Trade and the political economy of redistribution

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Abstract

This paper analyzes the effects of trade liberalisation on the political support for policies that redistribute income between workers in different sectors. We allow for worker heterogeneity and imperfect mobility of workers across sectors, giving rise to a trade-off between redistribution and the inefficiency of the labor allocation. We compare two environments, autarky and small open economy, and present three main findings. First, redistributive policies are more “likely” to arise in a small open than in a closed economy. Second, if a redistributive policy is adopted in both situations, its nominal level is higher in autarky than in the small open economy. Third, even though voters choose redistributive policies with lower nominal value in open economies, the actual extent of redistribution in equilibrium is larger in the open than in the closed economy. We discuss our results in the context of the debate about the effects of globalisation on government activity.

Keywords: International trade, redistribution, political economy, factor mobility

J.E.L. classification: F1, H2

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1 Introduction

A common source of inefficiency of redistributive policies stems from their heterogeneous impact on factor rewards across sectors and the resulting distortion in factor allocation between sectors (Saez (2004)). Subsidies to - or bailouts of - particular industries have this heterogeneity at their core. One of their typical aims, as far as their redistributive dimension is concerned, is to raise the wage or maintain employment of workers in declining sectors. Other policies, such as a progressive income tax, heterogeneous taxation of inputs or unemployment benefits (Wright (1986)) also distort factor allocation across sectors if these differ in their average wage, input mix\(^1\) or unemployment risk respectively.

The present paper analyzes how trade liberalisation changes the effect of and the political support for policies which redistribute income between workers in different sectors. Although these policies are generally considered inefficient ((Acemoglu and Robinson (2001)), they remain an important channel through which governments across the world redistribute income or support employment (e.g. Ford and Suyker (1990), OECD (2010), Rickard (2012)). In developing countries, Rickard (2012) shows that their prevalence increased over the 1980s and 1990s and that globalisation proved instrumental in driving this evolution. This may come as a surprise in light of the extensive academic literature and public debate, which stress that globalisation imposes new constraints on governments’ ability to redistribute income or protect their citizens through the welfare state\(^2\) (see Brady, Beckfield, and

\(^1\)Examples include among many others fuel, electricity, and water subsidies, the absence of kerosene tax, or differential taxation of capital and labor, which favor sectors intensive in energy, water or capital respectively.

\(^2\)For example, Wilson (1987) shows that the higher mobility of the tax base in an open

The present paper however argues that, for cross-sectoral redistributive policies, trade openness reduces the inefficiency associated with redistribution and makes these policies less costly to implement. This in turn translates to a stronger political support for redistribution in open economies and raises the likelihood that redistribution arises in a voting equilibrium. We therefore contribute to the positive analysis of the role of trade in shaping government interventions in two ways. First, we focus on cross-sectoral redistribution, a type of policies which, though widely used, has largely been ignored by the literature. Second, we recognize that conflicts of interests are at the core of redistributive policies and use a voting model to determine how the support for redistribution is affected by international trade.

Our theoretical framework offers a number of novel features which allow for a rich but tractable analysis. We assume that the economy consists of different sectors producing under perfect competition and using exclusively labor. The demand condition for each sector varies, thus setting the stage for redistribution towards workers in sectors with low demand. To capture the inherent trade-off of cross-sectoral redistribution, the key novelty is our parsimonious modeling of the imperfect mobility of workers between sectors, which builds on recent insights of the trade literature on comparative advantage\textsuperscript{3}. If workers were perfectly mobile, there would be no conflict of interest

\begin{footnotesize}
world limits the size of redistribution that a government can conduct, while Alesina and Perotti (1997) point to the negative effects of redistribution on a country’s competitiveness. Epifani and Gancia (2009) on the other hand argue that a terms of trade externality in the financing of public goods may create a positive effect of trade openness on the size of governments.

\textsuperscript{3}We apply the Eaton and Kortum (2002) framework on comparative advantage to the case of workers, see section 2.2. In line with earlier work on factor mobility (e.g. Grossman (1983)), we measure the degree of labor specificity by the percentage loss in productivity
\end{footnotesize}
as they would be made indifferent between sectors. If workers were tied to a sector on the other hand (perfect immobility), a policy redistributing income towards certain sectors would be very redistributive, but would carry no inefficiency as it would not affect the sectoral allocation of workers. Within this framework we assume that workers determine the level of intersectoral redistribution by majority voting. This creates a conflict of interest between workers in sectors with low demand, who benefit from redistribution, and workers in sectors with high demand, who lose. In this setup, redistribution only arises in equilibrium if the majority of workers choose to work in low-demand sectors, an outcome which depends among others on the relative number of low-demand sectors in the economy.

The main conclusions of our model rest on the observation that a given degree of cross-sectoral redistribution causes less inefficiency in an open than in a closed economy. Loosely speaking, the domestic distortion implied by redistributive policies is less costly when consumers can turn to foreign goods. If the world price of low-demand goods is not too low, these lower costs of redistribution translate into a stronger political support for redistribution, which manifests itself along two margins: (i) the median voter is “more likely” to vote for some redistribution in an open economy and (ii) if redistribution is implemented, its extent - measured by the equilibrium ratio of wages in low to wages in high demand sectors - is larger in an open economy. Although seemingly intuitive, the fact that lower costs of redistribution cause a broader political support for redistribution (the first margin) can only arise if there is some degree of worker mobility in the economy. Our model can accommodate that workers incur when changing sector.

4The minimum share of low demand sectors for a redistribution to arise is lower in an open economy.

5The contribution of our approach becomes clear when comparing our setup to a specific
in a tractable way the changing political support for redistribution when an economy opens to trade. We discuss a particularly interesting case of such changing support in section 5, where we consider an environment in which the world price of low demand good is much lower than in autarky. In such a case, opening up to trade not only reduces the wage in low demand sectors, but also induces workers to move to high demand sectors, thereby decreasing the political support for redistribution. If this causes redistributive policies to be abandoned, wage in low demand sectors further decrease.

Finally, our analysis shows that the nominal policy rate goes down when opening up to trade. The usual empirical measures of cross-sectoral redistributive policies - either the subsidy rate or the share of subsidies going to low demand sectors as a share of GDP - both decrease in our model when opening up to trade. The fact that actual redistribution (i.e. including the general equilibrium effect) increases when opening up to trade while nominal policies decrease can be reconciled by noting that opening to trade increases the price elasticity of demand for all goods. In autarky, subsidizing a sector raises the supply of its product and reduces its equilibrium price, thereby limiting both the increase in that sector’s factor use and factor rewards. In a small open economy, since prices are exogenous, the dampening effect of price changes is absent. Any given policy rate is therefore both more redistributive and more distortive in an open economy. These differences in redistributive effects explain both why voters in an open economy choose a redistributive factors model with two sectors and three types of workers (specific to low demand sectors, to high demand sectors, and mobile between sectors). In such a model, the support for redistribution is fixed: only workers specific to the low demand sectors favor redistribution. Although opening up to trade reduces the costs of redistribution, the number of workers supporting redistribution does not change and redistributive policies are only observed if the exogenous share of workers specific to low demand sectors exceeds 50%.
policy with lower nominal value than in autarky and why this policy can be more redistributive than the one chosen in autarky.

In addition to existing studies on globalisation and the welfare state which we mentioned earlier, the present paper relates to the literature on the distributive effects of international trade coming through a more elastic labor demand. Rodrik (1997) points out that, by increasing competition in product and factor markets, globalisation may raise the price elasticity of labor demand, with potentially adverse consequences for workers. Empirically, Slaughter (2001) finds strong evidence that the elasticity of labor demand has increased between the 1970s and 1990s in the U.S., although he cannot identify a strong effect of globalisation on this pattern (see also Krishna, Mitra, and Chinoy (2001) and Hasan, Mitra, and Ramaswamy (2007) for empirical evidence. Spector (2001) shows how changes in elasticity matters for redistributive polices in a standard income taxation model à la Mirrlees. In contrast to this literature, our approach shows that the more elastic labor demand does not in itself affect the extent of redistribution, as voters can choose a policy which cancels the real effect of a higher elasticity. Much more central to our results is that consumers have the possibility to import goods in an open economy. We also relate to the literature considering the effects of international trade when factors are imperfectly mobile between sectors or occupations (Kambourov (2009), Artuc, Chaudhuri, and McLaren (2010), Ohnsorge and Trefler (2007)).

Section 2 describes the setup of the model. Section 3 solves the model for a given redistributive policy, and describes the key differences between the closed and open economy. Section 4 introduces the political dimension of the model and endogenises the choice of policy. Section 5 conducts two extensions and discusses the robustness of our results to a setup with more
than two types of sectors and section 6 concludes.

2 The setup

2.1 Overview

The economy consists of a mass one of individuals who share the same Cobb-
Douglas utility function over $N + 1$ goods, indexed from 0 to $N$:

$$U = \prod_{n=0}^{N} q_n^{\alpha_n}$$

(1)

where $q_n$ denotes the consumption of good $n$ and $\sum_{n=0}^{N} \alpha_n = 1$. Individuals,
indexed by $j$, maximise utility subject to their income. Defining the economy-
wide income as $I$ and the price of good $n$ as $p_n$, the aggregate demand for
good $n$ is:

$$q_n^D = \alpha_n \frac{I}{p_n}.$$

(2)

We assume\footnote{We relax this assumption in section 5} that $x_L$ of the $N+1$ goods enter the utility with a weight $\alpha_n = \alpha_L$ (the “low demand” goods) while $x_H$ goods have a a parameter $\alpha_n = \alpha_H > \alpha_L$ (the “high demand” goods), where $x_L \alpha_L + x_H \alpha_H = 1$. We denote the set of sectors with low demand as $X_L$ and the set of sectors with high demand as $X_H$. Without loss of generality, we assume that sector 0 is a high demand sector.

Each good is produced in a separate sector under conditions of perfect
competition. Labor is the only factor of production in the economy, and
all individuals in the model are workers, who supply inelastically one unit
of labor. The productivity of a unit of labor is specific to a worker-sector
pair: each worker independently draws a productivity parameter for each
sector. The distribution of productivity draws determines the typical loss of productivity incurred by workers when changing sector, and indexes the degree to which workers are sector-specific. The government can subsidise sectors in which equilibrium wages are relatively low due to low demand parameters ($\alpha_L$).\footnote{With a Cobb-Douglas utility, differences in sector-wide productivity would not affect the share of total income spent on a particular sector. We concentrate on the Cobb-Douglas case and on demand heterogeneity for simplicity. All results of section 3 and 4 hold with a CES utility when sectors have heterogeneous productivity parameters. Redistribution in this case takes place towards sectors with a low productivity parameter.} We do not explicitly model how and why demand is low in some sectors. In a more general model, the demand parameters could be the result of the realisation of a stochastic process. In section 3 we analyze the effects of a given sectoral subsidy to low demand sectors, while in section 4 we assume that workers decide by majority voting on the size of these subsidies. We characterise and compare the political-economy equilibria for this economy when it is in autarky (all prices are endogenous) and when it is a small open economy (output prices are given from the world market).

The timing of the model can be summarised as follows. At time $t_0$, each individual observes his vector of productivity draws for each sector $n \in N$. At $t_1$, individuals decide by majority voting on the level of redistribution towards sectors with low demand. At $t_2$, workers decide on the sectors in which they want to work. We now turn in detail to each of the three steps of the model.

