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# Herding differently: A level- $k$ model of social learning 

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This paper proposes a behavioral model of social learning that unifies various forms of inferential reasoning in one hierarchy of types. Iterated best responses that are based on uninformative level-0 play lead to the following of the private information (level-1), to the following of the majority (level-2), to a differentiated view on predecessors (level-3), etc. I present evidence from three sources that these are the prevalent types of reasoning in social learning: a review of social learning studies, existing data from Çelen and Kariv (2004) as well as new experimental data that includes written accounts of reasoning from incentivized intra-team communication.

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[^0]How does the observation of others' choices influence a person's vote for a political candidate, a CEO's placing of a takeover bid or a doctor's choice of a medical treatment. In other words, how do people use information about others' decisions when they have to decide on a similar matter? Can the iterated belief structure of the level- $k$ model for strategic decisions usefully describe how others' decisions are fathomed in such a social learning context? This study aims to illuminate these questions with the use of experimental data.

A first theoretical perspective on social learning was given by models of Bikhchandani, Hirshleifer and Welch (1992) and Banerjee (1992). In a sequence of decision-makers that want to act according to the true but uncertain state of the world, players hold private information and observe their predecessors' decisions. A striking conformity and a wasteful neglect of private information are predicted to eventually emerge in form of an information cascade.

A large experimental literature, starting with Anderson and Holt (1997, AH henceforth), set out to test the assumption of Bayesian inference within errorrate or Quantal Response Equilibrium (QRE) models (for example, Çelen and Kariv, 2004; Drehmann, Oechssler and Roider, 2005; Goeree, Palfrey, Rogers and McKelvey, 2007). Kübler and Weizsäcker (2004) estimate the depth of reasoning in an error-rate model as they allow the error to depend on the level of reasoning. The studies generally find substantial deviations from Bayesian inference, mostly in that decision-makers overweight their own information.

Other studies try to explicitly model the deviations from Bayesian inference and propose various types of players. Hung and Plott (2001) consider "truthful, private-information revealers" and "naïve Bayesians", who think others are truthful. Eyster and Rabin (2005) analyze "cursed" players who underinfer from others actions and thus play truthful. Further, they propose "Best Response Trailing Naïve Inference", a best response to cursed players (Eyster and Rabin, 2010). Dominitz and Hung (2004) consider Bayesians that deem off-equilibrium play informative, an intuition already formulated by AH and predicted by the error-rate models.

In this paper, I propose a behavioral model of social learning that unites a variety of these types in one coherent framework. Inspired by the level of reasoning model of strategic thinking (Nagel, 1995; Stahl and Wilson, 1995; Costa-Gomes, Crawford and Broseta, 2001; Camerer, Ho and Chong, 2004), my model organizes types by the number of iterated best responses to uninformative level-0 play.In the resulting hierarchy, types are linked via the population belief so that lower
level types describe the model that higher types form of other decision-makers.
In particular, a level-1 player believes that all predecessors' actions stem from uninformative level-0 play; her best response is therefore to follow the private signal. Believing all others to play according to their signal, the level-2 player views all previous actions as equally informative as her own private signal. Level3 players realize that some level-2 actions do not reflect private signals and assess them as uninformative. So does level-4 for level-3 play, etc.

The model as a whole is related to the incomplete information level- $k$ model which Crawford and Iriberri (2007) introduce to analyze data on common value auctions. The most closely related model is by Brocas, Carillo, Wang and Camerer (2010) who provide a level- $k$ model in the context of zero-sum betting. Both applications feature inference of private information as well as strategic interaction. In contrast, looking at social learning puts the focus entirely on inference and abstracts from strategic interaction.

Most of the experimental literature in social learning relates to the seminal paper by Anderson and Holt (1997). In their experiment, subjects observe a binary private draw that reveals the state of the world correctly with a probability of $\frac{2}{3}$. Before subjects choose a binary action to match the state, they can observe the actions of all predecessors. Due to the coarse binary structure, this setting provides only limited information about the reasoning underlying the observed decisions. For example, level-2 or higher and Bayesian reasoning are predicted to always play the same actions. For Eyster and Rabin (2010, p. 236), "the existing experimental literature is generally not well-designed to differentiate among likely hypotheses about the nature of observational learning." In my analysis, I turn to a different framework of social learning and a new experimental method in order to be able to differentiate types.

First, I reconsider various experimental studies in the AH tradition and relate the findings to features of the proposed model. Second, I analyze the rich data of a social learning experiment with continuous actions from Çelen and Kariv (2004, ÇK henceforth). Finally, I use the original AH and ÇK setups in two experiments with intra-team communication as introduced by Burchardi and Penczynski (2012).

In all instances, I find evidence in favor of the modeled heterogeneity in inferential reasoning. In all analyses on the individual level, subjects of all types are detected and the modal behavior turns out to be level-2. The arguments observed in the team communication mirror the decision rules in the proposed hierarchy
of types.
I conclude that the level- $k$ framework is a valid behavioral model of social learning which explains occurring deviations from Bayesian play and rational expectations (see Weizsäcker, 2010). The extension of the level- $k$ concept to social learning has the potential to contribute a similarly good understanding of inferential reasoning as it achieves for strategic reasoning. Furthermore, this extension highlights the empirical importance of the cognitive processes that are modelled in the level- $k$ literature and provides another view on how people form models of others in their minds.

The paper is structured as follows. Section 1 introduces the general model. The empirical part is started with a new look at past studies in section 2. The analysis of ÇK's data (section 3) and of the experiments with intra-team communication (section 4) follows. Section 5 discusses the results before section 6 concludes.

## 1. The model

After a general introduction to the model, I apply it in sections 1.2 and 1.3 to the social learning frameworks used in the studies by AH and ÇK. ${ }^{1}$

### 1.1. The general model

Consider a general social learning framework, in which players take actions sequentially and aim to match an uncertain state of the world $\omega \in \Omega$. A player $t$ receives information about the state of the world via a private signal $s_{t} \in \mathcal{S}$. Furthermore, she observes the history of actions up to round $t$, denoted by $H_{t}=\left(a_{1}, a_{2}, \ldots, a_{t-1}\right)$. The strategy of a level- $k$ type at position $t$ in the sequence is $\sigma^{k}\left(s_{t}, H_{t}\right)$, mapping the observed history $H_{t}$ and the signal $s_{t}$ to an action $a \in \mathcal{A}$ in a way that the action maximizes the expected payoff under the information $s_{t}$ and $H_{t}$ and the belief that others follow the strategy $\sigma^{k-1}$. I introduce the model using a degenerate population belief on level $k-1, p^{k, l}=1$ for $l=k-1$, and discuss unconstrained population beliefs afterwards. In order to talk about the inference from the action to the observed signal under a given strategy, I denote the inverse image of a strategy as $\sigma^{k \leftarrow}\left(a_{t}, H_{t}\right)=\left\{s_{t} \in \mathcal{S}: \sigma^{k}\left(s_{t}, H_{t}\right)=a_{t}\right\}$, which maps histories and actions into sets of signals.

As is common in the level- $k$ literature, the model postulates a hierarchy of

[^1]heterogeneous types. In my model, the hierarchy's starting point is the level0 player that plays uninformatively and randomizes uniformly over the action space,
$$
\sigma^{0}\left(s_{t}, H_{t}\right) \sim U(\mathcal{A})
$$

Smith and Sørensen (2000) model a related, uninformative type which I view as nested in my specification. Their "crazy type" always chooses one and the same action independently of signal and history. Intuitively, a level-0 player lacks either an understanding of the game or the motivation to use any information for his action.

The level-1 player assumes others to be level- 0 players and best responds accordingly. Given the uninformativeness, the recovery of the private signals of preceding level-0 players is not possible, $\sigma^{0 \leftarrow}\left(a_{t}, H_{t}\right)=\mathcal{S}$ is not informative. Solely the own private signal is informative to level-1 players and enters their strategy,

$$
\sigma^{1}\left(s_{t}, H_{t}\right)=\sigma^{1}\left(s_{t}\right)
$$

This player understands how her private signal is informative but either expects others to play randomly or does not understand how others' decisions can be useful. This type has been discussed among others as "Truthful, private-information revealer" (Hung and Plott, 2001), "Follow your own signal"-type (Kübler and Weizsäcker, 2004, $\left.\lambda_{1} \rightarrow \infty, \lambda_{2}=0\right)^{2}$, and "Fully cursed player" (Eyster and Rabin, 2005).

While level-1 players disregard the history of random actions, level-2 strategies depend on it heavily since all earlier actions are believed to be informative of the private signals, with only a strict subset of $\mathcal{S}$ leading to each action, $\sigma^{1 \leftarrow}\left(a_{t}, H_{t}\right) \varsubsetneqq \mathcal{S}$. Call the set of predecessors whose actions are informative about the signal under a level- $k$ strategy $\mathcal{I}^{k}=\left\{i: \sigma^{k \leftarrow}\left(a_{i}, H_{i}\right) \varsubsetneqq \mathcal{S} \wedge i<t\right\}$. It follows that $\mathcal{I}^{1}=\{i: i<t\}$, the set of all predecessors. The level- 2 strategy simply aggregates the own signal and the detailed information about the other signals, ${ }^{3}$

$$
\sigma^{2}\left(s_{t}, H_{t}\right)=\sigma^{2}\left(s_{t},\left\{\sigma^{1 \leftarrow}\left(a_{i}, H_{i}\right)\right\}_{i \in \mathcal{I}^{1}}\right) .
$$

This player expects others to play according to their signal and does not see how they might be like him: influenced by their history. This type has been discussed

[^2]as "Naive Bayesian" (Hung and Plott, 2001), "counting heuristic"-type (Kübler and Weizsäcker, 2004, $\lambda_{1}=\lambda_{2} \rightarrow \infty, \lambda_{3}=0$ ) and BRTNI (Eyster and Rabin, 2010, Best Response Trailing Naïve Inference).

A level-3 player can infer the private signals from observed actions if the action of level-2 players is dependent of their signal, i. e. the signal is not made irrelevant by others' actions and $\sigma^{2 \leftarrow}\left(a_{i}, H_{i}\right) \varsubsetneqq \mathcal{S}$. The level-3 strategy is based on an inference from those predecessors' actions that are informative,

$$
\sigma^{3}\left(s_{t}, H_{t}\right)=\sigma^{3}\left(s_{t},\left\{\sigma^{2 \leftarrow}\left(a_{i}, H_{i}\right)\right\}_{i \in \mathcal{I}^{2}}\right) .
$$

Level-3 players understand that others are influenced by their predecessors and take this into account when inferring private signals. They are the lowest type in the hierarchy that necessarily differentiate individual predecessors in their inference, like Bayesian players do. Unlike Bayesians, they attribute previous play to level 2, and do not expect others to be like themselves. The level-3 type is somewhat related to the Bayesians that infer from off-equilibrium play as in Dominitz and Hung (2004) and AH's (p. 850) intuition.

The level-3 strategy will lead to a set of players $\mathcal{I}^{3}$ with informative actions which level-4 players will exclusively use to infer private signals. Depending on the informativeness of actions and signals, higher types' strategies are more or less different from lower types and strategies can be identical to Bayesian play. The strategy of a general level- $k$ type can be expressed as

$$
\sigma^{k}\left(s_{t}, H_{t}\right)=\sigma^{k}\left(s_{t},\left\{\sigma^{k-1 \leftarrow}\left(a_{i}, H_{i}\right)\right\}_{i \in \mathcal{I}^{k-1}}\right)
$$

The population beliefs in the level- $k$ literature are specified as either degenerate on the next lower level $k-1$ (e.g. Nagel, 1995; Costa-Gomes and Crawford, 2006) or as a non-degenerate distribution on levels $0, \ldots, k-1$ (e.g. Stahl and Wilson, 1995; Camerer et al., 2004). They usually are not specified to differentiate between individual opponents because in the common applications of the level- $k$ model players mostly face single individuals or homogeneous groups, like in the beauty contest game, normal form games, or multi-player auctions.

In social learning environments, differentiation enters naturally because players differ in their location in the sequence and thus in the information they hold. Since players know the entire public information available to their predecessors they will differentiate between them when they expect predecessors' strategies to depend on this information. It follows that level-1 and level- 2 players do not differentiate predecessors because they believe them to play independently of their
idiosyncratic history. However, from level-3 onwards, the population belief, $p_{i}^{k, l}$, depends on the position of the predecessors $i$ because, starting with level- 2 , the players' strategies depend on their history $H_{i}$.

For ease of exposition, the model is so far formulated with a degenerate and homogeneous population belief. In the non-degenerate model, the strategy $\sigma^{k}$ would simply depend on inferences from lower level players $\sigma^{l \leftarrow}, l=1, \ldots, k-1$ weighted by the population belief $p_{i}^{k, l}$. The complete specification that I want to put forward posits an initial homogeneous population belief (without subscript $i$ ), but allows that the observed history can be used to update the beliefs in a way that differentiates predecessors. Furthermore, I posit nearly degenerate population beliefs with most of the probability mass on level $k-1$ and small probabilities on the remaining levels. In particular, I assume that $p_{i}^{k, k-1}=1-\sum_{d=1}^{k-1} \varepsilon^{d}$ and $p_{i}^{k, l}=\varepsilon^{(k-1)-l}$ for $l=0, \ldots, k-2$, for a small and positive $\varepsilon$.

This specification is chosen for two reasons. First, if an observed action and history rules out that the player is of level $-k-1$, the update with Bayes' rule for this individual leads again to a nearly degenerate population belief on level $k-2{ }^{4}$ Second, believing with small probability that a predecessor plays independently of his signal and history, $p_{i}^{k, 0}=\varepsilon^{k-1}$, works as a tie-breaker and rationalizes that players follow their private information in case they would be indifferent between two actions under $p_{i}^{k, 0}=0$.

### 1.2. Anderson and Holt (1997)

The framework used by AH has a simple binary information structure with states of the world $\omega \in\{A, B\}$, private signals $s_{t} \in\{A, B\}$, and actions $a_{t} \in\{A, B\}$. Both states of the world are equally likely. The private signal is informative with $\operatorname{Pr}(s=\omega)=\frac{2}{3}$. The payoff of player $t$ is positive when $a_{t}=\omega$ and 0 otherwise. The level-0 player randomizes over the action space $\mathcal{A}=\{A, B\}$,

$$
\sigma^{0}\left(s_{t}, H_{t}\right)= \begin{cases}A, & \text { with } \operatorname{Pr}=0.5 \\ B, & \text { with } \operatorname{Pr}=0.5\end{cases}
$$

As in the general case, the level-1 player best-responds by simply following his signal,

$$
\sigma^{1}\left(s_{t}, H_{t}\right)=\sigma^{1}\left(s_{t}\right)=s_{t} \subset \mathcal{S} .
$$

[^3]The level- 2 player thus deems previous decisions similarly informative as his own signal and aggregates the information by counting the evidence $C^{1}$ for a given state of the world,

$$
C^{1}\left(\omega \mid s_{t}, H_{t}\right)=\mathbb{1}\left(s_{t}=\omega\right)+\sum_{i<t} \mathbb{1}\left(\sigma^{1 \leftarrow}\left(a_{i}, H_{i}\right)=\omega\right) .
$$

The level-2 player plays according to the state with the most evidence,

$$
\sigma^{2}\left(s_{t}, H_{t}\right)= \begin{cases}A, & \text { if } C^{1}\left(A \mid s_{t}, H_{t}\right)>C^{1}\left(B \mid s_{t}, H_{t}\right) \\ B, & \text { if } C^{1}\left(A \mid s_{t}, H_{t}\right)<C^{1}\left(B \mid s_{t}, H_{t}\right) \\ s_{t}, & \text { otherwise }\end{cases}
$$

Consequently, the level-3 player can infer the private signal from actions only if a different signal had caused the level-2 player to choose a different action. A level-3 player sums the evidence $C^{2}$ in favor of a given state of the world as follows

$$
C^{2}\left(\omega \mid s_{t}, H_{t}\right)=\mathbb{1}\left(s_{t}=\omega\right)+\sum_{i \in \mathcal{I}^{2}}\left(\mathbb{1}\left(\sigma^{2 \leftarrow}\left(a_{i}, H_{i}\right)=\omega\right)\right) .
$$

The level-3 strategy is then

$$
\sigma^{3}\left(s_{t}, H_{t}\right)= \begin{cases}A, & \text { if } C^{2}\left(A \mid s_{t}, H_{t}\right)>C^{2}\left(B \mid s_{t}, H_{t}\right) \\ B, & \text { if } C^{2}\left(A \mid s_{t}, H_{t}\right)<C^{2}\left(B \mid s_{t}, H_{t}\right) \\ s_{t}, & \text { otherwise }\end{cases}
$$

In this environment, level-3 actions and beliefs about the state of the world are identical to Bayesians' for a given history. Strategies $\sigma^{2}$ and $\sigma^{3}$ yield the same actions, hence higher level players only differ in the population belief and not in their actions or beliefs about the state of the world. ${ }^{5}$

## 1.3. Çelen and Kariv (2004)

The setting implemented in ÇK has discrete actions $a_{t} \in\{A, B\}$, but continuous, uniformly distributed, bounded signals $s_{t} \sim U[-10,10]$, with the payoff depending on the match of the action with the sign of the sum of all players' signals $\sum_{t=1}^{T} s_{t}$. For a positive sum, action $A$ yields a positive payoff and action $B$ nothing, and vice versa for a negative sum.

Usually, a strategy would map from the space of the observed history and the signal $\{A, B\}^{t-1} \times[-10,10]$ into the action space $\{A, B\}$. Since the expected payoffs are monotonic in the private signal, the optimal strategies are summarized

[^4]in a threshold $\theta \in[-10,10]$. In the experiment, subjects choose a threshold before seeing their own signal. After the realization of the signal, the computer derives actions from the thresholds as follows,
\[

\sigma\left(s_{t}, \theta\left(H_{t}\right)\right)= $$
\begin{cases}A & \text { if } s_{t} \geq \theta\left(H_{t}\right), \\ B & \text { if } s_{t}<\theta\left(H_{t}\right) .\end{cases}
$$
\]

Since the action only indicates that the signal is above or below the threshold, the inference about the signal from the action requires above all a belief about the threshold $\theta$. Then, in conjunction with the observed action, a range of the possible signal realization can be inferred and leads to an expected value of the signal.

