Consumer Debt and Default

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Florian Exler

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Abteilungssprecher: Prof. Dr. Jochen Streb
Referent: Prof. Michèle Tertilt, Ph.D.
Korreferent: Prof. Sang Yoon Lee, Ph.D.

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The behavior of individual consumptions, wealths, and portfolios is strongly at variance with the complete markets model implicit in the representative agent framework.

— Aiyagari (1994)

1 Preface

Whereas heterogeneous agent models with elaborate asset dynamics have claimed a prominent role in quantitative macroeconomics to better understand the link between wealth and income distributions and their impact on policy design, the role and evolution of debt has received relatively little attention. In the household sector, endogenous borrowing constraints, endogenous loan pricing and equilibrium default have only gained increasing attention fueled by the recent Subprime Crisis.

However, understanding how households use debt, why households default on their debt and how debt is priced holds genuine macroeconomic importance. In the absence of perfect financial markets, assets and debt are important tools for households to smooth consumption in the face of idiosyncratic risk. My work focuses on unsecured debt and – as presented in Table 1.1 – large fractions of households hold unsecured debt. In the U.S., more than 40% of households have some form of unsecured debt. This fraction is roughly 23% in Germany, but as high as two-thirds in Canada. Unsecured debt constitutes a significant fraction of total debt holdings. U.S. households hold roughly 13% of their debt as unsecured debt, Germans hold 24% as unsecured debt and Canadians hold nearly 30% of their debt as unsecured debt.

Default – potentially followed by debt relief – can not only provide valuable insurance for households in adverse situations; moreover, it also plays a central role when financial intermediaries price their debt. Future expected default rates are key to determining current credit price schedules. The possibility to default consequently influences all borrowers, not only those failing to service their debt.

The most important statutory way of debt relief granted to households is consumer bankruptcy. When designing consumer bankruptcy laws, policy-makers face a trade-off: on the one hand, if debts are easily forgiven, bankruptcy law grants partial insurance to those declaring bankruptcy; and on the other hand, default premia on interest rates rise in response to higher non-payment risk. This dissertation comprises three self-contained essays in which I analyze the effects of consumer bankruptcy laws on household borrowing, interest rates, default decisions, and welfare in different institutional settings.
1 Preface

Table 1.1: The Importance of Unsecured Debt

<table>
<thead>
<tr>
<th>Participation Rates</th>
<th>US</th>
<th>CA</th>
<th>FR</th>
<th>DE</th>
<th>IT</th>
<th>NL</th>
<th>ES</th>
<th>GB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit cards</td>
<td>39.4</td>
<td>24.8</td>
<td>n.a.</td>
<td>3.4</td>
<td>1.4</td>
<td>4.6</td>
<td>7.3</td>
<td>24.8</td>
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<tr>
<td>Overdrafts and credit lines</td>
<td>2.1</td>
<td>39.9</td>
<td>7.0</td>
<td>19.8</td>
<td>3.6</td>
<td>20.8</td>
<td>0.6</td>
<td>n.a.</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Fraction of Total Household Debt</th>
<th>US</th>
<th>CA</th>
<th>FR</th>
<th>DE</th>
<th>IT</th>
<th>NL</th>
<th>ES</th>
<th>GB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit cards</td>
<td>12.1</td>
<td>12.4</td>
<td>n.a.</td>
<td>2.3</td>
<td>3.0</td>
<td>1.1</td>
<td>3.4</td>
<td>9.8</td>
</tr>
<tr>
<td>Overdrafts and credit lines</td>
<td>0.7</td>
<td>15.9</td>
<td>8.3</td>
<td>21.7</td>
<td>6.2</td>
<td>13.4</td>
<td>0.5</td>
<td>n.a.</td>
</tr>
</tbody>
</table>

Source: Campbell (2016, Tables 1 and 2)

