Revenue management in an assemble-to-order production system

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Abstract

In this paper, we consider demand management decisions for an assemble-to-order production system in which both the availability of intermediate material and assembly capacity are limited. For each incoming order, the manufacturer must decide whether to accept it and what due date to quote for an accepted order. The actual assembly dates are still subject to change after these decisions, and a production schedule must be maintained to guarantee that the quoted due dates are met. Therefore, the decisions on accepting orders and quoting due dates must be made with incomplete knowledge of the actual resources used to fulfill the orders. To address these factors, we model this situation and develop a novel revenue management approach using bid prices. An extensive numerical study demonstrates the good performance of the proposed approach in comparison with benchmark algorithms and an ex-post optimal solution applied over a wide range of different supply and demand scenarios. Our results suggest that the consideration of assembly capacity constraints is more vital than the consideration of intermediate material constraints in our test cases.
1 Introduction

In an assemble-to-order (ATO) production system, intermediate materials are assembled into end products. Intermediate materials are held in stock because they often have long replenishment lead times. End products are assembled only for customer orders on hand.

ATO production systems are found in high-tech industries such as semiconductor manufacturing and the steel industry. In these industries, capital equipment costs are high and lead times for intermediate materials are long. Therefore, in the short-term, it is impossible to adjust capacity levels. The demand is typically fluctuating and uncertain. The production is order-driven due to a high risk of obsolescence or customer-specific configurations of the end products.

In companies with high capital equipment costs, the capacities are tightly planned because unused capacity is highly expensive. Therefore, if these companies face uncertain and fluctuating demand, resources may be insufficient to fulfill all incoming demand during peak phases. As customers are heterogeneous (i.e., they differ in their importance, time-sensitivity, willingness to pay, etc.), the question arises as to which customers to serve using the scarce resources. This scenario represents a typical revenue management problem.

Traditional revenue management approaches focus on the decision of which orders to accept (see, e.g., Talluri and van Ryzin (1999), Spengler et al. (2007)). However, in practice, the company typically must decide on the delivery time for each customer as well. Apart from giving discounts, failure to meet the customer’s preferred due date can lead to loss of goodwill of the affected customer. Ultimately, the company might even lose this customer to competitors, which is why it is important to meet the preferences for the quoted due dates of customers with high value to the company.

The actual assembly dates of accepted orders are still flexible up to a certain point. If preemption is not allowed, it might be beneficial to delay assembly for an order, even though assembly capacity is available, in anticipation of a rush order with higher value.

In this paper, we consider demand management decisions for a manufacturer using an ATO production system and facing stochastic demand. To maximize profit, the manufacturer decides which orders to accept and the due dates to quote for the accepted orders. A production schedule is maintained to guarantee that the quoted due dates are met. The availabilities of both assembly capacity and intermediate material are limited.

The main contributions of this paper are as follows:

- We model the described decision problem mathematically and develop a novel revenue management approach (ATO-RM) that uses bid prices.
• Using an extensive numerical study, we show that the approach works well by comparing it to benchmark algorithms and an ex-post optimal solution. We perform a sensitivity analysis demonstrating that the good performance is valid over a wide range of demand scenarios.

• From a managerial point of view, we show that the consideration of constraints relative to assembly capacity appears more important to obtaining good results than the consideration of constraints on intermediate material in our test cases.

This paper is organized as follows. After reviewing the literature in Section 2, we describe the assumptions of our model in Section 3. Next, we present our approach for quoting due dates and performing the scheduling in Section 4. The corresponding calculations require bid prices, which we derive in Section 5. A numerical study demonstrates the performance of our approach in Section 6, and Section 7 concludes the paper.

2 Literature review

In this section, we review the literature closely related to our research. First, we survey the stream of literature that considers decisions of production planning in ATO manufacturing. Next, we review the literature addressing online decisions of which orders to accept, what due dates to quote, and how to schedule the accepted orders. A growing body of literature exists in the area of revenue management in manufacturing, which is presented in this section. Finally, we examine the emerging literature on revenue management of flexible products.

Production planning in ATO manufacturing

Song and Zipkin (2003) give an overview of supply chain operations in ATO manufacturing. Kolisch (2001) describes a hierarchical planning approach used to model the decisions of an ATO manufacturer. An order selection process is placed in the first hierarchy level to decide on accepting orders and quoting due dates. It is assumed that the decisions are made in a batch-processing mode. The next hierarchy levels are manufacturing planning and operations scheduling. A model and solution approaches are described for each planning step. The actual production planning steps are modeled in much greater detail than in our research, but the previously described models are deterministic, and thus, order acceptance and due date quoting decisions are not made online as is the case in the current paper.
Benjaafar and ElHafsi (2006), ElHafsi (2009), and Cheng et al. (2011) contribute to a stream of literature that addresses production and inventory control in ATO manufacturing. Their models include various customer classes with different costs for lost sales, leading to an order selection problem. Only one end product that requires several intermediate materials for production is considered. The time required for assembly is assumed to be negligible, and thus these authors do not model assembly capacity; however, they decide when to produce the intermediate products. The production time for intermediate materials is stochastic, and orders cannot be backlogged in their setting. Therefore, in contrast to the model presented in this work, due date quoting decisions are not taken into account, replenishment of intermediate materials can be influenced, and assembly capacity for end products is not considered.

Order acceptance, due date quoting, and production scheduling

A wide field of literature exists on the topics of order acceptance, due date quoting, and production scheduling. However, research on combinations of all of these decisions is rare.

Slotnick (2011) gives a recent overview of the literature that addresses simultaneous order acceptance and scheduling decisions. She finds that most of the research in this area focuses on deterministic models. This study also suggests that resource constraints are rarely considered. Many models in this stream do not maximize profit; instead, they focus on other objectives, i.e., maximizing utilization or minimizing costs. Grigoriev et al. (2005) describe basic production scheduling problems that take into account raw material constraints. This work assumes that all orders must be processed and presents models that minimize make-span or lateness in a single machine environment. Voling and Spengler (2011) develop a model for simultaneous due date quoting and master production scheduling in make-to-order (MTO) automobile production. However, in this model, all orders must be accepted. Keskinocak and Tayur (2004) give a survey of due date management policies that also includes models with order acceptance decisions. Kolisch (2001) also contributes to this stream of literature by considering all of the previously mentioned decisions. However, in contrast to the model in this paper, these decisions are not made online.

Certain papers in this stream combine two of the decisions, but to the best of our knowledge, the current paper is the first work that takes into account all of the decisions online.

Revenue management in manufacturing

Revenue management approaches originally focused on applications in service industries such as the airline or hotel business. Recently, applications in manufacturing have been considered as
well. The literature concerned with revenue management in manufacturing can be categorized by the underlying production principle.

Important papers in the literature on revenue management in a make-to-stock (MTS) context are Meyr (2009) and Quante et al. (2009). In this context, only storable resources are considered, whereas one of the main characteristics of the model presented in the current paper is the combination of storable and non-storable resources.