A rational Bayesian player uses previous actions as well as beliefs about previous cutoffs to craft her optimal cutoff strategy, ${ }^{6}$

$$
\theta^{b}\left(H_{t}\right)=-E\left[\sum_{i=1}^{t-1} s_{i} \mid H_{t}, \theta^{b}\right] .
$$

This kind of inference assumes rational expectations with respect to the thresholds $\theta^{b}$ used by predecessors.

A natural choice for level-0 play is a uniform randomization of the cutoffs,

$$
\theta^{0}\left(H_{t}\right) \sim U[-10,10] .
$$

Since the cutoff and the signal are automatically translated into an action, this would be an atypical level-0 belief: the resulting action is as informative as a cutoff of 0 . As opposed to the level-0 action, I will therefore model the level-0 belief as randomizing uniformly ${ }^{7}$ over $\{-10,10\}$,

$$
\widehat{\theta^{0}}\left(H_{t}\right)=\left\{\begin{aligned}
-10, & \text { with } \operatorname{Pr}=0.5, \\
10, & \text { with } \operatorname{Pr}=0.5
\end{aligned}\right.
$$

This level- 0 belief reflects a uniform random choice of actions $a_{t} \in\{A, B\}^{8}$ and the best response for a level-1 player is to follow the private signal and therefore play the cutoff strategy

$$
\theta^{1}\left(H_{t}\right)=0 .
$$

[^5]For a level-2 reasoner, the observation of one action resulting from such a level1 strategy leads to an expected signal of 5 after action $A$ and -5 after action $B$. It follows that $E\left[\sum_{i}^{t-1} s_{i} \mid H_{t}, \theta^{1}\right]=5 \cdot \# A-5 \cdot \# B$, where $\# a$ gives the number of previously played $A$ or $B$ actions. It can be seen that level- 2 players reach the limits of the action space very quickly and play -10 or 10 once one action was played in two more occasions than the other one. Here, the level- $k$ model predicts the information cascades in which own signals are entirely disregarded $(\theta \in\{-10,10\})$ as analyzed in ÇK. Only five different actions are predicted to be observed from a level-2 player,

$$
\theta^{2}\left(H_{t}\right)=-E\left[\sum_{i}^{t-1} s_{i} \mid H_{t}, \theta^{1}\right] \in\{-10,-5,0,5,10\} .
$$

The best response to level-2 play fully discounts actions that resulted from an uninformative threshold of -10 or 10 and uses only the remaining actions to infer signals. Therefore, the level-3 uses her own signal in addition to the information from previous informative play,

$$
\theta^{3}\left(H_{t}\right)=-E\left[\sum_{i}^{t-1} s_{i} \mid H_{t}, \theta^{2}\right] .
$$

Due to the continuous strategy and signal space in this framework, higher level players can be differentiated. They form different beliefs about the informativeness of observed actions and therefore choose different thresholds. Denoting the strategy of a level- $k$ player by $\theta^{k}$, the strategies can be recursively expressed as

$$
\theta^{k}\left(H_{t}\right)=-E\left[\sum_{i}^{t-1} s_{i} \mid H_{t}, \theta^{k-1}\right] .
$$

## 2. A new look at the literature

The large amount of data available from various implementations of the AH game enables a first crude analysis of heterogeneity in terms of level- 1 vs. level- 2 and higher. For that purpose, using the meta-dataset of Weizsäcker (2010), I analyze decision situations in which the private signal $s_{t}$ goes against the majority in $\left(H_{t}, s_{t}\right)$. Thus, the choice is between "following the own signal" (level-1) and "following the majority" (level-2 or higher, $2+$ ). For experiments with $\operatorname{Pr}(\omega=$ $A)=\frac{1}{2}$ and $\operatorname{Pr}\left(s_{t}=\omega\right)=\frac{2}{3}$, the last column of Table 1 reports the fraction of players in the respective study that follow the own signal, from lowest to highest. This fraction varies between $36.9 \%$ and $63.4 \%$.

Such heterogeneity resembles the results of "beauty contest" games where the average level- $k$ estimated in a cognitive hierarchy model depends on the subject pool (Table II in Camerer et al., 2004). There, students from highly-ranked or technical universities or professors and graduate students achieve on average higher levels of reasoning than others. The same pattern is found here. The last column implies a ratio of level- 1 to level- $2+$ decisions between 0.59 and 1.74. For a Poisson-distributed level of reasoning, this implies average level estimates in the range $\tau \in[0.85,1.7] .{ }^{9}$ In Camerer et al. (2004), estimated average levels have a mean of 1.3 and a median of 1.61 , the obtained range is therefore a plausible one.

| Source | Subject pool | N | Level-1 (\%) |
| :--- | ---: | ---: | :---: |
| Goeree et al. (2007) | Caltech/UCLA | 1575 | 36.9 |
| Dominitz and Hung (2004) | Carnegie Mellon | 622 | 38.1 |
| Hung and Plott (2001) | Caltech | 266 | 41.0 |
| Ziegelmeyer et al. (2008) | Strasbourg | 202 | 43.1 |
| Nöth and Weber (2003) | Mannheim | 1479 | 43.1 |
| Kübler and Weizsäcker (2004) | Harvard | 139 | 51.1 |
| Alevy et al. (2007) | Maryland/ | 168 | 51.8 |
|  |  |  |  |
| Anderson (2001) | Professionals |  |  |
| Cipriani and Guarino (2005) | Virginia | 81 | 51.9 |
| Oberhammer and Stiehler (2003) | NYU, UCL | 34 | 55.9 |
| Drehmann et al. (2005) | Well-educated public | 95 | 62.1 |
| Anderson and Holt (1997) | U of Virginia | 80 | 62.5 |
| Willinger and Ziegelmeyer (1998) | Strasbourg | 93 | 63.4 |

Notes: Observations feature $\operatorname{Pr}(\omega=0)=0.5$ and $\operatorname{Pr}\left(s_{t}=\omega\right)=\frac{2}{3}$.
Exceptions are Ziegelmeyer et al. with $\operatorname{Pr}(\omega=0)=0.55$, Nöth and Weber with $\operatorname{Pr}\left(s_{t}=\omega\right)=0.6$, Cipriani and Guarino with $\operatorname{Pr}\left(s_{t}=\omega\right)=0.7$ and Willinger and Ziegelmeyer with $\operatorname{Pr}\left(s_{t}=\omega\right)=0.6$.

Table 1: Fraction of decisions following the own signal, not the majority

Apart from the subject pool, the specific experimental implementation seems to have an impact on the average level of reasoning as well. In particular, the two studies from Strasbourg occupy very different ranks. One major difference is that Ziegelmeyer, Koessler, Bracht and Winter (2008) elicit beliefs about true states of the world and predecessors' signals, while Willinger and Ziegelmeyer (1998) do not. It is possible that this procedure fosters a good understanding of

[^6]the situation and allows more subjects to evaluate and make use of the public information. ${ }^{10}$

Overall, this table provides first suggestive evidence of a heterogeneity in inferential reasoning analogue to the heterogeneity which the level $k$ literature has identified in strategic settings.

Some progress in terms of differentiating between various ways of reasoning behind one of the binary actions $A$ and $B$ has been made by collecting further information on beliefs. Oberhammer and Stiehler (2003), Dominitz and Hung (2004) and Ziegelmeyer et al. (2008) elicit beliefs and find that the beliefs become more extreme during a cascade, a behavior that - in my model - would be predicted by level-2 play but at odds with level- 1 , level- 3 and Bayesian play. This points to the important role of heterogeneity as established in my model and in particular to the potential relevance of level-2 play.

## 3. Data from Çelen and Kariv (2004)

ÇK implemented their social learning game as explained in section 1.3 in an experiment at New York University with 40 undergraduate subjects that played 15 independent rounds in sequences of 8 players. After each completed sequence, participants were informed whether their action coincided with the state of the world and were paid $\$ 2$ if it did.

I will use the 15 choices of each subject to investigate their individual type. Following Costa-Gomes and Crawford (2006), I compare the chosen cutoffs $\left\{\theta_{r}\right\}_{r=1}^{15}$ to the cutoffs $\left\{\theta_{r}^{k}\right\}_{r=1}^{15}$ that an ideal level- $k$ or rational type is predicted to choose in the exact same 15 situations. I distinguish between the 4 types: level-1, level2, level-3, and Bayesian. Figure 1 illustrates "fingerprints" of ideal types and selected subjects.

As the distance metric between fingerprints of subjects and ideal types I use the sum of squared differences, $S S D=\sum_{r=1}^{15}\left(\theta_{r}-\theta_{r}^{k}\right)^{2}$. I use this measure because calculating the expected payoff for a cutoff in a given situation is extremely complex due to the payoff function and the uniformly distributed signals. It can be observed, however, that the probability of playing the correct action is quadratic in the cutoff because the cutoff linearly influences i) the probability of playing a certain action and ii) the probability of playing the correct action given

[^7]

Figure 1: Subjects' and types' "fingerprints" in the data of ÇK
the action played.
Because of this complexity, it is difficult to craft an objective probability distribution of choosing the correct threshold for different ideal types. I can, however, calculate the distribution of SSDs resulting from random play and evaluate how likely it is that random play leads to smaller SSDs than the subject's SSD. With random play under the null hypothesis, this tests whether the observed play is significantly closer to the ideal type than random play. The distribution of SSDs is type-specific since, for example, random play achieves lower SSDs with respect to the level- 1 strategy of always playing the midpoint of the action space $\left(\theta^{1}=0\right)$ than under the level-2 strategy with frequent choices of $\theta^{2}=|10| .{ }^{11}$

Using this data, I classify subjects into the type that gives the lowest significant SSD ( $p$-value $\leq 0.05$ ). ${ }^{12}$ The six players that do not differ significantly from random play for any type are reported as unclassified (NA). The resulting type distribution is shown in Table 2a.

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| 1 | 6 | 15.0 |
| 2 | 20 | 50.0 |
| 3 | 5 | 12.5 |
| B | 3 | 7.5 |
| NA | 6 | 15.0 |
| Total | 40 | 100.00 |

(a) Classification by lowest significant SSD

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| 1 | 6.42 | 16.0 |
| 2 | 18.61 | 46.5 |
| 3 | 4.07 | 10.2 |
| B | 4.90 | 12.3 |
| NA | 6.00 | 15.0 |
| Total | 40 | 100.00 |

(b) Mixture classification

Table 2: Type overview for 40 subjects in ÇK

The table illustrates a pronounced heterogeneity of reasoning across the 40 subjects, with a distribution that has some similarities to commonly observed distributions in the level- $k$ literature. The levels 1 and 2 are present in significant shares and also level-3 and Bayesian behavior is detected. The level- $k$ distribution is hump-shaped, similar to estimated distributions in strategic interaction. By a large margin, the modal behavior turns out to be level-2 play, supporting the hypothesis that naïve inference is an important element in social learning. The results from my implementation of this framework in the experiment with intrateam communication are very similar (section 4.2).

[^8]Like subject 8 in Figure 1b, 7 players choose thresholds $\theta \in\{-10,-5,0,5,10\}$ in all 15 situations. Players classified as level-2 account for 103 of the 136 instances ( $75.7 \%$ ) of cascading $(\theta \in\{-10,10\}$ ). The model thus proposes a behavioral explanation for the information cascading detected in ÇK.

The experiment was not designed to discriminate between types in the way I do. The differences between types' fingerprints in the occurring sequence of situations are discriminating in the sense that the ideal fingerprints are never congruent and the SSDs are always distinct. Still, I need to account for cases in which subjects' fingerprints have a small SSD for more than one ideal type. To obtain a structure similar to a mixture model I classify subjects that are significantly close to, say, two types as partly one type and partly the other type. The share attributed to each type is chosen anti-proportionally to the $p$-value. Table 2 b shows that the main results remain unchanged when taking into account how well types can be discriminated.

Costa-Gomes and Crawford (2006) check for unknown types by testing whether subject's fingerprints are close to each other without being close to an ideal type. Such "clusters" of multiple subjects would suggest a common, but not predicted way of thinking about the situation. Unfortunately, such a specification test is not feasible with the original ÇK-data since each subject decides in different 15 situations. My implementation of this game with intra-team communication (section 4.2) confronts all subjects with the same situations. Further insights into the reasoning will thus be provided from the messages as well as the specification test.

## 4. Data from team communication

In order to put the results of the previous section under scrutiny and get a better insight into the reasoning process, I conducted two experiments with an intra-team communication protocol that yields incentivized written accounts of subjects' reasoning (see Burchardi and Penczynski, 2012). ${ }^{13}$ In these experiments, teams of two players are confronted with 6 decisions that arose in the original studies by AH and ÇK, respectively. In both cases, I chose decision situations that occurred later in the sequence in order for a substantial history of public information to exist.

The communication protocol incentivizes the individuals' messages within the

[^9]team as follows. Participants are randomly assigned into teams of two players. The two members are connected through the modified chat module of the experiment software. ${ }^{14}$ Once the situation is observed, each team member can state a so-called "suggested decision" and justify this in a written message. As soon as this is done for the first three situations, the suggestions and messages get exchanged simultaneously. In a next step, both team members individually state their "final decision" for the three first decisions. It is known to them that for each situation one of the two members' final decisions will be chosen randomly by the computer to count as the "team's action". This construction provides incentives to state the full reasoning underlying the suggested decision in a clear and convincing way.

In contrast to the original design in Burchardi and Penczynski (2012), subjects first write suggested decisions for three situations before both the suggested decisions and the messages are simultaneously exchanged. This ensures that the first three messages are written prior to any communication with the team partner and reflect only the individual's reasoning. The same procedure is repeated for the second three decisions. ${ }^{15}$

The message is entered freely without space or time limitations. In the AH experiment, apart from the suggested decision, another structured part of the communication consists of quantifying the confidence in the suggested decision. Subjects can put percentages between 50 and 100, indicating the subjective probability that their choice coincides with the true state of the world. As part of the incentivized communication stage, this is similar to eliciting individual beliefs.

The messages are classified independently by two research assistants and yield one main piece of information. For each individual message the RAs indicate the level of reasoning that the message corresponds to most closely. For this task, the RAs are introduced to the level- $k$ model and receive detailed instructions about characteristics of the individual types. ${ }^{16}$

The following features of reasoning are derived from the model and should be present for the message to be classified as a certain level. Random level-0 play results from misunderstanding the nature of the game or from putting arguments that are orthogonal to any reasonable inference from private signals and public

[^10]actions. Level-1 play features an open disregard of others' actions or a strong emphasis on the unambiguity of the own signal in contrast to others' decisions. Level-2 play requires that others' actions are taken at face value as a signal and predecessors are not differentiated. In contrast to this, level-3 play distinguishes individual predecessors and evaluates the information content of the action for each of them.

Both RAs first provide independent sets of classifications. After this, both are anonymously informed about all classifications of the other RA and have the possibility to revise their own classification. This iteration serves to reconsider diverging classifications and to screen errors or misperceptions. The information is merged by using only those classifications that coincide between the two RAs. The RAs agree in a large majority of classifications (AH: 567 (541) out of 636, $89.2 \%$ ( $85.1 \%$ ); ÇK: 218 (194) out of $252,86.5 \%$ ( $77.0 \%$ ); pre-revision in brackets). Burchardi and Penczynski (2012) provide further evidence for the robustness and replicability of this kind of classification. Overall, 17 out of 1272 individual RA level classifications imply a different decision than observed in the suggested decision. 10 of those come from 5 decisions in which the two RAs agree, suggesting an irregularity on the subject side.

### 4.1. The Anderson and Holt framework

The experiment was conducted in 7 experimental sessions at the LEEDR Laboratory of Cornell University. 106 subjects in teams of two were taking decisions in 6 social learning situations that arose in the original experiment of AH. ${ }^{17}$ In the experiment, the binary setting described in 1.2 has equally likely states $\omega \in\{A, B\}$ reflected by two urns $A$ and $B$. The signals $s \in\{$ White, Black $\}$ are informative draws from the urns, where urn $A$ contains 2 white and 1 black ball and urn $B$ contains 1 white and 2 black balls. Possible actions are $a \in\{A, B\}$.

Results. Table 3 gives an overview of the 6 urn choices. The first line gives the information (history and signal) available to subjects. The next two lines give means of the suggested decisions and confidences observed in the experiment. The suggested decisions take values $A$ or $B$ and the confidences take values between 50 and 100. Urn choices 1 and 3 were least controversial with a large

[^11]majority choosing $B$ and $A$, respectively, and high average confidences of nearly 70. Urn choice 4 was the most controversial with $53.8 \%$ choosing $B$. The lowest confidence, however, was indicated for urn choice 2 . The last four rows of the table indicate the expected behavior of the types proposed in the model.

|  | Urn choice |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |  |
| Information* $^{*}$ | $B B A b$ | $B A A B A b$ | $B A A a$ | $B B A B a$ | $A B A A A b$ | $B A A A b$ |  |
| Suggested decision** | 0.953 | 0.689 | 0.057 | 0.538 | 0.415 | 0.547 |  |
| Confidence | 69.76 | 60.75 | 69.14 | 64.27 | 67.17 | 64.68 |  |
| Probability | 0 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| of choice $B$ | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
|  | $2 / 3 / \mathrm{B}$ | 1 | 0.5 | 0 | 1 | 0 | 0 |

Notes: For information, $A$ and $B$ denote observed actions, $a$ and $b$ denote private signals favoring $A$ or $B$, respectively. Suggested decisions are reported in fractions of choice $B$.

