Capital-Labor Distortions in Project Finance\textsuperscript{1}

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Abstract

An entrepreneur needs a lender’s capital input to finance a project. The entrepreneur, who is privately informed about the project environment, provides a labor input (effort). Capital and labor are perfect complements. We show that the entrepreneur may optimally distort the project’s capital-labor ratio. The direction of the distortion in capital-labor ratio depends on contractibility of the entrepreneur’s labor input. If the entrepreneur’s labor input is contractible, in the optimal contract, the entrepreneur may provide an excessive amount of labor for the amount of capital funded by the lender. If, by contrast, the entrepreneur’s labor input is non-contractible, part of the physical asset funded by the lender may remain idle.

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1 Introduction

Privately held firms form the backbone of many economies and their success is seen as a main driver of economic growth. Typically, the entrepreneurs are the main equity holders in charge of managerial decisions for privately held firms,¹ and they have to obtain outside financing through a debt contract to carry out their projects. In such financial relationships, the lenders providing funds are at an informational disadvantage because the entrepreneurs often have private information about the project environment, such as idiosyncratic risk of failure leading to the liquidation of the firm.² As is well known, a privately informed entrepreneur may push a project to a lender by exaggerating its prospects.

The entrepreneur’s manipulating incentive is anticipated by the lender, and, thus, the entrepreneur may voluntarily introduce distortions in contractual terms so as to convince the lender that there will be no incentive problem.³ While contractual distortions in project financing have been extensively analyzed by previous contributions, the theoretical literature has mainly focused on the distortions in the lender’s capital input, or the size of the project. As recent empirical studies point out, however, input choices for a firm’s operation are often accompanied by “misallocation” of inputs.⁴ In other words, input compositions in a firm’s operation often appear to be distorted. To do so, we develop a framework in which capital and labor jointly determine the size of the project.

In a contractual relationship between an entrepreneur and a lender, this study provides a rationale for why the optimal capital-labor composition in carrying out a project can be distorted. The entrepreneur makes a contract offer to the lender who provides capital input for the project. The entrepreneur provides labor input by making an effort to implement the project. Capital and labor are perfect complements. Since the entrepreneur is privately informed on the project environment, she has an incentive to exaggerate the prospects of the project to the lender. In addition to the manipulating incentive, a further incentive problem may arise if the entrepreneur’s effort to implement the project is not verifiable. In some situations, the entrepreneur’s activity is well defined and can be closely monitored. In such cases, her effort level for the project is contractible. In other situations, the contract between the entrepreneur and the lender is subject to the entrepreneur’s hidden action.

¹As documented by Birtler et al. (2005) for the U.S. in the 1990’s, in the majority of privately held firms, the entrepreneur holds 100% of the equity. Furthermore, owners who are actively involved in the management of the firm hold on average 85% of private equity.
²For empirical support, see Moskowitz and Vissing-Jørgensen (2002).
³See Tirole (2006, chapter 6) for a survey.
⁴See, for example, George (2005) and Gilchrist et al. (2013).
To preview our results, when the entrepreneur’s effort for the project is verifiable (and, hence, contractible), our analysis suggests that the entrepreneur may exert more than necessary effort for the capital input provided by the lender. In other words, when the entrepreneur’s effort level (the labor input) can be closely monitored and contracted upon, the project’s capital-labor ratio may be distorted “downward” in the optimal contract. The intuition is as follows. In the optimal contract, the entrepreneur oversizes the project under the favorable environment. By doing so, the entrepreneur must increase the return-payment to the lender for his capital input, which mitigates her incentive to exaggerate the project environment. Because the entrepreneur can contract upon her effort level, committing to exert an excessive effort under the favorable project environment reduces her incentive to exaggerate the prospects of the project. This, in turn, allows the entrepreneur to reduce the distortion in project size. We show that the entrepreneur’s excessive effort takes place in the optimal contract if the cost of effort is not too large.

When, by contrast, the entrepreneur’s effort is not verifiable (non-contractible), part of the physical assets provided by the lender may remain idle—the capital-labor ratio may be distorted “upward” in the optimal contract. That is, when the entrepreneur cannot commit to her effort level, the distortion in capital-labor ratio arises in the opposite direction. While the project continues to be oversized in a favorable environment to mitigate the entrepreneur’s manipulating incentive, the entrepreneur now has an incentive to reduce her costly effort for the oversized project. This ex-post flexibility for her effort, however, makes it easier for the entrepreneur to misrepresent the project environment. Thus, to convince the lender that she is not exaggerating the prospects of the project, the entrepreneur may have to increase the size of the project even further by increasing the debt (the capital input). As a result, while all of the lender’s capital input is invested in the project, part of the physical asset may be idle in equilibrium.

While we model a simple debt contractual relationship (and, thus, the lender does not hold equity), our analysis also sheds some light on venture capital financing compared to angel financing, as our results are independent of whether the entrepreneur or the lender is the residual claimant. The case in which the entrepreneur’s labor is contractible can

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5The entrepreneur’s excessive effort in our paper contrasts with conventional findings in the literature that the entrepreneur’s effort is generally inefficiently low (e.g., Jensen and Meckling, 1976).

6According to Salman (1990), a venture capitalist’s close monitoring of the entrepreneur’s implementation include his involvement in recruiting and compensating the firm’s key employees, customer relationship management, and development of firm strategies.

7This is formally shown in Section 4.2.
be associated with venture capital financing. A venture capitalist often requires a serious commitment from the entrepreneur in implementing the project. As reported by Gorman and Sahlman (1989), venture capitalists frequently visit their entrepreneurs and spend hours for monitoring purposes. The case without the entrepreneur’s commitment to her labor input, on the other hand, can be viewed as angel financing. Angel financing, as pointed out by several studies, relies less on commitment mechanisms often adopted in venture capital financing—contractual relationships in angel financing are looser and rely more on social networks. Our results provide the testable implications that, with venture capital financing, a project’s capital-labor ratio is more likely to be distorted downward (excessive labor input), and with angel financing, the opposite is true (excessive capital input).

Several previous contributions have analyzed contracting problems when entrepreneurs are privately informed. Pioneering studies such as Bolton and Scharfstein (1990) and Holmström and Tirole (1997) analyze how the distribution of wealth across privately informed entrepreneurs and uninformed lenders affects contracting relationships. Ueda (2004) studies the entrepreneur’s source of funding, which determines whether her information is private or public. Dessi (2005) studies a contractual relationship under collusion between the entrepreneur and monitoring intermediary. Our paper contributes to this line of research by focusing on the input composition. This issue arises in contracting problems when a complementary input has to be provided by the entrepreneur.

While the settings are different from ours, other papers also demonstrate that projects or debt can be oversized due to private information. De Meza and Webb (1987) demonstrate that the inability of lenders to learn entrepreneurs’ information results in investment in excess of the socially efficient level. Unlike ours, however, only the pooling contract can be implemented in their model, and the authors do not consider distortions in composition of inputs. More closely related to ours is Khalil and Parigi (1998). They show that lack of contractibility makes the lender push the capital input beyond the efficient level by increasing the size of debt. In these papers, the entrepreneur’s labor input does not play a role, and thus there are no under-utilized inputs as a result of excessive provision.

Our paper also connects with the work on entrepreneur-lender relationships under lim-
ited contractibility.\textsuperscript{12} Hart and Moore (1994) show that when a debt contract allows the entrepreneur to walk away from providing her labor input, some profitable project may not be funded. Wang and Zhou (2004) show that dividing a project financing into different stages mitigates the entrepreneur’s incentive to shirk on implementing the project. Wang (2008) argues that the optimal contract with staged financing may not be separating according to the entrepreneur’s information. We add to this strand of literature the focus on capital-labor composition as the outcome of entrepreneur-lender relationships.

Finally, our paper is related to studies on agency theory that cope with the incentive of the party that makes the offer. While the current paper is related to studies on such issues, unlike in Maskin and Tirole (1990, 1992), we do not adopt a signaling approach in this paper. We employ a screening approach instead, because in our model, the entrepreneur learns her private information after her offer is accepted, as in Demski and Sappington (1993), Laffont and Martimort (2002), and Finkle (2005).

The remainder of the paper is organized as follows. The model is presented in the following section. In Section 3, we present the optimal outcome when the entrepreneur’s labor input is contractible, followed by the optimal outcome when the labor input is non-contractible. In Section 4, we provide further discussion and, in Section 5, we conclude. All proofs are relegated to the Appendix.

2 Model

We consider a contractual relationship between an entrepreneur (she) and a lender (he). The entrepreneur has the initiative for the project and makes a take-it-or-leave-it offer to the lender.\textsuperscript{13} The lender provides a capital input for the project, while the entrepreneur carries out the project by providing a labor input. The project size, denoted by $Q$, is determined by the Leontief function $Q = \min\{K, L\}$, where $K$ and $L$ are the lender’s capital input and the entrepreneur’s labor input for the project, respectively. Since providing inputs is costly, the first-best efficient capital-labor input combination satisfies $K = L$.