Chapter 2 analyzes whether and how to regulate small dollar lending in the U.S. For this purpose, I develop a structural model of unsecured lending where heterogeneous households can not only file for bankruptcy but also become delinquent. Introducing fixed cost of loan creation endogenously produces realistic interest rates of up to 300% for small loans. In the face of income and expenditure risk, households can partially insure through bankruptcy, which provides legally mandated debt relief. However, lump sum court fees and lawyer costs prevent low-income households from filing for bankruptcy protection. Without access to bankruptcy, these households become delinquent and pay late fees to avoid collection efforts from their banks. My quantitative results show that delinquency offers insufficient insurance against adverse events, granting room for welfare-improving policy interventions. In one such intervention, low-income households are allowed to repay bankruptcy filing costs after debt relief, making bankruptcy more affordable. I show that bankruptcy filings increase and delinquency decreases. Low-income households enjoy a 1% welfare increase, while aggregate welfare increases by 0.1%. The repayment plan proposed by the Consumer Financial Protection Bureau – which allows households to spread repayment over three periods – does not yield any welfare gains.

Chapter 3 evaluates the German bankruptcy system, which features relatively harsh wage garnishment. While the U.S. regime has a strong insurance component, many European systems are stricter in that they force delinquent households to repay (parts of) the outstanding debt through wage garnishment. Since wage garnishment raises the effective marginal tax rate, it exhibits adverse effects on labor supply. Explicitly modeling labor supply, this paper examines the optimal garnishment regime for the German economy and the resulting impact on credit prices. Under the optimal garnishment regime,
garnishment rates are reduced by more than 25%, while the duration of garnishment is increased from six to ten years. This results in an aggregate welfare increase of around 3.3%. Low-income households gain up to 7% since they have significantly better access to cheaper credit, which allows them to better smooth consumption over the life-cycle. High-income households already face favorable credit conditions and thus only gain 0.8%. Fully removing wage garnishment and moving to a “Fresh Start” regime similar to the U.S. raises the price of credit since lenders internalize the possibility of not being paid back. Young households suffer from restricted access to credit and the welfare of newborns decreases by roughly one percent.

Chapter 4 is joint work with Igor Livshits, James MacGee, and Michèle Tertilt. There is increased debate over whether the regulation of unsecured consumer lending products is required to protect some consumers from “over-borrowing.” To assess the quantitative benefits of regulating the cost of declaring consumer bankruptcy, we analyze a life-cycle model where some consumers have excessively optimistic beliefs about their exposure to unforeseen expenses. Building on Livshits, MacGee, and Tertilt (2010), we examine a heterogeneous agent incomplete market life-cycle model with bankruptcy. Over-optimists persistently believe that they face the same risks as realists, even though they are exposed to fundamentally higher expense risk. Competitive lenders are unable to directly distinguish between over-optimists and realists. However, they can observe a consumer’s present income, debt level and history of defaults. Lenders use this information to form a type score that represents the probability of whether a consumer is an over-optimistic or realist type. This results in partially pooling different types of households to whom lenders assign the same type score. Since lenders incorporate expected default risk in the bond price schedules that they quote consumers, over-optimistic households face lower interest rates and borrow more when they are pooled with realists. The opposite is true for realists. We calibrate the model to match aggregate bankruptcy filing rates, unsecured debt to income and average borrowing interest rates in the U.S. Using the calibrated model, we show that especially over-optimistic households suffer from a policy that introduces higher repayment requirements for households filing for bankruptcy.

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1Federal Reserve Bank of Philadelphia and BEROC. The views expressed here are those of the authors and do not represent those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.
2University of Western Ontario.
3University of Mannheim and CEPR.
Regulating Small Dollar Loans:
The Role of Delinquency

2.1 Introduction

Small, short-term loans are an important source of liquidity for many households. Especially sub-prime, low income households have little or no access to mainstream sources of credit such as credit cards or bank overdraft. Hence, financially constrained households turn to payday loans, deposit advance products, and other small dollar loans to gain access to credit. These loans are usually short-term, relatively small, and very expensive. Payday loans for example are due after two to four weeks and usually amount to less than $1,000. Interest rates as high as 20% per loan amount to annualized percentage rates (APRs) of up to 700%. Despite being very expensive, payday loans constitute a sizable market in the United States. Annual lending of $50 billion creates $8 billion in interest payments. About 12 million borrowers use payday loans each year. Although intended to provide short-term liquidity, payday loan users roll over increasing amounts of debt many times. More than 15% of loans are rolled over at least ten times.