Table 3: Suggested decisions and confidences as well as choice probabilities for different ideal types in the 6 urn choices $(N=106)$

The messages from the intra-team communication give direct information about individuals' reasoning. In the upcoming analysis, I focus on the first three decisions that were taken prior to any contact with the team partner. ${ }^{18}$ I pool the information for the first three decisions by forming the union of the level classifications. Table 4 summarizes the data by giving the subjects' lowest and highest level over these three decisions. ${ }^{19} 29$ subjects ( $27 \%$ ) are not classified, mostly because they did not write any message. $79 \%$ of the remaining 77 subjects are attributed a constant level.

The overall picture of this table reflects a large heterogeneity of reasoning. Focusing on the numbers of subjects on the diagonal with coinciding lower and upper bound, there is a pronounced heterogeneity with a hump-shaped distribution like in the previous analysis. Also, the marginal distributions are hump-shaped

[^12]|  | Highest level ( $\varnothing$ 1.66) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 1 | 2 | 3 | NA | Total |
| Lowest level$(\varnothing 1.38)$ | 0 | 8 | 3 | 3 | 1 | 0 | 15 |
|  | 1 |  | 12 | 6 | 0 | 0 | 18 |
|  | 2 |  |  | 40 | 3 | 0 | 43 |
|  | 3 |  |  |  | 1 | 0 | 1 |
|  | NA |  |  |  |  | 29 | 29 |
|  | Total | 8 | 15 | 49 | 5 | 29 | 106 |

Table 4: Level classification in first three decisions
and feature modes at level-2. The overall modal interval is $\{2\}$ with 40 subjects classified consistently as level-2.

Messages. With the classification being in accordance with the results in the ÇK-data, I want to illustrate each type's qualitative features in detail with the help of the messages. This will show how the subjects' views on the inference situation coincide with the kinds of reasoning that the model postulates. Tables 14 to 19 (pages 36 to 41 ) reproduce the messages of chosen subjects.

Level-0. The model is based on uninformative level-0 reasoning, which is modeled by random play. The messages in Table 14 show that subjects misunderstand the game or self-report to have no understanding of the game. Accordingly, they predominantly report the lowest possible confidence in their suggested decision of 50 .

Level-1. The messages that are categorized as level-1 reasoning illustrate an increment of understanding beyond level-0. Table 15 shows that subjects mostly explain their understanding as to why the private draw is informative. Often, a confidence of 66 or 67 is reported, which stems from the isolated understanding of the one private draw that indicates the true urn with a probability of $\frac{2}{3}$. Subjects follow their signal without exception. The reasoning frequently does not include the other players' actions. If it does, then it expresses confusion or ignorance as to how to make use of this information. In level- 1 players' minds, a model of other players is not existing, but the understanding suffices to process the information from the own private draw.

Level-2. Moving on to messages that were classified to reflect level-2 reasoning, Tables 16 and 17 show how the subjects take into account the history of others' actions in addition to their private signal. However, this perception of other players is not very differentiated as all observed actions $A$ and $B$ are given the same weight. Often, the observed history is summarized in statistics reporting, for example, that 3 out of 5 predecessors selected $A$, without taking into account the predecessors' choice situation. This information is mostly combined with the odds of the private draw being correct. Importantly, this kind of reasoning is exhibited irrespective of facing similar or mixed public information. As a result, subjects can become very confident in their own choice.

Level-3. Finally, Tables 18 and 19 present communication that was classified as reflecting level-3 reasoning. The messages show clearly how players differentiate between predecessors. They do so by putting themselves in the shoes of their predecessors, making use of the knowledge they have about their history and trying to back out their private draws. It becomes apparent that the population belief is updated using the public information available.

The model of their predecessors that these players have in mind takes into account that the signal is only one ingredient in the decision process. The other ingredient that influences the actions is the observed history. Thus, these players are sophisticated enough to model processes in other players' minds which - as shown in earlier messages - some of these other players exhibit and some do not achieve.

### 4.2. The Çelen and Kariv framework

The experiment was conducted in 4 experimental sessions at the Laboratory of the University of Mannheim. 42 subjects in teams of two were taking decisions in 6 social learning situations that arose in the original experiment of ÇK. In the experiment, the setting described in 1.3 has equally likely states $\omega \in\{A, B\}$ determined by the sign of the sum of 8 continuous signals that are uniformly distributed on $[-10,10]$. Subjects take decisions by entering a threshold in the interval $[-10,10]$. For each subject, the computer uses the threshold and the signal to determine the binary state that the subject chooses.

Results. Table 5 gives an overview of the 6 scenarios which were chosen to discriminate between the different types. The first line gives the information about
the history available to subjects. The following lines give mean and standard deviation of the choices in $[-10,10]$ as well as predicted play by level $-k$ and Bayesian players.

|  |  | Round |  |  |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 |  |  |
|  | History | $A A B$ | $B B A A$ | $A A A B A$ | $B B B$ | $A A B A$ | $B A B A$ |  |
| Sugg. decision (mean) | 0.17 | 0.18 | -3.52 | 4.29 | -2.50 | -0.48 |  |  |
| Sugg. decision (s.d.) | 4.20 | 3.95 | 5.26 | 5.43 | 4.84 | 3.02 |  |  |
| Predicted <br> choice | Level-1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
|  | Level-2 | -5 | 0 | -10 | 10 | -10 | 0 |  |
|  | Level-3 | -2.5 | -5 | -5 | 7.5 | -5 | -5 |  |
|  | Bayesian | 1.25 | -5.625 | -4.6875 | 8.75 | -4.375 | -3.125 |  |

Table 5: Empirical and theoretical decisions in the 6 rounds $(N=42)$
Like in section 3 , the 6 suggested decision per subject can be used to identify their type as illustrated in Figure 2. The analysis of the SSD to the ideal type of level- $k$ and Bayesian reasoning yields the results illustrated in Table 6. ${ }^{20}$ The individual weights in the mixture-type specification of Table 6b are reported in Table 23 on page 44. Similar to the results in sections 3 and 4.1, pronounced type heterogeneity and a mode behavior of level- 2 can be observed.

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| 1 | 10 | 23.8 |
| 2 | 15 | 35.7 |
| 3 | 3 | 7.1 |
| B | 4 | 9.5 |
| NA | 10 | 23.8 |
| Total | 42 | 100.00 |

(a) Classification by lowest significant SSD

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| 1 | 10.40 | 24.8 |
| 2 | 13.22 | 31.5 |
| 3 | 4.41 | 10.5 |
| B | 3.98 | 9.5 |
| NA | 10.00 | 23.8 |
| Total | 42 | 100.00 |

(b) Mixture classification

Table 6: Type overview for 42 subjects

Messages. Qualitatively, the messages in this experiment are very similar to the ones reported previously. The level classification yields the results shown in

[^13]

Figure 2
Subjects' and types' "fingerprints" in the team experiments à la ÇK

Table 7. The reasoning is found to be heterogeneous and the mode behavior is level-2 to a similar extent as before.

|  | Highest level ( $\varnothing$ 2.03) |  |  |  |  |  |  |
| ---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  | 0 | 1 |$)$

Table 7: Level classification in first three decisions

Specification Test. It is now possible to investigate whether types of reasoning are common which have not been considered in the theory. In contrast to section 3 , all subjects in this experiment face the same set of decisions. In a specification test as in Costa-Gomes and Crawford (2006), all fingerprints in the data are used as "pseudotypes" and compared to other subjects' fingerprints. This potentially identifies clusters of players that play similarly and therefore probably have a similarly structured reasoning.

The clusters are formed according to the conditions given in Costa-Gomes and Crawford (2006), who define a cluster "as a group of two or more subjects such that: (a) each subject's original estimated type has smaller likelihood than the pseudotypes of all other subjects in the group; and (b) all subjects in the group make 'sufficiently similar' guesses" (pp. 1761-1762). In my case, condition (a) translates into the pseudotypes having a lower $p$-value in the comparison with random play than the originally estimated ideal type. Condition (b) is implemented by defining "sufficiently similar" as significantly closer than random play ( $p<0.05$ ).

The analysis yields six clusters as shown in Table 8 (Table 23 reports cluster affiliation by subject). It will be seen that it is mostly some kind of decision error which is common among players in a cluster. Cluster D features four players who would be categorized based on the SSD if they had not persistently put the wrong sign on their thresholds, thus exhibiting probably the most simple and extreme case of decision error (see Figure 2e for an example). By the messages, they are indeed classified partially as level-1 or level-2. Clusters B, C, and F feature players who are significantly closer than random play only to level-2 play. In each
cluster, players' thresholds have a common feature which makes them closer to each other than to the ideal level-2 type. Players in cluster B and C start off with higher numbers, in cluster B they play a moderate fourth action. Players in cluster F start off with lower numbers.

Cluster E features four players who have mostly been classified as level-2 or level- 3 by the fingerprints and the messages and who share a pronounced tendency towards negative numbers which is not featured by other player of the same reasoning.

Although the decisions were chosen to discriminate as much as possible between level-2, level-3 and Bayesian, the specification test yields one large set A of overlapping and non-nested clusters of 19 players. Most of these players are classified by the fingerprints and messages as level-2, level-3 or Bayesian. This suggests that while the data is theoretically able to discriminate between types, due to decision noise this is only partially possible with the experimental data. This is a clear limitation of the data in the ÇK framework, which is somewhat alleviated in the original data due to a larger set of 15 decisions. In the setup with team communication, the messages allow to take a closer look at the reasoning and thus to investigate the validity of the fingerprint results.

| Cluster | Subjects | Characteristic |
| :---: | :--- | :--- |
| A | $2,6,7,9,12,13,14,16,21,22$, | Level-2, Level-3 or Bayesian |
|  | $23,27,29,33,34,37,38,39,42$ |  |
| B | 4,17 | Level-2 with higher round 1, |
|  |  | moderate round 4 |
| C | $5,28,31$ | Level-2 with higher round 1 |
| D | $11,19,35,40$ | Mistaken sign |
| E | $15,18,32,41$ | Negative tendency |
| F | 20,30 | Level- 2 with lower round 1 |

Notes: A is not a proper cluster, it is the union of overlapping, non-nested clusters.
Table 8: Clusters in the specification test

Level-3 and Bayesian types. Level-3 and Bayesians can be distinguished more easily in the ÇK than in the AH framework. While the classification of the messages was only investigating level reasoning, the level-3 messages can be checked for beliefs of rational play by others. To illustrate the functioning of the fingerprint analysis and the detailed insights of the messages, I want to discuss two players who could exhibit Bayesian reasoning. Table 24 reports their messages in the first three decisions translated from German.

The first example, subject 26, starts out in the first message clearly stating the assumption that predecessors played rationally. He is calculating correctly the expected signals following the first two $A$ 's in the history, but makes a rough guess for the $B$ in third position, which turns out too small a correction. Since the sequences in decisions 2 and 3 are even longer, the subject contents himself with guesses taking into account that later players in the sequence have a strong signal when they play against the majority. The fingerprint analysis identifies him as $80 \%$ level-3 and $20 \%$ Bayesian (Table 23). This follows from his guesses being closer to level-3 guesses, despite his assumption of rational preceding play (see Figure 1c).

The second example, subject 9 , starts out relatively similar as subject 26 . However, the forming of the expectation is biased towards 0 since a signal between 0 and 10 results in an expected threshold of 2 . In the third message, this is exhibited even stronger. In the second message, the reasoning is not differentiating predecessors at all and appears to be level-2. The very hesitant movements away from 0 make her being classified as $53 \%$ level-1, with the remaining weight distributed across level-2, level-3 and Bayesian (see Figure 2f).

## 5. Discussion

The fact that the level- $k$ structure known from strategic thinking successfully describes reasoning in social learning situations suggests that the model captures general features of decision making processes. The insights from this study help to flesh out some of these processes.

The results highlight how level-0, level-1, and level- 2 are fundamentally different: A level-0 player does not understand the game enough to even process her own signal. Level- 1 players understand the rules of the game and can use the signal, but do not understand how to learn from others since no model of others is conceived that tells what the history implies. Level-2 players then are able to model others in probably the least sophisticated way by assuming they play simply according to their own signal.

From level-2 onwards, the reasoning is characterized by the subjects' ability to model other peoples' reasoning in their head. The view that the step from level- 1 to level- 2 is a cognitively very important one has been put forward by Coricelli and Nagel (2009), who show in a neuroeconomic study that the level2 reasoning involves activity in other parts of the brain than level-1 reasoning
does. In particular, the additional activity is in the medial prefrontal cortex, an area that is connected with mentalizing, i. e. thinking about the mental activity of others. In the level- $k$ framework, the model of others is reflected by the population belief, which in the literature has been specified as degenerate on level $k-1$ or non-degenerate on levels lower than $k$.

The psychology literature suggests that a plausible starting point or anchor for a model of others' knowledge or reasoning is one's own knowledge or reasoning, which is then adjusted to the specific "other" (Tversky and Kahneman, 1974; Gordon, 1986; Nickerson, 1999). In this light, a degenerate population belief can be seen to reflect the first conceptualization of the other as someone that thinks in the same way as oneself thought until now. Best responding to this view of the other turns a level- $k-1$ player into a level- $k$ player who likens all others to his past level- $k-1$ self. This best response iteration might go on to level- $k+1$, etc. Alternatively, further deliberation might result in a more refined model of others in the sense that they might be less sophisticated than the player just was, making the (adjusted) population belief non-degenerate. This gives a cognitive perspective on the two competing assumptions of degenerate and non-degenerate population beliefs.

In any case, both the non-degenerate population belief and the additional best response can be viewed as a refinement of the initial model of others. Such refinements should occur in players of level-2 and higher. A possible reason for the rare observation of levels beyond 3 (Arad and Rubinstein, 2012) might be that eventually the correct specification of the population belief is deemed more fruitful than mechanically thinking through further best responses.

## 6. Conclusion

This paper proposes a level- $k$ model of social learning which introduces heterogeneous, hierarchical types in the context of inference. In contrast to much of the social learning literature, the experimental investigation is able to differentiate these various types thanks to two innovations. On the one hand, within the informative framework of social learning introduced by Çelen and Kariv (2004) the predicted behavior differs by type, enabling an analysis of individual reasoning on the basis of their data. On the other hand, independently of the framework, the nature of reasoning can be investigated with the help of written accounts obtained from incentivized intra-team communication as introduced by Burchardi
and Penczynski (2012).
The analyses find strong new evidence for predictions of the model, in particular the heterogeneity of reasoning across individuals and the presence of the particular types of reasoning. Level- 2 forms of reasoning are found to be the prevalent nature of social learning, implying that subjects have simplified models of others in their head and do not take into account others' specific situation. Rational expectations would imply that people hold the correct model of others in their minds. The model and the results explain why rational expectations fail to hold in a social learning context (Weizsäcker, 2010).

The model captures subjects' behavior well and gives a new interpretation of the hierarchical structure of reasoning. In particular, it reflects the gradual better understanding of the situation (level-0, 1 , and 2) as well as the stepwise refinement of the model held of others' reasoning (level-2, 3, etc.). Applying the concept of levels of reasoning to social learning is not only a useful extension of this theory's scope, it also highlights the importance and usefulness of iterated best responses as a framework to think about reasoning in social settings.