The project environment is denoted by state $i$ that can be either $i = G$ (good) with probability $\phi_G \in (0, 1)$, or $i = B$ (bad) with probability $\phi_B = 1 - \phi_G$. We assume that $\phi_i$ is neither too small nor too large such that the project size is strictly positive in either

\textsuperscript{12} There is a long literature on incomplete contracting. For seminal contributions, see Grossman and Hart (1986) and Hart and Moore (1990).

\textsuperscript{13} See, also, Holmstrom and Tirole (1997) and Neher (1999) for similar settings. As will be pointed out in Section 4.2, it is important for our results that some bargaining power rests with the entrepreneur.
environment. The probability distribution $\phi_i$ is publicly known, but, after signing the contract, the realized project environment $i \in \{G, B\}$ is privately observed only by the entrepreneur.

The project environment determines the probability of success. We denote by $\rho_i$ the probability that the project fails, in which case the entrepreneur defaults. A good environment has the feature that the project is less likely to fail; i.e., $\rho_G < \rho_B$. The lender’s cost of capital input is given by a convex cost function $C^k(K) = \psi^k K^2$. In case the project succeeds, the lender receives a lump-sum payment $R$; in case of failure he receives zero. In state $i \in \{G, B\}$, the lender’s payoff in expectation is:

$$\bar{u}_i = (1 - \rho_i)R - \psi^k K^2,$$

when providing $K$ to the entrepreneur for the project. Since $\rho_i$ is exogenous, we can divide the expression by $1 - \rho_i$ to transform the lender’s expected payoff to:

$$u_i = R - c_i^k K^2,$$

where $c_i^k \equiv \frac{\psi^k}{1 - \rho_i}$ and $c_G^k < c_B^k$. In what follows, we take $u_i$ as the lender’s payoff function.

In case the project succeeds, the project’s revenue is given by the function $v(Q)$ that is increasing and strictly concave in the project size $Q$ with $v(0) = 0$, and satisfies the Inada condition; in case the project fails, revenue is zero. The entrepreneur’s cost of labor input is given by $C^l(L) = \psi^l L^2$. In state $i \in \{G, B\}$, the entrepreneur’s payoff in expectation when providing $L$ for the project is:

$$\bar{\pi}_i = (1 - \rho_i)(v(Q) - R) - \psi^l L^2.$$

Again, $\rho_i$ is exogenous, so we can divide the expression by $1 - \rho_i$ to transform the entrepreneur’s expected payoff to:

$$\pi_i = v(Q) - R - c_i^l L^2,$$

where $c_i^l \equiv \frac{\psi^l}{1 - \rho_i}$ and $c_G^l < c_B^l$. In what follows, we take $\pi_i$ as the entrepreneur’s objective function.

We model state $i$ as being soft information—no verifiable evidence on the true state can be obtained, and, thus, it cannot be assessed by a court. We assume that all parties are protected by limited liabilities—the contract must ensure their reservation payoffs in expected terms in each state $i$. The reservation payoffs are normalized to zero.
Contracting when Labor Input is Contractible

When the entrepreneur’s labor input is contractible, the contract offered by the entrepreneur is contingent on her report to the lender about the project environment and specifies the lender’s capital input level $K_i$, the entrepreneur’s labor input level $L_i$, and the return payment to the lender $R_i$ where $i \in \{G, B\}$. The timing of the game under contractibility of the labor input is:

1. The entrepreneur offers a menu $\{(K_i, L_i, R_i)_{i \in \{G, B\}}\}$ to the lender, after which the lender decides whether to accept the contract menu.

2. If the offer is accepted, the entrepreneur observes the true state and announces a state $i \in \{G, B\}$.

3. The lender provides the capital input $K_i$ as specified in the contract for the announced state.

4. The entrepreneur carries out the project by providing the labor input $L_i$ as specified in the contract for the announced state.

5. The revenue is realized. The entrepreneur pays $R_i$ to the lender if the project has succeeded.

Contracting when Labor Input is Non-Contractible

When the entrepreneur cannot contract with the lender on her labor input level, the contract offered by the entrepreneur is contingent on her report to the lender about the project environment and specifies the lender’s capital input level $K_i$ and the return payment to the lender $R_i$ where $i \in \{G, B\}$. The entrepreneur determines her labor input level $L_i$ after contracting and learning state $i$. The timing of the game is as follows:

1. The entrepreneur offers $\{K_i, R_i\}_{i \in \{G, B\}}$ to the lender, after which the lender decides whether to accept the contract menu.

2. If the offer is accepted, the entrepreneur observes the true state and announces a state $i \in \{G, B\}$.

3. The lender provides the capital input $K_i$ as specified in the contract for the announced state.
4. The entrepreneur carries out the project by providing the labor input $L_i$ according to her best interest.

5. The revenue is realized. The entrepreneur pays $R_i$ to the lender if the project has succeeded.

**Capital-Labor Ratio**

In characterizing input choices, we express the entrepreneur’s labor input level as a function of the lender’s capital input level:

$$L_i = r_i K_i,$$

where $r_i \geq 0$, $i \in \{G, B\}$.

Here, $r_i = L_i/K_i$ is the inverse capital-labor ratio. For notational convenience, we let $q_i = K_i$ and $r_i q_i = L_i$. Then, the project size is $Q_i = \min\{q_i, r_i q_i\}$ since:

$$Q_i = \begin{cases} r_i q_i & \text{when } r_i \in [0, 1], \\ q_i & \text{when } r_i > 1. \end{cases}$$

(1)

As first-best efficiency requires $L_i = K_i$, a value of $r_i$ different from 1 stands for a distorted capital-labor ratio:

- $r_i > 1$: capital-labor ratio is distorted downward ($K_i/L_i < 1$).
- $r_i < 1$: capital-labor ratio is distorted upward ($K_i/L_i > 1$).

**The Full Information Benchmark**

When the project environment $i$ is publicly observed and verified, the efficient project size is characterized by:

$$v'(Q_i^*) = 2 \left(c_i^l + c_i^k \right) Q_i^*,$$

where $Q_i^* = q_i$ and the capital-labor ratio is efficient under full information: $r_i^* = L_i^*/K_i^* = 1$.

3 Analysis and Results

Since the entrepreneur is privately informed of the project environment, she may have an incentive to exaggerate the prospects of the project to the lender. As we will show, if labor input is contractible, under some conditions, the capital-labor ratio is distorted downward. Without such contractibility, by contrast, under some other conditions, the capital-labor ratio is distorted upward. We first analyze the case in which the entrepreneur’s labor input is contractible.
3.1 Capital-Labor Ratio when Labor Input is Contractible

In light of backward induction, we present constraints that the entrepreneur faces from the end of the time line to the beginning. As the revelation principle applies under complete contractibility, the following incentive compatibility constraints for the entrepreneur must be satisfied:

\[ v(Q_i) - c_i^j L_i^2 - R_i \geq v(Q_j) - c_j^i L_j^2 - R_j, \quad i, j \in \{G, B\}. \tag{2} \]

Inequalities (2) ensure that the entrepreneur’s payoff from a truthful report on the project environment is higher than her payoff from misreporting it. In addition, the entrepreneur’s offer must induce the lender’s participation:

\[ R_i - c_i^k K_i^2 \geq 0, \quad i \in \{G, B\}. \tag{3} \]

With \( K_i = q_i, \) \( L_i = r_i q_i, \) and the expressions in (1), the incentive compatibility constraints in (2) and the participation constraints in (3) can be respectively rewritten as:

\[
\begin{align*}
&v(r_i q_i) - c_i^j (r_i q_i)^2 - R_i \geq v(r_j q_j) + c_i^j (r_j q_j)^2 + R_j \quad \text{for } r_i, r_j \in [0, 1], \\
v(r_i q_i) - c_i^j (r_i q_i)^2 - R_i \geq v(q_j) + c_i^j (r_j q_j)^2 + R_j \quad \text{for } r_i \in [0, 1], \ r_j > 1, \tag{IC}
\end{align*}
\]

and

\[ R_i - c_i^k q_i^2 \geq 0, \tag{PC} \]

where \( i, j \in \{G, B\} \) with \( i \neq j \) and we recall that we have transformed our problem by letting: \( c_i^k \equiv \beta^k/(1 - \rho_i) \) and \( c_i^j \equiv \beta^j/(1 - \rho_i). \)

The entrepreneur maximizes her expected payoff:

\[
\sum_i \phi_i \left[ v(Q_i) - c_i^j L_i^2 - R_i \right] = \begin{cases} \\
\sum_i \phi_i \left[ v(r_i q_i) - c_i^j (r_i q_i)^2 - R_i \right] & \text{for } r_i \in [0, 1], \\
\phi_i \left[ v(r_i q_i) - c_i^j (r_i q_i)^2 - R_i \right] & \text{for } r_i \in [0, 1], \ r_j > 1, \\
\phi_j \left[ v(q_j) - c_j^i (r_j q_j)^2 - R_i \right] & \text{with } i \neq j, \\
\sum_i \phi_i \left[ v(q_i) - c_i^j (r_i q_i)^2 - R_i \right] & \text{for } r_i > 1,
\end{cases}
\]

subject to (IC) and (PC).