High prices and numerous roll-overs sparked a lively policy debate on how to regulate small dollar loans. Current state-level legislation spans from not regulating small dollar loans at all to prohibiting small dollar loans completely.\(^1\) Nationwide regulation is absent, but the recently established Consumer Financial Protection Bureau (CFPB) has just proposed a first attempt to regulate such loans.\(^2\) The CFPB argues that small dollar loans are “consumer debt traps” not only because of their exorbitantly high price but also because fees, interest payments and the principal are all due in one balloon payment, which is unlikely to be paid back in full.\(^3\) Some consumer agencies contend that unaffordable balloon payments form an essential part of the business model of small dollar lenders. To


\(^2\)Small dollar lenders are of course subject to other, more general regulation applicable to any commercial credit supplier such as the Fair Credit Reporting Act or the Truth in Lending Act.

that end, lenders allegedly provide loans that households can never repay in full. Keeping households in debt, these lenders are presumed to generate profit through repeatedly charging late fees for unpaid loans.\(^4\) In an other line of argument, Pew Charitable Trusts (2013) is concerned that borrowers do not fully understand the contracts they are offered. However, the rationale for regulation remains unclear: While prices may be high, small dollar loans are effectively the only tool to smooth consumption that is available to low-income households.

I provide a structural framework to quantify the trade-off between partial insurance obtained through bankruptcy or delinquency and increasing credit prices in the small dollar lending market, both for low-income households and the economy in aggregate. The calibrated model is used to answer the question of whether and how to regulate small dollar loans. Risk-averse households borrow from risk-neutral banks, who accept deposits at the risk-free rate but issue debt at state-contingent prices. Households cannot commit on repaying outstanding debt. Consequently, default arises in equilibrium. In particular, households can use two types of nonpayment to partially insure against idiosyncratic risk: bankruptcy or delinquency. When households cannot afford bankruptcy filing fees, they become delinquent on their debt. Unlike bankruptcy, delinquency does not offer debt relief. Furthermore, delinquent households are subject to collection efforts by their lenders. Delinquency thus provides only limited insurance which means little opportunity to smooth consumption across states.

Contrary to the ongoing policy discussion, the underlying inefficiency in the small dollar loan market does not stem from the size of repayments, but from insufficient risk sharing while in delinquency. In the spirit of Aiyagari (1994), Bewley (1986), and Huggett (1993), households face idiosyncratic risk that is not directly insurable in the market. By introducing some state contingency to debt contracts, default can potentially increase welfare (c.f. Zame, 1993). In the United States, Chapter 7 bankruptcy provides households with an opportunity of debt relief. But filing for bankruptcy involves paying significant filing fees such as lawyer fees and court fees. Low income households, who are typical borrowers in the small dollar loan market, consequently cannot afford to file for bankruptcy. For these households, the only way to insure against income or expenditure risk is to become delinquent. Some households are “trapped” in debt long enough to repay more than they originally owed. That means that the unluckiest households can insure the least.

In order to provide better insurance for low income households, I introduce a bankruptcy advance for those households unable to pay the filing fees. Households using a bankruptcy

advance can file for bankruptcy, gain debt relief and only later repay the lump sum costs. Since low income households gain the outside option of walking away from their debt, delinquency becomes less harsh along both the extensive and intensive margin: Delinquencies drop by two thirds, and conditional on delinquency, banks extract less resources. Both effects increase the amount of insurance low income households can access. As a result, these households enjoy a 1% gain in consumption-equivalent welfare. Over all income groups, welfare increases by 0.1% on average with no group becoming worse-off due to the reform.

