1999).

**Table 2:** Assumptions on the total fertility rate in the demographic model

<table>
<thead>
<tr>
<th>Year</th>
<th>Natives - West</th>
<th>Natives - East</th>
<th>Immigrants – West</th>
<th>Immigrants – East</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>1.35</td>
<td>1.12</td>
<td>1.9</td>
<td>1.55</td>
</tr>
<tr>
<td>2040</td>
<td>1.35</td>
<td>1.35</td>
<td>1.35</td>
<td>1.35</td>
</tr>
</tbody>
</table>

**Note:** A more detailed description of the transition paths is given in Birg and Börsch-Supan (1999).

**Source:** Birg and Börsch-Supan (1999).

**Table 3:** Assumptions on net migration in the demographic model

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>+ 47,098</td>
<td>+ 47,098</td>
<td>+ 47,098</td>
</tr>
<tr>
<td>since 2002 / 2005</td>
<td>+ 22,064</td>
<td>+ 120,000</td>
<td>+ 219,069</td>
</tr>
</tbody>
</table>

**Note:** A more detailed description of the transition paths is given in Birg and Börsch-Supan (1999).

**Source:** Birg and Börsch-Supan (1999).

**Table 4:** Assumptions on decreases in unemployment

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>10 %</td>
<td>10 %</td>
<td>10 %</td>
</tr>
<tr>
<td>2030</td>
<td>7 %</td>
<td>5 %</td>
<td>4 %</td>
</tr>
</tbody>
</table>

**Table 5:** Assumptions on the mean retirement age

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>2050</td>
<td>60</td>
<td>63</td>
<td>65</td>
</tr>
</tbody>
</table>
Table 6: Assumptions on an adjustment of labor force participation rates by 2050

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women to Men</td>
<td>0</td>
<td>0.6</td>
<td>0.9</td>
</tr>
<tr>
<td>East to West</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Immigrants to Natives</td>
<td>0</td>
<td>0.15</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Note: The difference between the age-specific labor force participation rates is reduced by the factor given in the table. A more detailed description of the procedure is given in Ludwig (2002).

Table 7: Calibration of parameters in the overlapping generations model

<table>
<thead>
<tr>
<th>Calibration parameter</th>
<th>Benchmark values</th>
<th>Sensitivity analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>α: output share of capital in the CES production function</td>
<td>0.4</td>
<td>0.3 – 0.4</td>
</tr>
<tr>
<td>β: elasticity of substitution in the CES production function</td>
<td>1</td>
<td>0.8 – 1</td>
</tr>
<tr>
<td>g: growth rate of labor augmenting technical change</td>
<td>0.015</td>
<td>0.01 – 0.02</td>
</tr>
<tr>
<td>δ: depreciation rate of capital</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>ρ: rate of time preference</td>
<td>0.01</td>
<td>0.01 – 0.03</td>
</tr>
<tr>
<td>σ: coefficient of relative risk aversion</td>
<td>2.7</td>
<td>2 – 4</td>
</tr>
</tbody>
</table>

Table 8: Sensitivity with regard to population and labor force scenarios

<table>
<thead>
<tr>
<th>Difference in...</th>
<th>Benchmark</th>
<th>Sensitivity analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>S/Y under CS</td>
<td>-0.092</td>
<td>-0.092</td>
</tr>
<tr>
<td>r under CS</td>
<td>-0.0067</td>
<td>-0.0072</td>
</tr>
<tr>
<td>S/Y between FR and CS</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td>r between FR and CS</td>
<td>-0.0047</td>
<td>-0.005</td>
</tr>
<tr>
<td>S/Y between RI and CS</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>r between RI and CS</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
</tbody>
</table>

Note: S/Y: savings rate, r: rate of return to capital, CS: pure PAYG system, FR: freezing model, RI: Riester reform. The differences between the savings rate and the rate of return to capital in the pure PAYG system are the effects of population aging and give the differences in the respective rates between the peak in about 2015 and the trough in about 2035. The differences in the respective rates between the pension systems are evaluated in 2050.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
### Table 9: Sensitivity with regard to the calibration parameters

<table>
<thead>
<tr>
<th>Difference in...</th>
<th>Benchmark</th>
<th>Sensitivity analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
</tr>
<tr>
<td>S/Y under CS</td>
<td>-0.092</td>
<td>-0.067</td>
</tr>
<tr>
<td>r under CS</td>
<td>-0.0067</td>
<td>-0.006</td>
</tr>
<tr>
<td>S/Y between FR and CS</td>
<td>0.013</td>
<td>0.0096</td>
</tr>
<tr>
<td>r between FR and CS</td>
<td>-0.0047</td>
<td>-0.0057</td>
</tr>
<tr>
<td>S/Y between RI and CS</td>
<td>0.002</td>
<td>0.0014</td>
</tr>
<tr>
<td>r between RI and CS</td>
<td>-0.001</td>
<td>-0.0012</td>
</tr>
</tbody>
</table>

**Note:** S/Y: savings rate, r: rate of return to capital, CS: pure PAYG system, FR: freezing model, RI: Riester reform. The differences between the savings rate and the rate of return to capital in the pure PAYG system are the effects of population aging and give the differences in the respective rates between the peak in about 2015 and the trough in about 2035. The differences in the respective rates between the pension systems are evaluated in 2050.