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A. Additional tables

| Subject ID | Level 1 |  | Level 2 |  | Level 3 |  | Bayesian |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | SSD | $p$ | SSD | $p$ | SSD | $p$ | SSD | $p$ |
| 1 | 549.13 | 0.671 | 1475.83 | 0.801 | 1143.33 | 0.750 | 1255.61 | 0.697 |
| 2 | 522.00 | 0.585 | 1022.00 | 0.183 | 870.75 | 0.199 | 850.50 | 0.202 |
| 3 | 714.61 | 0.964 | 946.61 | 0.095 | 777.61 | 0.224 | 823.28 | 0.140 |
| 4 | 331.00 | 0.068 | 131.00 | 0.000 | 134.75 | 0.000 | 99.08 | 0.000 |
| 5 | 1500.00 | 1.000 | 375.00 | 0.000 | 656.25 | 0.107 | 610.19 | 0.040 |
| 6 | 289.86 | 0.029 | 1020.36 | 0.237 | 796.35 | 0.306 | 721.46 | 0.178 |
| 7 | 725.00 | 0.970 | 200.00 | 0.000 | 368.75 | 0.004 | 441.56 | 0.012 |
| 8 | 700.00 | 0.954 | 25.00 | 0.000 | 268.75 | 0.003 | 170.86 | 0.000 |
| 9 | 255.00 | 0.012 | 320.00 | 0.000 | 203.75 | 0.000 | 298.23 | 0.001 |
| 10 | 627.00 | 0.862 | 97.00 | 0.000 | 245.75 | 0.001 | 177.84 | 0.000 |
| 11 | 687.17 | 0.943 | 275.17 | 0.000 | 156.42 | 0.000 | 168.43 | 0.000 |
| 12 | 423.00 | 0.260 | 818.00 | 0.027 | 426.75 | 0.009 | 595.65 | 0.016 |
| 13 | 661.25 | 0.914 | 51.25 | 0.000 | 117.50 | 0.000 | 97.09 | 0.000 |
| 14 | 400.01 | 0.197 | 450.01 | 0.002 | 363.01 | 0.003 | 494.64 | 0.006 |
| 15 | 838.61 | 0.997 | 394.61 | 0.001 | 310.11 | 0.002 | 318.56 | 0.001 |
| 16 | 883.00 | 0.999 | 158.00 | 0.000 | 289.25 | 0.002 | 305.77 | 0.001 |
| 17 | 0.02 | 0.000 | 723.65 | 0.059 | 536.24 | 0.034 | 529.73 | 0.033 |
| 18 | 100.00 | 0.000 | 475.00 | 0.014 | 243.75 | 0.001 | 269.21 | 0.002 |
| 19 | 186.39 | 0.001 | 548.09 | 0.023 | 555.54 | 0.041 | 575.62 | 0.055 |
| 20 | 224.69 | 0.005 | 485.09 | 0.004 | 316.89 | 0.001 | 328.71 | 0.002 |
| 21 | 524.25 | 0.593 | 120.25 | 0.000 | 140.75 | 0.000 | 112.44 | 0.000 |
| 22 | 391.00 | 0.175 | 656.00 | 0.035 | 528.50 | 0.042 | 587.05 | 0.042 |
| 23 | 421.00 | 0.254 | 141.00 | 0.000 | 137.25 | 0.000 | 124.54 | 0.000 |
| 24 | 242.25 | 0.008 | 52.25 | 0.000 | 162.25 | 0.000 | 115.79 | 0.000 |
| 25 | 308.25 | 0.043 | 743.25 | 0.012 | 415.75 | 0.007 | 607.40 | 0.013 |
| 26 | 1386.52 | 1.000 | 783.02 | 0.062 | 929.27 | 0.552 | 936.15 | 0.324 |
| 27 | 1171.15 | 1.000 | 1182.65 | 0.163 | 845.15 | 0.150 | 947.88 | 0.159 |
| 28 | 444.25 | 0.324 | 194.25 | 0.000 | 209.25 | 0.000 | 214.01 | 0.000 |
| 29 | 1028.33 | 1.000 | 258.33 | 0.000 | 490.58 | 0.006 | 389.41 | 0.001 |
| 30 | 514.66 | 0.561 | 602.66 | 0.019 | 455.91 | 0.019 | 456.18 | 0.010 |
| 31 | 893.26 | 0.999 | 537.26 | 0.003 | 538.26 | 0.030 | 522.84 | 0.008 |
| 32 | 619.25 | 0.847 | 939.25 | 0.043 | 684.25 | 0.073 | 834.31 | 0.064 |
| 33 | 5.00 | 0.000 | 885.00 | 0.093 | 652.50 | 0.054 | 740.49 | 0.071 |
| 34 | 396.98 | 0.190 | 1331.88 | 0.579 | 1110.53 | 0.705 | 1001.84 | 0.453 |
| 35 | 899.00 | 0.999 | 389.00 | 0.001 | 521.50 | 0.045 | 494.42 | 0.016 |
| 36 | 500.00 | 0.511 | 175.00 | 0.000 | 243.75 | 0.001 | 168.24 | 0.000 |
| 37 | 197.13 | 0.002 | 659.63 | 0.011 | 268.38 | 0.001 | 475.27 | 0.005 |
| 38 | 103.00 | 0.000 | 768.00 | 0.156 | 545.50 | 0.127 | 723.30 | 0.202 |
| 39 | 1500.00 | 1.000 | 575.00 | 0.013 | 881.25 | 0.444 | 669.35 | 0.066 |
| 40 | 807.11 | 0.995 | 505.11 | 0.023 | 727.31 | 0.309 | 665.61 | 0.174 |

Table 9: Type analysis in data of Çelen and Kariv (2004)

|  | Highest level ( $\varnothing$ 1.76) |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |  |
| Lowest level | 0 | 1 | 2 | 3 | NA | Total |  |
| $(\varnothing 1.50)$ | 1 |  | 13 | 6 | 0 | 0 | 11 |
|  | 2 |  |  | 42 | 3 | 0 | 18 |
|  | 3 |  |  | 2 | 0 | 45 |  |
|  | NA |  |  |  | 30 | 30 |  |
|  | Total | 5 | 13 | 53 | 5 | 30 | 106 |

Table 10: Level classification in second 3 decisions

|  | Urn choice |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Information | $B B A b$ | $B A A B A b$ | 3 | $4 A A a$ | $B B A B a$ | $A B A A A b$ |  |
| Level-0 | 5 | 7 | 7 | 8 | 6 |  |  |
| Level-1 | 5 | 10 | 3 | 12 | 16 | 17 |  |
| Level-2 | 40 | 29 | 44 | 39 | 40 | 36 |  |
| Level-3 | 3 | 2 | 3 | 1 | 4 | 4 |  |
| NA | 53 | 58 | 49 | 46 | 41 | 41 |  |
| Total | 106 | 106 | 106 | 106 | 106 | 106 |  |

Table 11: Level classification by urn choice

|  |  | Decision 1 (BBAb) |  |  |  |  | Decision 2 (BAABAb) |  |  |  |  | Decision 3 (BAAa) |  |  |  |  | Decision 4 (BBABa) |  |  |  |  | Decision 5 (ABAAAb) |  |  |  |  | Decision 6 (BAAAb) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { E } \\ 0 \\ \text { D } \\ 0 \\ 0 \\ \hline \end{gathered}$ | $\begin{aligned} & \text { A } \\ & \stackrel{U}{U} \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & \text { io } \\ & 00 \\ & 0 \\ & 0 . \end{aligned}$ | $\begin{aligned} & \text { U } \\ & \text { U } \\ & \text { U } \\ & \text { H } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $\stackrel{-1}{4}$ |  | $\begin{aligned} & \dot{0} \\ & \dot{0} \\ & \dot{0} \\ & \dot{0} \\ & 00 \\ & \dot{6} \end{aligned}$ | $\begin{aligned} & \text { U } \\ & \ddot{U} \\ & \text { U } \\ & \tilde{U} \\ & 0 \\ & \hline \end{aligned}$ |  | $\stackrel{\rightharpoonup}{4}$ | $$ | $\begin{gathered} \dot{\tilde{0}} \\ \tilde{0} \\ \dot{60} \\ 60 \\ 00 \end{gathered}$ | $\begin{aligned} & \text { U } \\ & \ddot{U} \\ & \text { U } \\ & \tilde{U} \\ & 0 \\ & \hline \end{aligned}$ |  | $\stackrel{\rightharpoonup}{4}$ | $$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & \dot{0} \\ & 60 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hline \stackrel{0}{U} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { E } \\ & \text { H } \\ & 0 \\ & 0 \\ & \text { D } \\ & \text { E } \end{aligned}$ | $\begin{aligned} & -7 \\ & 4 \end{aligned}$ | $\stackrel{N}{\sim}$ | $\begin{gathered} \dot{0} \\ \dot{0} \\ 00 \\ 60 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \text { U } \\ & \text { Z } \\ & \text { U } \\ & \text { H } \\ & 0 \\ & 0 \end{aligned}$ |  | $\underset{\sim}{4}$ | $\begin{gathered} N \\ \underset{\sim}{N} \end{gathered}$ |  | $\begin{aligned} & \text { U } \\ & \text { U } \\ & \text { U } \\ & \text { H } \\ & 0 \\ & 0 \end{aligned}$ |  | $\stackrel{\rightharpoonup}{4}$ | N |
| 1 | 1 | B | 75 | $\{0,1,2,3\}$ | ${ }_{2}$ | ${ }_{2}$ | B | 60 | \{0, 1, 2, 3\} | 2 | 2 | A | 75 | \{0, 1, 2, 3\} | 2 | 2 | A | 60 | \{0, 1\} | 2 | 1 | B | 75 | \{0, 1\} | 1 | 1 | B | 60 | $\{0,1\}$ | 1 | 1 |
| 1 | 2 | B | 0 | \{0, 1, 2, 3\} | 2 | 2 | A | 50 | \{0, 2, 3\} |  | 0 | A | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | \{0, 2, 3\} | 2 | 2 | A | 50 | \{0, 2, 3\} |  |  | A | 0 | \{0, 2, 3\} |  |  |
| 1 | 3 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 75 | \{0, 1, 2, 3\} | 2 | 2 | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 80 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 0 | 2 | A | 50 | $\{0,2,3\}$ |  |  |
| 1 | 4 | B | 70 | $\{0,1,2,3\}$ |  |  | B | 70 | $\{0,1,2,3\}$ |  |  | A | 70 | $\{0,1,2,3\}$ | 2 | 1 | B | 50 | $\{0,2,3\}$ |  |  | B | 70 | $\{0,1\}$ | 1 | 1 | A | 55 | $\{0,2,3\}$ | 2 | 2 |
| 1 | 5 | B | 80 | $\{0,1,2,3\}$ | 2 | 1 | B | 80 | $\{0,1,2,3\}$ | 1 | 1 | A | 80 | $\{0,1,2,3\}$ | 2 | 1 | A | 80 | $\{0,1\}$ | 1 | 1 | B | 80 | $\{0,1\}$ | 1 | 1 | B | 80 | \{0, 1\} | 1 | 1 |
| 1 | 6 | B | 75 | $\{0,1,2,3\}$ | 0 | 0 | B | 75 | \{0, 1, 2, 3\} |  | 1 | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 65 | \{0, 2, 3\} | 2 | 2 | A | 65 | \{0, 2, 3\} | 2 | 2 | A | 65 | \{0, 2, 3\} | 2 | 2 |
| 1 | 7 | B | 95 | $\{0,1,2,3\}$ | 2 | 2 | A | 50 | \{0,2,3\} | 0 | 2 | A | 90 | $\{0,1,2,3\}$ | 2 | 2 | B | 75 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,2,3\}$ | 2 | 2 | A | 70 | \{0, 2, 3\} | 2 | 2 |
| 1 | 8 | B | 70 | $\{0,1,2,3\}$ | 2 | ${ }_{2}$ | B | 55 | $\{0,1,2,3\}$ | 2 | 2 | A | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | $\{0,2,3\}$ | 2 | 2 | A | 55 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 |
| 1 | 9 | B | 67 | $\{0,1,2,3\}$ | 2 | 1 | B | 67 | \{0, 1, 2, 3\} |  | 1 | A | 67 | $\{0,1,2,3\}$ |  |  | A | 67 | \{0, 1\} | 1 | 1 | B | 67 | $\{0,1\}$ | 1 | 1 | B | 67 | $\{0,1\}$ | 1 | 1 |
| 1 | 10 | B | 0 | $\{0,1,2,3\}$ |  |  | A | 0 | $\{0,2,3\}$ | 0 | 2 | A | 0 | $\{0,1,2,3\}$ |  |  | B | 0 | \{0, 2, 3\} | 2 | 2 | A | 0 | $\{0,2,3\}$ | 2 | 2 | A | 0 | $\{0,2,3\}$ | 2 | 2 |
| 1 | 11 | B | 70 | $\{0,1,2,3\}$ | 2 | 1 | B | 50 | \{0, 1, 2, 3\} |  | 0 | A | 60 | \{0, 1, 2, 3\} | 2 | 2 | B | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 |
| 1 | 12 | B | 50 | $\{0,1,2,3\}$ |  |  | A | 50 | $\{0,2,3\}$ |  |  | B | 60 | \{0, $\{0\}$ |  |  | A | 50 | $\{0,1\}$ |  | 0 | B | 60 | $\{0,1\}$ | 1 | 1 | B | 50 60 | $\{0,1\}$ |  |  |
| 2 | 13 | B | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 0 70 | $\left\{\begin{array}{l}\{0,1,2,3\} \\ \{0,1,2,3\}\end{array}\right.$ | 2 | 1 | A | 100 70 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | ${ }_{2}^{2}$ | 1 | B | 55 70 | $\{0,2,3\}$ $\{0,1\}$ | 2 | 2 | A | 100 60 | $\{0,2,3\}$ $\{0,1\}$ | 2 | 2 | A | 60 60 | $\{0,2,3\}$ $\{0,1\}$ | 2 | 2 |
| 2 | 15 | B | 100 | \{0, 1, 2, 3\} | 3 | 3 | B | 70 | $\{0,1,2,3\}$ | 2 | 3 | A | 100 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | \{0, 2, 3\} | 2 | 2 | A | 100 | $\{0,2,3\}$ | 2 | 2 | A | 70 | \{0, 2, 3\} | 2 | 2 |
| 2 | 16 | B | 0 | $\{0,1,2,3\}$ | 2 | 1 | B | 65 | \{0, 1, 2, 3\} | 0 | 0 | A | 60 | $\{0,1,2,3\}$ | 0 | 0 | A | 67 | \{0, 1\} | 1 | , | B | 67 | \{0, 1\} |  |  | B | 67 | \{0, 1\} | 0 |  |
| 2 | 17 | B | 0 | $\{0,1,2,3\}$ |  |  | A | 50 | \{0,2,3\} |  |  | A | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 0 | \{0, 2, 3\} |  |  | B | 50 | $\{0,1\}$ |  | 0 | B | 50 | $\{0,1\}$ |  |  |
| 2 | 18 | B | 75 | \{0, 1, 2, 3\} | 2 | 2 | B | 50 | \{0, 1, 2, 3\} |  | 0 | A | 75 | \{0, 1, 2, 3\} |  |  | B | 50 | \{0, 2,3$\}$ | 2 | 2 | A | 50 | \{0, 2, 3\} | 2 | 2 | B | 50 | \{0, 1\} | 3 | 3 |
| 2 | 19 | B | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 | 2 | A | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | $\{0,2,3\}$ |  | 2 | A | 75 | $\{0,2,3\}$ | 2 | 2 | A | 75 | \{0, 2, 3\} | 2 | 2 |
| , | 20 | B | 85 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | $\{0,1,2,3\}$ |  | 2 | A | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 55 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 |
| 2 | 21 | B | 69 | $\{0,1,2,3\}$ |  |  | A | 69 | $\{0,2,3\}$ |  | 0 | A | 69 | $\{0,1,2,3\}$ |  |  | B | 69 | \{0, 2,3$\}$ |  |  | A | 99 | $\{0,2,3\}$ |  |  | A | 0 | $\{0,2,3\}$ |  |  |
| 2 | 22 | B | 80 | $\{0,1,2,3\}$ | . |  | A | 50 | \{0, 2, 3\} |  | . | A | 60 | $\{0,1,2,3\}$ | . | . | B | 70 | \{0, 2,3$\}$ |  | . | A | 60 | $\{0,2,3\}$ |  |  | A | 60 | $\{0,2,3\}$ |  |  |
| 2 | 23 | B | 67 | $\{0,1,2,3\}$ |  |  | B | 67 | \{0, $1,2,3\}$ |  |  | A | 67 | $\{0,1,2,3\}$ |  |  | A | 75 | $\{0,1\}$ |  |  | B | 70 | $\{0,1\}$ |  |  | B | 67 | \{0, 1\} |  |  |
| 2 | 24 | B | 70 | $\{0,1,2,3\}$ | 2 | 2 | A | 70 | $\{0,2,3\}$ | , | 0 | A | 70 | $\{0,1,2,3\}$ | ${ }_{2}$ | 2 | B | 60 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,2,3\}$ | 2 | 2 |
| 2 | 25 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 65 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 65 | $\{0,2,3\}$ | 2 | 2 | A | 85 | $\{0,2,3\}$ | 2 | 2 | A | 75 | $\{0,2,3\}$ | 2 | 2 |
| 2 | 26 | B | 66 | $\{0,1,2,3\}$ | 1 | 1 | A | 50 | \{0, 2,3$\}$ | 2 | 2 | A | 66 | $\{0,1,2,3\}$ |  | 2 | B | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 |
| 2 | 27 | B | 50 | $\{0,1,2,3\}$ | 0 |  | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | A | 50 | $\{0,1,2,3\}$ | 0 | 0 | A | 50 | $\{0,1\}$ | 0 | 0 | B | 50 | $\{0,1\}$ |  |  | B | 50 | $\{0,1\}$ | 0 | 0 |
| 2 | 28 | B | 50 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | \{0, 1, 2, 3\} |  |  | A | 50 | $\{0,1,2,3\}$ |  |  | A | 70 | $\{0,1\}$ |  |  | B | 70 | $\{0,1\}$ |  |  | B | 70 | $\{0,1\}$ |  |  |
| 2 | 29 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 70 | \{0, 2, 3\} | 2 | 2 | A | 70 | $\{0,2,3\}$ | 2 | 2 | A | 50 | $\{0,2,3\}$ | 0 | 0 |
| 2 | 30 | B | 70 | $\{0,1,2,3\}$ | . | . | B | 60 | $\{0,1,2,3\}$ |  | . | A | 65 | $\{0,1,2,3\}$ | . |  | A | 60 | \{0,1\} |  |  | A | 70 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 |
| 3 | 31 | B | 70 | $\{0,1,2,3\}$ | . |  | B | 70 | \{0, 1, 2, 3\} |  | . | B | O | \{0\} | . |  | B | 80 | \{0, 2,3$\}$ | 2 | 2 | A | 90 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,2,3\}$ |  |  |
| 3 | 32 | B | 0 | $\{0,1,2,3\}$ |  |  | B | 66 | \{0, 1, 2, 3\} |  | . | A | 66 | $\{0,1,2,3\}$ |  |  | A | 66 | $\{0,1\}$ |  |  | B | 67 | $\{0,1\}$ |  |  | B | 66 | $\{0,1\}$ |  |  |
| 3 | 33 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 80 | $\{0,1,2,3\}$ |  | . | A | 55 | $\{0,1,2,3\}$ | 0 | 0 | A | 65 | $\{0,1\}$ |  | . | A | 88 | $\{0,2,3\}$ |  |  | A | 75 | $\{0,2,3\}$ |  |  |
| 3 | 34 | B | 50 | $\{0,1,2,3\}$ | . |  | A | 60 | \{0,2,3\} |  | . | A | 50 | $\{0,1,2,3\}$ | . |  | A | 50 | $\{0,1\}$ | . | . | B | 50 | $\{0,1\}$ |  |  | B | 50 | $\{0,1\}$ |  |  |
| 3 | 35 36 | B | 67 90 | $\{0,1,2,3\}$ |  | 1 2 | B | 67 50 | $\{0,1,2,3\}$ |  |  | ${ }_{\text {A }}^{\text {A }}$ | 67 90 | $\{0,1,2,3\}$ |  |  | A | 67 | \{ $\{0,1\}$ |  |  | B | 67 | \{0, $\{0$, |  |  | B | 67 | \{0, 1\} | 2 |  |
| 3 | 36 | B | 90 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} |  | 2 | A | 90 | $\{0,1,2,3\}$ | 2 | 2 | B | 51 | $\{0,2,3\}$ | 2 |  | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 55 | $\{0,2,3\}$ | 2 | 2 |
| 3 | 37 | B | 50 | $\{0,1,2,3\}$ |  |  | A | 50 | $\{0,2,3\}$ |  |  | A | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,2,3\}$ | 2 | , | A | 50 | $\{0,2,3\}$ | 2 | , | A | 50 | $\{0,2,3\}$ |  | 2 |
| 3 3 | 38 39 | B | 66 0 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 2 | 2 | B | 50 0 | $\{0,1,2,3\}$ $\{0,2,3\}$ | 2 | 2 | A | 75 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 50 | $\{0,2,3\}$ $\{0,1\}$ | 2 | 2 | A | 70 50 | $\{0,2,3\}$ $\{0,2,3\}$ | 2 | 2 | A | 70 50 | $\underline{\{0,2,3\}}\left\{\begin{array}{l}0,1\}\end{array}\right.$ | 2 | 2 0 |
| 3 | 40 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 | 2 | A | 90 | $\{0,1,2,3\}$ | 2 | 1 | B | 75 | $\{0,2,3\}$ | 2 | 2 | A | 90 | $\{0,2,3\}$ | 2 | 2 | A | 60 | \{0, 2, 3\} | 2 |  |
| 3 | 41 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 60 | \{0,2,3\} | 2 | 2 | A | 60 | $\{0,1,2,3\}$ |  | 1 | B | 60 | \{0, 2, 3\} | 2 | 2 | A | 65 | \{0, 2, 3\} | 2 | 2 | B | 50 | $\{0,1\}$ | 0 | 0 |
| 3 | 42 | B | 75 | $\{0,1,2,3\}$ | 0 | 0 | B | 75 | \{0, 1, 2, 3\} |  |  | A | 75 | $\{0,1,2,3\}$ |  |  | A | 75 | \{0, 1\} | 1 | 1 | B | 75 | \{0, 1\} | 2 |  | B | 75 | $\{0,1\}$ |  |  |
| 3 | 43 | B | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | \{0, 1, 2, 3\} | 2 | 2 | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | B |  | \{0, 2, 3\} | 2 |  | A | 70 | \{0, 2, 3\} | 2 | 2 | B | 50 | $\{0,1\}$ | 3 | 2 |
| 3 | 44 | B | 75 | $\{0,1,2,3\}$ |  | 2 | B | 75 | $\{0,1,2,3\}$ | 1 | 1 | A | 75 | $\{0,1,2,3\}$ | ${ }_{2}^{2}$ | 1 | A | 50 | \{0, 1\} |  | 2 | B | 50 | $\{0,1\}$ | 1 | 2 | B | 50 | $\{0,1\}$ | 1 |  |
| 3 | 45 | B | 65 | $\{0,1,2,3\}$ |  |  | A | 55 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 70 | $\{0,2,3\}$ | 2 | ${ }^{2}$ | A | 70 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,2,3\}$ | 3 | 3 |
| 3 | 46 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 50 | \{0, 2, 3\} | 2 | 3 | A | 75 | $\{0,1,2,3\}$ |  |  | ${ }^{\text {B }}$ | 50 | $\{0,2,3\}$ | 2 | 3 | A | 50 | $\{0,2,3\}$ | 2 |  | A | 50 | $\{0,2,3\}$ |  |  |
| 3 | 47 | B | 50 | $\{0,1,2,3\}$ | 2 | 0 | B | 50 | $\{0,1,2,3\}$ | . | . | B | 100 | , 00$\}$ |  |  | A | 50 | $\{0,1\}$ |  | 1 | A | 50 | $\{0,2,3\}$ |  |  | B | 0 | $\{0,1\}$ |  |  |
| 3 | 48 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 75 | $\{0,1,2,3\}$ |  | . | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 75 | $\{0,1\}$ | 0 | - | B | 75 | $\{0,1\}$ | 0 | 0 | B | 75 | $\{0,1\}$ | 1 | 0 |
| 4 | 49 | B | 50 | $\{0,1,2,3\}$ | . |  | B | 100 | $\{0,1,2,3\}$ |  |  | A | 60 | $\{0,1,2,3\}$ |  |  | A | 50 | $\{0,1\}$ |  |  | A | 50 | $\{0,2,3\}$ |  |  | B | 75 | $\{0,1\}$ |  |  |
| 4 | 50 | B | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,1,2,3\}$ | - | 1 | A | 50 | $\{0,1,2,3\}$ | 0 | 1 | A | 75 | $\{0,1\}$ |  | 1 | B | 75 | $\{0,1\}$ | 1 | , | B | 75 | $\{0,1\}$ | 1 | 1 |
| 4 | 51 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 50 | \{0, 1, 2, 3\} | 0 |  | B | 50 | \{0\} | 0 | 0 | B | 0 | \{0, 2, 3\} | O | 0 | B | 50 | $\{0,1\}$ | 0 | 0 | B | 50 | $\{0,1\}$ | 0 | 0 |
| 4 | 52 | B | 0 | $\{0,1,2,3\}$ | 2 | 2 | A | 65 | $\{0,2,3\}$ |  | 2 | A | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 80 | $\{0,2,3\}$ | 1 | 1 | A | 100 | $\{0,2,3\}$ | 2 | 2 | A | 100 | $\{0,2,3\}$ |  | ${ }_{1}$ |
| 4 | 53 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 60 | \{0, 2, 3\} | ${ }_{0}$ | ${ }^{0}$ | A | 75 | $\{0,1,2,3\}$ | ${ }_{2}$ | 2 | A | 50 | $\{0,1\}$ | 1 | 1 | A | 50 | $\{0,2,3\}$ | 2 | 2 | B | 50 | $\{0,1\}$ | 1 | 1 |
| 4 | 54 | B | 80 | $\{0,1,2,3\}$ | 1 | 1 | B | 60 | $\{0,1,2,3\}$ | 1 | 1 | A | 70 | $\{0,1,2,3\}$ | 2 | 2 | A | 66 | $\{0,1\}$ | 1 | 1 | B | 60 | $\{0,1\}$ | 2 | 1 | B | 70 | \{0, 1\} | 1 | 1 |
|  | 55 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 75 | \{0, 1, 2, 3\} | 2 | 2 | A | 100 | $\{0,1,2,3\}$ | 2 | 2 | A | 75 | $\{0,1\}$ | 1 | 1 | A | 75 | $\{0,2,3\}$ | 2 | 2 | A | 75 | \{0, 2, 3\} | 2 | 2 |
| 4 | 56 | B | 75 | $\{0,1,2,3\}$ | . |  | A | 50 | \{0,2,3\} |  |  | A | 75 | $\{0,1,2,3\}$ | ${ }_{2}$ | 2 | B | 50 | $\{0,2,3\}$ | 2 | 2 | A | 50 | $\{0,2,3\}$ | 2 | 2 | A | 50 | $\{0,2,3\}$ | 2 | 2 |
| 4 | 57 | B | 75 | $\{0,1,2,3\}$ |  | 1 | B | 60 | $\{0,1,2,3\}$ | 1 | 1 | A | 85 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | A | 50 | $\{0,2,3\}$ | 2 | 2 |
| 4 | 58 | B | 50 | $\{0,1,2,3\}$ | 1 | 1 | B | 50 | \{0, 1, 2, 3\} |  |  | A | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | \{0, 2,3$\}$ |  |  | A | 50 | $\{0,2,3\}$ |  |  | A | 0 | \{0, 2, 3\} |  |  |
| 4 | 59 | B | 75 | $\{0,1,2,3\}$ | 2 | 2 | A | 50 | $\{0,2,3\}$ | 2 | 2 | A | 85 | $\{0,1,2,3\}$ | 2 | 2 | B | 60 | $\{0,2,3\}$ | 2 | 2 | A | 67 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | ${ }_{1}^{2}$ |
| 4 | 60 | B | 50 | $\{0,1,2,3\}$ | 2 | 1 | B | 50 | \{0, 1, 2, 3\} | 1 | 1 | A | 50 | $\{0,1,2,3\}$ |  |  | A | 0 | $\{0,1\}$ |  | . | B | 0 | $\{0,1\}$ |  |  | B | 0 | $\{0,1\}$ | 1 | 1 |
| 4 | 61 | B | 75 | \{0, 1, 2, 3\} |  |  | A | 50 | $\{0,2,3\}$ |  |  | A | 75 | $\{0,1,2,3\}$ |  |  | B | 60 | \{0, 2, 3\} |  |  | A | 75 | \{0, 2, 3\} |  |  | A | 55 | \{0, 2, 3\} |  |  |
| 4 | 62 | B | 66 | $\{0,1,2,3\}$ | 2 | 2 | A | 67 | \{0,2,3\} | 2 | 2 | A | 78 | $\{0,1,2,3\}$ | ${ }_{2}$ | 2 | B | 53 | $\{0,2,3\}$ | $\dot{0}$ | $\dot{0}$ | A | 50 | $\{0,2,3\}$ |  |  | A | 50 | $\{0,2,3\}$ | 2 | ${ }_{1}$ |
| 4 | 63 | B | 67 | \{0, 1, 2, 3\} | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 |  | A | 50 | $\{0,1,2,3\}$ | ${ }_{2}$ | 1 | A | 75 | $\{0,1\}$ | 0 | 1 | A | 50 | $\{0,2,3\}$ | 2 | 2 | B | 60 | $\{0,1\}$ | 1 | 1 |
| 4 | 64 | A | 50 | \{0\} |  | 0 | B | 100 | \{0, 1, 2, 3\} | 1 | 1 | A | 100 | $\{0,1,2,3\}$ | 1 | 1 | A | 100 | $\{0,1\}$ | 1 | 1 | B | 100 | $\{0,1\}$ | 0 | 1 | B | 100 | $\{0,1\}$ | 1 | 1 |