### 2.2 Worker heterogeneity

Workers differ in their labor productivity, which is sector-specific. This makes for a realistic situation and leads to heterogeneous interests when voting over...
sectoral subsidies takes place. At time $t_0$, each worker independently draws a productivity parameter $z$ for each sector from a Fréchet distribution\textsuperscript{8}:

$$F(z) = \exp(-z^{-\nu}).$$  \hspace{1cm} (3)

Worker $j$ obtains a vector of productivity draws, $\{z_{jn}\}_{n=0}^{N}$, which he observes. $z_{jn}$ denotes the number of efficiency units of labor that worker $j$ provides if he works in sector $n$. The parameter $\nu > 0$ affects the heterogeneity of productivity draws between sectors and provides a parsimonious way of capturing the degree of sector-specificity of workers. If $\nu$ is low, the heterogeneity of draws between sectors is large, and the percentage loss in productivity incurred by a worker changing sector is large.\textsuperscript{9} The parameter $\nu$ captures both technological and regulatory reasons for the sector specificity of workers\textsuperscript{10}.

2.3 Production and redistributive policies

Each sector consists of a large number of firms which behave in a perfectly competitive manner both on the product and on the labor market. Production in a sector equals the number of effective units of labor employed by the sector $(\Lambda)$\textsuperscript{11}:

$$y_n = \Lambda_n.$$ \hspace{1cm} (4)

\textsuperscript{8}For the sake of simplicity, we assume that the Fréchet distribution has only one parameter, $\nu$. The analysis can be generalised to allow the average $z$ to be sector specific.

\textsuperscript{9}This interpretation is the counterpart to that of comparative advantage made by Eaton and Kortum (2002) in a trade context. Artuc, Chaudhuri, and McLaren (2010) use the Fréchet distribution to model idiosyncratic shocks to the benefits of working in a particular sector.

\textsuperscript{10}In a dynamic perspective, the sector specificity of workers is similar to the concept of mobility of workers between sectors.

\textsuperscript{11}Allowing for productivity heterogeneity between sectors does not affect any of the results. See the working paper version Vannoorenberghe and Janeba (2013) for a production function of the form $y_n = \varphi_n \Lambda_n$ where $\varphi$ captures the sector-specific productivity.
To redistribute income towards workers in particular sectors, the government can use a sector-specific sales tax or subsidy. Profits in sector $n$ are given by:

$$\pi_n = [(1 - \tau_n)p_n - c_n]\Lambda_n$$  \hfill (5)

where $c_n$ denotes the wage paid per unit of effective labor in sector $n$. Anticipating the equilibrium solution of the model, the zero profit condition is given by:

$$c_n = (1 - \tau_n)p_n.$$  \hfill (6)

Sector specific taxes ($\tau_n > 0$) or subsidies ($\tau_n < 0$) thus affect the wage per unit of effective labor in $n$. For simplicity, and unless otherwise specified, we will refer to $c_n$ as the “wage” in sector $n$ in the rest of the analysis, which should be understood as the wage per unit of effective labor in that industry.

Before proceeding, it is worth discussing the nature of taxation and redistribution in our model. As argued in the introduction, intersectoral redistribution is a general phenomenon. Often, support to specific sectors is directly done, for example through price subsidies, bailouts or guarantees. Agriculture, coal mining, or the car industry are typical recipients of such policies. Subsidies to energy industries and the coal industry in particular are widespread, see Victor (2009) for an overview and Frondel, Kambeck, and Schmidt (2007) for Germany. Sometimes subsidies are more hidden and are not directly targeted to particular sectors, but are in practice when they are tied to characteristics of the production process (such as R&D, capital, energy or skill intensity), which vary across sectors. The sales tax $\tau_n$ in our model comes close to mimicking a price subsidy.

We assume that the government applies the same tax to all low demand sectors ($\tau_L$) and similarly to all high demand sectors ($\tau_H$) and define for simplicity $\beta_n$ as the policy parameter applying to sector $n$ compared to that...
of sector $0 \in X_H (\tau_0 = \tau_H)$:

$$\beta_n \equiv \frac{1 - \tau_n}{1 - \tau_0} = \begin{cases} 1 & \text{if } n \in X_H \\ \beta & \text{if } n \in X_L \end{cases}$$  \hspace{2cm} (7)

where $\beta \equiv (1 - \tau_L)/(1 - \tau_H)$. The parameter $\beta$ determines the extent to which low demand sectors are subsidized compared to high demand sectors. If $\beta > 1$, low demand sectors are taxed relatively less (or subsidized relatively more) than sectors with high demand.

Note that a policy $\beta$ fixes the ratio of $(1 - \tau_L)/(1 - \tau_H)$ but does not fix the level of taxation, which depends on $1 - \tau_H$. The latter is defined by the requirement that the policy be feasible, in the sense that the government runs a balanced budget:

$$\sum_{i=0}^{N} \tau_0 p_i \Lambda_i = 0 \iff \sum_{i=0}^{N} \beta_i p_i y_i = \frac{I}{1 - \tau_0} \hspace{2cm} (8)$$

where the second equation uses the zero profit condition (6).

### 2.4 Sectoral choice of workers

At $t_2$, individuals decide in which sector to work. They observe their idiosyncratic vector of sector-specific productivities $\{z_{jn}\}_{n=0}^{N}$ and the vector of sectoral wages $\{c_n\}_{n=0}^{N}$. Worker $j$ chooses to work in the sector which gives him the highest income, which is the product of the wage in the sector times the worker-sector specific productivity $z_{jn}c_n$. As shown in the appendix 7.1, the fraction of individuals deciding to work in sector $n$ is:

$$L_n = \frac{c_n^\nu}{\sum_{i=0}^{N} c_i^\nu}, \hspace{2cm} (9)$$

which is also the supply of labor as the number of individuals is normalised to one. $L_n$ is increasing in the wage paid in sector $n$. On the other hand, a higher
wage in other sectors makes employment in $n$ relatively less attractive and reduces $L_n$. The parameter $\nu$ represents a measure of the sector-specificity of labor and determines the sensitivity of employment to relative differences in wages between sectors. If $\nu$ is large, the productivity parameters drawn by individuals for different sectors are similar, making the choice of sector very dependent on the relative wages. The degree of sector specificity of workers is in this case very low. If $\nu \to 0$, on the other hand, workers are fully sector-specific and each sector employs $1/(N + 1)$ of the labor force regardless of differences in wages.

Using the sectoral supply of labor, we show in the appendix 7.1 that the supply of good $n$ as given by (4) is equal to:

$$
y_n = \Delta c_n^{\nu-1} \left( \sum_{i=0}^{N} c_i^{\nu} \right)^{1-\nu/\nu} \tag{10}
$$

where $\Delta \equiv \Gamma(1 - 1/\nu)$ and $\Gamma()$ denotes the gamma function. Sectors which pay higher wages have a higher supply curve since they attract more workers. For $y_n$ to be defined, we assume in the rest of the analysis that $\nu > 1$. If $N$ is large, $\nu - 1$ is the elasticity of the number of effective units of labor employed in a sector with respect to the wage paid in that sector.\textsuperscript{12} Solving for $c_n$ in (10) shows that the wage in $n$ is increasing in $y_n$ and that the total costs of production in sector $n$ are convex. To expand, sector $n$ needs to attract workers who may be relatively more productive in other sectors, and who therefore need to be paid a higher wage to accept working in sector $n$.

\textsuperscript{12}Note that, as $\nu \to 1$, the allocation of labor remains sensitive to $c_n$ (see (9)) but the amount of effective labor is not. For $\nu \to 1$, the differences in $z$ are such that the workers who join sector $n$ following an increase in $c_n$ are infinitely less productive than the average worker already in $n$, thereby increasing the production of $n$ by zero percent.
3 Economic equilibrium

In this section, we characterise the economic equilibrium for a given policy $\beta$, first under autarky and then in a small open economy. Although we assume that the vector $\beta$ is exogenous, $\tau_0$ is endogenous and ensures the feasibility of a particular policy $\beta$.

3.1 The autarkic equilibrium

In autarky, the market for each good must be in equilibrium, i.e. $y_n = q_n^D$ for all sectors $n$. Using (2) and (10), the goods market equilibrium implies\(^{13}\):

$$c_n^A = (1 - \tau_0)\beta_n p_n^A d_0 = (\alpha_n\beta_n)\frac{\frac{I^A}{\Delta}}{\left(\sum_{i=0}^{N} \alpha_i\beta_i\right)^{\frac{\nu-1}{\nu}}} (1 - \tau_0) \tag{11}$$

$$y_n^A = \Delta (\alpha_n\beta_n)^{\frac{\nu-1}{\nu}} \left(\sum_{i=0}^{N} \alpha_i\beta_i\right)^{\frac{1-\nu}{\nu}} \tag{12}$$

where $p_n^A$ and $y_n^A$ are the price and the production in sector $n$ in the autarkic equilibrium and where $\beta_n$ is as defined in (7). Equation (11) shows the wage obtained by workers in sector $n$. The wage and the production in a sector $n$ are increasing in the demand parameter ($\alpha_n$) of a sector and in the redistributive policy towards it. The degree of worker mobility ($\nu$) indexes the relative extent to which these parameters affect wages or quantities produced.

From (11) and (12) the balanced budget constraint (8) in autarky can be rewritten as:

$$1 - \tau_0 = \frac{1}{\sum_{i=0}^{N} \alpha_i\beta_i} \tag{13}$$

\(^{13}\)To obtain (11), we set $q_n^D$ of (2) and $y_n$ of (10) equal. We then solve for $c_n^A$ and add up over all $n$, which gives a solution for $\sum_i c_i^A$. Plugging the solution back in $q_n^D = y_n$ and rearranging gives (11).
This condition, combined with (11) and (12), pins down the vector of prices and of production in autarky for any given $I^A$.\footnote{Since we have not fixed any numeraire, total income - which is the sum of wages paid to all workers - can take any value.}

### 3.2 The equilibrium in a small open economy

We now consider a small open economy facing a vector of exogenous prices $\{p^T_n\}_{n=0}^N$ set on the world market. We define $I^T$ as the total nominal income of this economy. Since prices are fixed on the world market, the wage in sector $n$ is simply given by: $c^T_n = (1 - \tau_0) \beta_n p^T_n$. In a small open economy, domestic supply and demand of a good are not necessarily equal. The equilibrium production of a sector is determined by the supply equation given prices:

$$
y^T_n = \Delta (\beta_n p^T_n)^{\nu-1} \left( \sum_{i=0}^N (\beta_i p^T_i)^\nu \right)^{\frac{1-\nu}{\nu}}.
$$

Using (14) and rearranging, we can express the total nominal income of the small open economy ($I^T$) and the balanced budget condition of the government (8) respectively as:

$$
I^T = \sum_{i=0}^N p^T_i y^T_i = \Delta \left( \sum_{i=0}^N \beta_i^{\nu-1} (p^T_i)^\nu \right) \left( \sum_{i=0}^N (\beta_i p^T_i)^\nu \right)^{\frac{1-\nu}{\nu}}.
$$

$$
1 - \tau_0 = \left( \sum_{i=0}^N \beta_i^{\nu-1} (p^T_i)^\nu \right) \left( \sum_{i=0}^N (\beta_i p^T_i)^\nu \right)^{-1}.
$$

To conduct a meaningful comparison between the small open and the closed economy, we assume in the following that the world price distribution is given by:

$$
p^T_n = \frac{I^A}{\Delta} \beta_n^\nu \forall n,
$$

which implies that prices in the open economy are equal to autarky prices with no redistribution ($\beta = 1$). Under this assumption, trade is “unbiased”
in the sense that (a) the efficient allocation of resources is identical in the closed and open economy, (b) the total real income with no redistribution is the same under the closed and the open economy and (c) if $\beta = 1$ in the open economy, no international trade takes place.