In MTO revenue management, the scarce resource is production capacity. Spengler et al. (2007) apply revenue management to the iron and steel industry in which unique orders arrive over time. This group models the decision of which orders to accept in a setting with multiple resources. However, the production time does not exceed one planning period as opposed to the setting in the current paper in which resources are required over multiple planning periods. Their decisions are based on bid prices, which are computed via a multi-dimensional knapsack problem formulation. Barut and Sridharan (2005) develop a heuristic approach to compute the capacity contingents for several customer classes to decide which orders to accept in an order-driven production system, but they do not model raw material requirements. Gallien et al. (2004) describe a model for admission control in a single server queue with preemption allowed at no cost. This group makes their decisions by computing a three-dimensional acceptance region consisting of price, quantity, and lead time. Raw material requirements are not taken into account. Kuhn and Defregger (2005) present a revenue management approach for a MTO manufacturer with limited end-product inventory. In each period, the company decides whether the incoming order is accepted. If the order is accepted, it must be determined to what extent the order is satisfied from stock. Additionally, in each period, the company must decide if new end products should be produced and put into stock. Raw material restrictions are not modeled.

In this literature stream, the availability of raw material is mostly disregarded. Additionally, due date quoting and scheduling decisions are rarely taken into account. Harris and Pinder (1995) are the first to mention revenue management in an ATO context. They validate that the requirements for successfully applied revenue management are fulfilled in ATO manufacturing. This paper also describes a pricing and capacity allocation approach for a simplified model. Gao et al. (2012) model an available-to-promise assembly system. They use pseudo-orders to model uncertainty in the orders and develop a Markov chain model to obtain insights into optimal order acceptance decisions. This model also includes inventory and capacity constraints, but the raw material does not have to be available at the time of the production start (availability within the given lead time is sufficient). Accepted orders are processed in a first-come-first-served manner.
The approaches in this literature stream rely on dynamic programming. Order processing, including due date quoting and scheduling, is not modeled because it would render these approaches intractable due to the large size of the resulting state space.

**Revenue management of flexible products**

*Gallego and Phillips (2004)* introduce the concept of revenue management of flexible products. Their model includes both specific and flexible products. Although the capacity requirements are fixed for specific products, every flexible product has a set of possible execution modes. Each execution mode can lead to a different level of resource consumption and profit margin. To save capacity for the higher-margin specific products that are yet to come, orders can be rejected and the right execution modes for the flexible products must be chosen. At a fixed point in time, the so-called notification date, the execution modes of all flexible products must be fixed. After that, only demand for specific products occurs. In our model, in contrast, we quote a due date for each accepted order immediately. At the beginning of each time period, we fix the assembly date (which corresponds to the possible execution modes) of those orders for which assembly begins in this time period.

*Gallego and Phillips (2004)* look at a two-period/two-product case, and due to the small size of the problem, they are able to compute optimal booking limits using dynamic programming. *Gallego et al. (2004)* extend this approach to a network setting with an arbitrary number of periods, which vastly increases the size of the dynamic program such that they introduce a bid price approach to solve the problem heuristically. *Petrick et al. (2012)* further extend this concept by introducing a novel model formulation that allows for reallocation of the execution mode for flexible products and a more general choice of the notification date.

This literature stream is the one closest related to this paper. However, the time structure of our model is different, which makes the described approaches not applicable in the setting of the current paper. In the papers in this stream, service is provided after all flexible orders have arrived, but these events cannot be strictly separated in our model. In the previous models, it is possible to decide the execution modes using batch processing, but in our approach, these decisions have to be made online. This also implies that in our model, the possible execution modes for an order diminish over time. Additionally, the approaches in the literature do not make independent decisions on due dates and scheduling.
3 Model formulation

In our problem setting, orders arrive over time according to a known probability distribution. The task is to decide on the acceptance and quoted due dates for each incoming order. A feasible schedule must be maintained to guarantee that the quoted due dates are met. The limiting resources are assembly capacity and intermediate materials.

In this section, we describe the basic assumptions of our model. The used assembly model, the properties of the orders, and the order processing are explained. Finally, the decisions that must be made by the model are presented.

The model presented in this paper is based on the following assumptions. The considered planning horizon is divided into $T$ time periods ($t = 1, \ldots, T$). Incoming orders are denoted by $d = 1, \ldots, D$.

**Assumption 3.1 (Assembly process).** Assembly is modeled as a deterministic one-stage process. The constrained resources are assembly capacity $a_t$ and the availability of intermediate materials $m = 1, \ldots, M$ in each time period $t$. Each order $d$ requires processing in $cap_d$ consecutive periods on one of the $a_t$ identical parallel machines. Additionally, at the start of the assembly, $use_{dm}$ units of intermediate material $m$ are required, and preemption is not allowed.

We model a simplified assembly process because the actual production scheduling is not the main focus of this model. The main decisions are whether to accept an incoming order and the choice of a quoted due date. For these decisions, the remaining assembly capacity must be estimated.

**Assumption 3.2 (Resources).** A short-term planning horizon is considered. Within this time horizon, it is impossible to extend the company’s maximum capacity levels. A replenishment amount $r_{mt}$ is exogenously given for each intermediate material $m$ in each time period $t$.

This assumption reflects the fact that capacity adjustments are mid- to long-term decisions and intermediate materials often have long replenishment lead times, whereas our model is used for short-term decisions. It follows that $a_t$ and $r_{mt}$ are not decisions in the model presented in this work.

**Assumption 3.3 (Orders).** In each time period, orders arrive according to a known probability distribution. Each order $d$ has the following characteristics:

- Arrival period ($arr_d$)
- Preferred due date ($pref_d$)
• Amount of required intermediate material \( (\text{use}_{dm}) \) for \( m = 1, \ldots, M \)

• Required number of consecutive periods of assembly capacity \( (\text{cap}_{d}) \)

• Contribution margin \( (\text{contr}_{d}) \)

• Holding cost rate \( (c_{d}^{H}) \)

• Backlog cost rate \( (c_{d}^{B}) \)

Future demand is stochastic, but forecasts are available and are represented by probability distributions for the incoming orders. The contribution margin does not include backlog and holding costs.

**Assumption 3.4 (Order processing).** The customer requests delivery on a preferred due date \( \text{pref}_{d} \). The company immediately either quotes a due date \( \text{quo}_{d} \) to the customer, which must be met, or rejects the order. The actual delivery date is not required to correspond with the quoted due date because the company also can begin the assembly earlier. The quoted due date is always accepted by the customer. If the quoted due date is beyond the preferred due date, backlog costs occur. Finishing assembly before the preferred due date results in holding costs.

Orders can only be fully accepted. However, it is possible to fulfill the order via partial deliveries over different time periods.