|  |  | Decision 1 (BBAb) |  |  |  |  | Decision 2 (BAABAb) |  |  |  |  | Decision 3 (BAAa) |  |  |  |  | Decision 4 (BBABa) |  |  |  |  | Decision 5 (ABAAAb) |  |  |  |  | Decision 6 (BAAAb) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 苟 | $\begin{aligned} & \theta \\ & \dot{U} \\ & \stackrel{0}{0} \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & \text { D } \\ & \text { bo } \\ & 00 \\ & 0 \\ & 4 \end{aligned}$ | $\begin{aligned} & \text { U } \\ & \text { y } \\ & 0 \\ & \text { E } \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & -7 \\ & 4 \end{aligned}$ | $\begin{gathered} N \\ \sim \end{gathered}$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & \dot{0} \\ & 60 \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 00 \\ & \text { U } \\ & 0 \\ & \text { U } \\ & \text { U } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 4 \\ & 4 \end{aligned}$ |  | $\begin{aligned} & \dot{0} \\ & \dot{0} \\ & \dot{0} \\ & \dot{0} \\ & \dot{6} \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \text { U } \\ & \text { U } \\ & \text { U } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $$ | $$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & \dot{0} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { U. } \\ & \text { U } \\ & 0 \\ & \text { U } \\ & \text { U } \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & -1 \\ & 4 \\ & 0 \end{aligned}$ | $\begin{gathered} N \\ \sim \end{gathered}$ |  | $\begin{aligned} & \text { O } \\ & \text { U } \\ & \text { U } \\ & \text { H } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \stackrel{1}{4} \\ & \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \sim \end{aligned}$ | $\begin{aligned} & \dot{0} \\ & 0 \\ & 0 \\ & \dot{0} \\ & \dot{6} \\ & \tilde{0} \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & \text { O } \\ & 0 \\ & \text { U } \\ & \text { U } \\ & \text { U } \\ & 0 \\ & \hline \end{aligned}$ |  | $\begin{aligned} & 2 \\ & \stackrel{y}{2} \end{aligned}$ | N |
| 5 | 65 | B | 75 | \{0, 1\} |  |  | B | 67 | \{0, 1\} |  |  | A | 67 | \{0, 1\} |  |  | A | 67 | \{0, 1, 2, 3\} | 2 |  | B | 67 | \{0, 1, 2, 3\} |  |  | B | 67 | \{0, 1, 2, 3\} |  |  |
| 5 | 66 | A | 65 | \{0, 2, 3\} | 2 | 2 | A | 80 | $\{0,2,3\}$ | 2 | 2 | B | 60 | \{0,2,3\} | 2 | 2 | A | 100 | $\{0,1,2,3\}$ |  |  | B | 100 | $\{0,1,2,3\}$ | 2 | 2 | B | 100 | $\{0,1,2,3\}$ | 2 | 2 |
| 5 | 67 | B | 70 | $\{0,1\}$ |  |  | B | 70 | $\{0,1\}$ |  |  | A | 70 | $\{0,1\}$ |  |  | A | 60 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,1,2,3\}$ |  |  | B | 60 | $\{0,1,2,3\}$ |  |  |
| 5 5 | 68 | B ${ }_{\text {B }}$ | 75 75 | $\{0,1\}$ | 0 | 0 | B ${ }_{\text {B }}$ | 75 75 | $\{0,1\}$ | 0 | 0 | A | 75 75 | $\{0,1\}$ | 1 | 1 | A | 75 50 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 2 | 1 | B | 75 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | ${ }_{1}^{2}$ | 1 | B B | 75 63 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ |  |  |
| 5 | 69 70 | B | 75 60 | $\{0,1\}$ $\{0,1\}$ | 1 | 1 | B | 75 65 | $\{0,1\}$ $\{0,1\}$ | 1 | 1 | A | 75 75 | $\{0,1\}$ $\{0,1\}$ | 0 | 0 | A | 50 60 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 2 | 1 | B B | 63 55 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 1 | 1 | B | 63 60 | \{0, $1,2,3\}$ | 2 | ${ }_{0}^{1}$ |
| 5 | 71 | B | 67 | $\{0,1\}$ | 1 | 1 | B | 66 | $\{0,1\}$ | 1 | 1 | A | 67 | $\{0,1\}$ | 1 | 1 | A | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 67 | $\{0,1,2,3\}$ |  |  | B | 80 | \{0, $1,2,3\}$ | 2 | 2 |
| 5 | 72 | B | 67 | $\{0,1\}$ | 1 | 1 | B | 67 | $\{0,1\}$ |  |  | A | 67 | $\{0,1\}$ |  |  | A | 67 | $\{0,1,2,3\}$ |  |  | B | 67 | \{0, 1, 2, 3\} |  |  | B | 67 | \{0, 1, 2, 3\} |  |  |
| 5 | 73 | A | 60 | \{0, 2, 3\} | 2 | 2 | A | 60 | \{0, 2, 3\} | 2 | 2 | B | 60 | \{0, 2, 3\} | 2 | 2 | A | 60 | \{0, 1, 2, 3\} | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 | 2 | B | 60 | \{0, 1, 2, 3\} | 2 | 2 |
| 5 | 74 | A | 50 | $\{0,2,3\}$ |  |  | A | 50 | \{0, 2,3$\}$ | 3 | 3 | A | 50 | $\{0,1\}$ | 0 | 0 | A | 70 | \{0, 1, 2, 3\} | 3 | 3 | A | O | \{0,2,3\} | 2 | 2 | B | 70 | \{0, 1, 2, 3\} | 3 | 3 |
| 5 | 75 | B | 50 | $\{0,1\}$ |  |  | B | 60 | \{0, 1\} |  |  | B | 50 | $\{0,2,3\}$ |  |  | A | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | \{0, 1, 2, 3\} |  |  | B | 60 | $\{0,1,2,3\}$ |  |  |
| 5 | 76 | B | 70 | $\{0,1\}$ | 3 | 3 | B | 70 | $\{0,1\}$ | 3 | 3 | B | 50 | \{0, 2, 3\} | 3 | 3 | A | 66 | $\{0,1,2,3\}$ | 3 | 3 | B | 66 | \{0, 1, 2, 3\} | 3 | 3 | B | 66 | \{0, 1, 2, 3\} |  |  |
| 5 | 77 | B | 80 | $\{0,1\}$ | 0 | 0 | B | 85 | \{0, 1\} |  |  | A | 80 | \{0, 1\} |  | . | A | 65 | \{0, 1, 2, 3\} | 2 | 2 | A | 65 | \{0, 2, 3\} |  |  | B | 65 | \{0, 1, 2, 3\} | 2 | 2 |
| 5 | 78 | B | 50 | $\{0,1\}$ | 1 | 1 | A | 50 | \{0, 2, 3\} | 2 | 2 | B | 50 | $\{0,2,3\}$ |  |  | A | 75 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 | 2 | B | 75 | \{0, 1, 2, 3\} | 2 | 2 |
| 6 | 79 | B | 50 | $\{0,1\}$ | 0 | 0 | B | 50 | $\{0,1\}$ | 0 | 0 | A | 50 | $\{0,1\}$ | 0 | 0 | A | 50 | \{0, 1, 2, 3\} | 0 | 0 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 |
| 6 | 80 | A | 90 | \{0, 2, 3\} | 2 | 2 | A | 98 | \{0, 2, 3\} | 2 | 2 | B | 85 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 1 | 1 | B | 95 | \{0, 1, 2, 3\} | 2 | 2 |
| 6 | 81 | B | 66 | $\{0,1\}$ | 1 | 1 | B | 66 | $\{0,1\}$ | 1 | 1 | A | 66 | $\{0,1\}$ | , | 1 | A | 50 | \{0, 1, 2, 3\} | 2 | 2 | A | 50 | \{0, 2, 3\} |  |  | B | 50 | \{0, 1, 2, 3\} | 2 | 2 |
| ${ }_{6}^{6}$ | 82 | A | 75 | $\{0,2,3\}$ | 2 | 2 | A | 60 | $\{0,2,3\}$ | 2 | 2 | B | 55 | $\{0,2,3\}$ | 2 | 2 | A | 75 | $\{0,1,2,3\}$ | . |  | B | 50 | $\{0,1,2,3\}$ | 2 | 2 | B | 75 | $\{0,1,2,3\}$ |  |  |
| 6 | 83 | B | 50 | $\{0,1\}$ |  |  | B | 50 | $\{0,1\}$ |  |  | $\stackrel{B}{8}$ | 50 | $\{0,2,3\}$ |  | . | A | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,1,2,3\}$ |  |  |
| 6 | 84 | B | 67 | $\{0,1\}$ | 1 | 1 | B | 67 | $\{0,1\}$ | 1 | 1 | A | 67 | $\{0,1\}$ |  |  | A | 75 | \{0, 1, 2, 3\} | 2 | 2 | B | 67 | $\{0,1,2,3\}$ | 2 | 2 | B | 70 | $\{0,1,2,3\}$ | 2 | 1 |
| ${ }_{6}$ | 85 | B | 66 | \{0, 1\} | 1 | 1 | B | 66 | \{0, 1\} | 1 | 1 | A | 66 | $\{0,1\}$ | 1 | 1 | A | 66 | $\{0,1,2,3\}$ | 1 | 1 | B | 66 | $\{0,1,2,3\}$ | 1 | 1 | B | 66 | $\{0,1,2,3\}$ | 1 | 1 |
| 6 | 86 | A | 0 | \{0, 2, 3\} | 2 | 2 | A | 60 | \{0, 2, 3\} | 2 | 2 | A | 50 | $\{0,1\}$ |  | . | A | 50 | \{0, 1, 2, 3\} | . |  | A | 50 | \{0, 2, 3\} |  |  | B | 50 | \{0, 1, 2, 3\} |  |  |
| 6 | 87 | B | 50 | \{0, 1\} |  |  | A | 50 | $\{0,2,3\}$ |  |  | B | 50 | $\{0,2,3\}$ |  |  | A | 70 | $\{0,1,2,3\}$ |  |  | B | 50 | \{0, 1, 2, 3\} |  |  | B | 70 | $\{0,1,2,3\}$ |  |  |
| 6 | 88 | A | 75 | $\{0,2,3\}$ | 2 | 2 | A | 75 | $\{0,2,3\}$ | 2 | 2 | B | 70 | \{0, 2, 3\} | 2 | 2 | A | 70 | \{0, 1, 2, 3\} | 2 | 2 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 75 | $\{0,1,2,3\}$ |  | 1 |
| 6 | 89 | B | 67 | $\{0,1\}$ |  |  | B | 67 | \{0, 1\} |  |  | A | 50 | \{0, 1\} |  |  | B | 50 | \{0\} |  |  | B | 50 | $\{0,1,2,3\}$ |  |  | A | 60 | , \{0, |  |  |
| 6 | 90 | A | 50 | $\{0,2,3\}$ | 2 | 2 | A | 50 | \{0,2,3\} | 0 | 0 | ${ }^{\text {B }}$ | 50 | $\{0,2,3\}$ | 2 | 2 | A | 70 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | $\{0,1,2,3\}$ | 0 | 0 | B | 60 | $\{0,1,2,3\}$ | 2 | 2 |
| 6 | 91 | ${ }^{\text {B }}$ | 60 | $\{0,1\}$ | . |  | B | 60 | \{0, $\{0,3\}$ | 3 | 3 | B | 70 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,1,2,3\}$ | 3 | , | B | 70 | $\{0,1,2,3\}$ | 3 | 2 | B | 90 75 | $\{0,1,2,3\}$ | 3 | 3 |
| 6 | 92 | B | 50 50 | $\{0,1\}$ $\{0,1\}$ | 3 | 0 3 | A | 50 50 | $\{0,2,3\}$ $\{0,2,3\}$ | 3 | 3 | B B | 50 70 | $\{0,2,3\}$ $\{0,2,3\}$ | 2 | 2 | A | 70 50 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 2 | 2 | A | 65 50 | $\{0,2,3\}$ $\{0,1,2,3\}$ | ${ }_{2}^{2}$ | 2 | B B | 75 80 | $\{0,1,2,3\}$ $\{0,1,2,3\}$ | 2 | 2 |
| 7 | 94 | B | 80 | $\{0,1\}$ | 1 | 1 | A | 75 | \{0, 2,3$\}$ | 2 | 2 | B | 50 | \{0,2,3\} |  | . | A | 90 | \{0, 1, 2, 3\} | 2 | 2 | A | 80 | \{0,2,3\} | 2 | 2 | B | 95 | \{0, 1, 2, 3\} | 2 | 2 |
| 7 | 95 | B | 75 | $\{0,1\}$ |  | 1 | B | 75 | $\{0,1\}$ |  |  | A | 75 | $\{0,1\}$ |  |  | B | 50 | \{0\} | . | . | A | 50 | \{0, 2, 3\} |  |  | A | 50 | \{0\} |  |  |
| 7 | 96 | B | 100 | $\{0,1\}$ | 1 | 1 | B | 75 | $\{0,1\}$ | 1 | 1 | A | 75 | $\{0,1\}$ | 0 | 0 | A | 55 | $\{0,1,2,3\}$ |  |  | B | 50 | $\{0,1,2,3\}$ |  |  | B | 57 | \{0, 1, 2, 3\} |  |  |
| 7 | 97 | B | 0 | $\{0,1\}$ | 1 | 1 | B | 50 | $\{0,1\}$ |  | 1 | A | 100 | $\{0,1\}$ | 1 | 1 | A | 50 | $\{0,1,2,3\}$ | 1 | , | ${ }^{\text {B }}$ | 100 | $\{0,1,2,3\}$ | ${ }_{2}$ | 1 | A | 80 | \{0, 0 \} |  |  |
| 7 | 98 | B | 60 | \{0, 1$\}$ |  | 1 | B | 60 | \{0, 1\} | 1 | , | A | 60 | \{0, 1\} |  |  | A | 60 | $\{0,1,2,3\}$ | 2 | 2 | A | 50 | \{0, 2, 3\} | 2 | 2 | B | 50 | \{0, 1, 2, 3\} |  |  |
| 7 | 99 | A | 80 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,2,3\}$ |  | 2 | B | 70 | $\{0,2,3\}$ |  | 2 | A | 80 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | 2 | 2 | B | 80 | $\{0,1,2,3\}$ | 2 |  |
| 7 | 100 | A | 80 | $\{0,2,3\}$ | 2 | 2 | A | 80 | $\{0,2,3\}$ | 2 | 2 | B | 80 | $\{0,2,3\}$ | 2 | 2 | A | 85 | $\{0,1,2,3\}$ | 2 | 2 | A | 50 | \{0,2,3\} | ${ }_{2}$ | 3 | B | 85 | $\{0,1,2,3\}$ | 2 | 2 |
| 7 | 101 | A | 75 | $\{0,2,3\}$ | 2 | 2 | A | 90 | $\{0,2,3\}$ | 2 | 2 | B | 80 | $\{0,2,3\}$ | 2 | 2 | A | 95 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | $\{0,1,2,3\}$ | 2 | 2 | B | 80 | $\{0,1,2,3\}$ | 2 | 2 |
| 7 | 102 | B | 75 | $\{0,1\}$ | . | . | B | 75 | \{0, 1$\}$ |  |  | A | 90 | $\{0,1\}$ |  | . | A | 50 | $\{0,1,2,3\}$ |  |  | B | 65 | $\{0,1,2,3\}$ |  |  | B | 100 | $\{0,1,2,3\}$ |  |  |
| 7 | 103 | B | 50 | $\{0,1\}$ |  | . | A | 80 | $\{0,2,3\}$ |  |  | A | 80 | $\{0,1\}$ |  |  | A | 50 | $\{0,1,2,3\}$ |  |  | A | 75 | $\{0,2,3\}$ |  |  | B | 75 | $\{0,1,2,3\}$ |  |  |
| 7 | 104 | B | 80 | \{0, 1$\}$ |  |  | A | 80 | $\{0,2,3\}$ |  |  | A | 75 | \{0, 1\} |  |  | A | 70 | $\{0,1,2,3\}$ | . |  | B | 85 | $\{0,1,2,3\}$ |  |  | B | 80 | $\{0,1,2,3\}$ |  |  |
| 7 | 105 | A | 80 | $\{0,2,3\}$ | 2 | ${ }_{2}^{2}$ | A | 80 | $\{0,2,3\}$ |  |  | B | 80 | $\{0,2,3\}$ |  | - | A | 80 | $\{0,1,2,3\}$ |  |  | B | 80 | $\{0,1,2,3\}$ |  |  | B | 80 | $\{0,1,2,3\}$ |  |  |
| 7 | 106 | A | 90 | \{0, 2, 3\} | 2 | 2 | A | 90 | $\{0,2,3\}$ | . |  | B | 80 | $\{0,2,3\}$ | . | . | A | 95 | $\{0,1,2,3\}$ | 2 | 2 | B | 50 | \{0, 1, 2, 3\} | . | 0 | B | 90 | $\{0,1,2,3\}$ | 2 |  |