The following proposition compares the optimal project size and the associated capital-labor ratio when the entrepreneur’s labor input is contractible to the full-information benchmark.
Proposition 1 When the entrepreneur’s labor input is contractible, the optimal outcome entails that:

- In state $G$, project size is inflated ($Q^c_G > Q_G^*$) and, in state $B$, it is at the same level as under full information ($Q^c_B = Q_B^*$).

- In state $G$, the capital-labor ratio is inefficient and distorted downward ($r^c_G > 1$) when the entrepreneur’s labor costs are sufficiently small and efficient otherwise. In state $B$, the capital-labor ratio is efficient ($r^c_B = 1$).

The entrepreneur makes the project oversized when the project environment is good. If the entrepreneur reports about the project environment, the lender questions the validity of the report, because the entrepreneur may have an incentive to misreport the project environment. In particular, when the project environment is bad (true state is $B$), the entrepreneur may benefit by exaggerating the prospects of the project to the lender. Since the lender anticipates such an incentive problem, the contract offered by the entrepreneur must convince the lender that the entrepreneur’s report will be true. For this purpose, the entrepreneur increases the project size in the good environment from the first-best level and, thereby, increases the return payment to the lender accordingly. This prevents the entrepreneur from exaggerating the prospects of the project when the true state is $B$.

Note that, while the project size is distorted upward, the capital-labor ratio can be distorted downward in state $G$. In our setting with perfect complements, this implies that a fraction of the entrepreneur’s labor input is wasted when implementing the project in the good environment. In other words, the entrepreneur’s effort level is higher than technologically required to reach project size $Q$ determined by the lender’s input level. Recall that the entrepreneur may have an incentive to misrepresent the project environment. It is costly for the entrepreneur to provide the labor input to implement the project and, therefore, committing to exert more than the required effort to reach $Q$ in the announced good environment discourages the entrepreneur from claiming that the project environment is good when it is bad. It is optimal for the entrepreneur to distort the capital-labor ratio by making an excessive effort when the entrepreneur’s labor costs are sufficiently small. To see this, consider the binding incentive constraint for $i = B$:

$$
\pi^*_B = v(q_G) - c^l_B (r_G q_G)^2 - c^k_B q^2_G, \tag{4}
$$

where $q_G = Q_G$. Absent a distortion of the capital-labor ratio, the project size $Q_G$ would need to be heavily oversized to be incentive compatible. Making an excessive effort in the
favorable environment ($r_G > 1$) allows the entrepreneur to reduce the distortion in $q_G$, which is optimal when doing so is not very costly ($c_G^l$ is small).

**Numerical Example:** Suppose that $v(Q_i) = 1000 \ln(q_i + 1)$, $\phi_G = 0.4$ (i.e., $\phi_B = 0.6$), $c_G^l = 1$, $c_B^l = 5$, $c_G^r = 0.2$, and $c_B^r = 1$ (thus the underlying parameters are $\psi^k = 0.8$, $\psi^l = 0.2$, $\rho_G = 0.2$, and $\rho_B = 0.4$. The first-best outcome then is: $r^*_G = r^*_B = 1$, $Q^*_G = 19.91$, $Q^*_B = 8.64$, and first-best profits are $E[\pi^*_i] = \phi_G \pi^*_G + \phi_B \pi^*_B = 0.4 \times 2564.54 + 0.6 \times 1818.02 = 2116.63$. When the entrepreneur is privately informed about the project environment, the optimal contract features a downward distortion of the capital-labor ratio for $i = G$ ($r^c_G = 1.32 > 1$ and $r^c_B = 1$). The optimal input levels and the project sizes then are:

- $Q^c_G = q^c_G = 21.86$ and $r^c_G q^c_G = 28.85$,
- $Q^c_B = Q^*_B = 8.64$,
- $E[\pi_i] = \phi_G \pi_G + \phi_B \pi_B^* = 0.4 \times 2484.83 + 0.6 \times 1818.02 = 2084.75$.

If the entrepreneur could not distort the capital-labor ratio ($r_G = 1$), then $Q_G = 27.8$ and her expected payoff would be 2064 ($< 2084.75$).

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14 In terms of the underlying parameters in the model, this means that the project is unlikely to fail in the good state; i.e., $\rho_G$ is small.
To illustrate the two regimes, it is useful to plot the optimal outcome as a function of the virtual cost of labor input in the good state, \( c_{lG} \). Figure 1 does so in the case when the labor input is contractible. Notice from Figure 1 that \( r^c_G \) and \( Q^c_G \) have two regimes (\( r^c_G > 1 \) and \( r^c_G = 1 \)), and in each regime they are independent of the entrepreneur’s virtual cost of labor \( c_{lG} \). When \( c_{lG} \) is small enough, the optimal capital input \( q^c_G \) (i.e., the optimal project size \( Q^c_G \) since \( Q_G = q_G \) for \( r_G > 1 \)) and the optimal inverse capital-labor ratio \( r^c_G \) are characterized by (see proof of Proposition 1 in Appendix A):

\[
v'(q^c_G) - 2c^k_Gq^c_G = 0 \quad \text{and} \quad r^c_G = \sqrt{\frac{v(q^c_G) - c^k_G(q^c_G)^2 - \pi^*_B}{c^l_B(q^c_G)^2}}.
\]

The reason that in each regime \( r_G \) and \( Q_G \) are independent of \( c_{lG} \) is that these variables are determined taking the entrepreneur’s misreporting incentive into account before she announces the state, when the true state is \( B \).

### 3.2 Capital-Labor Ratio when Labor Input is Non-contractible

We now proceed to the case in which the entrepreneur’s labor input is non-contractible. Because the entrepreneur has no incentive to understate the project environment as \( B \) when the true state is \( G \), the equilibrium outcome for \( i = B \) is again the first best and the same as the one under contractibility of the labor input: \( Q_B = Q^*_B \) and \( r_B = 1 \). For convenience, we again use the following notation for the first-best surplus in the state \( i = B \):

\[
\pi^*_B = v(Q^*_B) - (c^l_B + c^k_B)Q^2_B.
\]

Since \( Q_B = Q^*_B \) and \( r_B = 1 \), we can focus on the variables for \( i = G \). The contract offered to the lender specifies the capital input level provided by the lender, \( q_i = K_i \), and the return payment, \( R_i \), to him. The entrepreneur chooses the capital-labor ratio that determines her labor input level (\( L_G = r_Gq_G \)) according to her best interest at the point of carrying out the project. That is, \( r_i \) is chosen to maximize the entrepreneur’s payoff after the capital input \( K_i = q_i \) has been committed.

Note that, since the entrepreneur cannot commit to her labor input level, she will optimally choose \( L_G \leq K_G \). Therefore, using the expression in (1), we obtain the following lemma.

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15 Alternatively, we could plot the optimal outcome as a function of one of the underlying parameters. However, for the purpose of interpretation, the present comparative statics are more helpful.

16 As will be discussed below, when the entrepreneur’s labor input is non-contractible, the optimal \( r_G \) and \( Q_G \) is not independent on \( c_{lG} \) in the regime where the capital-labor ratio is distorted.
Lemma 1 When labor input is non-contractible, \( r_G \leq 1 \) in equilibrium.

The lemma implies that, in equilibrium,

\[
Q_G = \min\{r_G q_G, q_G\} = r_G q_G.
\]

Therefore, in contrast to the case in which the entrepreneur’s effort is contractible, a downward distortion in the capital-labor ratio \( (r_G > 1) \) cannot occur in the optimal contract.

Even though the entrepreneur’s choices are not fully contractible, the revelation principle holds, and thus the optimal contract always induces the entrepreneur’s truthful report. The reason is that only the entrepreneur learns the state and has an action, and therefore no relevant beliefs are affected under limited commitment—-in Appendix B, we show that it is optimal for the entrepreneur to report truthfully.