3.3 Redistributive policies and distortion

From the equilibrium $c^A_n(\beta)$ and $c^T_n(\beta)$ derived above, the ratio of wages between low and high demand sectors in autarky and in an open economy is respectively given by:

$$\frac{c^A_n(\beta)}{c^H_n(\beta)} = \left(\frac{\alpha_L}{\alpha_H}\right)^{\frac{1}{\nu}} \quad \frac{c^T_n(\beta)}{c^H_n(\beta)} = \beta \left(\frac{\alpha_L}{\alpha_H}\right)^{\frac{1}{\nu}},$$  \hspace{1cm} (18)

where, with a slight abuse of notation, we use the subscripts $L$ and $H$ to denote a sector $n \in X_L$ and $n \in X_H$ respectively. If $\beta = 1$, it is immediate that the wage is higher in high demand sectors. The set of policies that we consider are those which redistribute income towards sectors with lower wages, i.e. we restrict attention to $\beta \geq 1$. Redistributive policies should also not make high demand sectors worse off than low demand sectors. We therefore impose $\beta < (\alpha_H/\alpha_L)$ under autarky and $\beta < (\alpha_H/\alpha_L)^{\frac{1}{\nu}}$ under an open economy. The closer to one is the ratio $c_L/c_H$, the more redistribution there is in the economy.

Before proceeding with the politico-economic equilibrium, we derive the indirect utility of a worker $j$ in sector $n$ from (1) and (2):

$$V_{jn}^S(\beta) = z_j u_n^S(\beta) = z_j D_n^S(\beta) R_n^S(\beta)$$  \hspace{1cm} (19)

$$D_n^S(\beta) \equiv \frac{c_n^S(\beta)}{I^S(\beta)} = \frac{(1 - \tau_0) \beta_n p_n^S(\beta)}{I^S(\beta)}$$  \hspace{1cm} (20)

$$R^S(\beta) \equiv I^S(\beta) \prod_{i=0}^{N} \alpha_i^{\alpha_i}(p_i^S(\beta))^{-\alpha_i}$$  \hspace{1cm} (21)
where $S \in \{A,T\}$ indexes whether we are considering the autarkic or small open economy case (“trade”). From the equilibrium derived in the previous sections, $p^A(\beta)$ and $I^T(\beta)$ are functions of the policy $\beta$.

Equation (19) decomposes the indirect utility of worker $j$ in sector $n$ between a common component to all workers in sector $n$, $u^S_n(\beta)$, and an idiosyncratic parameter representing the productivity draw of $j$ in $n$, $z_{jn}$. The common component $u^S_n(\beta)$, which is the real wage per effective unit of labor in sector $n$ (henceforth “real wage”) can further be decomposed into two parts. The first, $D^S_n(\beta)$, is the wage in sector $n$ as a fraction of total income. The impact of the policy on $D^S_n(\beta)$, which captures the extent to which workers in $n$ benefit from the policy relative to others, is the redistributive effect of the policy. The higher the $D^S_n(\beta)$, the more redistribution there is in the economy. If $D^S_L(\beta) > D^S_L(\beta')$, policy $\beta$ is more redistributive than $\beta'$. The second component, $R^S(\beta)$, is the total real income in the economy. The impact of the policy on this second component captures the distortive effect of the policy.

We first turn to the redistributive effect of the policy. Using (12), (14) and (20), $D^S_n$ is:

$$D^S_n(\beta) = (y^S_n(\beta))^{\frac{1}{\nu-1}} \Delta^{-\frac{1}{\nu-1}},$$

where $y^S_n$ is increasing in $\beta$ for low demand sectors and decreasing in $\beta$ for high demand sectors. A higher $\beta$ (more redistributive policy) is thus equivalent to a transfer from workers in high demand sectors to workers in low demand sectors. As evident from (22), this redistribution happens by sustaining output in low demand sectors and by decreasing output in high demand sectors compared to laissez faire. This compression of output distribution

\[\Delta\left(x_L \left(c^S_L(\beta)\right)^\nu + x_H \left(c^S_H(\beta)\right)^\nu\right)^{\frac{1}{2}} \quad \text{and} \quad D^S_L(\beta) = \Delta^{-1} \left(x_L + x_H \left(c^S_H(\beta) \over c^S_L(\beta)\right)\right)^{-\frac{1}{2}}.\]
across sectors implies a misallocation of resources in the economy.

$D^S_n$ depends on the output of sector $n$ in the same way in autarky and in the small open economy. This implies that if the equilibrium output vector is the same in autarky and in the open economy, the extent of redistribution is also identical in the two cases. From (12) and (14), however, an increase in $\beta$ raises output in low demand sectors proportionately more in a small open economy than in autarky. In autarky, the increased supply of good $n \in X_L$ puts a downward pressure on its price - and therefore on the wage in $n$ - thereby limiting the inflow of workers to $n$. This dampening effect of prices does however not occur in a small open economy, inducing more workers to work in $n$ than in autarky. A given nominal policy $\beta$ is therefore more redistributive in a small open economy as it has a stronger effect on the output vector.

**Lemma 1** Consider two policies, $\beta$ and $\beta'$, such that $\beta' = \beta^\nu$. The output vector and the share of total income accruing to each sector (the vector $D^S_n$) are the same under policy $\beta$ in a small open economy and under policy $\beta'$ in autarky.

We now express the total real income of the economy as a function of exogenous parameters and of the policy $\beta$, which is considered exogenous in the present section. In autarky and in a small open economy, these are respectively:

$$
R^A(\beta) = \prod_{i=0}^{N}(y^A_i(\beta))^{\alpha_i} = \Delta \zeta (x_L \alpha_L \beta + x_H \alpha_H)^{\frac{1}{\nu}} \beta^{\alpha_L x_L \frac{\nu-1}{\nu}}
$$

(23)

$$
R^T(\beta) = \zeta \left( \sum_{i=0}^{N} \alpha_i^{\frac{1}{\nu}} y^T_i(\beta) \right) = \Delta \zeta \frac{\alpha_L x_L \beta^{\nu-1} + \alpha_H x_H^\nu}{(\alpha_L x_L \beta^{\nu} + \alpha_H x_H)^{\frac{\nu-1}{\nu}}}.
$$

(24)

where $\zeta \equiv \prod_i \alpha_i^{\frac{\nu-1}{\nu}}$. In both cases, there is no other distortion than the redistributive policy and real income is maximised when $\beta = 1$. Any devi-
ation from the efficient allocation distorts the output vector and creates a misallocation of effective labor. $R^S$ captures the cost of this misallocation in terms of real national income. The first part of equations (23) and (24) shows that the mapping from a given vector of output to real income differs between autarky and the small open economy. The reason is that a change in the vector of production only distorts the supply side in the open economy while it also distorts the demand side in the closed economy\textsuperscript{16}.

To assess the distortive costs of a given effective redistribution in autarky and in the open economy, we compare the (log) real income which obtain for the same output vector in both situations:

$$\log(R^A(\beta)) - \log(R^T(\beta^{\frac{1}{\nu}})) = \alpha_Lx_L\left(\frac{\nu - 1}{\nu}\right)\log(\beta) - \log\left(\alpha_Lx_L\beta^{\frac{\nu - 1}{\nu}} + \alpha_Hx_H\right)$$

(25)

The derivative of the above expression with respect to $\beta$ - which corresponds to changing the output vector in the same manner for the autarkic and the open economy cases - is negative if $\beta > 1$. Since the right hand side of (25) is equal to zero for $\beta = 1$, $\log(R^A(\beta)) - \log(R^T(\beta^{\frac{1}{\nu}})) < 0$. The following Lemma summarizes these results:

**Lemma 2** A given increase in $y_L/y_H$ over its laissez-faire level decreases real income proportionately more in autarky than in an open economy.

Loosely speaking, Lemma 2 states that a given domestic output distortion is less costly for welfare when consumers do not only consume domestic goods.

\textsuperscript{16}This effect can best be seen by taking an extreme example. Assume that the labor allocation is such that no worker produces good $n$. In autarky, it implies that consumers cannot buy good $n$, driving their utility to zero. In an open economy on the other hand, consumers can still buy good $n$ at the world market price, ensuring that their utility remains positive.
4 Political Equilibrium

In the previous section we characterised the economic effects of government intervention via the nominal policy vector $\beta$. In this section, we endogenise the choice of policy via a political process (requiring economic equilibrium given policy choice) and define the winning policy as the one which beats all other policies in a pairwise comparison. We assume that all individuals in the economy vote on the policy $\beta$ and that voting takes place at time $t_1$, prior to the sectoral work choice by individuals. Redistributive policies therefore affect the sectoral choice of workers and give rise to economic distortions.

4.1 Pairwise policy comparison

The pairwise comparison of two policies is a non-trivial exercise as workers are heterogeneous after obtaining their distribution of sectoral productivity draws (even though the draws come from the same (Fréchet) distribution). When considering his utility under a given policy vector, each worker, knowing his own vector of sectoral productivity and the distribution of productivity in the population, correctly solves the economic equilibrium described in the previous section. In other words, each worker correctly anticipates in which sector he would work given a policy $\beta$, as well as the prices which would obtain under that policy. When comparing two policies, each individual therefore votes for the one giving him the highest utility, knowing that a deviation from the policy $\beta$ to some other policy $\beta'$ involves a change of worker allocation across sectors including his or her own choice. Formally, worker $j$, with productivity draw $z_{jn}$ in sector $n$ votes for policy $\beta$ over policy $\beta'$ if:

$$\max_n \{z_{jn} u_n(\beta)\} > \max_n \{z_{jn} u_n(\beta')\}$$

(26)
where \( u_n(\beta) \), defined in (19), is the real wage in sector \( n \) given policy \( \beta \). We aggregate this condition across all workers to determine the condition under which policy \( \beta \) is preferred by half of the population against policy \( \beta' \) in the following Proposition.

**Proposition 1** Define \( I_1 \) as the set of sectors \( i \) for which \( u_i(\beta) < u_i(\beta') \) and \( I_2 \) as its complement. Policy \( \beta \) wins over policy \( \beta' \) if and only if:

\[
\sum_{i \in I_2} (u_i(\beta))^{\nu} > \sum_{i \in I_1} (u_i(\beta'))^{\nu} \tag{27}
\]

**Proof:** See Appendix \( \blacksquare \)

Proposition 1 reduces the problem of determining which of two policies wins a vote to a condition involving only the sector-wide real wages \( (u_n(\beta)) \). This considerably reduces the dimension of the problem, as it allows to abstract from tracking the individual productivity draws \( (z_{ji}) \) and sectoral decisions.

Two elements play a role in determining which of two policies win. First, the number of sectors in which policy \( \beta \) is preferred to \( \beta' \) matters (the relative size of the sets \( I_1 \) and \( I_2 \)), as would be the case in models where voters are fully specific to a sector. Second, since workers have some degree of mobility between sectors, the mass of workers deciding to work in a particular sector depends on the real wage \( (u_n(\beta)) \) in that sector. If the sectors preferring \( \beta \) to \( \beta' \) offer a relatively high real wage, more workers are likely to work in these sectors and to vote for \( \beta \). To further clarify the importance of mobility in (27), consider the workers who choose sector \( i \in X_H \) under \( \beta' \). When deciding to vote for \( \beta \) to \( \beta > \beta' \), these workers not only consider whether sector \( i \) would benefit from policy \( \beta \), but also whether they should switch to a sector \( n \in X_L \), which benefits from \( \beta \) more than sector \( i \). If their expected
net income in $n$ under $\beta$ is higher than in sector $i$ under $\beta'$, they favor policy $\beta$ over $\beta'$. Equation (27) aggregates these choices to determine which policy wins. It is worth noting that a worker choosing sector $i$ under $\beta'$ would only switch to $n$ under $\beta$ if his draw of $z_{jn}$ is not too far from $z_{ji}$. The lower the heterogeneity of $z$ between sectors (the higher the $\nu$), the more the mobility matters and the more sensitive is the comparison between sectors. This is reflected by the exponents in (27).