To evaluate the policy proposal put forward by the CFPB, a repayment plan is introduced in the economy. It is supposed to make repaying small dollar loans easier. Instead of repaying all debt in one large balloon payment, the plan spreads repayment over three periods. When using this plan, households cannot borrow on other small dollar loans. The proposal does not work; I find that households never opt into the repayment plan. In good times, households prefer to repay directly in order to retain the flexibility of choosing future debt or asset positions after learning about future income and unforeseen expenditures. In bad times, the repayment plan is not affordable and low income households still resort to delinquency.

I contribute to the consumer bankruptcy literature along two dimensions: Firstly, I add delinquency as a realistic nonpayment choice when bankruptcy is not affordable. Subject to limited commitment, banks try to optimally extract resources from delinquent borrowers. This mechanism makes delinquency especially harsh on households that do not have the outside option of officially filing for Chapter 7 bankruptcy protection. Secondly, I include per loan fixed cost when banks originate loans in a competitive market. These fixed costs generate realistically high interest rates for small loans.

To my knowledge, this paper is the first to provide a quantitative analysis of the small dollar loan market. There are some papers, however, that attempt to document the impact that payday loans have on households’ financial well being. Payday loans are the most prevalent form of small dollar loans. On the one hand, Morgan, Strain, and Seblani (2012) and Zinman (2010) document that payday loans help households to smooth consumption. On the other hand, Melzer (2011) and Skiba and Tobacman (2011) provide evidence that using payday loans makes households less likely to repay outstanding financial obligations. The payday lending market seems to be an alternative to mainstream lending through credit cards and overdraft. Using data from a payday lender matched to credit histories, Bhutta, Skiba, and Tobacman (2015) find that mainly financially constrained households take out payday loans. For a proper welfare analysis, however, it is necessary to structurally model how household choices and equilibrium outcomes influence household utility.
Notwithstanding, there exists a large empirical literature documenting important facts of the payday lending market. Although intended to provide short-term liquidity, payday loan users roll over increasing amounts of debt many times. More than 60% of newly created loans are rolled over at least once, while 15% of loans are rolled over at least ten times (Burke, Lanning, Leary, and Wang, 2013).

Flannery and Samolyk (2005) and Skiba and Tobacman (2007) provide evidence on the importance of fixed cost of loan creation. Even though prices are very high, payday lenders do not earn excess profits when compared to other lenders such as credit card companies. Rather than market power, per-loan fixed cost drive up prices for small short-term loans. Ernst and Young (2004) calculates that of the total cost in the payday lending industry, 75% are fixed cost while 20% are due to nonpayment.

Despite typically carrying three digit interest rates, small dollar loans are generated employing similar technology as larger unsecured loans. When comparing variable costs, small dollar loan businesses face costs of funds very similar to credit card lenders. In terms of per loan fixed cost, Stango (2012a) highlights that while absolute fixed costs per loan are comparable, fixed costs relative to loan size are much larger in the small dollar lending market simply because of smaller loan sizes. Mechanically, fixed costs per dollar lent decrease in the size of the loan.

Bankruptcy filing costs can be prohibitively high for low-income households. Using the increase in bankruptcy filing costs after the 2005 BAPCPA reform, Albanesi and Nosal (2015) document that low-income households remain delinquent longer and file for Chapter 7 bankruptcy less often. High-income households are not affected by the increase in filing cost. Similarly, Mann and Porter (2009) document that liquidity constraints bar low income households from filing for Chapter 7 bankruptcy. Gross, Notowidigdo, and Wang (2014) show that increased liquidity from tax rebates increases bankruptcy filings.