Note that backlog costs can occur even if the order is actually delivered on the preferred due date because these costs depend only on the quoted due date. If \( t \) is the quoted due date for order \( d \), the resulting backlog costs are

\[
bc_{dt} := (t - \text{pref}_{d})^{+} \cdot c_{d}^{B},
\]

where \( c_{d}^{B} \) is the backlog cost rate. \(^1\)

If the assembly process is finished before the preferred due date, holding costs are incurred because early delivery is not desired by the customer. If \( t \) is the actual start of assembly for order \( d \), the resulting holding costs are

\[
hc_{dt} := (\text{pref}_{d} - (t + \text{cap}_{d}))^{+} \cdot c_{d}^{H},
\]

where \( c_{d}^{H} \) is the holding cost rate. It follows that it is never beneficial to quote a due date that is earlier than the preferred due date.

\(^{1}\)In this paper, the notation \( x^{+} \) is used as short for \( \max\{0, x\} \).
Assumption 3.5 (Time structure within a period). The time structure within a period is shown in Figure 1. The assembly of end products is finished at the beginning of a time period. Preferred and quoted due dates refer to the point in time after assembly of end products is finished. Intermediate material replenishment occurs before the assembly process for the orders begins in a time period. In every period, orders arrive after the assembly processes have been started.

According to this assumption, orders do not require assembly capacity in the period in which they are shipped. Furthermore, the delivered intermediate material can be used for assembly in the same time period. Orders arriving within a period cannot begin their assembly before the beginning of the next time period.

Assumption 3.6 (Decisions). The company makes the following decisions in each time period:

1. Due date quoting: For each incoming order, decide the quoted due date (or reject it).

2. Scheduling: At the beginning of every time period, decide which accepted orders must begin assembly in the current time period.

These decisions are based on knowledge of the available resources, the previously accepted orders, and the probability distribution for future incoming orders.

To summarize, we present an example that illustrates the entire order process, as depicted in Figure 2. An order arrives in period 2. The company can begin the assembly process for this order at the beginning of period 3. If the preferred due date of the order falls in period 9 and the company quotes a due date in period 13, this decision leads to backlog costs for 4 periods, independent from the actual finish of assembly. It follows that the assembly must begin in period 10 at the latest because the assembly process requires three periods. However, the company can begin assembly at any time between periods 3 and 10. In this example, the company chooses to begin assembly in period 5, leading to holding costs for 1 period because the order is ready for shipment in period 8, but the customer requested delivery in period 9. Hence, both backlog and holding costs occur for this order.
4 Revenue management approach

In this section, we describe how to make the decisions stated in Assumption 3.6. The general approach is summarized in Algorithm 1. We base the decisions on opportunity cost estimates for the used resources. As is common in the literature, bid prices are used as an estimate for the real opportunity costs; see, e.g., Simpson (1989), Williamson (1992) or Talluri and Van Ryzin (2004). We use the bid prices in this section but postpone their computation until Section 5. We start by describing how to decide on due date quoting. Next, we explain how to schedule the accepted orders.

Compute bid prices;
for $t = 1$ to $T$ do
    Schedule accepted orders (using aggregated bid prices);
    Update bid prices;
    foreach order $d$ arriving in $t$ do
        Quote due date for $d$ or reject $d$ (using aggregated bid prices);
    end
end

Algorithm 1: General approach

4.1 Due date quoting

For each incoming order, we search for a feasible due date that maximizes the profitability of the order. The decision on accepting orders is integrated into the due date quoting decision. Profitable orders are accepted with the respective due date, and non-profitable orders are rejected. In the following, the profitability metric of an order is defined, and we explain how to check the feasibility of a schedule.
We measure the *profitability* of an order $d$ with quoted due date $t$ by

$$\left(\text{contr}_d - b_c d_t\right) - b_{p_{dt}^\text{due}},$$  

which is the difference between the immediate profit earned and $b_{p_{dt}^\text{due}}$, the bid price for accepting $d$ with due date $t$. Note that holding costs, which depend on the actual assembly date, are included in $b_{p_{dt}^\text{due}}$ as explained in the bid price computation in Section 5.

To check *feasibility*, we maintain a preliminary schedule. To test if order $d$ arriving in period $t_{\text{fix}}$ is feasible with the quoted due date $\text{quo}_d$, we set $\text{acc}_d$ to 1, which indicates that $d$ is accepted. Next, we check if there is a schedule satisfying the following inequalities: \(^2\)

\[
\sum_{d=1}^{D} \sum_{i=0}^{\text{cap}_d - 1} x_{d(t-i)} \leq a_t \quad \text{for } t = t_{\text{fix}} + 1, \ldots, T
\]

\[
\text{ATP}_{mt} - \text{ATP}_{m(t-1)} + \sum_{d=1}^{D} \text{use}_{dm} \cdot x_{dt} = r_{mt} \quad \text{for } t = t_{\text{fix}} + 1, \ldots, T; \ m = 1, \ldots, M
\]

\[
\sum_{t = \text{arr}_d + 1}^{\text{quo}_d - \text{cap}_d} x_{dt} = 1 \quad \text{for } d = 1, \ldots, D \text{ with } \text{acc}_d = 1
\]

\[
x_{dt} = x_{dt}^{f_{\text{fix}}} \quad \text{for } d = 1, \ldots, D; \ t \leq t_{\text{fix}}
\]

\[
\text{ATP}_{mt} = \text{ATP}_{m(t-1)} \quad \text{for } m = 1, \ldots, M; \ t \leq t_{\text{fix}}
\]

Table 1 summarizes the applied notation. We model the decisions by $x_{dt}$, which indicates for what fraction of $d$ the assembly begins in period $t$. Variables $x_{dt}$ exist only for $\text{arr}_d < t \leq T - \text{cap}_d$ to prevent the case in which assembly for an order begins before its arrival and to ensure that assembly is finished at the end of the planning horizon.

The available assembly capacity cannot be exceeded in any time period. To this end, the number of orders for which assembly begins in a given period or has begun in a previous period, but is not yet finished is compared with the available assembly capacity (cf. (2)). Balance equation (3) models the inventory of intermediate materials and must apply for each time period and each intermediate material type. In this work, $\text{ATP}_{mt}$ is the available-to-promise quantity of type $m$ at the end of period $t$. By constraint (4), each accepted order must be scheduled such that the quoted due date is met. Note that due to this equation, the entire order must be processed. Constraints (5) and (6) ensure that decisions can only be undertaken for future time periods. The

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\(^2\)We follow the convention that undefined variables are equal to 0.
Indices:
\[ t = 1, \ldots, T \] Periods of the planning horizon
\[ d = 1, \ldots, D \] Orders
\[ m = 1, \ldots, M \] Intermediate material types