Under limited commitment, the contract offered by the entrepreneur must respect the later choices of the entrepreneur. Because \( r_G \), which represents the entrepreneur’s labor input level under truth-telling, may be different from the one under misrepresenting the state, we denote by \( r^B_G \) the entrepreneur’s labor input ratio (given capital input) when she claims that \( i = G \) while the true state is \( B \). Then, \( r_G \) and \( r^B_G \) must satisfy:

\[
\begin{align*}
  r_G &\in \arg \max_{\tilde{r}_G} v(\tilde{r}_G q_G) - c_G(\tilde{r}_G q_G)^2 \quad \text{with} \quad r_G \leq 1, \\
r^B_G &\in \arg \max_{\tilde{r}^B_G} v(\tilde{r}^B_G q_G) - c^B_B(\tilde{r}^B_G q_G)^2 \quad \text{with} \quad r^B_G \leq 1,
\end{align*}
\]

(\( EX_G \))

where \( r^B_G \leq 1 \) is implied by \( r_G \leq 1 \); if \( r_G \) as the maximizer of \( v(r_G q_G) - c_G(r_G q_G)^2 \) is less than one, then the same must be true for \( r^B_G \) as the maximizer of \( v(r^B_G q_G) - c^B_B(r^B_G q_G)^2 \). These conditions represent the entrepreneur’s choice of labor input according to her objectives after announcing that the project environment is good: \( (EX_G) \) represents her choice of labor input in the case of truth-telling, and \( (EX^B_G) \) represents her choice in the case of misreporting. The entrepreneur’s offer in equilibrium must satisfy all these constraints.

Recall that the optimal outcome associated with the bad environment is the first best with the payoff: \( \pi^*_B = v(Q^*_B) - (c^B_B + c^B_B)Q^*_B \). When the labor input is non-contractible, the entrepreneur maximizes her expected payoff:

\[
E[\pi_i] = \phi_G[v(r_G q_G) - c_G(r_G q_G)^2 - R_G] + \phi_B \pi^*_B ,
\]

subject to

\[
\pi^*_B \geq v(r^B_G q_G) - c^B_B(r^B_G q_G)^2 - R_G ,
\]

(\( IC^*_B \))

\(^{17}\)Since the capital input is contracted, the lender does not take any action after accepting the contract.
\[ R_G - c_G^k q_G^2 \geq 0, \tag{PC_G^n} \]

where \( r_G \) and \( r_B^G \) are given by \((EX_G)\) and \((EX_B^G)\).

The following proposition provides results on the optimal project size and the associated capital-labor ratio when the entrepreneur’s labor input is non-contractible.

**Proposition 2** When the entrepreneur’s labor input is non-contractible, the capital input in equilibrium is larger than when it is contractible \((q_G^n > q_G^c)\). The optimal outcome entails that:

- In state \( G \), project size is inflated \((Q_G^n > Q_G^c)\) and, in state \( B \), it is at the same level as under full information \((Q_B^n = Q_B^c)\).

- In state \( G \), the capital-labor ratio is inefficient and distorted upward \((r_G^n < 1)\) when the entrepreneur’s labor costs are sufficiently large and efficient otherwise. In state \( B \), the capital-labor ratio is efficient \((r_B^n = 1)\).

As in the case in which the entrepreneur’s labor input is contractible, the project is oversized in state \( G \) to discourage the entrepreneur from exaggerating the prospects of the project. However, because her labor input is not contractible, the entrepreneur can save on her labor cost ex post for the oversized project by leaving a proportion of the capital input provided by the lender idle. This possibility makes it less costly for the entrepreneur to misrepresent a bad project environment at the announcement stage.

The entrepreneur’s ex post shirking incentive, however, is anticipated by the lender. Therefore, to convince the lender to accept the entrepreneur’s offer, the capital input (as the contractible variable) needs to be increased compared to the case of contractibility. The increased capital input, while convincing the lender that the entrepreneur will not misrepresent the project environment, exacerbates the entrepreneur’s ex post incentive to under-provide effort. As a result, when the entrepreneur’s labor costs are large, the capital-labor ratio for the project is distorted upward since the entrepreneur does not utilize the entire physical asset financed by the lender.

In summary, when her labor input is contractible, the entrepreneur may have to work excessively, resulting in an downward distortion in the capital-labor ratio. By contrast, when the labor input cannot be contracted upon, the entrepreneur will borrow more than necessary, resulting in an upward distortion in the capital-labor ratio.
Figure 2: Capital-labor ratio and project size when the entrepreneur’s labor input is non-contractible

**Numerical Example:** Consider the same parameters for the entrepreneur’s value function, as well as \( \phi_G = 0.4 \) (\( \therefore \phi_B = 0.6 \)). Suppose now that \( c_G^k = 1.8, c_B^k = 2, c_G^l = 9, \) and \( c_B^l = 10 \) (thus the underlying parameters are \( \psi^k = 0.8, \psi^l = 4, \rho_G = 0.55, \) and \( \rho_B = 0.6 \)). The first-best outcome then is: \( r_G^* = r_B^* = 1, Q_G^* = 6.32, Q_B^* = 5.97, \) and first-best profits are \( E[\pi_i] = \phi_G \pi_G^* + \phi_B \pi_B^* = 0.4 \times 1559.23 + 0.6 \times 1513.92 = 1532.05 \). When the entrepreneur is privately informed about the project environment \( i \in \{G, B\} \), in the optimal contract the capital-labor ratio is distorted upward for \( i = G \) (\( r_G^\text{opt} = 0.98 < 1 \) and \( r_B^\text{opt} = 1 \)). The optimal input levels and the project sizes then are:

\[
Q_G^\text{opt} = r_G^\text{opt} q_G^\text{opt} = 6.92 \quad \text{(where } q_G^\text{opt} = 7.13), \]
\[
Q_B^\text{opt} = Q_B^* = 5.97
\]
\[
E[\pi_i] = \phi_G \pi_G + \phi_B \pi_B^* = 0.4 \times 1546.91 + 0.6 \times 1513.92 = 1527.12.
\]

If the entrepreneur could contract upon her labor input, in the optimal contract, \( r_G = 1 \) and \( Q_G = 6.51 \) (\( < 7.13 \)). Hence, in this example, the entrepreneur optimal would not distort the capital-labor ratio if the labor input were contractible. Her expected payoff would then be \( E[\pi_i] = 1531.77 \) (\( > 1527.12 \)).

Figure 2 illustrates the optimal outcome as a function of \( c_G^l \) when the entrepreneur’s labor input is non-contractible. Recall that, when labor input is contractible, neither \( r_G^c \)
nor $Q^c_G$ varies according to the entrepreneur’s virtual cost of labor $c^l_G$ within any regime ($r^c_G > 1$ or $r^c_G = 1$)—see Figure 1. The reason was that $r^c_G$ and $Q^c_G$ are determined by taking the entrepreneur’s misreporting incentive into account before she announces the state, when the true state is $B$. By contrast, when the labor input is non-contractible, both $r^n_G$ and $Q^n_G$ vary with $c^l_G$ in the regime of $r^n_G < 1$. Unlike when the labor input is contractible, $r_G$ is chosen after the entrepreneur truthfully reported that the state is $G$. Since $Q_G = r_Gq_G$ under non-contractibility, the optimal project size $Q^n_G$ is also affected by $c^l_G$.

4 Discussion and Extensions

4.1 Mandatory Monitoring of the Labor Input

Several implications of our results are worth further discussion. Since the capital-labor ratio may be optimally distorted and thus valuable inputs may be wasted, a possible policy recommendation is to make monitoring the entrepreneur’s effort mandatory. In view of our framework, this means that prior to such a policy intervention labor input is non-contractible and becomes contractible thereafter. According to our result, as a general rule, such a policy is problematic if the objective is to restore the efficient capital-labor ratio. Recall that when the entrepreneur’s labor input is non-contractible, part of the physical assets may remain idle—the entrepreneur’s low labor input level relative to the lender’s capital input level distorts capital-labor ratio upwards. At first glance, one may suggest that mandatory monitoring may solve the problem. When the entrepreneur’s labor input is observed and, thus, contractible, however, distortion in capital-labor ratio may be changed to the opposite direction. As we have shown, given the capital input level, the entrepreneur may exert more effort than required to implement the project in its full size—i.e., a downward distortion of the capital-labor ratio may be the consequence of mandatory monitoring.

For illustration, we revisit our numerical examples in Section 3.1, the case in which the entrepreneur’s effort is contractible: $v(Q_i) = 1000\ln(q_i + 1)$, $\phi_G = 0.4$ (.$\phi_B = 0.6$), $c^k_G = 1$, $c^k_B = 5$, $c^l_G = 0.2$ and $c^l_B = 1$. We have $r^c_G = 1.32$ at the optimum. In our model, this means that 0.32 of the labor input is not needed for the project size in state $G$. For the same $c^l_G$ and $c^l_B$, if the entrepreneur’s effort is not contractible, then $r^n_G = 1$. Thus, mandatory monitoring of labor input introduces a distortion of the capital-labor ratio.

When the entrepreneur’s cost of labor input is large, by contrast, the requirement that the lender monitors the entrepreneur’s effort may remove the distortion in capital-labor
ratio (and improve the project’s outcome). From the example in Section 3.2 with the same parameters as before but $c^k_G = 1.8$, $c^k_B = 2$, $c^l_G = 9$, and $c^l_B = 10$, when the entrepreneur’s effort is non-contractible, we have $r^n_G = 0.98$ at the optimum. That is, $0.02q^n_G$ of the capital input remains idle. If the entrepreneur’s effort is contractible, then $r_G = 1$ for the same parameters. Hence, mandatory monitoring is a mixed blessing if policy is aimed at removing distortions of the capital-labor ratio.