**Source:** Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 1: Projections of the old-age dependency ratio

Notes: This figure shows projections of the number of persons older than 65 as a percentage of the working age population for three demographic scenarios.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 2: Projections of the economic dependency ratio

Notes: This figure shows projections of the number of pensioners as a percentage of the number of workers for combinations of demographic and labor force scenarios.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
**Figure 3:** Projections of the comprehensive contribution rate to the public pension system under alternative pension systems

Notes: This figure shows projections of the comprehensive contribution rate to the public pensions system derived under benchmark assumptions for demographic change and the development of the labor force and alternative pension systems.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 4: Projections of the replacement rate of the public pension system under alternative pension systems

Notes: This figure shows projections of the comprehensive contribution rate to the public pensions system derived under benchmark assumptions for demographic change and the development of the labor force and alternative pension systems.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 5: Projections of the aggregate saving rate under alternative pension systems

Notes: This figure shows projections of the aggregate saving rate derived from an overlapping generations model under benchmark assumptions for demographic change and the development of the labor force and alternative pension systems.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 6: Projections of the aggregate capital stock under alternative pension systems

Notes: This figure shows projections of the aggregate capital stock derived from an overlapping generations model under benchmark assumptions for demographic change and the development of the labor force and alternative pension systems.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
Figure 7: Projections of the rate of return to capital under alternative pension systems

Notes: This figure shows projections of the rate of return to capital derived from an overlapping generations model under benchmark assumptions for demographic change and the development of the labor force and alternative pension systems.

Source: Own calculations, based on demographic projections by Birg and Börsch-Supan (1999).
# Discussion Paper Series

Mannheim Research Institute for the Economics of Aging Universität Mannheim

To order copies, please direct your request to the author of the title in question.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Autoren</th>
<th>Titel</th>
<th>Jahr</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-02</td>
<td>Axel Börsch-Supan, Alexander Ludwig, Joachim Winter</td>
<td>Aging and International Capital Flows</td>
<td>02</td>
</tr>
<tr>
<td>11-02</td>
<td>Axel Börsch-Supan</td>
<td>Labor market effects of population aging</td>
<td>02</td>
</tr>
<tr>
<td>12-02</td>
<td>Axel Börsch-Supan</td>
<td>Kann die Finanz- und Sozialpolitik die Auswirkungen der Bevölkerungsalterung auf den Arbeitsmarkt lindern?</td>
<td>02</td>
</tr>
<tr>
<td>13-02</td>
<td>Florian Heiß, Jens Köke</td>
<td>Dynamics in ownership and firm survival: Evidence from corporate Germany</td>
<td>02</td>
</tr>
<tr>
<td>15-02</td>
<td>Axel Börsch-Supan</td>
<td>Nach der Reform ist vor der Reform: Weitere Schritte für eine nachhaltige Reform der Altersvorsorge in Deutschland</td>
<td>02</td>
</tr>
<tr>
<td>16-02</td>
<td>Florian Heiss</td>
<td>Specification(s) of Nested Logit Models</td>
<td>02</td>
</tr>
<tr>
<td>17-02</td>
<td>Axel Börsch-Supan</td>
<td>What We Know and What We Do NOT Know About the Willingness to Provide Self-Financed Old-Age Insurance</td>
<td>02</td>
</tr>
<tr>
<td>18-02</td>
<td>Axel Börsch-Supan, Agar Brugiavini</td>
<td>Savings: The Policy Debate in Europe</td>
<td>02</td>
</tr>
<tr>
<td>19-02</td>
<td>Axel Börsch-Supan, Lothar Essig</td>
<td>Stockholding in Germany</td>
<td>02</td>
</tr>
<tr>
<td>20-02</td>
<td>Axel Börsch-Supan, Simone Kohnz, Reinhold Schnabel</td>
<td>Micro Modeling of Retirement Decisions in Germany</td>
<td>02</td>
</tr>
<tr>
<td>21-02</td>
<td>Joachim Winter</td>
<td>The impact of pension reforms and demography on stock markets</td>
<td>02</td>
</tr>
<tr>
<td>22-02</td>
<td>Axel Börsch-Supan</td>
<td>Mehr Zuwanderung? Zur Rolle des Auslands bei der Stabilisierung der gesetzlichen Rentenversicherung in Deutschland</td>
<td>02</td>
</tr>
<tr>
<td>23-02</td>
<td>Axel Börsch-Supan, Florian Heiss, Alexander Ludwig, Joachim Winter</td>
<td>Pension reform, capital markets, and the rate of return</td>
<td>02</td>
</tr>
</tbody>
</table>