Parameters:
- \( \text{contr}_d \) Contribution margin of \( d \)
- \( \text{prof}_{dt} \) Profit of \( d \) if assembly starts in \( t \)
- \( \text{pref}_d \) Preferred due date for \( d \)
- \( \text{use}_{dm} \) Amount of intermediate material \( m \) required for \( d \)
- \( \text{arr}_d \) Arrival period of \( d \)
- \( \text{cap}_d \) Required assembly capacity (in time periods) for \( d \)
- \( e_d^2 \) Holding cost rate (per time period) for \( d \)
- \( c_d^2 \) Backlog cost rate (per time period) for \( d \)
- \( h_{cd_t} \) Holding costs for \( d \) if assembly starts in \( t \)
- \( b_{cd_t} \) Backlog costs for \( d \) if \( t \) is quoted due date
- \( \text{acc}_d \) 1 if \( d \) was accepted in a previous period, 0 else
- \( \text{quo}_d \) Quoted due date for \( d \) if \( d \) is accepted order
- \( a_t \) Available assembly capacity in \( t \)
- \( r_{mt} \) Replenishment amount of \( m \) in \( t \)
- \( t_{\text{fix}} \) Time period up to which the schedule is fixed
- \( x_{dt}^{\text{fix}} \) Fixed decision variables of the previous time periods
- \( \text{ATP}_{mt} \) Fixed decision variables of the previous time periods

State variables:
\[ \text{ATP}_{mt} \geq 0 \] Available-to-promise quantity of \( m \) at the end of \( t \)

Decision variables:
\[ 1 \geq x_{dt} \geq 0 \] Indicates for what fraction of \( d \) the assembly starts in \( t \)
(for \( d = 1, \ldots, D \) and \( t = \text{arr}_d + 1, \ldots, T - \text{cap}_d \))

Bid prices:
- \( \text{bp}_{cap}^t \) Bid price for assembly capacity in \( t \)
- \( \text{bp}_{mat}^m \) Bid price for intermediate material \( m \) in \( t \)
- \( \text{bp}_{\text{start}}^d \) Bid price for \( d \) with given actual start of assembly in \( t \)
- \( \text{bp}_{\text{due}}^d \) Bid price for \( d \) with given due date in \( t \)

| Table 1: Notation |  |  |
decision variables for the periods before period $t^{\text{fix}}$ are set to the previously fixed values $x_{dt}^{\text{fix}}$ and $ATP_{mt}^{\text{fix}}$, respectively.

### 4.2 Scheduling

In this section, we explain how to schedule the accepted orders. At the beginning of each time period, the orders that begin their assembly in the current period must be determined. Therefore, we set up a linear program that includes all accepted orders. As the schedule must be feasible, inequalities (2)-(6) are included. Next, we need to define a suitable objective function. The decisions that this linear program suggests for the current period are taken, and decisions for later periods are still subject to change.

As the orders are already accepted with given due dates, cost wise, scheduling can only influence the resulting holding costs. However, if the schedule is optimized only with respect to the holding costs, orders tend to begin their assembly late, which leads to unused capacities in the upcoming periods.

A better measure for the quality of a solution can be found by also taking into account the incoming future demand with the use of bid prices, which are used to minimize the usage of the most profitable resources. Let $b_{dt}^{\text{start}}$ denote the bid price for order $d$ with given actual start of assembly in period $t$. Next, we define the objective function as follows:

$$
\sum_{d=1}^{D} \sum_{t=\text{arr}_d+1}^{T-cap_d} \left( b_{dt}^{\text{start}} + hc_{dt} \right) \cdot x_{dt}.
$$

Minimizing (7) with respect to constraints (2)-(6) gives the preliminary schedule. The decisions made in the current period $(t^{\text{fix}} + 1)$ are fixed from this point on:

- $x_{d,t^{\text{fix}}+1}^{\text{fix}} := x_{d,t^{\text{fix}}+1}$ for $d = 1, \ldots, D$
- $ATP_{m,t^{\text{fix}}+1}^{\text{fix}} := ATP_{m,t^{\text{fix}}+1}$ for $m = 1, \ldots, M$

After fixing these values, $t^{\text{fix}}$ is increased by 1 because the schedule for the current period is fixed from this point on.
5 Bid price computation

In this section, we explain how to derive the bid prices used in the previous section. The computation can be split into the following three steps, each of which is discussed in detail in this section.

1. **Single Resource Bid Prices**: Compute bid prices for the single resources ($bp_{i}^{cap}$ for assembly capacity in period $t$, $bp_{m}^{mat}$ for intermediate material type $m$ in period $t$).

2. **Aggregate**: Aggregate bid prices for single resources to bid prices that reflect the opportunity costs of accepting an order with given start of assembly ($bp_{dt}^{start}$) and with given due date ($bp_{dt}^{due}$), respectively.

3. **Update Bid Prices**: Update bid prices for single resources if new information is available.

The approach is summarized in Figure 3.

5.1 Single resource bid prices

First, we describe how to compute bid prices for single resources. As is common practice in revenue management literature, bid prices are determined using linear programming. The problem is formulated as a profit-maximizing deterministic linear program. Next, the dual variables in an optimal solution corresponding to the resource inequalities determine the bid prices for the respective resources (cf., e.g., Talluri and Van Ryzin (2004)).

In our case, orders arrive according to a known probability distribution. Thus, to use a deterministic linear program, we must determine the set of incoming orders. In the literature (see,
e.g., Talluri and Van Ryzin (2004)), one common method is deterministic linear programming. In this approach, the number of arriving orders in each time period is simply the expected value.

Another approach is to generate \( I \) \( (i = 1, \ldots, I) \) demand scenarios \( D_i \) according to the probability distributions. Next, the bid prices are computed for each \( D_i \). The used bid prices are determined by taking the mean of the bid prices computed with the demand scenarios \( D_i \). This approach is known as randomized linear programming \((RLP)\); see, e.g., Talluri and van Ryzin (1999). We modify this approach and do not use a fixed number of demand scenarios. Instead, we iteratively update the actual used bid prices by integrating the bid prices computed with the new randomly generated demand scenarios until they converge in a certain sense. To the best of our knowledge, this approach has not yet been described in literature. In Appendix A, we prove a theorem that supports the choice of taking the mean of the resulting bid prices in the given problem setting.

If the set of arriving orders is known, the problem at hand reduces to the problem of which orders to accept and how to schedule them. Quoting the due dates is trivial in this case because the optimal due dates correspond with the completion times of the orders; they are known after scheduling has been performed for all orders. Therefore, we only need to schedule the start of assembly for each order. Let

\[
\textit{prof}\_{dt} := \textit{contr}_d - \textit{hc}_d t - \textit{bc}_d (t + \textit{cap}_d)
\]

define the profit that an order \( d \) generates if its assembly begins in period \( t \). In this definition, holding and backlog costs are subtracted from the contribution margin of the order. Next, we define the objective function as follows:

\[
\sum_{d=1}^{D} \sum_{t=\text{arr}_d+1}^{T-\text{cap}_d} \textit{prof}\_{dt} \cdot x_{dt}.
\]

The restrictions (2)-(6) must apply to guarantee feasibility. The following additional set of inequalities ensures that more will not be produced than is demanded:

\[
\sum_{t=\text{arr}_d+1}^{T-\text{cap}_d} x_{dt} \leq 1 \quad \text{for } d = 1, \ldots, D \text{ with } \text{acc}_d = 0.
\]
the assembly capacity in the respective time period. Similarly, the values of the dual variables corresponding to the constraints (3) in an optimal solution (i.e., $b_{mat}^m$) give the bid prices for each type of intermediate material in each time period.