However, the policy objective of removing distortions in capital-labor ratio can be questioned. Despite the possible distortion of the capital-labor ratio under contractibility, making the entrepreneur’s labor input contractible by monitoring the entrepreneur’s effort always Pareto-dominates the outcome in the entrepreneur-lender relationship.\[^\text{18}\] Hence, based on joint surplus, the outcome under contractibility is always superior and, thus, mandatory monitoring should be chosen if not too costly. As long as monitoring costs are sufficiently low and funding for the monitoring activity can be arranged efficiently, contractibility of the labor input would be privately arranged in the entrepreneur-lender relationship. Thus, any justification of a policy intervention has to rely on a wedge between private and social benefits arising from affected third parties, which is not present in our model, or the inability to allocate the monitoring costs efficiently between both parties.

### 4.2 Robustness

We also shortly elaborate on four modifications. First, we reverse the bargaining power. Following previous contributions, such as Holmström and Tirole (1997) and Neher (1999), we assumed that the entrepreneur takes the initiative, and, therefore, is the one who designs and offers a take-it-or-leave-it contract to the lender. If the bargaining power shifts to the lender—i.e., the lender offers the contract to the privately informed entrepreneur—then the optimal outcome in such case is not sensitive to the contractibility of the entrepreneur’s labor input.\[^\text{19}\] In addition, although the project will be under-sized (instead of over-sized) in such a case, there will be no distortion of the capital-labor ratio. The reason is as follows. The lender’s contract offer maximizes his expected payoff, $\sum_i \phi_i \left[ R_i - c^k_i q^2_i \right]$, subject to the

\[^{18}\text{To illustrate this result, we return to the two examples. In the first of the two examples from above, the entrepreneur’s payoff is } E[\pi_i] = 2084.75 \text{ under contractibility, and } E[\pi_i] = 2064 \text{ under non-contractibility. In the second of the two examples, the entrepreneur’s payoff is } E[\pi_i] = 1527.12 \text{ under non-contractibility, and } E[\pi_i] = 1531.77 \text{ under contractibility. The lender’s rent is zero in either case.}\]

\[^{19}\text{For studies assuming that the lender offers the contract to the entrepreneur, see Wang and Zhou (2004) and Kaplan and Stromberg (2001).}\]
following incentive compatibility and participation constraints for the entrepreneur:

\[
\begin{align*}
    v(Q_i) - c_i^l (r_i q_i)^2 - R_i & \geq v(Q_j) - c_j^l (r_j q_j)^2 - R_j, \\
v(Q_i) - c_i^l (r_i q_i)^2 - R_i & \geq 0,
\end{align*}
\]

where \( Q_i = \min\{q_i, r_i q_i\} \) and \( i, j \in \{G, B\} \) with \( i \neq j \). Notice first that \( r_i \) does not enter the lender’s objective function, and, therefore, the optimal \( Q_i \) and \( R_i \) for the lender will be the same with or without the contractibility of \( r_i \) (which represents the entrepreneur’s effort level). Also, the entrepreneur’s manipulating incentive changes its direction. When the contract is offered by the lender, the entrepreneur has an incentive to understate the project’s prospects so that she can “pocket” the revenue as much as possible—unlike in the case where the entrepreneur makes the offer, she has an incentive to understate the project environment. Therefore, as in the standard screening problem, the project becomes undersized when the environment is unfavorable, \( Q_B < Q_B' \). If \( r_B < 1 \) (i.e., \( L_B < K_B \)), the lender could always gain by decreasing his capital input level. If \( r_B > 1 \) (i.e., \( L_B > K_B \)), the lender would simply give up extra rent to the entrepreneur. Hence, we must have \( r_B = 1 \). To summarize, distortions of the capital-labor ratio can only occur if the entrepreneur makes the contract offer with positive probability.

Second, we make the lender, instead of the entrepreneur, the residual claimant. Our result is robust to which party is the residual claimant, as long as the entrepreneur offers the contract. To see this, suppose that the entrepreneur offers the contract, but the lender takes \( v(Q_i) \) and pays \( R_i \) to the entrepreneur. In such case, the entrepreneur’s contract offer maximizes her expected payoff: \( \sum_i \phi_i \left[R_i - c_i^l (r_i q_i)^2\right] \), subject to the incentive compatibility condition for herself,

\[ R_i - c_i^l (r_i q_i)^2 \geq R_j - c_j^l (r_j q_j)^2, \]

and the participation constraint for the lender,

\[ v(Q_i) - c_i^k q_i^2 - R_i \geq 0, \]

where \( Q_i = \min\{q_i, r_i q_i\} \) and \( i, j \in \{G, B\} \). Since the participation constraints for the lender are binding in the optimal contract, we must have \( R_i = v(Q_i) - c_i^k q_i^2 \). Substituting for \( R_i \) in the objective function and the incentive compatibility constraint, the problem becomes the same as our original problem.

Third, we implicitly assumed that the entrepreneur cannot use invested capital for alternative internal or external use. If, by contrast, this is the case, our mechanism still applies as long as this internal or external alternative use is sufficiently unattractive. Then, in
the environment in which the entrepreneur’s labor input is non-contractible, the distortion of the capital-labor ratio can be further exacerbated.

Fourth, we assumed that capital and entrepreneurial effort are perfect complements. This led to a tractable framework with the specific feature that in the optimal contract a fraction of one of the two inputs is wasted. If some substitution is possible between the two factors, we will no longer observe that some units of input remain idle; however, as long as the two factors are not perfect substitutes, the same forces as in our model are at work such that, under some conditions, the optimal input allocation for the project is accompanied by a distorted capital-labor ratio.

5 Conclusion

In this paper, we have investigated the optimal capital-labor ratio of a project when the entrepreneur who needs to finance the project is privately informed about the project environment. The entrepreneur obtains the capital input for the project through a debt contract with a lender, and provides her labor input to implement the project. We analyzed whether and why the capital-labor ratio in the optimal contract can be distorted. Our result suggests that the capital-labor ratio may be distorted in either direction, depending on the contractibility of the entrepreneur’s effort. Due to her private information about the project environment, the entrepreneur must make the project oversized when contracting with the lender. If the labor input is contractible, the entrepreneur may optimally exert an excessive effort under a good project environment. This pushes the capital-labor ratio down below the efficient level. If the entrepreneur’s labor input is not contractible, the capital input under a good environment is further increased, while the use of labor relative to capital may be optimally reduced by the entrepreneur. This gives rise to an excessive capital-labor ratio.

Our results suggest that private information possessed by entrepreneurs is an important determinant of capital-labor ratios in industries with privately held firms. Whether these ratios lie above or below what would be observed under full information depends on the contracting environment. The observed differences of capital-labor ratios across industries and time may therefore reflect not only differences in technology, but also differences with respect to the contractibility of the complementary labor input.
Appendix A: Proofs

Proof of Proposition 1.

Notice from \( (IC) \) that there are four possible cases: (i) \( r_G, r_B \in [0, 1] \), (ii) \( r_G, r_B > 1 \), (iii) \( r_G \in [0, 1] \) and \( r_B > 1 \), and (iv) \( r_G > 1 \) and \( r_B \in [0, 1] \). As will become clear, however, by allowing the corner solutions in case (ii), (iii) and (iv), i.e., \( r_G \geq 1 \) and \( r_B \geq 1 \), it is sufficient to check cases (i) and (ii). Below we first consider case (i) where \( r_i, r_j \in [0, 1] \) to show that \( r_G = 1 \) with \( Q_G > Q_G^* \) and \( r_B = 1 \) with \( Q_B = Q_B^* \) (the outcome associated with state \( B \) is the first best). Since the outcome associated with state \( G \) is distorted, we then proceed to case (ii) by allowing the corner solution in the ratio, \( r_i, r_j \geq 1 \) to check if the ratios from case (i) is indeed the optimal solution.