|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

Table 14: Examples of messages classified as level-0

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

Table 15: Examples of messages classified as level-1

|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  | Message |  |  |

Table 16: Examples of messages classified as level-2 (continued next table)

|  | $\left(H_{t}, s_{t}\right)$ | Message |  |  | \% |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | BAAAb ABAAAb | since 3 of the teams before us chose A, we might assume that they got a white ball. we may have just gotten a black ball by chance from urn A since most teams before us have chosen A, we may assume that most of them got the white ball from their drawing. using probability, it's more likely that we got a black by with $1 / 3$ chance than the previous teams getting a white ball from urn B with much lower probability |  |  |  |
|  | BBABa | we can assume that team 1 got the black ball, team 2 probably also got the black ball, and team 3 probably got the white ball. probability wise, it's more likely that its from urn B |  |  |  |
| 101 | BAAAb | Okay so the teams before us chose A more frequently, which means they private draw must have been white, with one black. We got a black one, but since its a $50 \%$ chance that both urns could be the true urn, we can attribute ours to simple probability Thus, we have a $75 \%$ chance of it being A More frequency-better chance it comes from one urn than the other | A $75 \quad\{2\}$ |  |  |
|  | ABAAAb | Frequency of a is greater than B, thus, it's a good chance its urn A because although we picked black 4 whites to two blacks shows a 2 to 1 ratio meaning that for everty 3 balls, 2 are white and one is black this reflects the probabilty of urn A |  |  |  |
|  | BBABa | The first team must have seen a black ball and chose B second team probably got black as well so chose $B$ third team probably got white and decided to go with A 4th team got black again and went with B Thus if we assume all the choices before are rational then the choices are more black than white thus B MUST be the rationally right answer as in if you don't pick $B$, it would be quite irrational |  |  |  |
| 106 | BAAAb | Given that we were shown a black ball, there is a $2 / 3$ chance that it is in B. However, the other teams chose A, which means that they were given a white ball, meaning that there is a greater chance of A |  |  | $\{2\}$ |

Table 17: Examples of messages classified as level-2

|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | Message | The first team definitely saw a black ball, they went in <br> blind and chose B The second team must have also saw <br> a black ball, and corroborated with what team 1 saw, <br> the 3rd team saw a white ball, which contradicted what | 100 |

Table 18: Examples of messages classified as level-3 (continued next table)

|  | $\left(H_{t}, s_{t}\right)$ | Message |  | 0 0 0 0 0 0 | 長 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 93 | BAAAb | Well, from our observations it seems that we would be choosing Urn B. However, if we look at previous guesses, a desire may be made to choose Urn A from belieivng that other people must be drawing white balls. We have to consider the fact that other teams are also being influenced by previous decisions. For example, team 4 may have drawn a black ball and wanted to choose Urn B, but didn't in order to match the previous people. Therefore, I still think we should choose Urn B. | B |  | $3\}$ |
|  | ABAAAb | Even though we drew a black ball, feel that for $80 \%$ of people to have chosen A, this may indicate that white is the majority. However, I'm not sure at all, due to previous people also being influenced by previous decisions. We know that the first decision is not based off of previous decisions, therefore we can assume that the first person drew a white ball. The second person contrasted the first person so we can assume they went by their ball and chosen B. The third person was going off of a balanced group where half of the people had chosen whtie and half of the people had chosen black, therefore their decision is valid too. After that it was based on how people analyzed it somewhat. I believe the fourth person would still have stayed with what they got so think A is better. | A |  | \{3\} |
|  | BBABa | We know that the first person would stick to what ball they drew, therefore it can be decided to be B. The second person would have no reason to follow the first person since its just one statistic so they are probably accurate too. The third person needed a reason to deviate so they were definitely A. The last person probably would have stayed with what they got so I think it was B , regardless of our drawing. | B |  | \{2\} |

Table 19: Examples of messages classified as level-3

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| $(0)$ | 17 | 40.5 |
| 1 | 12 | 28.6 |
| 2 | 10 | 23.8 |
| 3 | 2 | 4.8 |
| B | 1 | 2.4 |
| Total | 42 | 100.00 |

(a) Classification by lowest significant SSD

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| $(0)$ | 17.00 | 40.5 |
| 1 | 12.00 | 28.6 |
| 2 | 10.00 | 23.8 |
| 3 | 1.89 | 4.5 |
| B | 1.11 | 2.6 |
| Total | 42 | 100.00 |

(b) Mixture classification

Table 20: Type overview for 42 subjects in decisions 1-3

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| $(0)$ | 18 | 42.9 |
| 1 | 9 | 21.4 |
| 2 | 8 | 19.0 |
| 3 | 0 | 0.0 |
| B | 7 | 16.7 |
| Total | 42 | 100.00 |

(a) Classification by lowest significant SSD

| Type | Frequency | Fraction (\%) |
| :---: | ---: | ---: |
| $(0)$ | 18.00 | 42.9 |
| 1 | 9.00 | 21.4 |
| 2 | 8.55 | 20.4 |
| 3 | 0.33 | 0.8 |
| B | 6.11 | 14.6 |
| Total | 42 | 100.00 |

(b) Mixture classification

Table 21: Type overview for 42 subjects in decisions 4-6

|  | Highest level ( $\varnothing$ 2.11) |  |  |  |  |  |  |
| ---: | :---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  | 0 | 1 | 2 | 3 | NA | Total |
|  | 0 | 1 | 0 | 1 | 2 | 0 | 4 |
| Lowest level | 1 |  | 2 | 2 | 0 | 0 | 4 |
| $(\varnothing 1.68)$ | 2 |  |  | 15 | 2 | 0 | 17 |
|  | 3 |  |  |  | 3 | 0 | 3 |
|  | NA |  |  |  | 14 | 14 |  |
|  | Total | 1 | 2 | 18 | 7 | 14 | 42 |