### 5.2 Aggregate bid prices

In this section, we show how to aggregate the bid prices for single resources to a bid price that reflects the opportunity costs of accepting an order with a given actual start of assembly and with a given due date.

First, we define the bid price for accepting an order with a *given actual start of assembly*. To this end, we add up the bid prices of the single resources required by the order. These values are known if the assembly date is fixed. This approach is an analogue to the process in network revenue management in which more than one resource is also required to fulfill an order; see, e.g., Talluri and van Ryzin (1999). Therefore, we compute the bid price of an order $d$ that begins its assembly in period $t$ as follows:

$$
bp_{start}^{dt} := \sum_{m=1}^{M} use_{dm} \cdot b_{mat}^m + \sum_{i=0}^{cap_d-1} b_{cap}^{t+i}.
$$

Even with given due date, it is not clear which resources will be used to produce an order because the actual assembly date is not fixed yet. To cope with this problem, we follow the ideas of Gallego et al. (2004) and Petrick et al. (2012). Let the periods in which it is feasible to start assembly for an order $d$ with given due date $t$ form the set

$$
\mathcal{T}_{dt} := \{ t' \in \{arr_d + 1, \ldots, t - cap_d \} \mid t' \text{ is feasible start of assembly} \}.
$$

We compare the resulting bid price with the immediate profit that an order generates. Holding costs are not included in the immediate profit because they depend on the actual assembly date, which is not yet determined. However, if the assembly date is fixed, the holding costs are known, and therefore, we choose to include them in the bid price. As the best feasible period for the start of the assembly can be chosen, we take the minimum over all feasible periods:

$$
bp_{due}^{dt} := \min_{t' \in \mathcal{T}_{dt}} \{ bp_{start}^{t'} + hc_{dt'} \}.
$$
5.3 Update bid prices

As demand is stochastic, updating the bid prices if new information becomes available can improve the results. We use the following methods to do so.

A simple approach to updating bid prices is to re-compute them after a certain number of periods. In this work, we use the knowledge of the already accepted orders and their quoted due dates. Additionally, information on orders that are currently in the process of assembly and the resources required by them in future periods is available.

Another approach to updating bid prices is to adapt the already computed bid prices. Let $n_{bp}^t$ represent the mean number of accepted orders up to period $t$ in the bid price computation. Let $n_t$ denote the actual number of accepted orders up to period $t$. At the end of period $t$, the base bid prices are adjusted by multiplying them with $n_t/n_{bp}^t$, the ratio between the actual number of accepted orders and the expected number of accepted orders. Therefore, if fewer orders than expected were accepted, the bid prices decrease, and vice versa.

Additionally, at the beginning of each time period, we set the bid price for the assembly capacity in the same period to 0 because no new arriving orders can begin assembly in this period.

6 Numerical study

In the following numerical study, we evaluate the performance of the previously described revenue management approach, which we refer to as ATO-RM. After presenting the test bed and the used benchmark algorithms, we compare the results of ATO-RM and the benchmark algorithms with an ex-post optimal solution. Next, a sensitivity analysis shows the influence of the varied parameters on the performance of the tested algorithms and on the structure of the results.

The simulation environment corresponds with the description of the approach in Algorithm 1. The algorithms were implemented in C++ using a Gurobi 5.5 solver on a 3.20 GHz Intel Core i7 machine with 32GB of RAM.

6.1 Test bed

First, we present the scenarios and algorithms used in the numerical study.

In each scenario, three different order types are available. In each time period, the number of arriving orders of each of these order types is drawn according to a negative binomial distributed random variable $NB(\mu_t, cv)$ with mean $\mu_t$ and coefficient of variation $cv$. We choose this distribu-
Fixed parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of periods in the planning horizon $T$</td>
<td>60</td>
</tr>
<tr>
<td>Number of intermediate materials $M$</td>
<td>1</td>
</tr>
<tr>
<td>For all orders $d$:</td>
<td></td>
</tr>
<tr>
<td>Coefficient for required int. material $use_{dm}$</td>
<td>1</td>
</tr>
<tr>
<td>Required assembly capacity $cap_d$</td>
<td>7</td>
</tr>
<tr>
<td>Pref. due date $pref_d$ dependent on order type</td>
<td>$arr_d + (8/11/14)$</td>
</tr>
<tr>
<td>Holding cost rate $c^H_d$</td>
<td>1% of $contr_d$</td>
</tr>
<tr>
<td>Backlog cost rate $c^B_d$</td>
<td>10% of $contr_d$</td>
</tr>
</tbody>
</table>

Varied parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit heterogeneity</td>
<td>${(500/550/600), (300/550/800), (100/550/1000)}$</td>
</tr>
<tr>
<td>$contr_d$ dependent on order type</td>
<td></td>
</tr>
<tr>
<td>Available assembly capacity $a_t$</td>
<td>${205, 175, 155, 135}$</td>
</tr>
<tr>
<td>Int. mat. replenishment $r_{mt}$</td>
<td>${1600, 1360, 1180, 1040}$</td>
</tr>
<tr>
<td>(in periods $t = 1, 16, 31, 46$)</td>
<td></td>
</tr>
<tr>
<td>Demand arrival pattern (peaks)</td>
<td>${\text{early, middle, no peak}}$</td>
</tr>
</tbody>
</table>

Table 2: Parameters used in the numerical study with three order types

For the sake of clarity of analysis, we model a basic situation. Each order requires one unit of the only required intermediate material and seven consecutive periods of assembly capacity. High-value customers request shorter preferred lead times than low-value customers. This situation reflects the fact that customers generally must pay a premium to obtain shorter lead times, and short lead times are provided for important customers. The holding (backlog) cost rate for an order is 1% (10%) of its contribution margin.

Varied parameters

One of the prerequisites for successful application of revenue management is that demand is heterogeneous. It is interesting to observe whether applying a revenue management approach
is worthwhile even if the profits gained from different customer classes are nearly the same. Therefore, the profit heterogeneity that reflects the different importance of customers is varied.

To reflect an adequate range of variability in the test cases, the coefficient of variation $cv$ is also varied.

The ratio of the congestion of resources is especially interesting because we model scarcity in both resources. To be able to adjust capacity and intermediate material congestion independently, we keep the expected number of order arrivals fixed over the entire time horizon and vary the number of available parallel machines $a_t$ and the replenishment amount for the intermediate material $r_{mt}$, respectively. In this way, we generate scenarios with approximately 85%, 100%, 115%, and 130% congestion for each resource, respectively.