**Case (i) \( r_G, r_B \in [0, 1] \):** The Lagrangian of the entrepreneur’s problem is as follows:

\[
\mathcal{L} = \sum_i \phi_i \left[ v(r_i q_i) - c_i'(r_i q_i)^2 - R_i \right] + \lambda_1 \left[ R_G - c_G'^2 q_G \right] + \lambda_2 \left[ R_B - c_B'^2 q_B \right] + \lambda_3 \left[ v(r_G q_G) - c_G'(r_G q_G)^2 - R_G - v(r_B q_B) + c_G'(r_B q_B)^2 + R_B \right] + \lambda_4 \left[ v(r_B q_B) - c_B'(r_B q_B)^2 - R_B - v(r_G q_G) + c_B'(r_G q_G)^2 + R_G \right], \text{ with } 1 \geq r_i \geq 0.
\]

First-order conditions of maximizing the Lagrangian are

\[
\frac{\partial \mathcal{L}}{\partial R_G} = -\phi_G + \lambda_1 - \lambda_3 + \lambda_4 = 0, \quad (A1)
\]

\[
\frac{\partial \mathcal{L}}{\partial R_B} = -\phi_B + \lambda_2 + \lambda_3 - \lambda_4 = 0, \quad (A2)
\]

\[
\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[ r_G v'(r_G q_G) - 2c_G'^2 q_G \right] - 2\lambda_1 c_G' q_G + \lambda_3 \left[ r_G v'(r_G q_G) - 2c_G'^2 q_G \right] - \lambda_4 \left[ r_G v'(r_G q_G) - 2c_B'^2 q_G \right] = 0, \quad (A3)
\]

\[
\frac{\partial \mathcal{L}}{\partial q_B} = \phi_B \left[ r_B v'(r_B q_B) - 2c_B'^2 q_B \right] - 2\lambda_2 c_B' q_B - \lambda_3 \left[ r_B v'(r_B q_B) - 2c_B'^2 q_B \right] + \lambda_4 \left[ r_B v'(r_B q_B) - 2c_B'^2 q_B \right] = 0, \quad (A4)
\]

From (A1), \( \phi_G + \lambda_3 = \lambda_1 + \lambda_4 \). Therefore, (A3) gives:

\[
(\lambda_1 + \lambda_4) \left[ r_G v'(r_G q_G) - 2c_G'^2 q_G \right] - 2\lambda_1 c_G' q_G - \lambda_4 \left[ r_G v'(r_G q_G) - 2c_B'^2 q_G \right] = 0.
\]

The equation, after a simple rearrangement becomes:

\[
\lambda_1 \left[ r_G v'(r_G q_G) - 2(c_G'^2 q_G + c_G^2) q_G \right] = 2\lambda_4 \left( c_G' - c_B' \right) r_G q_G. \quad (A5)
\]
In (A5), if \( \lambda_1 = 0 \) then it must be that \( \lambda_4 = 0 \). Then, (A1) gives \( \phi_G = -\lambda_3 \) and we have a contradiction. Therefore, \( \lambda_1 > 0 \) and thus \( R_G = c_G^k q^2_G \).

From (A2), \( \phi_B + \lambda_4 = \lambda_2 + \lambda_3 \). Therefore, (A4) gives:

\[
(\lambda_2 + \lambda_3) [r_B v'(r_B q_B) - 2c_B r_B^2 q_B] - 2\lambda_2 c_B q_B - \lambda_3 [r_B v'(r_B q_B) - 2c_G r_B^2 q_B] = 0.
\]

The equation, after a simple rearrangement becomes:

\[
\lambda_2 [r_B v'(r_B q_B) - 2(c_B r_B^2 + c^k_B)q_B] = 2\lambda_3 (c_B^i - c_G^i) r_B^2 q_B. \tag{A6}
\]

In (A6), if \( \lambda_2 = 0 \) then it must be that \( \lambda_3 = 0 \). Then, (A1) gives \( \phi_B = -\lambda_4 \) and we have a contradiction. Therefore, \( \lambda_2 > 0 \) and thus \( R_B = c_B^k q_B^2 \).

We now show that \( \lambda_3 = 0 \). Suppose that \( \lambda_3 > 0 \). Then, since \( \lambda_2 > 0 \), (A6) implies that \( r_B v'(r_B q_B) - 2(c_B r_B^2 + c^k_B)q_B > 0 \). This implies that the project size is distorted downward: \( Q_B < Q_B^* \). For \( r_B \in [0, 1] \), however, the entrepreneur can always increase \( q_B \), by an arbitrary small amount to increase her expected payoff. Thus, it must be that \( \lambda_3 = 0 \) in the optimum.

Since \( R_G = c_G^k q_G^2 \), \( R_B = c_B^k q_B^2 \) and \( \lambda_3 = 0 \), we can rewrite the Lagrangian as:

\[
\mathcal{L} = \sum_i \phi_i \left[ v(r_i q_i) - c_B^k (r_i q_i)^2 - c^k q_i^2 \right] + \lambda_4 \left[ v(r_B q_B) - c_B^k (r_B q_B)^2 - c_B^k q_B^2 - v(r_G q_G) + c_B^i (r_G q_G)^2 + c^k G^2 \right],
\]

with \( 1 \geq r_i \geq 0 \). First-order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[ r_G v'(r_G q_G) - 2(c_B^k r_G^2 + c_G^k)q_G \right] \tag{A7}
- \lambda_4 \left[ r_G v'(r_G q_G) - 2(c_B^k r_G^2 + c_G^k)q_G \right] = 0.
\]

\[
\frac{\partial \mathcal{L}}{\partial q_B} = (\phi_B + \lambda_4) \left[ r_B v'(r_B q_B) - 2(c_B^k r_B^2 + c_B^k)q_B \right] = 0 \tag{A8}
\]

From (A8), we have \( r_B v'(r_B q_B) - 2(c_B^k r_B^2 + c_B^k)q_B = 0 \) implying that:

\[
r_B \left[ v'(r_B q_B) - 2c_B r_B^2 q_B \right] = 2c_G^k q_B > 0. \tag{A9}
\]

Differentiating with respect to \( r_B \) gives:

\[
\frac{\partial \mathcal{L}}{\partial r_B} = (\phi_B + \lambda_4) \left[ v'(r_B q_B) - 2c_B^k r_B q_B \right] q_B > 0,
\]

since \( v'(r_B q_B) - 2c_B^k r_B q_B > 0 \) from (A9). Therefore, \( r_B = 1 \), and (A8) implies that \( q_B = Q_B^* \). Consequently, the optimal outcome for \( i = B \) is the first best.
From (A7), we have:

$$
\phi_G \left[ r_G v'(r_G q_G) - 2(\ell_G^2 + c_G^k)q_G \right] = \lambda_4 \left[ r_G v'(r_G q_G) - 2(\ell_B r_G^2 + c_G^k)q_G \right],
$$

(A10)

and on the left-hand side of (A10), \( r_G v'(r_G q_G) - 2(\ell_G^2 + c_G^k)q_G \) is non-negative, and on the right-hand side of (A10) it must be negative, and, since \( r_G v'(r_G q_G) - 2(\ell_B r_G^2 + c_G^k)q_G \) must be negative, and, since \( r_G v'(r_G q_G) - 2(\ell_G^2 + c_G^k)q_G \) is non-negative, it must be that \( \lambda_4 > 0 \) and \( \phi_G > \lambda_4 \). Also, from (A10), we have:

$$
\left\{ \phi_G \left[ v'(r_G q_G) - 2(\ell_G r_G q_G) \right] - \lambda_4 \left[ v'(r_G q_G) - 2(\ell_B r_G q_G) \right] \right\} r_G = 2(\phi_G - \lambda_4)c_G^k q_G > 0. \tag{A11}
$$

Differentiating the Lagrangian with respect to \( r_G \) gives:

$$
\frac{\partial \mathcal{L}}{\partial r_G} = \left\{ \phi_G \left[ v'(r_G q_G) - 2(\ell_G r_G q_G) \right] - \lambda_4 \left[ v'(r_G q_G) - 2(\ell_B r_G q_G) \right] \right\} q_G > 0,
$$

where the strict inequality is implied by (A11). Therefore, \( r_G = 1 \) and since \( r_G v'(r_G q_G) - 2(\ell_G^2 + c_G^k)q_G \) must be negative, and, since \( r_G v'(r_G q_G) - 2(\ell_B r_G^2 + c_G^k)q_G \) must be negative, it must be that \( q_G = Q_G > Q^*_G \).

**Case (ii) \( r_G, r_B \geq 1 \):** Recall that the solution for \( i = B \) in case (i) where \( r_i, r_j \in [0, 1] \) is the first best. It can be shown in a similar way that the solution for \( i = B \) in the case of \( r_B \geq 1 \) is the first-best: \( r_B = 1 \) and \( q_B = Q^*_B \) (since the first-order condition with respect to \( r_B \) is strictly negative, implying that \( r_B = 1 \)). For convenience, we define the entrepreneur’s first-best payoff in the state \( i = B \):

$$
\pi^*_B = v(Q^*_B) - (\ell^i_B + c^i_B)Q^*_B^2.
$$

Regarding \( r_G \), if we have the corner solution in the case of \( r_i \geq 1 \), the optimal outcome is the one that we obtained for the case of \( 1 \geq r_i \geq 0 \). If we have an interior solution here, then \( r_G = 1 \) for the case of \( 1 \geq r_i \geq 0 \) cannot be optimal.