Households in my model behave rationally and are fully aware of the high costs of small loans. While this assumptions abstracts from problems when borrowers do not fully understand the contracts they are offered, there is little evidence for this. Bertrand and Morse (2011) find that only 10% of borrowers react to information treatments right before taking out payday loans. All other borrowers understand the cost of borrowing and do not adapt loan sizes at all. Using administrative data on an experiment conducted by a large American bank, Agarwal, Chomsisengphet, Liu, and Souleles (2015) find that borrowers correctly choose the credit contract that minimizes their cost on average.

I set up a quantitative limited commitment model of unsecured debt that features both, official Chapter 7 bankruptcy and delinquency. My model extends quantitative models of consumer bankruptcy, most notably Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007). Most models focus on bankruptcy
as the only nonpayment option, while I include the additional option of delinquency. Quantitative models of bankruptcy have been used to analyze many important policy questions: Mitman (2016) analyzes the interplay between bankruptcy and mortgage default regulation. Chatterjee and Gordon (2012) compare the welfare effects of bankruptcy protection vis-à-vis creditors directly garnishing income. Exler (2016) analyses how to reform income garnishment as a part of official consumer bankruptcy in Germany. Bankruptcy also provides insurance to potential entrepreneurs, see Akyol and Athreya (2011) and Mankart and Rodano (2015) for a setup with secured credit.

Athreya, Sánchez, Tam, and Young (2015) are most closely related to my setup. They also allow borrowers to informally default by simply not repaying and, hence, becoming delinquent on outstanding debt. Athreya, Sánchez, Tam, and Young document that a model with formal default through bankruptcy and informal default through delinquency does well in matching observed consumer credit patterns. I depart from their setup in two ways: (1) I introduce fixed cost in loan creation which produce realistic credit prices for small dollar loans. (2) I allow banks to optimally exploit the hold-up situation that arises after a household becomes delinquent. Banks can not only restructure the loan but also charge optimal late fees. This is not only a realistic assumption but also crucial to the amount of insurance delinquency provides. Households with no (or low) outside options are treated more harshly in delinquency than households that might simply file for bankruptcy. Furthermore, Herkenhoff (2012) employs a search framework to analyze partial repayment as a means of informal unemployment insurance. In Herkenhoff’s setup, households choose which fraction of debt to repay in order to insure against separation shocks.

The remainder of this paper is structured as follows: Section 2.2 presents the model. The calibration is discussed in Section 2.3. Section 2.4 describes the trade-off between bankruptcy and delinquency and documents that delinquency might “trap” unlucky households in debt. Subsequently, a repayment plan is introduced and the results of this policy experiment are discussed (Section 2.5). Section 2.6 describes the effects of introducing a bankruptcy advance. Section 2.7 concludes.

### 2.2 Model

The economy is populated by a continuum of infinitely lived households. Subject to individual earnings and expenditure risk, households maximize utility by choosing consumption and savings. Besides just repaying outstanding debt, individuals can choose to file for Chapter 7 bankruptcy or simply choose not to repay. In the latter case, they
become delinquent on their debt.

Financial intermediaries operate in a perfectly competitive market. They offer loan contracts dependent on loan size and household characteristics, subject to variable cost and fixed cost of loan creation. In the case of delinquency, banks maximize expected recovery by optimally choosing to levy late fees and restructure the loan contract.

### 2.2.1 Household Problem

The household state is fully described by individual asset holdings ($a_t$, where $a_t < 0$ is debt), individual earnings ($e_t$) and individual expenditure shock ($\kappa_t$). Earnings consist of a persistent component ($z_t$, modeled as an AR(1) process) and a transitory component ($\epsilon_t$, modeled as white noise). See Equation (2.8) for details. Expenditure shocks represent unforeseen expenditures that strain a household’s budget. They represent expenditures as caused by marital disruptions, the replacement of durables and large health care bills. These shocks are assumed to be uncorrelated across time.\(^5\) For brevity of notation, I will summarize the exogenous household states as $s_t = (z_t, \epsilon_t, \kappa_t)$ such that the full state simply reads $(a_t, s_t)$.