To observe how the algorithms perform in different demand arrival scenarios, scenarios with three different demand arrival patterns are generated:

- **No peak**: $\mu_t = 10$ for $t = 1, \ldots, 46$
- **Early peak**: $\mu_t = \begin{cases} 
9 & \text{for } t = 1, \ldots, 10 \\
13.5 & \text{for } t = 11, \ldots, 46 
\end{cases}$
- **Middle peak**: $\mu_t = \begin{cases} 
9 & \text{for } t = 1, \ldots, 18 \text{ and } t = 30, \ldots, 46 \\
13.5 & \text{for } t = 19, \ldots, 29 
\end{cases}$

In the last 14 periods, no orders arrive ($\mu_t = 0$ for $t = 47, \ldots, 60$) such that all orders can be fulfilled within their preferred due dates.

Overall, this leads to $3^3 \cdot 4^2 = 432$ scenario settings. For each scenario setting, 20 instances are randomly generated, resulting in 8640 instances.

**Algorithms**

For ATO-RM, the following settings are used to compute single resource bid prices. We begin by computing bid prices using ten demand scenarios generated according to the known probability distributions. In Section 5.1, we did not specify how we define convergence of the bid prices. In this numerical study, we check convergence by looking at the sets of bid prices generated by the five most recent integrated demand scenarios. We add the bid prices one by one and test if one of the mean bid prices changes by more than 1 absolute unit and 5% relative to the previous mean bid price. If this is not the case for all of the bid prices, we state that the bid prices have converged. Single resource bid prices are recomputed every 12 time periods and adapted in every time period in which the bid prices have not been recomputed; cf. Section 5.3.
We compare ATO-RM with the following benchmark algorithms:

- **First-come-first-served (FCFS):** An order is accepted and is quoted its preferred due date if there are sufficient resources available to fulfill the order within the preferred due date, otherwise it is rejected. Accepted orders are scheduled as early as possible. Although the FCFS approach does not differentiate between different customers, it is still used in practice because it is a clear and easy to use strategy.

- Revenue management approaches that ignore one of the two resources in the bid price computation:
  - **ONLYCAP** ignores the intermediate material availability, i.e., inequality (3) is not included in the linear program when computing single resource bid prices and $bp_{t}^{mat} = 0$ for all $t$.
  - **ONLYMAT** ignores the assembly capacity restrictions, i.e., inequality (2) is not included in the linear program when computing single resource bid prices and $bp_{t}^{cap} = 0$ for all $t$.

These algorithms are proxies for processes using existing MTO or MTS revenue management approaches.

We would prefer to compare the heuristic ATO-RM approach with an optimal policy. However, this comparison is impossible because computing an optimal policy is intractable for problems of the given size. Instead, we compare the results to an ex-post optimal solution: If all arriving orders are known, one can compute the maximum attainable profit by solving the deterministic linear program described in Section 5.1. Note that in practice this ex-post algorithm (say POSTOPT) cannot be implemented because the demand is unknown. POSTOPT gives an upper bound on the maximum attainable profit.

### 6.2 Simulation results

Next, we compare the results of ATO-RM and the benchmark algorithms with POSTOPT and explain the reasons for these results.

The main result is shown in Figure 4, which illustrates the mean relative difference to the optimal ex-post profit over all 8640 tested instances for ATO-RM and the benchmark algorithms. We emphasize three observations:
ATO-RM is close (3.96%) to the ex-post optimal solution indicating that the proposed approach works quite well. This result is even more impressive because the ex-post solution was created with full knowledge of all incoming demand, whereas ATO-RM used only information from the probability distributions, which means ATO-RM will produce a result even closer to a solution computed with an optimal policy, which also has only information from the probability distribution.

ATO-RM clearly dominates the benchmark algorithms. It follows that order discrimination and taking into account both scarce resources are important to obtaining good results.

ONLYCAP (11.17%) performs much better than ONLYMAT (21.99%), which indicates that in our test cases, considering the assembly capacity constraints appears to be more critical than considering the intermediate material constraints.

Figure 5 shows the distribution of the relative differences of the resulting profits to the ex-post optimal solution over all 8640 tested instances for ATO-RM and the benchmark algorithms. The above-described results do not only apply in the mean; ATO-RM also performs well in most of the tested scenarios, leading to an upper quartile of 5.28%. The benchmark algorithms are not only worse in terms of the mean, but there are also a significant amount of instances in which the resulting profits are located rather far away from the ex-post solution. The sharp bend in the curve for FCFS stems from the fact that, for low profit heterogeneity, 93.8% of the instances show a relative difference to PostOpt of 9 – 15%. In Section 6.3, we illustrate the behavior of the algorithms in different scenarios in greater detail.

In the following, we provide additional insights to explain these results. The main decisions of the algorithms are which orders are accepted and when they are scheduled. Thus, we further
Figure 5: Distribution of the relative differences to the optimal ex-post profit over all instances

examine the fill rate for the different order types and the resulting backlog and holding costs for
each algorithm.

POSTOPT accepts almost all high-value orders and only approximately half of the low-value
orders as shown in Figure 6a. The revenue management approaches exhibit similar behavior.
Still, ATO-RM accepts a larger amount of orders from high-value customers than ONLYCAP and
ONLYMAT. One reason for this observation may be that the bid prices computed by ATO-RM
are generally higher than those of ONLYCAP and ONLYMAT because they ignore the opportunity
costs for one of the two scarce resources. FCFS accepts a larger amount of low- than high-value
orders because low value orders have longer preferred lead times. Therefore, in case of scarce
resources, it is still feasible to accept a low-value order but not to accept a high-value order.

Figure 6b shows the resulting backlog and holding costs. The POSTOPT solution results in
relatively low backlog and holding costs, perhaps because this algorithm uses information on
all future incoming demand and thus quotes no due dates later than the actual delivery date.
ATO-RM generates lower costs than ONLYCAP and ONLYMAT. Again, one reason for this
observation might be that these approaches underestimate the opportunity costs because they
only take into account the opportunity costs of one of the two resources. This situation can
lead to accepting orders with high backlog costs, whereas ATO-RM rejects these orders after
comparing the resulting profits to the corresponding bid price. As expected, FCFS creates no
backlog costs.

The bid prices for a given order computed by ONLYMAT at a certain point in time are es-
sentially a decreasing step function of the quoted due date. The steps originate from the points in time at which new intermediate material becomes available. Therefore, late points in time are viewed as more appealing (cf. Section 4), which might be a reason why ONLYMAT generates much more backlogging than ONLYCAP. Additionally, this situation might lead ONLYMAT to accept a larger amount of low-value orders due to low bid prices that do not take into account that not all orders can be assembled at that point in time because of the assembly capacity constraints. Thus, assembly capacity is not saved for high-value orders, which in turn must be rejected.

Computation time is not a critical factor in our tests. For each instance, ATO-RM takes 30 seconds on average for bid price computation and 1 second for feasibility checks, order selection, etc.