Thus, the Lagrangian of the entrepreneur’s problem for \( r_i \geq 1 \) is:

$$
\mathcal{L} = \phi_G \left[ v(q_G) - \ell_G (r_G q_G)^2 - R_G \right] + \phi_B \pi^*_B
+ \lambda_5 \left[ R_G - c_G^k q_G \right]
+ \lambda_6 \left[ \pi^*_B - v(q_G) + \ell_B (r_G q_G)^2 + R_G \right], \text{ with } r_G \geq 1.
$$

Differentiating with respect to \( R_G \) and \( q_G \) gives respectively:

$$
\frac{\partial \mathcal{L}}{\partial R_G} = -\phi_G + \lambda_5 + \lambda_6 = 0, \tag{A12}
$$

$$
\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[ v'(q_G) - 2(\ell_G^2 + c_G^k)q_G \right] - 2\lambda_5 c_G^k q_G - \lambda_6 \left[ v'(q_G) - 2(\ell_B r_G^2 + c_G^k)q_G \right] = 0. \tag{A13}
$$

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From (A12), we have $\phi_G = \lambda_5 + \lambda_6$ and, therefore, (A13) can be rewritten as:

$$\lambda_5 \left[ v'(q_G) - 2(c_{GR}^2 G + c_k^k q_G) \right] = 2\lambda_6 \left[ c_G^1 - c_B^1 \right] r_G^2 q_G. \tag{A14}$$

In (A14), if $\lambda_5 = 0$ then it must be that $\lambda_6 = 0$, which leads to a contradiction since $\lambda_5 + \lambda_6 = \phi_G > 0$ from (A12). Therefore $\lambda_5 > 0$, and thus $R_G = c_G^2 q_G^2$. If we have the corner solution, $r_G = 1$, then the optimal outcome is the solution that we obtained in the case of $r_i \in [0,1]$, and (A5) with $r_G = 1$ gives $v'(q_G) - 2(c_G^2 + c_k^k)q_G < 0$. This means, in (A14), that $\lambda_5 > 0$ implies that $\lambda_6 > 0$, if we have the corner solution. If we have an interior solution, $r_G > 1$, then we still have $r_G v'(r_G q_G) - 2(c_G^2 + c_k^k)q_G < 0$, and hence in (A14) $\lambda_5 > 0$ implies that $\lambda_6 > 0$. Since $\lambda_5 > 0$ and $\lambda_6 > 0$, it is implied that $\phi_G > \lambda_6$ from $\phi_G = \lambda_5 + \lambda_6$ in (A12).

Since $R_G = c_G^2 q_G^2$, we can rewrite the Lagrangian as:

$$L = \phi_G \left[ v'(q_G) - c_G^1 (r_G q_G)^2 - c_k^k q_G^2 \right] + \phi_B \pi_B^* + \lambda_6 \left[ \pi_B^* - v(q_G) + c_G^1 (r_G q_G)^2 + c_k^k q_G^2 \right], \text{ with } r_G \geq 1.$$  The first-order condition with respect to $q_G$ is:

$$\frac{\partial L}{\partial q_G} = \phi_G \left[ v'(q_G) - 2 \left( c_G^1 r_G^2 + c_k^k \right) q_G \right] - \lambda_6 \left[ v'(q_G) - 2 \left( c_B^1 r_G^2 + c_k^k \right) q_G \right] = 0.$$  Rearranging this equation we have:

$$2 \left[ \phi_G c_G^1 - \lambda_6 c_B^1 \right] r_G^2 q_G = (\phi_G - \lambda_6) \left[ v'(q_G) - 2 c_k^k q_G \right]. \tag{A15}$$

Note that $\phi_G > \lambda_6$ as shown above. Differentiating with respect to $r_G$ gives:

$$\frac{\partial L}{\partial r_G} = 2 \left( \lambda_6 c_B^1 - \phi_G c_G^1 \right) r_G q_G^2. \tag{A16}$$

To have an interior solution for $r_G$ (i.e., $\partial L / \partial r_G = 0$), (A16) requires that:

$$\lambda_6 = \frac{\phi_G c_G^1}{c_B^1}, \tag{A17}$$

and if (A17) holds, then the left-hand side of (A15) is zero. Then from the right-hand side of (A15), it must be that:

$$v'(q_G) - 2 c_k^k q_G = 0, \tag{A18}$$

and $q_G$ is characterized by (A18). Since $\lambda_6 > 0$, the associated binding constraint gives:

$$r_G = \sqrt{\frac{v(q_G) - c_k^k q_G^2 - \pi_B^*}{c_B^1 q_G^2}}. \tag{A19}$$
If \( r_G = 1 \), then \( q_G = Q_G \), and (A19) gives: \( \pi_B^* - v(Q_G) + c_B^1 Q_G^2 + c_G^k Q_G^k = 0 \). This equation implies that the optimal project size is always distorted upward and independent of \( c_G^1 \). If \( r_G > 1 \) (i.e., it is an interior solution), from the expression in (A17), \( \lambda_6 \) is decreasing in \( c_G^1 \), implying that the distortion of project size is smaller than if the entrepreneur were forced to set \( r_G = 1 \). Therefore, for \( c_G^1 \) small enough, the entrepreneur’s effort is distorted and \( r_G \) has an interior solution, which is independent of \( c_G^1 \), as follows from (A19).

**Case (iii)\( r_G \in [0, 1] \), \( r_B \geq 1 \) and Case (iv)\( r_G \geq 1 \), \( r_B \in [0, 1] \):** In both cases, as in the previous cases, the solution for \( i = B \) is the first best: \( r_B = 1 \) and \( q_B = Q_B^* \). Since we always have \( r_B = 1 \) it is then sufficient to consider cases (i) and (ii). ■

**Proof of Lemma 1.**

Suppose, by contradiction, that \( r_i > 1 \) is optimal when chosen ex post. For \( r_i \geq 1 \), the entrepreneur’s problem with respect to \( r_i \) is \( \max r_i \, v(q_i) - c_i(r_i q_i)^2, \, i \in \{G, B\} \). It is clear from the problem that the entrepreneur will set \( r_i \) as small as possible, implying that the optimal \( r_i \) cannot be larger than 1. ■

**Proof of Proposition 2.**

The Lagrangian of the entrepreneur’s problem is:

\[
\mathcal{L} = \phi_G[v(r_G q_G) - c_G^1 (r_G q_G)^2 - R_G] + \phi_B^* \pi_B^*
+ \lambda_7 \left[ R_G - c_G^k q_G^2 \right]
+ \lambda_8 \left[ \pi_B^* - v(r_G q_G) + c_B^1 (r_G q_G)^2 + R_G \right], \text{ with } (EX_G) \text{ and } (EX_B^B).
\]

First-order conditions are:

\[
\frac{\partial \mathcal{L}}{\partial R_G} = -\phi_G + \lambda_7 + \lambda_8 = 0, \quad (A21)
\]

\[
\frac{\partial \mathcal{L}}{\partial q_G} = \phi_G \left[ r_G v'(r_G q_G) - 2c_G^1 r_G q_G \right] - 2\lambda_7 c_G^k q_G
- \lambda_8 \left[ r_G^B v'(r_G q_G) - 2c_B^1 (r_G^B)^2 q_G \right] = 0. \quad (A22)
\]

Suppose that \( \lambda_7 > 0 \). Then, (A21) implies that \( \phi_G > \lambda_8 \). Also, since \( \lambda_7 = \phi_G - \lambda_8 \) from (A21), we can rewrite (A22) as:

\[
\phi_G [r_G v'(r_G q_G) - 2 \left( c_G^1 r_G^2 + c_G^k \right) q_G] = \lambda_8 [r_G^B v'(r_G q_G) - 2 (c_B^1 (r_G^B)^2 + c_G^k) q_G]. \quad (A23)
\]
Since \( \phi_G > \lambda_8 \), (A23) implies that:

\[
r_G v'(r_G q_G) - 2c_G^1 r_G^2 q_G < r_G^B v'(r_G^B q_G) - c_B^1 (r_G^B)^2 q_G. \tag{A24}
\]

From \((EX_G)\) and \((EX_G^B)\), \( r_G \geq r_G^B \). If \( r_G = 1 \) and \( r_G^B < 1 \), then \((EX_G)\) and \((EX_G^B)\) imply that the left-hand side of (A24) is strictly positive and the right-hand side of (A24) is zero. This contradicts the inequality in (A24). If \( r_G = r_G^B = 1 \), then we have \( v'(q_G) - 2c_G^1 q_G < v'(q_G) - 2c_B^1 q_G \) from (A24), which is a contradiction. If \( r_G < 1 \) and \( r_G^B < 1 \), then both sides of (A20) are zero, which contradicts the inequality. Thus, it must be that \( \lambda_7 = 0 \), which implies that \( \lambda_8 = \phi_G(> 0) \) from (A21).