Table 22: Level classification in second 3 decisions

| $\begin{aligned} & \text { A } \\ & \stackrel{U}{0} \\ & \stackrel{0}{\vdots} \\ & \text { in } \end{aligned}$ | $\begin{gathered} \text { Decision } 1 \\ (\mathrm{AAB}) \\ \hline \end{gathered}$ |  |  | $\begin{gathered} \text { Decision 2 } \\ (\mathrm{BBAA}) \\ \hline \end{gathered}$ |  |  | $\begin{aligned} & \hline \text { Decision } 3 \\ & \text { (AAABA) } \\ & \hline \end{aligned}$ |  |  | $\begin{gathered} \text { Decision } 4 \\ (\mathrm{BBB}) \\ \hline \end{gathered}$ |  |  | $\begin{gathered} \text { Decision } 5 \\ (\mathrm{AABA}) \\ \hline \end{gathered}$ |  |  | $\begin{gathered} \hline \text { Decision } 6 \\ (\mathrm{BABA}) \end{gathered}$ |  |  | Fingerprint Analysis <br> Decisions 1-6 |  |  |  | Cluster |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | نٍ |  |  | نٍ |  |  | نٍ |  |  | $\begin{aligned} & \dot{0} \\ & \hline 0 \end{aligned}$ |  |  | $\begin{gathered} \dot{0} \\ 0 \end{gathered}$ |  |  | $\begin{gathered} \dot{0} \\ 0 \end{gathered}$ |  |  |  |  |  |  |  |
|  | $\begin{aligned} & \dot{\infty} \\ & \dot{60} \\ & \dot{5} \end{aligned}$ | $$ | $\begin{aligned} & N \\ & \sim \end{aligned}$ | $\begin{aligned} & \dot{6} \\ & 60 \\ & \dot{60} \\ & \dot{n} \end{aligned}$ | $$ | $\begin{aligned} & \text { N } \\ & \text { «゙ } \end{aligned}$ | $\begin{aligned} & \dot{6} \\ & \dot{6} \\ & \stackrel{0}{n} \end{aligned}$ | $\begin{aligned} & \square \\ & \stackrel{y}{2} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \stackrel{y}{c} \end{aligned}$ | $\begin{aligned} & \dot{8} \\ & \dot{6} \\ & \dot{6} \end{aligned}$ | $\begin{aligned} & -1 \\ & 4 \\ & \sim \end{aligned}$ | $\begin{aligned} & \sim \\ & \sim \end{aligned}$ | $\begin{aligned} & \dot{6} \\ & \dot{60} \\ & \stackrel{0}{n} \end{aligned}$ | $\stackrel{\rightharpoonup}{4}$ | $\begin{aligned} & \sim \\ & \sim \\ & \sim \end{aligned}$ | $\begin{aligned} & \dot{6} \\ & \dot{60} \\ & \stackrel{0}{n} \end{aligned}$ | $\stackrel{\rightharpoonup}{4}$ | $\begin{aligned} & \sim \\ & \sim \\ & \sim \end{aligned}$ | Level-1 | Level-2 | Level-3 | Bayesian |  |
| 1 | 2 | . | . | 5 | . | . | 0 | . | . | -1 | . | . | 0 | 1 | 1 | -1 | . | . | 1.00 | 0.00 | 0.00 | 0.00 |  |
| 2 | 8 | . | . | 2 | . | . | 2 | . | . | 4.32 | . | . | 7.31 | . | . | -1.11 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | A |
| 3 | 0 | 1 | 1 | 0 | . | . | 0 | . | . | 0 | . | . | 0 | . | . | 0 | . | . | 1.00 | 0.00 | 0.00 | 0.00 |  |
| 4 | 4 | 0 | 2 | 0 | 2 | 1 | -8 | 2 | 2 | 2 | 2 | 0 | -10 | 2 | 2 | 0 | 2 | 2 | 0.00 | 1.00 | 0.00 | 0.00 | B |
| 5 | 5 | . | . | 0 | . | . | -10 | . | . | 10 | 2 | 2 | -10 | . | . | 0 | . | . | 0.00 | 1.00 | 0.00 | 0.00 | C |
| 6 | 0 | . | . | 0 | . | . | -5 | 2 | 2 | 7 | . | . | 0 | . | . | 3 | . | . | 0.27 | 0.32 | 0.00 | 0.41 | A |
| 7 | -2 | . | . | -2 | 3 | 3 | -4 | . | . | 7 | . | . | 3 | 3 | 3 | 0 | . | . | 0.27 | 0.00 | 0.32 | 0.41 | A |
| 8 | -4 | 2 | 2 | -3 | . | . | -10 | . | . | 10 | . | . | 0 | . | . | -10 | . | , | 0.00 | 0.00 | 1.00 | 0.00 |  |
| 9 | -2 | 2 | 3 | 0 | 2 | 2 | -3 | 3 | 3 | 5 | 2 | 2 | -2 | 2 | 2 | 0 | 2 | 2 | 0.53 | 0.12 | 0.18 | 0.17 | A |
| 10 | 5 | 0 | 0 | 0 | . | . | 0 | . | . | 0 | 0 | 0 | 1.11 | . | . | -2 | . | . | 1.00 | 0.00 | 0.00 | 0.00 |  |
| 11 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | -10 | . | . | 0 | . | . | 0 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | D |
| 12 | 1 | 2 | 1 | -2 | 3 | 3 | -1 | . | . | 7 | 2 | 2 | -1 | 3 | 3 | 1 | 0 | 0 | 0.33 | 0.00 | 0.16 | 0.51 | A |
| 13 | -5 | 2 | 2 | 0 | . | . | -8 | . | . | 4 | 2 | 2 | -4 | . | . | 0 | . | . | 0.00 | 0.84 | 0.16 | 0.00 | A |
| 14 | 4 | 0 | 0 | 4 | 0 | 0 | 2 | 2 | 2 | 9 | 2 | 2 | 2 | . | 0 | 4 | 0 | 0 | 0.00 | 0.00 | 0.00 | 0.00 | A |
| 15 | -8 | . | 0 | -8 | . | . | -8 | . | . | 0 | 2 | 2 | -9 | 2 | 0 | -1 | . | . | 0.00 | 1.00 | 0.00 | 0.00 | E |
| 16 | -2 | 2 | 0 | 0 | 2 | 2 | -10 | 2 | 2 | 8 | 2 | 0 | -1 | 2 | 2 | 0 | 2 | 2 | 0.00 | 0.67 | 0.16 | 0.17 | A |
| 17 | -1 | . | . | 0 | 2 | 2 | -8 | 2 | 2 | 0 | 2 | 1 | -8 | 2 | 2 | 0 | 2 | 2 | 0.00 | 1.00 | 0.00 | 0.00 | B |
| 18 | 0 | . | . | -10 | . | . | -5 | . | . | 0 | . | . | -6 | . | . | -6 | . | . | 0.00 | 0.00 | 0.59 | 0.41 | E |
| 19 | 3 | . | . | 5 | . | . | 6 | . | . | -2 | . | . | 3 | . | . | 2 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | D |
| 20 | -10 | . | . | 0 | 2 | 2 | -10 | 2 | 2 | 8 | 2 | 2 | -10 | 2 | 2 | -1 | 2 | 0 | 0.00 | 1.00 | 0.00 | 0.00 | F |
| 21 | 4 | 2 | 2 | 3 | . | . | 2 | 2 | 2 | 9 | 2 | 2 | 3 | 2 | 2 | 7 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | A |
| 22 | -5 | 2 | 2 | 0 | 2 | 2 | -8 | 2 | 2 | 9 | 2 | 2 | -2 | 2 | 2 | -2 | 3 | 3 | 0.00 | 0.70 | 0.21 | 0.09 | A |
| 23 | 1 | 2 | 2 | 0 | 2 | 2 | -4 | 2 | 2 | 8 | 2 | 2 | -1 | 2 | 2 | 0 | 2 | 2 | 0.10 | 0.14 | 0.19 | 0.57 | A |
| 24 | -1 | . | . | 1 | . | . | -1 | , | . | -3 | . | . | 1 | . | . | -1 | . | . | 1.00 | 0.00 | 0.00 | 0.00 |  |
| 25 | 7 | 0 | 2 | 7 | 0 | 2 | 7 | . | . | 10 | . | . | 0 | . | . | -9 | . | . | 0.00 | 0.00 | 0.00 | 0.00 |  |
| 26 | -2 | 3 | 3 | -3 | 3 | 3 | -5 | 3 | 3 | 10 | , | . | -5 | 3 | 3 | -4 | 2 | 3 | 0.00 | 0.02 | 0.80 | 0.19 |  |
| 27 | 0 | 1 | 1 | 0 | 1 | 1 | -9 | . | . | 8 | 2 | 2 | -0.5 | 2 | 1 | 0 | . | . | 0.00 | 0.44 | 0.21 | 0.35 | A |
| 28 | 4 | . | . | 6 | . | . | -10 | 2 | 2 | 8 | 2 | 2 | -10 | 2 | 2 | 5 | . | 0 | 0.00 | 1.00 | 0.00 | 0.00 | C |
| 29 | 3 | 2 | 0 | 2 | 2 | 2 | -1 | 2 | 2 | 3.5 | 3 | 3 | -1 | 2 | 2 | 2.5 | 0 | 0 | 1.00 | 0.00 | 0.00 | 0.00 | A |
| 30 | -8 | 2 | 2 | 0 | 2 | 2 | -10 | 2 | 2 | 10 | 2 | 2 | -8 | 2 | 2 | 0 | 2 | 2 | 0.00 | 1.00 | 0.00 | 0.00 | F |
| 31 | 0 | . | . | 0 | 2 | 1 | -10 | 2 | 2 | 10 | 2 | 2 | -10 | 2 | 2 | 0 | 1 | 1 | 0.00 | 1.00 | 0.00 | 0.00 | C |
| 32 | 0 | . | . | -10 | . | . | -10 | 3 | 2 | 5 | . | . | -10 | . | . | 0 | . | . | 0.00 | 0.59 | 0.00 | 0.41 | E |
| 33 | 3.1 | . | . | 0 | . | . | 1 | . | . | 4.25 | . | . | 3.1 | . | . | 0 | . | . | 1.00 | 0.00 | 0.00 | 0.00 | A |
| 34 | 0 | . | . | 8 | . | . | 7 | . | . | 10 | . | . | -8 | . | . | 0 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | A |
| 35 | 8 | 0 | 2 | 1 | 2 | 0 | 3 | 2 | 2 | -10 | 2 | 2 | 3 | 0 | . | 0 | 1 | 1 | 0.00 | 0.00 | 0.00 | 0.00 | D |
| 36 | 0 | 0 | 1 | 0 | . | 1 | 0 | , | . | 0 | 1 | 1 | 0 |  | . | 0 | . | . | 1.00 | 0.00 | 0.00 | 0.00 |  |
| 37 | -2 | 3 | 3 | 0 | . | 2 | -4 | 2 | 2 | 2 | 2 | . | -1.5 | 2 | 0 | 1 | . | . | 0.91 | 0.03 | 0.03 | 0.03 | A |
| 38 | 2 | 2 | 0 | 4 | . | . | 0 | 2 | 2 | 8 | 2 | 2 | -4 | 2 | 2 | 0 | 2 | 2 | 0.00 | 0.00 | 0.00 | 0.00 | A |
| 39 | 0 | 2 | 2 | -1 | 3 | 3 | -2 | . | 0 | 2 | 2 | 3 | -1.5 | 3 | 3 | -0.5 | . | . | 0.99 | 0.00 | 0.00 | 0.00 | A |
| 40 | 4 | . | . | 6 | . | . | 3 | 2 | 0 | -7 | 2 | 2 | 5 | . | . | -3 | . | . | 0.00 | 0.00 | 0.00 | 0.00 | D |
| 41 | -8 | 2 | 2 | -8 | . | . | -10 | . | . | 6 | . | . | -10 | . | . | -4 | . | . | 0.00 | 0.76 | 0.24 | 0.00 | E |
| 42 | -1 | 2 | 2 | 0.5 | 2 | 0 | -7 | 2 | 2 | 9 | 2 | 2 | -3 | 3 | 3 | 0 | . | . | 0.00 | 0.61 | 0.15 | 0.24 | A |


|  | $\left(H_{t}\right)$ | Message | UoỊS!̣əə |  |
| :---: | :---: | :---: | :---: | :---: |
| 26 | AAB | Hello. I propose that we assume that all participants before decided rationally. The sequence suggests that at action $1 y>0$. Expected value would so far be 5. Was this considered in action 2 , the decision for all numbers greater than -5 would be A too. Expected value $(5+2,5) / 2=3,75$. Was this considered in action 3 , b was chosen only for strongly negative numbers. This is why I would propose b as well, that is to say a negative number. | 2 | \{3\} |
|  | BBAA | Decider 3 and 4 had more information, expected value consequently positive. Would therefore tend to A. At each $x>-3$ jump to A. | $-3$ | \{3\} |
|  | AAABA | Last participant has again most information. Apparently participant 4 had a strongly negative number. Still, I would be to chose A at negative numbers up to -5 . | 5 | \{3\} |
| 9 | AAB | Purely mathematically assumed: ;-) I am fourth and have information A,A,B. The first should have set 0 as marginal point to have a 5050 chance. The second can thus be sure that the first had a positive number. He thus sets his point a bit higher at 2. With two A decisions the third can assume that both previous numbers were positive, so he can courageously bet on A with -2 . This might be too courageous, he has maybe a negative number. Long speech, short meaning A with still high risk -2 | -2 | $\{2,3\}$ |
|  | BBAA | I am fifth with $\mathrm{B}, \mathrm{B}, \mathrm{A}, \mathrm{A}$ as information. This give a 50:50 chance so I would guess a 0 . | 0 | $\{2\}$ |
|  | AAABA | 6. with information $\mathrm{A}, \mathrm{A}, \mathrm{A}, \mathrm{B}, \mathrm{A}$ assume: 1 . chooses $0 \rightarrow$ positive number, 2. chooses 0 as well $\rightarrow$ positive number, 3 . becomes more courageous chooses $-2 \rightarrow$ possibly small negative number but in sum still everything positive, 4. still more courageous -3 , now we would be back in the area around zero, so 5 chooses 0 again $\rightarrow$ has a positive number. Means we should be more courageous :D hehe I think this makes little sense | -3 | $\{3\}$ |

Table 24: Examples of messages potentially reflecting Bayesian reasoning

## A. The model in the Eyster and Rabin framework

The social learning environment used in Eyster and Rabin (2010) has continuous signals $s_{t} \in[0,1]$ and actions $a_{t} \in[0,1]$. The signal obeys $s=\operatorname{Pr}[\omega=1 \mid s]$. Given an assumed payoff function $g_{t}\left(a_{t} ; \omega\right)=-\left(a_{t}-\omega\right)^{2}$, player $t$ maximizes her expected payoff by setting $a_{t}=E\left[\omega \mid s_{t}, H_{t}\right]$.

Level-0 play is random and not connected to the private signal. The level-0 strategy is hence

$$
\sigma^{0}\left(s_{t}, H_{t}\right) \sim U[0,1] .
$$

In the notation of Eyster and Rabin (2010), random play over $[0,1]$ leads to an expectation of the log odds of $\ln \left(\frac{a_{t}}{1-a_{t}}\right)=0$.

The best response to uninformative preceding play is to follow the private information. A level-1 player $t$ thus follows strategy

$$
\sigma^{1}\left(s_{t}, H_{t}\right)=s_{t}
$$

Due to the equivalence of level- 1 actions and signals, a level- 2 player combines the private information with predecessors' actions. The strategy $\sigma^{2}\left(s_{t}, H_{t}\right)$ is implicitly defined as

$$
\ln \left(\frac{\sigma^{2}}{1-\sigma^{2}}\right)=\left[\sum_{i=1}^{t-1} \ln \left(\frac{a_{i}}{1-a_{i}}\right)\right]+\ln \left(\frac{s_{t}}{1-s_{t}}\right) .
$$

Given $\sigma^{1}\left(s_{t}, H_{t}\right)$, the actions of predecessors $a_{i}$ are taken at face value and processed as if they reflected the private information. This level-2 specification is Eyster and Rabin's (2010, p. 230) specification of BRTNI play. If all players $t$ in a sequence follow this strategy and take previous actions at face value, the early signals will be overweighted as follows

$$
\ln \left(\frac{\sigma^{2}}{1-\sigma^{2}}\right)=\left[\sum_{i=1}^{t-1} 2^{t-1-i} \ln \left(\frac{s_{i}}{1-s_{i}}\right)\right]+\ln \left(\frac{s_{t}}{1-s_{t}}\right)
$$

The best response to such behavior is to undo the overweighting, back out the true signals of predecessors, and combine them with the own signal. Given the public history and the informative action space, the individual signal $s_{i}, \forall i<t$ can be backed out perfectly. Level-3 ends up playing like a rational Bayesian player that combines all information in strategy $\sigma^{3}\left(s_{t}, H_{t}\right)$ so that

$$
\ln \left(\frac{\sigma^{3}}{1-\sigma^{3}}\right)=\sum_{i=1}^{t} \ln \left(\frac{s_{i}}{1-s_{i}}\right)
$$

Of course, this hinges on the assumption that the population belief of this level3 player is indeed correct and would deliver different results if the predecessors
were actually level-0 or level- 1 players. In this rich-information setup it is always possible to back out the private signals of non-level-0 players. The prerequisite for doing this correctly is to have a correct population belief of player $i$, that is, to know how player $i$ used her signal when choosing her action. Then, the signal can be backed out by simply observing her action and all prior actions. This framework features the possibility to back out any signal that was used in a deterministic strategy. In other words, any level- $k$ play, $k>0$, is fully informative given the correct population belief about it.

Like in the Anderson and Holt setup, higher levels of reasoning are conceivable but are only interesting because of their population belief. In particular, level-4 might believe that players' actions simply summarize the previous signals in an optimal fashion (level-3). Beyond this self-awareness of level-3, no further insights are obtained when considering higher levels.

## B. Experiment instructions (AH framework)

## Welcome to the experiment!

## Introduction

You are about to participate in an experiment in team decision making. The experiment is funded by Cornell University. Please follow the instructions carefully.

In addition to the participation fee of $\$ 5$, you may earn a considerable additional amount of money. Your decisions determine the additional amount. You will be instructed in detail how your earnings depend on your decisions. All that you earn is yours to keep, and will be paid to you in private, in cash, after today's session.

It is important to us that you remain silent and do not look at other people's screens. If you have any questions or need assistance of any kind, please raise your hand, and an experimenter will come to you. If you talk, shout out loud, etc., you will be asked to leave. Thank you.

The experiment consists of a test round and 2 parts. Part I is designed as a warm-up with 3 trivia questions. Part II consists of 2 rounds of 3 decisions each. In all 6 decisions your task will be identical and success is identically rewarded.

Since this is a team experiment, you will be randomly matched with another participant in this room, to form a team that plays as one entity. Your teammate will change every round, so please do not assume content of previous communication to be known by your new partner. The way you interact as a team to take decisions will be the same throughout all rounds.

Now, let me explain how your Team's Action is determined. In fact, both your teammate and you will enter a Final Decision individually and the computer will choose randomly which one of your two final decisions counts as your team's action. The probability that your teammate's final decision is chosen is equal to the probability that your final decision will be chosen (i. e. your chances are 50:50). However, you have the possibility to influence your partner's final decision in the following way: Before you enter your final decision, you can propose to your partner a Suggested Decision and send him one and only one text Message. Note that this message is your only chance to convince your partner of the reasoning behind your suggested decision. Therefore, use the message to explain your suggested decision to your teammate. After you finish entering your suggested decision and your message, these will be shown to your teammate. Simultaneously, you will receive your partner's suggested decision and message. Both of you will then make your final decision. As outlined above, once you both enter your final decision, the computer chooses randomly one of your final decisions as your team's action.

If you have any questions at this point, please raise your hand. In order for you to get familiar with the messaging system, you will now try it out in a Test Period. Please turn the page for further instructions.

## Test period

A participant in this room is now randomly chosen to be your teammate. The Test Period has two rounds, with communication in each round. Since this is only a test, your earnings will not depend on anything that happens now. In both test rounds you will need to send and receive pieces of information. The information consists of the answer to a question and one given phrase. After the successful exchange, you will enter the number again. This way, the communication structure is identical to the one in the experiment rounds.

The messenger allows Messages of any size. However, you have to enter the message line by line since the input space is only one line. Within this line you can delete by using the usual "Backspace" button of your keyboard. By pressing "Enter" on the keyboard, you add the written sentence to the message. Please note that only added sentences will be sent and seen by your partner. The words in the blue input line will not be sent. You can always delete previously added sentences by clicking the "Clear Input" button. The number of lines you send is not limited. You can therefore send messages of any length. You finally send the message to your partner by clicking the "Send Message" button.

If you have any questions at this point, please raise your hand. When you are ready, please click the "Ready" button to start the Test Period.

## Experiment - Part II

You are about to start Part II of the experiment. ${ }^{21}$ This part consists of 2 periods of 3 decisions each. ${ }^{22}$ In each period you will team up with a different, randomly chosen participant. Therefore you will make 3 urn choices with a given partner.