6.3 Sensitivity analysis

In the following, we examine the influence of the varied parameters on the performance of the algorithms as well as on the solution characteristics. All simulation results are summarized in Table 3, which shows the relative difference to POSTOPT and gives the fill rates for high-, medium-, and low-value orders in parentheses. Additionally, the \( \text{hcbc\_share} := \frac{\text{holding costs} + \text{backlog costs}}{\text{holding costs} + \text{backlog costs} + \text{profit}} \) is presented, which gives the relationship between the costs and the contribution margin. The results are generated by fixing one (or in case of different congestion, two) of the varied parameters and taking the mean over all scenarios with these fixed parameter values.
<table>
<thead>
<tr>
<th>Instance</th>
<th>PostOpt</th>
<th>ATO-RM</th>
<th>OnlyCap</th>
<th>OnlyMat</th>
<th>FCFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>0(99.5,94.3,47.4,2.0)</td>
<td>4.0(95.7,94.3,46.2,4.7)</td>
<td>11.2(89.2,93.4,59.0,11.0)</td>
<td>22.0(81.5,92.5,64.5,19.0)</td>
<td>26.1(38.6,86.1,198.1,2.0)</td>
</tr>
<tr>
<td>ProfHet</td>
<td>Low</td>
<td>0(98.5,92.3,53.0,3.0)</td>
<td>3.0(90.0,90.6,59.9,4.0)</td>
<td>7.0(81.3,90.2,72.7,8.6)</td>
<td>16.6(72.9,89.3,76.5,15.8)</td>
</tr>
<tr>
<td></td>
<td>Med</td>
<td>0(99.9,94.8,46.3,1.9)</td>
<td>4.5(98.1,95.6,43.2,5.1)</td>
<td>12.8(91.4,94.1,57.0,12.0)</td>
<td>23.6(83.3,93.7,62.2,20.0)</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0(100,95.7,42.9,1.2)</td>
<td>4.4(99.1,96.6,35.4,4.9)</td>
<td>13.6(94.8,95.8,47.3,12.1)</td>
<td>25.7(88.3,94.6,56.4,21.0)</td>
</tr>
<tr>
<td>cv</td>
<td>1/3</td>
<td>0(99.9,93.5,45.7,1.8)</td>
<td>2.3(98.3,95.6,44.7,3.3)</td>
<td>11.1(90.5,94.4,59.1,10.7)</td>
<td>21.3(87.4,92.0,57.9,11.1)</td>
</tr>
<tr>
<td></td>
<td>2/3</td>
<td>0(99.6,94.9,48.3,1.9)</td>
<td>4.5(96.1,94.7,46.9,4.8)</td>
<td>11.3(89.5,93.8,60.1,11.0)</td>
<td>21.9(81.8,92.7,65.4,18.9)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0(99.0,91.8,48.2,2.3)</td>
<td>5.5(92.8,92.3,47.0,6.1)</td>
<td>11.2(87.4,92.0,57.9,11.1)</td>
<td>22.8(79.7,91.0,65.2,19.9)</td>
</tr>
<tr>
<td>Congestion</td>
<td>85-100</td>
<td>0(100,99.9,93.9,0.9)</td>
<td>3.5(98.4,99.4,94.7,4.1)</td>
<td>4.0(98.7,99.6,96.4,5.0)</td>
<td>14.1(97.6,99.3,97.9,14.8)</td>
</tr>
<tr>
<td></td>
<td>115-150</td>
<td>0(99.6,94.5,42.2,1.7)</td>
<td>4.2(95.6,96.6,46.4,4.4)</td>
<td>4.2(95.6,96.7,46.4,5.4)</td>
<td>32.8(75.1,91.4,74.8,28.1)</td>
</tr>
<tr>
<td></td>
<td>130-130</td>
<td>0(99.0,87.0,27.0,2.6)</td>
<td>4.8(94.1,87.6,25.6,5.4)</td>
<td>6.6(92.0,89.9,28.6,8.1)</td>
<td>17.5(82.6,87.3,35.6,15.1)</td>
</tr>
<tr>
<td>Peaks</td>
<td>no peak</td>
<td>0(100,99.4,74.2,1.6)</td>
<td>3.1(97.0,98.6,74.1,4.1)</td>
<td>15.2(92.3,98.7,95.7,17.4)</td>
<td>6.7(95.6,98.5,80.8,8.3)</td>
</tr>
<tr>
<td></td>
<td>early</td>
<td>0(100,99.7,44.0,2.1)</td>
<td>3.5(95.9,96.6,45.8,4.7)</td>
<td>14.8(81.5,94.7,71.2,16.1)</td>
<td>8.5(92.9,96.3,54.7,10.0)</td>
</tr>
<tr>
<td></td>
<td>middle</td>
<td>0(100,98.0,47.4,2.2)</td>
<td>2.5(97.3,96.9,49.5,4.1)</td>
<td>27.5(76.5,91.5,89.9,25.3)</td>
<td>4.3(95.7,96.8,54.5,6.2)</td>
</tr>
<tr>
<td></td>
<td>115-130</td>
<td>0(99.8,90.1,28.3,2.8)</td>
<td>3.7(95.1,90.9,29.6,5.3)</td>
<td>13.0(83.6,92.4,47.7,14.7)</td>
<td>8.7(90.9,90.6,35.6,9.9)</td>
</tr>
<tr>
<td></td>
<td>100-130</td>
<td>0(99.9,91.2,31.9,2.8)</td>
<td>3.0(95.9,91.2,31.7,4.9)</td>
<td>23.1(74.4,88.3,64.2,15.2)</td>
<td>5.5(93.3,91.4,36.2,7.3)</td>
</tr>
</tbody>
</table>

Table 3: Results from sensitivity analysis [DiffOpt(Fill rates, hcbc_share)]
Impact of profit heterogeneity

Figure 7a shows the influence of profit heterogeneity on the performance of the tested algorithms. It can be observed that using ATO-RM is worthwhile even if the profit heterogeneity is low. Increasing profit heterogeneity in our tests leads to worse results for the benchmark algorithms, whereas ATO-RM is influenced to a lesser degree.

With the increasing difference of the profits from orders of different customer classes it becomes increasingly more important to select which orders are accepted. For ATO-RM, it is easier to differentiate between the order types with increasing profit heterogeneity, which means that even though the bid prices might be not completely correct, they can lead to the correct decisions. Therefore, the fill rates of high- and medium-value orders are notably high. However, the overall number of accepted orders is relatively low due to the rather low acceptance rate for low-value orders. FCFS suffers most from increasing profit heterogeneity because the fill rates for all customer types remain the same regardless of the profit heterogeneity. As mentioned above, the decision of which orders to accept made by ONLYCAP and ONLYMAT are also not as good, which might explain their worse results for higher profit heterogeneity. Due to the high backlog costs, FCFS performs better for low profit heterogeneity than ONLYMAT.