4.2 Politico-economic equilibrium

In a first step, we ask what is the preferred policy of a worker given that it works in a low (respectively high) demand sector, i.e. what policy $\beta$ maximizes $u_L(\beta)$ (respectively $u_H(\beta)$), where we use the subscripts $L$ and $H$ to denote a sector $n \in X_L$ and $n \in X_H$ respectively. In a second step, we show that there exists a unique policy beating all others in a pairwise comparison.

First, we note that, both in autarky and in a small open economy, workers in high demand sectors favor policy $\beta = 1$ (no redistribution) over any other policy. Workers in high demand sectors are harmed from redistributive policies as they are net contributors to the government’s budget and lose from their distortive effect. Formally, this is obtained by showing that the first derivative of $u^S_H(\beta)$ is negative for any feasible $\beta$.

We then turn to workers in low demand sectors. The real wage in these sectors is given by $u^S_L = D^S_L R^S$ for $S \in \{A, T\}$, where $D^S_L$ obtains by plugging (12) and (14) in (22) and where $R^S$ is given by (23) and (24). In contrast to high demand sectors, workers choosing to work in low demand sectors benefit from the redistributive effect of the policy, although they lose from its distortive effect. For $\beta$ close to 1, the marginal redistribution is first-
order while the distortive effect is second-order, meaning that workers in low demand sectors want at least some redistribution. Two factors however limit the size of the redistributive policy from the perspective of workers in low demand sectors. First, the distortive effect of redistribution is convex in $\beta$, limiting the redistribution they wish to implement. Second, the wage in low demand sectors should not surpass that in high demand sectors. Setting $\partial u_s^R(\beta)/\partial \beta = 0$ shows the unique value of $\beta$, defined as $\beta^T_L$, that maximizes income per efficiency unit in low demand sectors under autarky and trade\textsuperscript{17}:

$$
\beta^A_L = \min \left( 1 + \frac{1}{(\nu - 1)\alpha_L x_L}, \frac{\alpha_H}{\alpha_L} \right)
$$

(28)

$$
x_H \alpha_H = x_L \alpha_L (\beta^T_L)^{\nu - 1} (\beta^T_L (\nu - 1) - \nu)
$$

(29)

where $\beta^T_L$ is defined as the minimum between the implicit solution to (29) and $((\alpha_H/\alpha_L)^{\nu - 1})^{1/\nu}$.

The above equations highlight two characteristics of the preferred redistributive policy for workers in low demand sectors. First, redistribution is less attractive the higher the mobility of labor. The reason is twofold: (i) the distortion of the output vector is stronger the larger the labor mobility and (ii) the redistribution is less strong under higher mobility as the inflow of workers into subsidised sectors limits the possible wage increase in these sectors. Second, $\beta^A_L$ is weakly decreasing in $\alpha_L x_L$. From the perspective of workers in $X_L$, the redistributive gain of the policy decreases in the number of workers in $X_L$, as the fraction of net contributors to the policy decreases.

The following Lemma offers a comparison of the preferred policy of workers in low demand sectors between autarky and the open economy.

\textsuperscript{17}To show that this value is unique, take the second derivative of $u_s^R(\beta)$ with respect to $\beta$ and plug $\beta^T_L$ in. The sign of this expression is negative, which implies that if the first derivative is equal to zero, the second derivative is negative and that $u_s^R(\beta)$ is single peaked.
Lemma 3 The bliss policy in low demand sectors is such that \( \beta_L^{A_L} \leq \beta_L^T < \beta_A^L \) where the first inequality is strict as long as \( \beta_A^L < \alpha_H/\alpha_L \).

**Proof:** See Appendix

The above Lemma shows that, in an open economy, workers in low demand sectors prefer a policy with a smaller nominal level but a greater redistribution (by Lemma 1) than in autarky. As shown in Lemma 2, a given redistribution is less costly to attain under a small open economy as the distortive effect of redistribution is dampened by the availability of foreign goods. By Lemma 1, on the other hand, since a smaller nominal policy is needed to obtain a given redistribution, workers in low demand sectors prefer a lower nominal policy rate than in autarky. Observing a drop in the subsidy rate towards certain sectors when an economy opens up to trade does therefore not necessarily mean that redistribution is less intense, and may in fact be associated with stronger redistribution\(^{18}\).

We now turn to the second step, which is to prove that there is a unique equilibrium policy given the parameters of the model and that this policy must be either \( \beta = 1 \) (the preferred policy of high demand sectors) or \( \beta = \beta_L^S \), the preferred policy in low demand sectors. The proof consists of two steps. First, consider the case where policy \( \beta_L^S \) beats policy \( \beta = 1 \), i.e. \( x_L(u_L(\beta_L^S))^{\nu-1} = -x_LU_L^2y_L^2/T^S \) evaluated at \( \beta_L^S \) rather than the subsidy rate \( \beta \). Using the equilibrium \( \beta_L^S \) as derived in (28) and (29) as well as the price, output and income functions derived in section 3 shows that \( \Theta^A(\beta_L^A) = x_H\alpha_H\nu \) and \( \Theta^T(\beta_L^T) = x_H\alpha_H\nu \left(x_L\alpha_L(\beta_L^T)^{\nu-1} + x_H\alpha_H\right)^{-1} < \Theta^A(\beta_L^A) \) if \( \beta_L^T \) and \( \beta_L^A \) are interior, and that \( \Theta^T(\beta_L^T) < \Theta^A(\beta_L^A) \) also holds if the policy rates are maximum. Just as the nominal rate, subsidies as a share of GDP are therefore larger in autarky than in the open economy although redistribution is less strong in autarky.

\(^{18}\)Empirical studies on the prevalence of industrial subsidies (e.g. Ford and Suyker (1990)) typically look at the amount of sectoral subsidies as a share of GDP (which is given in our model by \( \Theta^S(\beta_L^S) \equiv -x_LU_L^2y_L^2/T^S \) evaluated at \( \beta_L^S \) rather than the subsidy rate \( \beta \). Using the equilibrium \( \beta_L^S \) as derived in (28) and (29) as well as the price, output and income functions derived in section 3 shows that \( \Theta^A(\beta_L^A) = x_H\alpha_H\nu \) and \( \Theta^T(\beta_L^T) = x_H\alpha_H\nu \left(x_L\alpha_L(\beta_L^T)^{\nu-1} + x_H\alpha_H\right)^{-1} < \Theta^A(\beta_L^A) \) if \( \beta_L^T \) and \( \beta_L^A \) are interior, and that \( \Theta^T(\beta_L^T) < \Theta^A(\beta_L^A) \) also holds if the policy rates are maximum. Just as the nominal rate, subsidies as a share of GDP are therefore larger in autarky than in the open economy although redistribution is less strong in autarky.
$x_H(u_H(1))^\nu > 0$ by Proposition 1. Since $x_H(u_H(1))^\nu > x_H(u_H(\beta))^\nu$ for any $\beta > 1$ ($\beta = 1$ is the bliss policy in high demand sectors), policy $\beta_S^L$ strictly beats any other policy. A similar reasoning shows that if policy $\beta = 1$ beats $\beta_S^L$, it strictly beats any other policy. It remains to check under which condition $\beta_S^L$ wins over $\beta = 1$, a step conducted in the following Proposition, where $\chi_S$ is the unique value of $\alpha_L x_L$ for which $x_L(u_S^L(\beta_S^L))^\nu = x_H(u_S^H(1))^\nu$.

**Proposition 2** Equilibrium policy

For $S \in \{A, T\}$, the equilibrium policy is given by:

$$\beta_S = \begin{cases} 
1, & \text{if } \alpha_L x_L < \chi_S \\
\beta_L^S, & \text{if } \alpha_L x_L \geq \chi_S
\end{cases}$$

(30)

where $\beta_L^S$ is defined by (28) and (29).

**Proof:** See Appendix □

The product $\alpha_L x_L$ is a direct determinant of the mass of workers in sectors with low demand and needs to be large enough for the redistributive policy to win by majority voting. If this is the case, the winning policy is $\beta_S^L$ while there is no redistribution otherwise. We now turn to our main result, which compares the equilibrium policies under trade and autarky.

**Proposition 3** Comparison of policies under autarky and small open economy.

1. For any $\nu$, $\chi_T < \chi_A$, that is, redistribution is more “likely” to occur in the small open economy;

2. Conditional on redistribution taking place under autarky and in the open economy, the nominal policy is higher in autarky but redistribution is stronger in the open economy.
Proof: See Appendix ■

Under a small open economy, redistribution is more likely to take place and, conditional on redistribution happening, redistribution is stronger \( (D_L^T(\beta^T_L) > D_L^A(\beta^A_L)) \) than in autarky. As emphasized earlier, the inefficiency attached to a given redistribution is smaller under an open economy as the domestic distortion is less important when goods can be imported (Lemma 2). Proposition 3 shows that this lower inefficiency in an open economy raises equilibrium redistribution through two margins: (i) it translates into a broader political support for redistribution and (ii) it raises the level of redistribution conditional on it being supported by the median voter. Although seemingly intuitive, Part 1 of Proposition 3 would not obtain in a standard model with specific factors as it requires some degree of labor mobility across sectors. If workers were immobile, the fraction of workers supporting redistribution would be exogenously given by the fraction of workers in low demand sectors. Moving from autarky to trade would in this case not affect the likelihood that redistribution wins by majority voting. In our setup with labor mobility, however, some workers who vote against redistribution and work in high demand sectors under autarky prefer to vote for redistribution - as it is less inefficient - in an open economy and work in low demand sectors. This mechanism, which is stronger the higher the \( \nu \) is key to obtain changes in the political support for redistribution when opening to trade.

5 Extensions

5.1 Comparative advantage

We assumed in the previous sections that trade is “neutral” in the sense that the world prices in the open economy are equal to the equilibrium autarky
prices without redistribution ($\beta = 1$). In the present section, we relax this assumption and allow some sectors to benefit more than others from trade openness. We replace (17) by:

\[ p^T_L = \frac{I^A}{\Delta} \delta \alpha_L \quad p^T_H = \frac{I^A}{\Delta} \alpha_H \quad \forall n, \]  

(31)

where the previous analysis assumed that $\delta = 1$. To keep the analysis tractable, we assume that all low demand sectors have the same parameter $\delta$. If $\delta > (\leq) 1$, the country has a comparative advantage\(^{19}\) in the sectors for which it has a low (high) domestic demand. We further impose for simplicity that $\alpha_L \delta < \alpha_H$, which guarantees that $p^T_L < p^T_H$. In other words, sectors with a relatively high price under autarky still obtain a relatively high price in a small open economy. The ratio of wages in low and high demand sectors becomes:

\[ \frac{c^T_L(\beta)}{c^T_H(\beta)} = \beta \delta \left( \frac{\alpha_L}{\alpha_H} \right)^{\frac{1}{\nu}}. \]  

(32)

Replicating the previous analysis shows that the bliss policy for low demand sectors is $\beta^T_L(\delta)$, implicitly defined by:

\[ x_H \alpha_H = x_L \alpha_L \delta \left( \beta^T_L \right)^{\nu-1} \left( \beta^T_L (\nu - 1) - \nu \right). \]  

(33)

or by $\beta^T_L(\delta) = \delta^{-1} (\alpha_H / \alpha_L)^{\frac{1}{\nu}}$ if the value implied by (33) is larger. An increase in $\delta$ has a direct and an indirect effect on the relative wage in low demand sectors. The direct effect of trade is to raise the price of the low demand good, increasing the relative wage of low demand sectors. The indirect effect, obtained by totally differentiating (33), is to make policies less redistributive ($\beta^T_L(\delta) < 0$). The reason is that workers in low demand sectors

\(^{19}\)We define “comparative advantage” based on a comparison of world and autarky prices for $\beta = 1$, i.e. the home country has a comparative advantage in low demand goods if $p^A_L(1)/p^A_H(1) < p^T_L/p^T_H$.  