With \( \lambda_7 = 0 \) and \( \lambda_8 = \phi_G \), the equation in (A22) becomes:

\[
r_G v'(r_G q_G) - 2c_G^1 r_G^2 q_G = r_G^B v'(r_G^B q_G) - 2c_B^1 (r_G^B)^2 q_G.
\]

Again from \((EX_G)\) and \((EX_G^B)\), \( r_G \geq r_G^B \). If \( r_G = r_G^B = 1 \), then we have a contradiction in the above equation since \( v'(q_G) - 2c_G^1 q_G > v'(q_G) - 2c_B^1 q_G \). There are two possible cases: \( r_G = 1 \) and \( r_G^B < 1 \), and \( r_G < 1 \) and \( r_G^B < 1 \). That is, it must then hold that \( r_G^B < 1 \) in any case. Since \( \lambda_8 > 0 \), it is implied that: \( \pi_B^* = v(r_G^B q_G) - c_B^1 (r_G^B q_G)^2 - R_G \). There is a leeway in this equation since \( \lambda_7 = 0 \), i.e., \( R_G - c_G^k q_G^2 \geq 0 \) is automatically satisfied. One way is to set \( R_G = c_G^k q_G^2 \) to have:

\[
\pi_B^* = v(r_G^B q_G) - c_B^1 (r_G^B q_G)^2 - c_G^k q_G^2. \tag{A21}
\]

The value of \( q_G \) and \( r_G^B \) are determined by solving (A21) and \((EX_G^B)\) simultaneously. Since \( r_G^B < 1 \), equation (A21) implies that the level of \( q_G \) is distorted upward even further compared to the level when the entrepreneur’s effort is contractible, i.e., \( q_G^* > q_G^C \). Finally, with the values of \( q_G \) and \( r_G^B \) determined, \( r_G \) is computed from \((EX_G)\), which implies that, for \( c_G^1 \) large enough, \( r_G < 1 \). ■

Appendix B: Optimality of Truthful Reporting When the Labor Input is Non-contractible

We denote by \( \alpha \) the probability that the entrepreneur makes a truthful report when the true state is \( i = B \), and by \( \beta \) the probability that the lender accepts the entrepreneur’s offer. We show that \( \alpha = \beta = 1 \) in equilibrium.

Under limited commitment, the contract offered by the entrepreneur must respect the later choices of the entrepreneur and the lender according to each party’s objective functions
at the corresponding stages. Because \( r_G \), which represents the entrepreneur’s labor input level under truth-telling, may be different from the one under misrepresenting the state, we let \( r^B_G \) represent the entrepreneur’s labor input level when she claims that \( i = G \) when the true state is \( B \). In equilibrium, \( \alpha, \beta, r_G \) and \( r^B_G \), must satisfy:

\[
\alpha \in \arg\max_{\tilde{\alpha}} \tilde{\alpha} \pi_B^* + (1 - \tilde{\alpha}) \left[ v(r^B_G q_G) - c^B_B (r^B_G q_G)^2 - R_G \right],
\]

\[
\beta \in \arg\max_{\tilde{\beta}} \tilde{\beta} \left\{ \phi_G \left[ R_G - c^k_G q^2_G \right] + \phi_B (1 - \alpha) \left[ R_G - c^k_B q^2_G \right] \right\},
\]

\[
r_G \in \arg\max_{\tilde{r}_G} v(\tilde{r}_G q_G) - c^G_G (\tilde{r}_G q_G)^2 \text{ with } r_G \leq 1,
\]

\[
r^B_G \in \arg\max_{\tilde{r}^B_G} v(\tilde{r}^B_G q_G) - c^B_B (\tilde{r}^B_G q_G)^2 \text{ with } r^B_G \leq 1,
\]

where \( r^B_G \leq 1 \) is implied by \( r_G \leq 1 \) (Lemma 1). The first constraint, \( (B1) \), represents the entrepreneur’s choice regarding truth-telling versus misreporting after learning that the true state is \( B \). The second constraint, \( (B2) \), represents the lender’s choice of accepting or rejecting the offer. Notice from \( (B2) \) that, when the entrepreneur claims that the state is \( G \), there are two possibilities from the lender’s point of view: The report is true with probability \( \phi_G \), and the report is false with \( \phi_B (1 - \alpha) \). With probability \( \phi_B \alpha \), the outcome is first best, and the lender’s rent is zero. The last two constraints, \( (B3) \) and \( (B4) \), represent the entrepreneur’s choice of labor input level according to her objectives after announcing that the project environment is good; \( (B3) \) represents her choice of labor input in the case of truth-telling, and \( (B4) \) represents her choice in the case of misreporting. The entrepreneur’s offer in equilibrium must satisfy all these constraints.

There are three candidates for an outcome. First, the entrepreneur induces herself to truthfully report the project environment to the lender when the true state is \( B \), and the lender rationally anticipates that the report will be truthful and accepts the offer (\( \alpha = 1 \) and \( \beta = 1 \)). Second, the entrepreneur induces herself to exaggerate the prospects of the project when the true state is \( i = B \), and the lender anticipates this and rejects the offer (\( \alpha = 0 \) and \( \beta = 0 \)). Finally, the entrepreneur induces herself to randomize between reporting the truth and misrepresenting the state when the true state is \( i = B \), and anticipating this, the lender randomizes between accepting and rejecting. Then, both, entrepreneur and lender, may use mixed strategies (\( 1 > \alpha > 0 \), and \( 1 > \beta > 0 \)). We show that the contract that induces \( \alpha = 1 \) and \( \beta = 1 \) dominates those that induce \( \alpha = 0 \) and \( \beta = 0 \), or \( 1 > \alpha > 0 \), and \( 1 > \beta > 0 \).
We define:

\[ \Omega \equiv \pi_B^* - \left[ v(r_G^B q_G) - c_B^I(r_G^B q_G)^2 - R_G \right], \]

where \( \pi_B^* \equiv v(Q_B^*) - (c_B^I + c_B^k)Q_B^2 \). Then, the entrepreneur’s decision regarding a truthful report follows the rule:

\[
\alpha \in \begin{cases} 
0 & \text{if } \Omega < 0, \\
1 & \text{if } \Omega > 0, \\
(0, 1) & \text{if } \Omega = 0.
\end{cases}
\]

For \( \Omega < 0 \), the optimal contract with \( \alpha = 0 \) will induce the lender’s participation only when the true state is \( G \) with the first-best production levels: \( q_G = q_G^* \) and, thus, \( r_G = 1 \). This case prevails when \( \phi_B \) is very small. We rule this case out because, as mentioned in the model section, we are focusing on the situation in which \( \phi_i \) is not too small that the entrepreneur wants to realize the project in either state.

For \( \Omega > 0 \), \( \alpha = 1 \), and thus the participation constraint for the lender when \( i = G \), \( R_G - c_G^k q_G^2 \geq 0 \), implies that \( \beta = 1 \). Then, the entrepreneur’s incentive constraint becomes:

\[ \pi_B^* \geq v(r_G^B q_G) - c_B^I(r_G^B q_G)^2 - R_G. \] (B5)

The inequalities are weak since these constraints may be binding. The strictness of \( \Omega > 0 \) follows from the usual argument in a model of this type that, by choosing the level of \( q_G \) slightly higher than the level that satisfies (B5) with equality, the entrepreneur strictly prefers to truthfully report that the state is \( B \). This means that \( q_G = \tilde{q}_G + \epsilon \), where \( \tilde{q}_G \) satisfies (B5) with equality, and \( q_G \) approaches \( \tilde{q}_G \) in the limit as \( \epsilon \to 0 \). We restrict attention to the case that (B5) is binding, and show in the proof of Proposition 2 that (B5) must be binding. Then, the entrepreneur solves:

\[
\max_{q_G, r_G} \phi_G[v(r_G^B q_G) - c_G^I(r_G^B q_G)^2 - R_G] + \phi_B \pi_B^*,
\]

subject to

\[
R_G - c_G^k q_G^2 \geq 0,
\]

\[
\pi_B^* = v(r_G^B q_G) - c_B^I(r_G^B q_G)^2 - R_G,
\]

(B3), and (B4).

For \( \Omega = 0 \), the entrepreneur’s objective function is:

\[
\beta \left\{ \phi_G[v(r_G^B q_G) - c_G^I(r_G^B q_G)^2 - R_G] + \phi_B [\alpha \pi_B^* + (1 - \alpha)(v(r_G^B q_G) - c_B^I(r_G^B q_G)^2 - R_G)] \right\},
\]

26
Since $\Omega = 0$, we have $\pi_B^* = v(r_B^q q_G) - c_B^d (r_B^q q_G)^2 - R_G$ and simplify the objective function further:

$$\beta \left\{ \phi_G [v(r_G q_G) - c_B^d (r_G q_G)^2 - R_G] + \phi_B \pi_B^* \right\}.$$  \hfill (B6)

The entrepreneur maximizes her payoff in (B6), subject to $\pi_B^* = v(r_B^q q_G) - c_B^d (r_B^q q_G)^2 - R_G$, $R_G - c_G q_G^2 \geq 0$, (B2), (B3) and (B4). It is clear that, for any $\beta < 1$, the outcome from this problem gives a strictly lower payoff to the entrepreneur than the one with $\Omega > 0$ (for $\beta = 1$, the outcome of the problem is the same as the one with $\Omega > 0$).
References