The 6 scenarios for your urn choices are taken from another experiment. They are randomly chosen and simply allow me to confront you with more scenarios than if we played it out ourselves. I will now explain the setup to you in the same fashion the participants of the original experiment were instructed. The original experiment was not done by teams, but I will explain it with teams for simplicity.

In each urn choice your task is the following:
Your team is asked to predict from which randomly chosen urn a ball was drawn. It is equally likely that urn A or urn B will be the true urn. That is, there is a 50 percent chance that Urn A is the true urn, and a 50 percent chance that Urn B is the true urn. Urn A contains 2 white balls, and 1 black ball. Urn B contains 1 white ball and 2 black balls. Therefore, there is a $\frac{2}{3}$ chance that a white ball comes from urn A and a $\frac{2}{3}$ chance that a black ball comes from urn B.

To help determine which urn is the right one in a given scenario, you will see one ball, drawn at random, from the urn. Only your team will see the outcome of this private draw. And your team will only see this one draw, you will get the same information as your teammate. After each draw, the ball is returned to the container before making the next draw. Therefore, each team will have one private draw, with the ball being replaced after each draw. This way each team draws from an urn that contains 3 balls in total.

[^14]When it is your turn to decide, bold text in the top of your screen will give you information about your draw. If the ball the computer has randomly drawn for you is white, your window will read, "Your team's private draw from the urn is WHITE." Your window will read, "Your team's private draw from the urn is BLACK.", if the ball the computer has randomly drawn for you is black.

In each scenario, decisions are made sequentially, i.e. one team after the other. The order in which teams decide in a given scenario is determined randomly. Once the first team has agreed on its action based on its private draw, the second team will be asked to make its decision. The members of team 2 will see their private draw, and also which urn was chosen by the first team. Until the end of the round, all following teams will be informed about the chosen urns of all earlier teams, but not about the other teams' draw from the urn.

In each round, you see the three scenarios and will write down your Suggested Decision and Message for each of them. After that you see your partner's Suggested Decision and Message for each of the three scenarios and will make your Final Decisions. Finally, you will be informed about your Team's Action and the true urn. The order of events is illustrated in the table below. For each of the 6 scenarios, if your team's action and the chosen urn coincide, you will individually earn $\$ 1.00$ (your teammate will get $\$ 1.00$ as well).

As described earlier, you will send your teammate a Suggested Decision and a Message. Remember to explain in the message your reasoning behind your suggested decision. (And note again that the words in the blue input line will not be sent. Press "Enter" to add them to the message.) After this information is exchanged, both of you enter your Final Decision, from which the computer randomly chooses the Team's Action.

As part of the communication, you can quantify your Confidence for each choice you make. You can put numbers between $50 \%$ and $100 \%$, where $50 \%$ implies that you think both urns are equally likely and higher numbers reflect your higher confidence in your choice, up to certainty (100\%).

Let me summarize the main points: (1) In each scenario, it is equally likely that urn $A$ or urn $B$ is the true urn. (2) There is a $\frac{2}{3}$ chance that a white ball came from urn $A$ and a $\frac{2}{3}$ chance that a black ball came from urn B. (3) In the knowledge of the previous teams' actions and your draw, choose either urn A or urn B. (4) Like you, other teams saw one draw from the true urn and their predecessors' urn choice. (5) If your team chooses the true urn, you will earn \$1.00.

If you have any questions at this point, please raise your hand. When you click the "Ready" button, you will start the first round of the experiment.

## C. Classification instructions (AH framework)

In the following I will describe the classification process for the analysis of an experiment. Subjects play a game of incomplete information. Their reasoning will be classified along the lines of a model of level of reasoning, which will be laid out in the first section. It is set up in analogy to the level- $k$ model in complete information settings as introduced by Nagel (1995) and Camerer et al. (2004).

Follow the instructions of this booklet. Read them entirely to get an overview and then start the classification. The game is the social learning experiment as implemented by Anderson and Holt (1997), which I assume you are fully familiar with. The subjects were put in 6 different situations that occurred in the original study by Anderson and Holt (1997) and decided in a team about their action, which would be remunerated according to the true urn in Anderson and Holt (1997).

The model will introduce four different ways of reasoning in the context of this game. The aim of the classification is to take a look at the 6 messages per player and connect him/her with one or

| Round | Urn choice | Action |
| :---: | :---: | :---: |
| 1 | 1 | Suggested Decision |
|  | 2 |  |
|  | 3 |  |
|  | 1 | Final Decision |
|  | 2 |  |
|  | 3 |  |
|  | 1 | Get result |
|  | 2 |  |
|  | 3 |  |
| 2 | 4 | Suggested Decision |
|  | 5 |  |
|  | 6 |  |
|  | 4 | Final Decision |
|  | 5 |  |
|  | 6 |  |
|  | 4 | Get result |
|  | 5 |  |
|  | 6 |  |

Table 25: Order of events in Part II.
more types. Details will be explained below. Please limit yourself to making inferences only from what can clearly be derived from the message stated, i.e. do not try to think about what the player might have thought.

## Reasoning types in the model

In the context of social learning, reasoning types differ in the ways they process private information (in form of draws from an urn) and public information (in form of actions of predecessors). A natural model of level of reasoning starts out with a random level-0 type and builds a hierarchy based on the number of iterated best responses. In this game, this hierarchy turns out as follows:

Level-0 Randomizing between $A$ and $B$, irrespective of own private signal.
Level-1 If others' signal is not informative, like level-0 random play, then the best response is to get informed by the own signal. Therefore, level-1 players are following their own private signal.

Level-2 Since level-1 play is fully informative about the private signals, the best response in light of this is to follow the majority if it is ahead by more than one signal difference. A level-2 player will thus only follow his signal if the previous actions were split equally between the two urns or one urn was chosen just one time more often.

Level-3 Since level-2 play is as informative as Bayesian play, level-3 play, a best-response, is like original Bayesian play. This implies that the play and beliefs of level-3 players are identical to play and beliefs in the Bayesian equilibrium if everybody is level-3.

Starting from this model, I will now explain some expected message contents for the individual levels of reasoning

Level-0 Random play should result from messages that clearly do not understand the nature of the game or put reasons for play that are orthogonal to any reasonable inference using private signal and previous actions.

Level-1 Level-1 is characterized by disregarding others' signal. In the message, this can be an open disregard of others' actions or an emphasis on the unambiguity of the own signal.

Level-2 A level-2 players with a degenerate population belief (as introduced in the model above) implies that all others' actions are taken at face value like a signal. It follows that the ratio of As vs. Bs in the previous actions might weighted against the $2 / 3$ chance of the own signal to be correct. In any case, a level-2 player never engages in a differentiation of individual predecessors. If everybody is assumed to play their signal, this implies that others are regarded as a homogeneous crowd that simply differs in the signals they received not in their position in the sequence. Alternatively to a degenerate population belief, a level-2 player might think that some players played random, like level- 0 . Then, the signals are not taken at full face value and probably the own signal is valued more than the observed actions.
Level-3 heterogeneity Under a degenerate population belief, only level-3 players distinguish individual predecessors in the extent to which their actions reflect their private signals. It is therefore a characteristic of level-3 players to differentiate informative and uninformative observed actions depending on the position of predecessors in the sequence. One example is that a level-3 player will note in a history of AAAA that the later players might just have followed the majority, he therefore does not infer those players signal with certaint. Put starkly, observing BBBA implies that a level-3 player rules out the fourth player to be level-2, an observation only level-3 will make.

## The data

You will see different situations in which the players decided to choose urn A or urn B.
Each situation is explained in brackets by the information available to the subject. The capital letters indicate previous players' actions. The small letters indicate the private signal received by the player at hand. In addition to the sent message, you see the suggested decision, which is 1 for urn $A$ and 2 for urn B. The communication was structured in the sense that it gave the players the possibility to indicate the confidence in the own urn choice. This number is given to you as well.

## The classification

I would like you to classify messages into one of the 4 levels described above. Indicate the closest type under 'level' in the according space.

Indicate whether the population belief is degenerate on the next lower level or non-degenerate. If it is non-degenerate, denote which types (in terms of levels) are assumed to be present.

In addition to this classification, I ask you to indicate any difference between the observed reasoning and the type given. This might include differences in the belief about when others start imitating others' actions, the weight the public information receives compared to the private information, the way the population updating is done by a level-3 player, etc. Also, if you think the potential level could be more than one, indicate your considerations here.

## D. Experiment instructions (ÇK framework)

The experiment instructions in German combine the communication instructions (see Introduction and Test period above) with instructions from Çelen and Kariv (2004) translated to German and adapted to the team setting. A translation is available upon request.

## E. Classification instructions (ÇK framework)

In the following I will describe the classification process for the analysis of an experiment. Subjects play a game of incomplete information. Their reasoning will be classified along the lines of a model of level of reasoning, which will be laid out in the first section. It is set up in analogy to the level- $k$ model in complete information settings as introduced by Nagel (1995) and Camerer et al. (2004).

Follow the instructions of this booklet. Read them entirely to get an overview and then start the classification. The game is the social learning experiment as implemented by Celen and Kariv (2004), which I assume you are fully familiar with. The subjects in my experiment were put in 6 different situations that occurred in the original study by Çelen and Kariv (2004) and decided in a team about their action, which would be remunerated according to the true signal in Celen and Kariv (2004).

The model will introduce five different ways of reasoning in the context of this game. The aim of the classification is to take a look at the 6 messages per player and connect him/her with one or more types. Details will be explained below. Please limit yourself to making inferences only from what can clearly be derived from the message stated, i.e. do not try to think about what the player might have thought.

## Reasoning types in the model

In the context of social learning, reasoning types differ in the ways they process private information (in form of draws from an urn) and public information (in form of actions of predecessors). Çelen and Kariv (2004) analyse Bayesian behavior, which is fully rational and believes others to be fully rational. Please consult the paper to get a thorough understanding of this kind of reasoning in this context.

A natural model of level of reasoning starts out with a random level-0 type and builds a hierarchy based on the number of iterated best responses. In this game, this hierarchy turns out as follows:

Level-0 Playing a random threshold on the action space.
Level-1 If others' signals are not informative, like level-0 random play, then the best response is to get informed by the own signal. Therefore, level-1 players are following their own private signal and set a threshold of 0 .

Level-2 With a threshold of 0 , the observed action of a predecessor indicates whether the signal was in $[-10,0)$ or $[0,10]$. With this information, the best response is to add 5 or subtract 5 for each previous action $A$ or $B$ in the expected value of the sum of the signals. A level- 2 player will thus expect a sum of signals of 0 if the history has equally many A's as B's and, e.g., 5 if one more B's than A's have been observed and 10 if two more B's than A's have been observed. The threshold will be set as the negative of the expected sum.

Level-3 Thresholds of -10 or 10 lead to uninformative play since the action is decided irrespective of the signal. Level-3 players hence only use information from level- 2 players that are not cascading, i.e. play $-5,0$, or 5 .
Level-4 Level-4 players only use information from level-3 players that are not cascading. Although level-4 play is distinct from level-3, we will not pursue this or higher levels in our classification.

Starting from this model, I will now explain some expected message contents for the individual levels of reasoning.

Level-0 Random play should result from messages that clearly do not understand the nature of the game or put reasons for play that are orthogonal to any reasonable inference using private signal and previous actions.

Level-1 Level-1 is characterized by disregarding others' action. In the message, this can be an open disregard of others' actions or an emphasis on the unambiguity of the own signal.
Level-2 A level-2 player with a degenerate population belief (as introduced in the model above) implies that all others' actions are equally informative and indicative of the sign of the signal and thus each one changes the expected sum of signals by +5 or -5 . It follows that the relative number of As vs. Bs in the previous actions determine the threshold. In any case, a level-2 player never engages in a differentiation of individual predecessors. If everybody is assumed to play their signal, this implies that others are regarded as a homogeneous crowd that simply differs in the signals they received not in their position in the sequence.

Level-3 heterogeneity Level-3 players are the first in the hierarchy to distinguish individual predecessors in the extent to which their actions are informative about their private signals. In particular, level- 2 actions might be completely uninformative if they result from a threshold of 10 or -10 . At the same time, actions that result from a signal below a threshold of -5 or above a threshold of 5 induce a strong change in the expected sum of signals. Therefore, sum actions might have a stronger influence on the level-3's threshold than others. It is therefore a characteristic of level-3 players to differentiate more or less informative observed actions depending on the position of predecessors in the sequence and the likely threshold they had set. One example is that a level-3 player will note in a history of $A B$ that the second player's signal must have been very low since a threshold of -5 was still undercut. This informativeness changes to uninformativeness when the threshold is expected to be 10 or -10 . Then, the action is uninformative.

Bayesian More than level-3 players, Bayesian players realize that every predecessor has most likely set a different threshold. This way, particular later actions in the sequence can have a strong influence on the expected sum of signals and thus the set threshold.

## The data

You will see different situations in which the players observed histories of A's and B's and set a Suggested Decision between -10 and 10 .

Each situation is described by the information available to the subject. In addition to the sent message, you see the suggested decision.

## The classification

I would like you to classify messages into one of the 5 levels described above. Indicate the closest type under 'level' in the according space.

Indicate whether the population belief is degenerate on the next lower level or non-degenerate. If it is non-degenerate, denote which types (in terms of levels) are assumed to be present.

In addition to this classification, I ask you to indicate any difference between the observed reasoning and the type given. This might include differences in the belief about what threshold is assumed to be behind others actions, the way the population updating is done by a level-3 player, etc. Also, if you think the potential level could be more than one, indicate your considerations here.

Please also note that some messages contain considerations regarding the threshold to set etc. which are orthogonal to the inferential considerations. Please make sure to only classify according to the strategic content.


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[^1]:    ${ }^{1}$ The application to the framework of Eyster and Rabin (2010) is shown in section A of the supplementary material.

[^2]:    ${ }^{2} \lambda_{i}$ is the precision in a logistic choice function for the $i$ th iterated best response.
    ${ }^{3}$ With slight abuse of notation, the argument $H_{t}$ is replaced by the inferred set of signals to highlight the informational input of the strategy.

[^3]:    ${ }^{4}$ For example, a level-3 player observes an action that cannot come from a level-2 player. Then the updated population belief is again nearly degenerate: $p_{i}^{3,0}=\frac{\varepsilon}{\varepsilon+1}$ and $p_{i}^{3,1}=\frac{1}{\varepsilon+1}$. This example reflects the intuition that breaks in a cascade are most likely informative, i. e. they result from level-1 play.

[^4]:    ${ }^{5}$ This shows why action data in this framework is not well-suited to distinguish certain forms of behavior, as noted by Eyster and Rabin (2010).

[^5]:    ${ }^{6}$ See equations 1 and 2 in ÇK.
    ${ }^{7}$ Any strategy is uninformative that exclusively involves the thresholds -10 and 10 .
    ${ }^{8}$ Since the information content of the level- 0 action is not obvious, it is potentially not uncovered by a higher level player. Then, the level-0 action has the same effect in the minds of higher level players as the modeled level-0 belief.

[^6]:    ${ }^{9}$ This range is in accordance with the limited depth of reasoning found in Kübler and Weizsäcker (2004, $\lambda_{3}=0$ ).

[^7]:    ${ }^{10}$ The experiments in Drehmann, Oechssler and Roider (2005) and Cipriani and Guarino (2005) are framed in a more complex financial market context. This might be a reason for the relatively low ranks despite their highly educated subject pool.

[^8]:    ${ }^{11}$ Table 9 on page 32 reports the data for each subject and type.
    ${ }^{12}$ In the following, I will use the $p$-value not only in its original meaning in the hypothesis tests, but with slight abuse also as my best type-independent measure of closeness between fingerprints.

[^9]:    ${ }^{13}$ Instructions are reprinted in sections $B$ and $D$ of the supplementary material.

[^10]:    ${ }^{14}$ The experiments were programmed and conducted with the software z-Tree (Fischbacher, 2007).
    ${ }^{15}$ Having messages exchanged after three rather than one suggested decision has the advantage that all three messages are not "contaminated" by the team partner's arguments. In the original setup with an immediate exchange, this would only be true for the very first message.
    ${ }^{16}$ Instructions for the RAs are reprinted in sections $C$ and $E$ of the supplementary material.

[^11]:    ${ }^{17}$ In particular, subjects are told at the beginning that they will experience situations that arose in a previous experiment which was played by individuals. Subsequently, the game is explained in the instructions and finally they see the information about the specific situation on the screen.

[^12]:    ${ }^{18}$ The classification for the last 3 decisions is reported in Table 10 on page 33. A Wilcoxon signed-rank test does not reject the hypothesis that the level lower bounds or the level upper bounds are different between decisions 1-3 (Table 4) and decisions 4-6 (Table 10, $\left.p_{\text {lower }}=0.515, p_{\text {upper }}=0.470\right)$. To control for order effects, the order of choices was reversed in the last three sessions. If the choices are not numbered, the analysis presents the temporal order of the decisions, i.e.the "first three decisions" will always refer to the decisions taken without prior conversation with the team partner.
    ${ }^{19}$ The union excludes any outcome of a non-classification (NA). If all three decisions are classified differently or as NA by both RAs, the subject appears under NA. For more details, Table 11 on page 33 reports this data by decision. Tables 12 and 13 on pages 34 and 35 report the data of both RAs on an individual level.

[^13]:    ${ }^{20}$ The analysis of the first three decisions is less powerful and yields significant non-random play only for slightly more than $50 \%$ of the players. Results for the first 3 and second 3 decisions are reported in Tables 20 and 21. Like in the AH experiment (section 4.1), the communication has no detectable effect on the sophistication since the results are not significantly different from each other (Wilcoxon rank-sum, $p=0.703$.)

[^14]:    ${ }^{21}$ The experiment consists of two parts, part I being independent of part II.
    ${ }^{22}$ Table 25 was shown to give an overview.