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are benefiting less from redistribution if the laissez-faire wage differences are smaller, which also explains why \( \beta^S_L \) in (28) and (29) is decreasing in \( \alpha_L \). As shown in Proposition 4, the direct effect dominates and an increase in \( \delta \) reduces the wage difference between high and low demand sectors. Although a higher \( \delta \) reduces the size of a winning redistributive policy, we show in Proposition 4 that it raises the likelihood that redistribution occurs. A high \( \delta \), by increasing the price \( p^T_L \), raises both the wage and employment in sectors \( n \in X_L \), which translates into more support for redistributive policies. The following Proposition summarizes these results.

**Proposition 4 Comparative advantage**

1. A higher \( \delta \) reduces the equilibrium policy \( (\partial \beta^T_L(\delta)/\partial \delta < 0) \), but raises the relative wage of low demand sectors in equilibrium \( (\partial (\delta \beta^T_L(\delta))/\partial \delta > 0) \)

2. Redistribution in a small open economy is more “likely” the larger the \( \delta \) \( (\chi^T_T(\delta) < 0) \).

3. There is a \( \delta^* < 1 \) below which redistribution is less “likely” in an open economy than in autarky \( (\chi^T_T(\delta) > \chi_A \) for \( \delta < \delta^* \)).

Part 3 of Proposition 4 makes for a particularly interesting case. Consider a country with a strong comparative advantage in the high demand sectors \( (\delta << 1) \) but a relatively large number of low demand sectors such that \( \chi^T_T(\delta) > \alpha_L x_L > \chi_A \). In autarky, the equilibrium policy is \( \beta^A_L \) and redistribution takes place. When opening to trade, such a country sees the relative wage in low demand sectors drop for two reasons: (i) a low \( \delta \) directly reduces the wage and employment in the low demand sectors, and (ii) the decrease in employment from the direct effect implies that the median voter does not
support redistributive policies anymore, creating a further decrease in wage and employment of low demand sectors.

5.2 More than two types of sectors

Although the previous sections depicted a world with two types of sectors, a large part of the analysis in sections 3 and 4.1 directly applies to more general distributions of $\alpha_n$. First, equations (1) to (24), which describe the setup and the economic equilibrium, hold for any distribution of $\alpha_n$ when defining $\beta$ as the vector of $\beta_n = (1 - \tau_n)/(1 - \tau_0)$ ($L$ and $H$ in (18) should also be replaced by $n$ and $n'$). An extended version of Lemmas 1, 2 and 3 also holds\(^{20}\) for any distribution of $\alpha_n$. Second, the condition derived in Proposition 1 for a policy to win over another by majority voting also holds for any distribution of $\alpha_n$.

Showing that there exists a unique equilibrium policy however requires additional assumptions once we allow for more than two types of sectors. If there are three types of sectors or more, a policy is a vector of $\beta_n$ which cannot be characterized by a unique parameter. To circumvent that problem but allow for more than two types of sectors, we assume that a redistributive policy takes the following form:

$$\beta_n = \left(\frac{\alpha_0}{\alpha_n}\right)^b,$$

where individuals vote over the policy parameter $1 > b > 0$. When voting on $b$, voters choose their preferred level of redistribution across sectors in the

\(^{20}\)The concept of redistribution has to be made more precise when considering an arbitrary distribution of $\alpha$, and we think of redistribution as a compression of the income distribution in the sense that the ratio of wages between any two sectors becomes closer to one. A precise description of how section 3 extends to any distribution of $\alpha$ can be found in Vannoorenberghe and Janeba (2013).
economy without being able to specifically target one sector. Plugging (34) in (11) and in $\beta_n p^T_n$ with $p^T_n$ defined by (17) gives:

$$\frac{c^n_A(b)}{c^n_A'(b)} = \left( \frac{\alpha_n}{\alpha_{n'}} \right)^{1-b} \quad \frac{c^n_T(b)}{c^n_T'(b)} = \left( \frac{\alpha_n}{\alpha_{n'}} \right)^{\frac{1}{\nu} - b}.$$ (35)

The larger the $b$, the more compressed is the distribution of effective wages across sectors, i.e. the more redistributive the policy. Maximum redistribution is attained for a parameter $b = 1$ in autarky and $b = 1/\nu$ in a small open economy, in which case the wages in all sectors are equalized. As in the previous sections, a given $b$ causes a stronger redistribution in an open economy than in autarky.

Assuming a policy function as in (34) is however not sufficient for the existence of a unique political equilibrium. To better understand the conditions under which a political equilibrium exists, consider two feasible policies $b_1$ and $b_2$ with $b_2 > b_1$ and denote the distribution of $\alpha$ by $F(\alpha)$ with support $[\underline{\alpha}, \overline{\alpha}]$. From (27), the condition for policy $b_2$ to win over $b_1$ is:

$$\int_{\overline{\alpha}}^{\underline{\alpha}} \left( u^S(\alpha, b_2) \right) \nu dF(\alpha) - \int_{\underline{\alpha}}^\alpha \left( u^S(\alpha, b_1) \right) \nu dF(\alpha) > 0 \quad (36)$$

where $\hat{\alpha}^S(b_1, b_2)$ is implicitly defined by $u^S(\hat{\alpha}, b_1) = u^S(\hat{\alpha}, b_2)$ if it is interior. For ease of exposition, we denote in the following low (high) demand sectors as those sectors with $\alpha < \hat{\alpha}^S(b_1, b_2)$ ($\alpha > \hat{\alpha}^S(b_1, b_2)$). An important condition to determine the existence and nature of an equilibrium is whether policy $b_2$ beats any policy lower than $b_1$ if (36) holds. Starting from a situation where (36) holds, it is sufficient for $b_2$ to beat a policy lower than $b_1$ that the derivative of the left hand side with respect to $b_1$ be negative, i.e. that:

$$2 \frac{\partial \hat{\alpha}^S(b_1, b_2)}{\partial b_1}(u^S(\hat{\alpha}, b_1))^\nu f(\hat{\alpha}) - \int_{\hat{\alpha}(b_1, b_2)}^{\overline{\alpha}} \frac{\partial (u^S(\alpha, b_1))^\nu}{\partial b_1} dF(\alpha) < 0. \quad (37)$$

**Intensive margin $> 0$**
When $b_1$ decreases, workers in sectors with high demand ($\alpha > \tilde{\alpha}(b_1, b_2)$) are more strongly in favor of $b_1$ as they on average gain from lower redistribution. Keeping $\tilde{\alpha}(b_1, b_2)$ constant (i.e. keeping the set of sectors in which $b_2$ is preferred to $b_1$ constant), a lower $b_1$ raises the average utility from working in a high demand sector, and induces some workers previously in low demand sectors to switch to high demand sectors and support $b_1$. We denote this effect the “intensive margin” as it keeps $\tilde{\alpha}_S(b_2, b_1)$ constant. The second effect arising after a decrease in $b_1$ is a change in the set of sectors favoring $b_2$ and $b_1$ (the “extensive margin”). For (37) to hold, a decrease in $b_1$ should raise $\tilde{\alpha}_S(b_2, b_1)$ sufficiently, i.e. increase the number of sectors preferring $b_2$ to $b_1$ sufficiently for $b_2$ to win.

In section 4, the inequality (37) is violated as the extensive margin is zero in the relevant\(^{21}\) range of policy $[1, \beta_S^L]$; for any two policies in that range, workers in high demand sectors prefer the less redistributive while workers in low demand sectors choose the more redistributive one. In that case, if $\beta_S^L$ beats $\beta = 1$, it beats any intermediate policy $1 < \beta < \beta_S^L$. This explains the form of the equilibrium in section 4, where the selected policy is either no redistribution or the bliss point of the low demand sectors. With a different assumption on the distribution of $\alpha$, however, we can impose that the extensive margin is negative enough to guarantee that if $b_2$ beats $b_1$, it beats any policy $b_0 < b_1$\(^{22}\).

**Proposition 5** Assume that the demand parameter $\alpha \in [(\kappa - 1)/\kappa, \infty)$ is Pareto distributed with density $f(\alpha) = \kappa^{1-\kappa}(\kappa-1)^{\kappa-1} \alpha^{-\kappa-1}$ and a shape parameter $\kappa > 2$. If $\nu \geq 1.101$, there exists a unique equilibrium policy which beats

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\(^{21}\)Policies larger than $\beta > \beta_S^L$ are strictly dominated by $\beta_S^L$ as all workers would prefer $\beta_S^L$ to them.

\(^{22}\)In a similar way, we check in the appendix 7.7 that when the extensive margin is negative enough, if $b_1$ beats $b_2 > b_1$, it beats any $b_3 > b_2$.  

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all others pairwise. It is given in autarky and in the small open economy respectively by:

\[ b^A = \min \left( (1 - \log(2))^\frac{\kappa - 1}{\nu - 1}, 1 \right) \] (38)

\[ b^T = \min \left( \frac{1 - \log(2) \kappa - 1}{\log(2) \frac{1}{\nu - 1}}, \frac{1}{\nu} \right) \] (39)

**Proof:** See Appendix  ■

The support of the Pareto distribution in Proposition 5 is chosen to ensure that \( E[\alpha] = 1 \) and preserve constant returns to scale in utility. The conditions on \( \kappa \) and \( \nu \) are sufficient to ensure that the extensive margin is negative enough and that (37) holds for any two feasible policies \( b_1 \) and \( b_2 \). A few remarks are in order. First, for the same reasons as in the case with two types of sectors, the equilibrium redistributive policy is smaller when workers are more mobile between sectors (higher \( \nu \)). Second, the larger the \( \kappa \), the lower the dispersion of \( \alpha \) across sectors and the larger the fraction of sectors below a certain demand parameter. A high \( \kappa \) means that low demand sectors are strong in the population, and choose a more redistributive policy. In section 4, with two types of sectors, a higher prevalence of low demand sectors (higher \( \alpha_L x_L \)) had two opposite effects (see (28) and (29)): (i) it tended to decrease the policy level \( \beta_{IL}^S \), but (ii) made it more likely that a redistributive policy was chosen. Under the Pareto assumption, both these effects are combined and reflected in the parameter \( \kappa \). The second effect dominates if the extensive margin is strong, meaning that a higher \( \kappa \) raises redistribution. Third, \( b^T > b^A \), which implies that, as with two sector types, redistribution is stronger in an open economy, i.e. the ratio \( c_n^S(b^S)/c_n^S(b^S) \) in (35) with \( b^S \) as defined in (38) and (39) is closer to one in an open economy than in autarky. The structure of the equilibrium policy is here different from section 4, as some redistribution takes place for any \( \kappa \) and the “likelihood”
that a policy is chosen is now one in a closed and an open economy. It should however be noted that the endogenous shifts in political support, which were governing the changes in likelihood in section 4 are still contributing to the increase in redistribution in the present case. Fourth, although the nominal policy $b^T$ is larger than $b^A$ for some parameters, it can be shown$^{23}$ that the share of GDP spent on transfers to subsidy recipients is weakly lower in an open than in a closed economy.

The results of Proposition 5 offer an interesting robustness test to our main results of section 4 and confirms that our model extends to more advanced settings, and in particular to cases where the “extensive margin” dominates the “intensive margin” on the whole range of feasible policies. For many distributions of $\alpha$, however, the “extensive margin” dominates only on some range, which can give rise to well-known issues of circularity and non-existence of equilibria when there are more than two types of sectors. A case by case approach is then needed to determine whether an equilibrium exists$^{24}$.