Impact of coefficient of variation

Figure 7b shows the influence of the coefficient of variation of the arriving demand on the performance of the algorithms. Only ATO-RM appears to suffer from an increasing coefficient of variation, whereas the benchmark algorithms are relatively indifferent in this case. However,
even in the worst tested case, ATO-RM still clearly dominates the benchmark algorithms. The reason for the decreasing performance of ATO-RM may be because the bid prices computed by ATO-RM must be good for many different demand situations. In the case of a high variance, these situations differ to a greater extent. Therefore, a scenario in which the computed bid prices do not fit the actual demand situation might occur more often, which might be the reason for increasing $hcbo_share$ and a decreasing fill rate for high-value orders, leading to worse results of ATO-RM. ONLYCAP and ONLYMAT do not use the correct bid prices in the first place because they ignore one of the limiting resources. It is possible that this is why a higher variance does not significantly influence their performance. FCFS makes no use of forecasts at all. As the mean demand remains the same, the solution quality is also nearly the same. For all tested algorithms except FCFS, the fill rates for high- and medium-value orders decrease with higher variance, which may be related to the arriving demand being less uniform over time.
Impact of congestion

Figure 8 depicts the influence of the different supply scenarios on the performance. Note the different scale in the diagram for ATO-RM. ATO-RM performs well under all circumstances, but its performance worsens with increasing scarcity of assembly capacity. This situation corresponds with the observation that consideration of assembly capacity appears to be more important than consideration of intermediate material (cf. Section 6.2). Note that ATO-RM performs better than ONLYMAT even if intermediate material is the clear bottleneck resource and as good as ONLYCAP if assembly capacity is the clear bottleneck resource. As expected, if the bottleneck is not taken into account in the benchmark algorithm, the respective algorithm performs poorly. FCFS performs worse the more congested the system becomes.

The reason why ATO-RM performs worse if assembly capacity is the bottleneck resource might be because intermediate material is required only at the beginning of assembly, whereas assembly capacity is needed over several time periods, and it is thus harder to manage. Additionally, assembly capacity is not storable, and therefore unused capacity is ultimately lost. For ONLYCAP and ONLYMAT, it can be observed that in the cases that ignore the bottleneck resource, the algorithms do not differentiate between order types anymore, perhaps because they do not see a bottleneck and thus do not reject orders due to the bid prices. This also may be an explanation why the backlog costs drastically increase in these cases. If the congestion is still low, additional orders can be accepted, and thus FCFS also accepts a large amount of high-value orders, which can be a reason for its performance curve.

Impact of demand arrival patterns

From the results, no significant influence of different demand arrival patterns on the tested algorithms can be observed.

7 Conclusions and further research

In this paper, we model the decisions of an ATO manufacturer. In addition to the decisions on accepting orders, we also include scheduling and due date quoting decisions. The intermediate material and assembly capacity are explicitly modeled in this work. We present a novel revenue management approach that uses bid prices to make the decisions.

As shown from the numerical study, ATO-RM works quite well. On average, the profit obtained is close to that of the ex-post solution, which contains full knowledge of all incoming
demand. The proposed approach clearly outperforms the other tested benchmark algorithms. In our tests, the capacity constraints appear more important than the intermediate material constraints. This observation is illustrated by the better performance of \textsc{OnlyCap} in comparison with \textsc{OnlyMat} and by the decreasing performance of ATO-RM with a higher scarcity of assembly capacity. Computation time is no issue in the tested instances.

The sensitivity analysis shows that increasing the variance and scarcity of the assembly capacity lead to decreasing quality of ATO-RM. However, even in these situations, ATO-RM still performs quite well. The proposed approach outperforms the benchmark algorithms in all of the tested settings.

However, the modeled production environment is still rather basic. We assume one-stage deterministic production and deterministic replenishments of the intermediate materials. Relaxing these assumptions offers a good opportunity for further research. It also would be of interest to analyze how the algorithms perform under more complex material structures or in a system with additional stochastic influences.

\section*{A Appendix}

To support the RLP approach in Section 5.1, we present the following result. Let

\begin{equation}
\text{profit}^{\text{imm}}_{dt} := \text{contr}_d - \text{bc}_{dt},
\end{equation}

represent the immediate profit that \( d \) generates with quoted due date \( t \). Let \( S \) represent the set of orders that arrive in the given scenario. We define the opportunity costs of accepting an order \( d \in S \) with due date \( t \) as

\begin{equation}
\text{opp}_{S}(d, t) := \text{profit}(S \setminus \{d\}) - (\text{profit}(S_{d,t}) - \text{profit}^{\text{imm}}_{dt}),
\end{equation}

where \( S_{d,t} \) is the set of orders \( S \) with order \( d \) accepted with quoted due date \( t \), and \( \text{profit}(S) \) is the maximum attainable profit if orders \( S \) arrive. In this definition, the maximum reachable profit if \( d \) is rejected \( (\text{profit}(S \setminus \{d\})) \) is compared with the maximum profit that can be gained if order \( d \) is accepted with quoted due date \( t \) and subtracting the immediate profit gained.

The following theorem supports the approach that takes the mean over the computed bid prices in every scenario.
**Theorem A.1.** Let order $d$ arrive and $profit_{dt}^{imm}$ indicate the immediate profit of quoting due date $t$ for order $d$. Let $S$ be a collection of possible sets of orders with $p(S)$ as the probability that set $S \in S$ will occur. Let $opp_{S}(d, t)$ represent the opportunity costs of accepting $d$ with quoted due date $t$ if the set of orders $S \in S$ arrives. Therefore

i) The expected marginal profit of accepting order $d$ is positive iff

$$profit_{dt}^{imm} \geq \min_{t' = arr_{d} + 1, \ldots, T - cap_{d}} \sum_{S \in S} p(S) \cdot opp_{S}(d, t').$$

ii) This quantity is maximized by quoting due date $t$ with

$$t \in \arg\max_{i = arr_{d} + 1, \ldots, T - cap_{d}} \left\{ profit_{dt}^{imm} - \sum_{S \in S} p(S) \cdot opp_{S}(d, i) \right\}.$$

**Proof.** By (14) with $S':= S \cup \{d\}$ we get

$$profit(S'_{d, t}) - profit(S) = profit_{dt}^{imm} - opp_{S'}(d, t).$$

Hence, the expected marginal profit of accepting order $d$ with quoted due date $t$ over rejecting $d$ is

$$\sum_{S \in S} p(S) \cdot (profit_{dt}^{imm} - opp_{S'}(d, t))$$

$$= \sum_{S \in S} p(S) \cdot profit_{dt}^{imm} - \sum_{S \in S} p(S) \cdot opp_{S'}(d, t)$$

$$= profit_{dt}^{imm} - \sum_{S \in S} p(S) \cdot opp_{S'}(d, t).$$

\[ \square \]

**References**


Kuhn, H. and F. Defregger (2005). Revenue management for a make-to-order company with limited inventory capacity. OR spectrum 29(1), 137–156.