\section{Conclusion}

In this paper we have adopted a general equilibrium perspective on sectoral redistribution policies, which are prevalent in many countries and situations. The key theoretical tool was the modelling of partial labor mobility or sectoral specificity, which means that heterogenous workers respond to wage

\footnote{Proofs are available from the authors upon request.}

\footnote{Unreported numerical simulations show for example that, assuming three types of sectors (i.e. a distribution of $\alpha$ with three mass points), a unique equilibrium exists for a very large parameter range but that issues of non-existence can arise for some particular values of $\alpha$.}
differences across sectors, but not completely and not in the same way. Redistribution policies thus favor some workers, but at a cost of allocational inefficiency, which workers need to take into account when they vote over policies. This flexible framework allows us to compare policies at two extreme ends of openness to international trade: no trade and the small open economy with given world market prices. We expect that results for an open economy with some pricing power would lie between these two situations, although this claim would have to be proven in future research.

Our main findings make clear that the relationship between openness and redistribution is complex and multi-dimensional. We show that in political equilibrium redistribution is more likely to occur in the open economy in the sense that it takes fewer sectors with low demand parameters to make redistribution attractive compared to no trade. Yet, when redistribution takes place in both situations, the nominal or face value in the small open economy is smaller, even though the redistributive effect - once general equilibrium effects are taken into account - is larger. International trade increases the elasticity of labor demand, which magnifies the effects of a given redistribution policy, and reduces the inefficiency of a given labor misallocation because consumer prices are not (or less) distorted.

References


ARTUC, E., S. CHAUDHURI, AND J. MCLAREN (2010): “Trade shocks and


7 Appendix

7.1 Derivation of $L_n$ and $y_n$ in (9) and (10)

If worker $j$ receives a productivity draw $z$ in sector $n$, the probability that it is best to work in $n$ is the probability that the draws of $z_i$ in all others sectors are lower than $c_n z / c_i$:

$$G \left( \frac{c_n z}{c_i} \right) = \prod_{i \neq n} F \left( \frac{c_n z}{c_i} \right) = \exp \left( - (c_n z)^{-\nu} \left( \sum_{i \neq n} (c_i)^\nu \right) \right).$$

The supply of workers in sector $n$ is given by the integral over all $z$ of the probability that a draw of $z$ makes it optimal to work in $n$, i.e.:

$$L_n = \int_0^\infty G \left( \frac{c_i z}{c_n} \right) dF(z) = \frac{c_n^\nu}{\sum_{n=1}^N c_i^\nu}. \quad (41)$$

The supply of goods from sector $n$ is the total effective labor in $n$, i.e.:

$$y_n = \int_0^\infty zG \left( \frac{c_i z}{c_n} \right) dF(z) = \Delta c_n^{\nu-1} \left( \sum_{i=1}^N c_i^{\nu} \right)^{\frac{1-\nu}{\nu}}. \quad (42)$$

7.2 Proof of Proposition 1

The proof of Proposition 1 consists of three steps. In the first step, we derive the probability ($p_i(z_{ji})$) that worker $j$ prefers policy $\beta$ to $\beta'$ conditional on (i) the fact that sector $i$ is his preferred sector under policy $\beta'$ (ii) his
idiosyncratic productivity draw in $i$ is $z_{ji}$. In a second step, we relax the second part of the conditional exercise above and ask what is the probability ($p_i$) that a worker prefers policy $\beta$ given that he works in sector $i$ under policy $\beta'$. Finally, the third step derives the unconditional probability ($p$) that a worker prefers policy $\beta$ to $\beta'$. Since the economy consists of a continuum of agents, this is also the fraction of workers who prefer $\beta$ to $\beta'$.

- **Step 1**

Assume that, for worker $j$, $z_{ji}u_i(\beta') = \max_n\{z_{jn}u_n(\beta')\}$, meaning that under policy $\beta'$, worker $j$ works in sector $i$. In Step 1, we derive the probability that worker $j$ prefers policy $\beta$ over $\beta'$.

If $u_i(\beta) > u_i(\beta')$, worker $j$ prefers policy $\beta$ for sure (even if $i$ may not be the best choice of sector under policy $\beta$). If $u_i(\beta') < u_i(\beta)$ on the other hand, he finds policy $\beta$ better than $\beta'$ if for at least one sector $n \neq i$: $z_{jn}u_n(\beta) > z_{ji}u_i(\beta')$. Conditional on $i$ being the best sector for worker $j$ under policy $\beta'$, the probability that $j$ prefers policy $\beta_2$ to $\beta_1$ is:

$$p_i(z_{ji}) = \mathbb{I}\{u_i(B_2) < u_i(B_1)\} \left(1 - \prod_{n \neq i}^N \text{Prob}_c(u_n(B_2)z_{jn} < u_i(B_1)z_{ji})\right) + \mathbb{I}\{u_i(B_2) > u_i(B_1)\}$$

(43)

with $\mathbb{I}$ the indicator function and $\text{Prob}_c()$ the conditional probability operator $\text{Prob}(z_{jn}u_n(\beta_1) < z_{ji}u_i(\beta_1))$. $\text{Prob}_c(u_n(\beta)z_{jn} < u_i(\beta')z_{ji})$ is the probability that $z_{jn} < z_{ji}u_i(\beta')/u_n(\beta)$ given that $z_{jn} < z_{ji}u_i(\beta')/u_n(\beta')$. It is equal to one for all sectors $n \in I_1$. For sectors $n \in I_2$ it is:

$$\text{Prob}_c(u_n(\beta)z_{jn} < u_i(\beta')z_{ji}) = \exp\left(-(z_{ji}u_i(\beta'))^{-\nu}\left(\sum_{n \in I_2} (u_n(\beta))^\nu - (u_n(\beta'))^\nu\right)\right)$$

(44)
which implies:

\[ p_i(z_{ji}) = 1 - \mathbb{I}\{u_i(\beta') < u_i(B_1)\} \exp \left( -(z_{ji} u_i(\beta'))^{-\nu} \left( \sum_{n \in I_2} (u_n(\beta'))^\nu - (u_n(\beta'))^\nu \right) \right). \]  

(45)

• Step 2

In the second step, we ask what is the probability \( (p_i) \) that a worker prefers policy \( \beta \) given that he chooses to work in sector \( i \) under policy \( \beta' \). For a given \( z \) in \( i \), the probability that \( i \) is the best choice under \( \beta' \) is the probability that for all \( n \neq i \), \( z_{jn} u_n(B_1) \leq z u_i(B_1) \), which is \( \exp[-(z u_i(B_1))^{-\nu} \sum_{n \neq i} (u_n(B_1))^{\nu}] \). Integrating all \( z \) gives the probability that \( i \) is the best sectoral choice under \( \beta' \). We define the density function \( h_i(z) \) as the probability that a worker has productivity \( z \) in \( i \), conditional on \( i \) being its choice of sector under \( \beta' \):

\[ h_i(z) = \frac{\nu z^{-\nu-1} \exp(-z^{-\nu}) \exp \left( -(z u_i(\beta'))^{-\nu} \sum_{n \neq i} (u_n(\beta'))^{\nu} \right)}{\int \nu \zeta^{-\nu-1} \exp(-\zeta^{-\nu}) \exp \left( -\zeta u_i(\beta'))^{-\nu} \sum_{n \neq i} (u_n(\beta'))^{\nu} \right) d\zeta}. \]  

(46)

The conditional density \( h_i(z) \) shows that, if \( i \) is the best sector for a worker under \( \beta' \), the likelihood that the worker has drawn a given \( z \) depends on both (i) the unconditional likelihood to draw \( z \) (given by \( \nu z^{-\nu-1} \exp(-z^{-\nu}) \)) and (ii) the likelihood that \( z \) is sufficient to make \( i \) the best choice under \( \beta' \), which is larger the higher the \( z \). The probability \( (p_i) \) that a worker prefers policy \( \beta \) given that he chooses to work in sector \( i \) under policy \( \beta' \) is:

\[ p_i = \int p_i(z) h_i(z) dz. \]  

(47)

Using (45) and integrating gives:

\[ p_i = 1 - \mathbb{I}\{u_i(\beta') > u_i(\beta)\} \frac{\sum_{n=1}^{N} (u_n(\beta'))^{\nu}}{\sum_{n \in I_2} (u_n(\beta'))^{\nu} + \sum_{n \in I_1} (u_n(\beta'))^{\nu}}. \]  

(48)
• Step 3

To obtain the fraction ($p$) of workers who find policy $\beta$ better than $\beta'$, we take the sum of $p_i$ over all $i$ weighted by the likelihood that sector $i$ is the best choice of policy under $\beta'$\textsuperscript{25}. This yields:

$$p = \frac{\sum_{n \in I_2}(u_n(\beta))^\nu}{\sum_{n \in I_2}(u_n(\beta))^\nu + \sum_{n \in I_1}(u_n(\beta'))^\nu}.$$  \tag{49}

Setting $p \geq 1/2$ generates Proposition 1.

### 7.3 Proof of Lemma 3

Equation (29) can be rewritten as:

$$\left[\frac{1}{x_L\alpha_L(\nu - 1)} + 1 - (\beta_L^T)^\nu\right] + \frac{\nu}{\nu - 1}\left((\beta_L^T)^{\nu - 1} - 1\right) = 0. \tag{50}$$

By definition of a redistributive policy, $\beta_L^T \geq 1$, which implies that the left hand side above is weakly decreasing in $\beta_L^T$. At $\beta_L^T = 1$, the left hand side is positive. It must therefore be the case that $\beta_L^T > 1$ for the above equation to hold. If the square bracket were equal to zero, with $\beta_L^T$ and $\beta_L^A$ interior, we would have that $\beta_L^T = (\beta_L^A)^{\frac{1}{\nu}}$. In this case, the redistribution and the output vector would be the same in the open and in the closed economy case. The additional term to the square bracket on the left hand side is positive for $\beta_L^T > 1$ and reflects the fact that a given distortion of the output vector is less costly in an open economy. It must therefore be the case for the above to hold that\textsuperscript{26} $\beta_L^T > (\beta_L^A)^{\frac{1}{\nu}}$ if they are interior, while $\beta_L^T = (\beta_L^A)^{\frac{1}{\nu}} = (\alpha_H/\alpha_L)^{\frac{1}{\nu}}$ if they are not. To show that $\beta_L^T < \beta_L^A$, evaluate (29) at the value $\beta_L^T = \beta_L^A$ for $\beta_L^A$ interior and note that it is negative. Since the left hand side of (29) is decreasing in $\beta_L^T$, it must be the case that $\beta_L^T < \beta_L^A$.

\textsuperscript{25}This likelihood is equal to $(u_i(\beta'))^\nu/\sum_{n=1}^N(u_n(\beta'))^\nu$.

\textsuperscript{26}Note that if $\beta_L^A = \alpha_H/\alpha_L$, the inequality also holds as $\beta_L^T$ is larger than the shadow value of $\beta_L^A$ to the power $1/\nu$, and is therefore larger than $(\alpha_H/\alpha_L)^{\frac{1}{\nu}}$. 
7.4 Proof of Proposition 2

- Autarky

We define the function:

\[ G^A(\chi) = \frac{\chi^{\frac{1}{2}} (\beta^A_L)^{\frac{1}{2} + \frac{\chi - 1}{\nu}}}{\chi \beta^A_L + 1 - \chi} - (1 - \chi)^{\frac{1}{2}} \]  

(51)

where \( \beta^A_L \) is defined by (28) and is itself a function of \( \chi \). \( G^A(\chi) \) is equal to \( x^L u^A_L(\beta^A_L) - x^H u^A_H(1) \) divided by \( \zeta \) and is such that if \( G^A(\chi) \geq 0 \), policy \( \beta^A_L \) wins the majority of votes, while policy \( \beta = 1 \) wins if \( G^A(\chi) < 0 \). Since \( \beta^A_L \in (1, \alpha_H/\alpha_L] \), \( G^A(0) = -1 \) and that \( G^A(1) = 1 \). Differentiating \( G^A(\chi) \) gives:

\[ \frac{\partial G^A(\chi)}{\partial \chi} = \frac{1}{\nu} \frac{\chi^{1/2} (\beta^A_L)^{\frac{1}{2} + \frac{\chi - 1}{\nu}}}{(\chi \beta^A_L + 1 - \chi)^2} \left[ 1 + \chi (\beta^A_L - 1)(1 - \nu) \right] + \frac{1}{\nu} (1 - \chi)^{1/2 - \nu} \]

\[ + \frac{\chi^{1/2} \beta^A_L}{\chi \beta^A_L + 1 - \chi} \ln(\beta^A_L) \frac{\nu - 1}{\nu} (\beta^A_L)^{\frac{\chi - 1}{\nu}} + \frac{\partial G^A(\chi)}{\partial \beta^A_L} \frac{\partial \beta^A_L}{\partial \chi}. \]  

(52)

If \( \beta^A_L \) is interior, \( \partial G^A(\chi)/\partial \beta^A_L = 0 \) and the last product is equal to zero. If \( \beta^A_L \) is constrained by the upper bound \( \alpha_H/\alpha_L \), \( \partial \beta^A_L/\partial \chi = 0 \) and the last product is also equal to zero. The second line above is therefore positive. By definition of \( \beta^A_L \), we know that:

\[ \chi (\nu - 1)(1 - \beta^A_L) + 1 \geq 0 \]  

(53)

where the inequality is strict if \( \beta^A_L = \alpha_H/\alpha_L \), which ensures that \( \partial G^A(\chi)/\partial \chi > 0 \). \( G^A(\chi) \) is thus monotonically increasing on \( \chi \in [0, 1] \) with \( G^A(0) < 0 \) and \( G^A(1) > 0 \). This completes the proof of Proposition 2 for the autarkic case.

- Small open economy

Define:

\[ G^T(\chi, \delta) = (\chi \delta)^{\frac{1}{2}} \chi^\delta (\beta^T_L)^{\nu} + (1 - \chi) \beta^T_L \]

\[ - (1 - \chi)^{\frac{1}{2}} \]  

(54)
where $\beta^T_L$ is defined by (29) and is itself a function of $\chi$. $\delta$ is defined as in section 5.1 and allows to anticipate on the proof of the model extensions.

For the purpose of Proposition 2, $\delta = 1$. $G^T(\chi, \delta)$ is equal to $x^T_L u^T_L(\beta^T_L) - x^T_H u^T_H(1)$ divided by $\zeta$ and is such that if $G^T(\chi, \delta) \geq 0$, policy $\beta^T_L(\delta)$ wins the majority of votes, while policy $\beta = 1$ wins if $G^T(\chi, \delta) < 0$. Since $\beta^T_L(\delta) \in (1, (\alpha_H \delta_H / (\alpha_L \delta_L))^{1/\nu})$, $G^T(0, \delta) = -1$ and $G^T(1, \delta) = \delta^{1/\nu}$. Differentiating $G^T(\chi, \delta)$ with respect to $\chi$ gives:

$$\frac{\partial G^T(\chi, \delta)}{\partial \chi} = \frac{1}{\nu} \chi^{1-\nu} \beta^T_L \frac{\chi \delta \left( (\beta^T_L)^\nu - (\beta^T_L)^{\nu+1} \right)}{\delta \left( (\beta^T_L)^\nu + (1 - \chi) \right)} + \frac{(\chi \delta)^{\frac{1}{\nu}} \beta^T_L \left( (\beta^T_L)^\nu - (\beta^T_L)^{\nu+1} \right)}{(\chi \delta \left( (\beta^T_L)^\nu + (1 - \chi) \right)^2}$$

$$+ \frac{1}{\nu} (1 - \chi)^{\frac{1}{\nu}} \frac{\partial G^T(\chi, \delta)}{\partial \beta^T_L} \frac{\partial \beta^T_L}{\partial \chi}. \tag{55}$$

If $\beta^T_L$ is interior, $\partial G^T(\chi, \delta) / \partial \beta^T_L = 0$ and the last term drops out. If $\beta^T_L$ is constrained by its upper bound, on the other hand, $\partial \beta^T_L / \partial \chi = 0$. In both cases, the last term above drops out. By definition of $\beta^T_L(\delta)$:

$$\chi \delta \left( (\beta^T_L)^\nu - (\beta^T_L)^{\nu+1} \right) \geq 0 \iff \frac{(\beta^T_L)^\nu - (\beta^T_L)^{\nu+1}}{\chi \delta \left( (\beta^T_L)^\nu + (1 - \chi) \right)} \geq - \beta^T_L.$$  

Plugging the above inequality in (55) and rearranging, we obtain:

$$\frac{\partial G^T(\chi, \delta)}{\partial \chi} \geq \frac{1}{\nu} (\chi \delta)^{\frac{1}{\nu}} \frac{\delta \left( (\beta^T_L)^\nu - (\beta^T_L)^{\nu+1} \right)}{\chi \delta \left( (\beta^T_L)^\nu + (1 - \chi) \right) \left( (\beta^T_L)^\nu - (\beta^T_L)^{\nu+1} \right)} + \frac{1}{\nu} (1 - \chi)^{\frac{1}{\nu}}$$

$$\geq \frac{1}{\nu} \chi \delta \left( (\beta^T_L)^\nu + (1 - \chi) \right) \left( (\beta^T_L)^\nu - (\delta \chi \left( (\beta^T_L)^\nu + (1 - \chi) \right)^{\frac{1}{\nu}} \beta^T_L + 1 \right). \tag{57}$$

where the square bracket (of the form $c^\nu - c + 1$) is positive. $G^T(\chi, \delta)$ is therefore a monotonically increasing function in $\chi$ from $G^T(0, \delta) < 0$ to $G^T(1, \delta) > 0$ and there exists therefore a unique cutoff $\chi_T(\delta)$ such that policy $\beta = 1$ wins if $\chi < \chi_T(\delta)$ while policy $\beta^T_L(\delta)$ wins otherwise.
7.5 Proof of Proposition 3

The first part of Proposition 3 requires to show that $\chi_T < \chi_A$. Noting that, under the price assumption (17), $u_A^n(1) = u_T^n(1)$, we have: $G^T(\chi, 1) - G^A(\chi) = x_T^n \left( u_A^n(\beta_L^A) - u_T^n(\beta_L^T) \right)$. We now show that the following inequalities hold:

$$u_T^n(\beta_T^T) \geq u_T^n \left( (\beta_L^A)^{\frac{1}{\nu}} \right) > u_A^n (\beta_L^A), \quad n \in X_L.$$ (58)

The first inequality holds by definition of $\beta_T^T$. The second inequality holds by combining Lemma 1 with Lemmas 3 and 2. Policy $\beta_L^A$ in autarky and policy $\left( \beta_L^A \right)^{\frac{1}{\nu}}$ in a small open economy give rise to the same output and $D_n$ vector. Since the cost of the distortion is lower in a small open economy, utility is higher in that case. This implies that for any $\chi$, $G^T(\chi) > G^A(\chi)$ and in particular: $G^T(\chi_A) > G^A(\chi_A) = 0$. Since $G^T$ is increasing in $\chi$, it proves that $\chi_T > \chi_A$.

7.6 Proof of Proposition 4

From the Proof of Proposition 2, we know that the equilibrium policy in Proposition 2 also applies for $\delta \neq 1$, where $\chi_T$ and $\beta_T^T$ are functions of $\delta$ and where $\beta_T^T(\delta)$ is given by (33).

The first part of Proposition 4 is shown by totally differentiating (33), which gives:

$$\frac{d\beta_T^T}{d\delta} \frac{\delta}{\beta_T^T} = - \frac{\beta_T^T (\nu - 1) - \nu}{\nu (\nu - 1) (\beta_T^T - 1)} < 0.$$ (59)

The right hand side of the above equation is decreasing in $\beta_T^T$. For $\beta_T^T \to \infty$, the right hand side is $-1/\nu$, meaning that the left hand side is larger than $-1/\nu$. Since $\nu > 1$, it implies that $\partial(\delta \beta_T^T)/\partial \delta > 0$.

For the second and third parts of Proposition 4, we differentiate $G^T(\chi, \delta)$
with respect to $\delta$:

$$\frac{\partial G^T(\chi, \delta)}{\partial \delta} = \frac{(\chi \delta)^{1/2}}{\chi \delta \beta_L^{1/\nu} + (1 - \chi)} \left[ \frac{1}{\nu}(\chi \delta \beta_L^{1/\nu} + (1 - \chi) \beta_L^T) + \frac{(1 - \chi) \chi \beta_L^{1/\nu} (1 - \beta_L^T)}{\chi \delta \beta_L^{1/\nu} + 1 - \chi} \right],$$

where we used the fact that $\partial G^T(\chi, \delta)/\partial \beta_L^T = 0$ if $\beta_L^T = (\alpha H/\alpha L \delta)^{1/2}$ and $\partial \beta_L^T / \partial \delta = 0$ otherwise. Plugging the implicit definition of $\beta_L^T(\delta)$ given by (33) and rearranging, we obtain:

$$\frac{\partial G(\chi, \delta)}{\partial \delta} = \frac{(\chi \delta)^{1/2} + 1}{(\chi \delta \beta_L^{1/\nu} + 1 - \chi)^2} \beta_L^{2\nu - 1}(\beta_L^T - 1) > 0. \tag{61}$$

Combined with the fact that $\partial G(\chi, \delta)/\partial \chi > 0$, we obtain $\chi^T(\delta) < 0$. Using (54), we then show that for $\delta = 0$, $\chi^T = 1$, implying that there is a $\delta$ below which $\chi_T(\delta) > \chi_A$.

7.7 Proof of Proposition 5

Using $E$ to denote an expectation over $\alpha$, we write the real income per efficient unit of labor in sector $n$ under autarky and trade as:

$$u^A(\alpha_n, b) = \zeta^{1-b} \frac{\alpha_n}{E[\alpha^{1-b}]} = \zeta^{1-b} \frac{\alpha_n^{1-b}}{\kappa b (\kappa - 1)^{1-b}} \tag{62}$$

$$u^T(\alpha_n, b) = \zeta \frac{\alpha_n^{1-b}}{E[\alpha^{1-b}]} \left( E[\alpha^{1-b} + b] \right) = \zeta \alpha_n^{1-b} \left( \frac{\kappa - 1}{\kappa} \right)^b \frac{\kappa + b \nu - 1}{\kappa + b(\nu - 1) - 1} \tag{63}$$

where the second equality corresponds to the case of the Pareto distribution as defined in Proposition 5. Setting $u^S(\tilde{\alpha}^S, b_1) = u^S(\tilde{\alpha}^S, b_2)$ defines the function $\tilde{\alpha}^S(b_1, b_2)$ where the second equality corresponds to the Pareto case:

$$\tilde{\alpha}^A(b_1, b_2) = \zeta^{-\nu} \left( E[\alpha^{1-b_1}] \right)^{1/\nu_2 - \nu_1} = \zeta^{-\nu} \left( \frac{\kappa - 1}{\kappa} \right) \left( \frac{\kappa + b_2 - 1}{\kappa + b_1 - 1} \right)^{1/\nu_2 - \nu_1} \tag{64}$$

$$\tilde{\alpha}^T(b_1, b_2) = \left( E[\alpha^{1-b_1} \nu] / E[\alpha^{1-b_2} \nu - 1] \right)^{1/\nu_1 - \nu_2} = \frac{\kappa - 1}{\kappa} \left( \frac{\kappa + b_2 \nu - 1}{\kappa + b_1 (\nu - 1) - 1} \right)^{1/\nu_1 - \nu_2} \tag{65}$$
The proof of Proposition 5 requires to show that if a policy \( b_2 \) beats a policy \( b_1 \), it beats any \( b_0 < b_1 \), and that if \( b_1 \) beats \( b_2 > b_1 \), it beats any \( b_3 > b_2 \). These imply that the following two conditions should hold:

\[
2 \frac{\partial \tilde{\alpha}(b_1, b_2)}{\partial b_1} (u^S(\alpha, b_1))^{\nu} f(\tilde{\alpha}) - \int_{\tilde{\alpha}(b_1, b_2)} \frac{\partial (u^S(\alpha, b_1))^{\nu}}{\partial b_1} dF(\alpha) < 0 \tag{66}
\]

\[
2 \frac{\partial \tilde{\alpha}(b_1, b_2)}{\partial b_2} (u^S(\alpha, b_1))^{\nu} f(\tilde{\alpha}) + \int_{\alpha} \frac{\partial (u^S(\alpha, b_2))^{\nu}}{\partial b_2} dF(\alpha) < 0. \tag{67}
\]

In a first step, we show that both inequalities (66) and (67) hold under the assumptions of Proposition 5. In a second step, we derive the unique equilibrium policy (38) and (39).

- Step 1

To simplify notation, we define:

\[
\Delta^A = \kappa^{1-b_1} (\kappa - 1)^{\kappa} \left( \frac{\kappa + b_1 - 1}{(\kappa - 1)^{b_1 b_1}} \right)^{\nu} (\tilde{\alpha}^A)^{1-b_1-\kappa} \tag{68}
\]

\[
\Delta^T = \kappa^{1-\kappa} (\kappa - 1)^{\kappa} \zeta^{\nu} \left( \frac{\kappa - 1}{\kappa} \right)^{b_1 \nu} \left( \frac{\kappa + b_1 \nu - 1}{\kappa + b_1 (\nu - 1) - 1} \right)^{\nu} (\tilde{\alpha}^T)^{1-b_1-\kappa} \tag{69}
\]

where, from the definition of \( \tilde{\alpha}^S \), the \( b_1 \) on the right hand side of the above equations can equivalently be replaced by \( b_2 \). From the definition of \( \zeta \), furthermore:

\[
\text{Log}(\zeta) = \frac{\nu - 1}{\nu} \left( \int_{\kappa - 1}^{\infty} \alpha \text{Log}(\alpha) dF(\alpha) \right) = \frac{\nu - 1}{\nu} \left( \text{Log} \left( \frac{\kappa - 1}{\kappa} \right) + \frac{1}{\kappa - 1} \right) \tag{70}
\]

where the second equality obtains after integration by parts.

The extensive margin \( EX_i^S \), for \( i \in \{1, 2\}, S \in \{A, T\} \) are equal to:

\[
EX_i^A = 2 \frac{\Delta^A}{b_j - b_1} \left[ \text{Log} \left( \tilde{\alpha}^A \right) + \nu \text{Log} \left( \frac{\zeta \kappa}{\kappa - 1} \right) - \frac{\nu}{\kappa + b_1 - 1} \left( \kappa - 1 \right) \right] \tag{71}
\]

\[
EX_i^T = 2 \frac{\Delta^T}{b_j - b_1} \left[ \text{Log} \left( \frac{\kappa \tilde{\alpha}^T}{\kappa - 1} \right) - \frac{\nu}{(\kappa + b_1 (\nu - 1) - 1)(\kappa + b_1 \nu - 1)} \right] \tag{72}
\]
where \( j = 2 \) if \( i = 1 \) and \( j = 1 \) if \( i = 2 \).

To obtain the intensive margin in autarky, we differentiate \( u^A(\alpha, b) \) with respect to \( b \) and use integration by parts to show:

\[
INT_i^A \leq \frac{\Delta^A}{\kappa + b_i - 1} \left[ \frac{1 - \nu}{\kappa + b_i - 1} + \nu \log \left( \frac{\zeta \kappa}{\kappa - 1} \right) + \log(\hat{\alpha}^A) \right],
\]

(73)

where the inequality is weak for \( i = 1 \) and strict\(^{27}\) for \( i = 2 \).

To simplify notation, define \( \theta_i = \kappa - 1 + b_i \nu \), \( \tau_i = \kappa - 1 + b_i(\nu - 1) \) and \( \rho_i = b_j - b_i \). The sum of the extensive and intensive margin are therefore:

\[
EX_i^A + INT_i^A \leq \frac{\Delta^A \nu}{(b_j - b_i)^2} \left[ (2 + x) \log(1 + x) - 2x - \frac{\nu - 1}{\nu} x^2 \right].
\]

(74)

Under the assumption that \( \kappa > 2 \), \(-1 < x < 1\) and the right hand side above is negative as long as \( \nu > 1/(3(1 - \log(2))) = 1.0863 \), which is the condition for the right hand side of the equation above to be negative when \( x = 1 \).\(^{28}\)

Differentiating \( u^T(\alpha, b) \) with respect to \( b \) and using integration by parts for the small open economy:

\[
INT_i^T \leq \frac{\nu \Delta^T}{\kappa + b_i \nu - 1} \left[ \log \left( \frac{\hat{\alpha}^T \kappa}{\kappa - 1} \right) + \frac{b_i(\nu - 1)}{(\kappa + b_i \nu - 1)(\kappa + b_i(\nu - 1) - 1)} \right].
\]

(75)

To simplify notation, we denote: \( \theta_i = \kappa - 1 + b_i \nu \), \( \tau_i = \kappa - 1 + b_i(\nu - 1) \) and \( \rho_i = b_j - b_i \). The sum of the extensive and intensive margin is therefore:

\[
EX_i^T + INT_i^T \leq \frac{\Delta^T}{\rho_i^2} \left[ (2 + \frac{\nu \rho_i}{\theta_i}) \log \left( \frac{\theta_j \tau_i}{\theta_i \tau_j} \right) - 2 \frac{(\kappa - 1) \rho_i}{\theta_i \tau_i} + \frac{\nu b_i(\nu - 1)}{\theta_i^2 \tau_i} \right]
\]

(76)

\(^{27}\)The strict inequality for \( i = 2 \) comes from the fact that \( INT_2^A \) is equal to the right hand side of (73) plus a negative term equal to \( \Delta^A (\frac{\kappa - 1}{\kappa \alpha})^{1-\kappa} \) where we use (70).

\(^{28}\)If the right hand side of the inequality is negative for \( x = 1 \), it is negative for all \( x \). To see this, differentiate the square bracket 3 times with respect to \( x \), and note that (i) the third derivative is positive for any \( x \leq 1 \), (ii) the second derivative is negative for \( x = 0 \) and (iii) the first derivative is equal to zero for \( x = 0 \). We repeat a similar exercise for the small open economy with more details in the following.

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To show under which conditions the right hand side is negative\textsuperscript{29}, we differentiate the square bracket with respect to $\rho_i$ (which enters $\tau_j = \tau_i + \rho_i(\nu - 1)$ and $\theta_j = \theta_i + \rho_i\nu$) for a given $b_i$ and show that it is equal to zero for $\rho_i \to 0$. The second derivative of the square bracket with respect to $\rho_i$ can be shown to be negative at $\rho_i \to 0$ if $\kappa > 2$, which, by l’Hopital’s rule, shows that $EX_i^T + INT_i^T < 0$ for $\rho_i \to 0$. Finally, the third derivative of the square bracket with respect to $\rho_i$ is positive on the whole range $-b_1 < \rho_i < 1/\nu - b_1$ if $\kappa > 2$. This implies that $EX_i^T + INT_i^T < 0$ if $\rho_i < 0$ (the second derivative is negative at zero and, by the third derivative, also for any $\rho_i \leq 0$ meaning that the square bracket is smaller for any $\rho_i < 0$ than for $\rho_i = 0$, i.e. it is negative). On the range $0 \leq \rho_i < 1/\nu$ on the other hand, the square bracket is maximized either at $\rho_i = 0$ or at $\rho_i = 1/\nu - b_1$. It remains to check under what condition the square bracket is negative if $\rho_i = 1/\nu - b_1$. At $b_1 = 0$, the condition for the square bracket in (76) to be negative boils down to:

\[(2\kappa - 1) \log \left( \frac{\kappa}{\kappa - \frac{1}{\nu}} \right) - \frac{2}{\nu} < 0\]  

(77)

It can be shown that if the above inequality holds for $\kappa = 2$, it holds for any $\kappa > 2$. A sufficient condition for the above to be negative therefore boils down to $3\log(2/(2 - 1/\nu)) - 2/\nu < 0$, i.e. $\nu \geq 1.101$. This concludes step 1.

- Step 2

Step 2 showed that if a policy $b_2$ beats $b_1 < b_2$, it beats any policy $b_0 < b_1$ while if $b_2$ beats $b_3 > b_2$, it beats any $b_4 > b_3$. If there exists a policy $b$ which beats $b + \delta$ and $b - \delta$ for $\delta$ arbitrarily small, it must therefore be the unique policy equilibrium. A policy $b^S$ is therefore the unique policy equilibrium if,

\textsuperscript{29}We here only present the steps taken without carrying them out for the sake of space. All the steps of the analytical proof can be obtained from the authors upon request.
for $\delta \to 0$:

$$\int_{\frac{1}{\kappa}}^{\frac{\kappa}{1}} (u^S(\alpha, b^S))^{\nu} dF(\alpha) \geq \int_{\frac{1}{\kappa}}^{\frac{\kappa}{1}} (u^S(\alpha, b^S - \delta))^{\nu} dF(\alpha)$$

$$\int_{\frac{1}{\kappa}}^{\infty} (u^S(\alpha, b^S + \delta))^{\nu} dF(\alpha) \leq \int_{\frac{1}{\kappa}}^{\infty} (u^S(\alpha, b^S))^{\nu} dF(\alpha).$$

Plugging (62) and (63) in the above conditions and imposing that $\delta \to 0$, these boil down to:

$$\int_{\frac{1}{\kappa}}^{\kappa} \alpha^{1-b} dF(\alpha) = \int_{\frac{1}{\kappa}}^{\kappa} \alpha^{1-b} dF(\alpha)$$

and

$$\int_{\frac{1}{\kappa}}^{\infty} \alpha^{1-b\nu} dF(\alpha) = \int_{\frac{1}{\kappa}}^{\kappa} \alpha^{1-b\nu} dF(\alpha)$$

where $\tilde{a}^S (S \in \{A, T\})$ above corresponds to $\lim_{\delta \to 0} \tilde{a}^S(b, b + \delta)$. Using (64), (65), (70) and applying L'Hopital's rule shows that:

$$\lim_{\delta \to 0} \text{Log} (\alpha^A(b, b + \delta)) = \text{Log} \left( \frac{\kappa - 1}{\kappa} \right) - \nu - 1 + \frac{\nu}{\kappa + b - 1}$$

$$\lim_{\delta \to 0} \text{Log} (\alpha^T(b, b + \delta)) = \text{Log} \left( \frac{\kappa - 1}{\kappa} \right) + \frac{\kappa - 1}{(\kappa + b\nu - 1)((\kappa + b(\nu - 1) - 1))}$$

Plugging these values in (80) gives (38) and (39).