Households choose the sequence $\{c_t, p_t, a_{t+1}\}_{t=0}^{\infty}$ of consumption $c_t$, repayment mode $p_t$ and next period asset holdings $a_{t+1}$ to maximize the discounted sum of expected utilities

$$E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \mid a_0, s_0 \right].$$

Here, $u(c) = c^{1-\rho}/(1 - \rho)$ is a standard utility function featuring constant relative risk aversion. The problem is presented in recursive formulation, where $x'$ denotes the next period value of a variable $x$.

In each period, solvent households

1. observe their idiosyncratic earnings $e$, expenditure shock $\kappa$ and assets $a$,

2. optimally choose whether to stay solvent ($p(s, a) = S$), file for Chapter 7 bankruptcy ($p(s, a) = B$) or become delinquent ($p(s, a) = D$) and

3. choose consumption ($c(s, a)$) and savings (i.e. next period’s asset holdings, $a'(s, a)$) optimally.

---

\(^5\)This specification is standard in the consumer bankruptcy literature, see Livshits, MacGee, and Tertilt (2007, 2010). Unforeseen expenditures are frequently quoted as an important reason for default. Not paying unsecured loans (e.g. through filing for bankruptcy) thus partially insures households against these risks.
2.2 Model

Solvency

The value of solvency \((V^S)\) is presented in Equation (2.1):

\[
V^S(a, s) = \max_{c, a'} \left[ u(c) + \beta E_s V(a', s') \right]
\]

\[s.t. \quad c + q(a', s)a' = e + a - \kappa,\]

(2.1)

where \(E_s\) denotes the expectation of next period’s value conditional on the current household state \(s\). The budget constraint in solvency simply states that expenditures (consumption plus next period wealth) cannot exceed earnings \((e)\) plus initial wealth \((a)\) minus the expenditure shock \((\kappa)\). \(q(\cdot)\) denotes the bond price households are offered for saving or borrowing. \(q(\cdot)\) represents the inverse of one plus the interest rate.

Bankruptcy

If choosing to file for Chapter 7 bankruptcy, households have to pay lump-sum costs \(F\) and additionally suffer utility cost \(\zeta_B\). Since individuals can neither save nor borrow in the period of filing for Chapter 7 bankruptcy, they simply consume their endowment minus monetary filing costs. In the period following bankruptcy, all debt is forgiven \((a' = 0)\) and there are no further repercussions. Hence, next period’s value reads \(V(0, s')\). The value from filing for bankruptcy protection thus is

\[
V^B(a, s) = u(e - F) - \zeta_B + \beta E_s V(0, s').
\]

(2.2)

Assuming that individuals do not face negative consequences following a Chapter 7 bankruptcy abstracts from possible effects of bankruptcy on a household’s credit report. This is not a strong assumption, as Bhutta, Skiba, and Tobacman (2015) show that credit scores typically are not affected by filing for bankruptcy (but rather by previous failures to repay). Even low income bankrupts are not excluded from unsecured lending; Cohen-Cole, Duygan-Bump, and Montoriol-Garriga (2009) document that specialized lenders target unsecured credit at lower income households just after Chapter 7 bankruptcy. Additionally, Han and G. Li (2011) document that households still use small dollar loans after filing for bankruptcy. These loans are generated using sub-prime credit scores that do not respond to bankruptcy flags, either (Bhutta, Skiba, and Tobacman, 2015).

All remaining negative consequences of declaring bankruptcy are assumed to be captured by \(\zeta_B\). It is supposed to represent stigma of filing for bankruptcy and other adverse effects outside the model such as difficulties when renting an apartment or signing up for phone contracts.
Filing costs $F$ represent out-of-pocket expenses necessary to cover lawyer fees and court fees and have to be paid upfront when filing for Chapter 7. Due to these fees, filing for bankruptcy is very painful if not infeasible for low income households. Even if filing fees do not exceed labor income, the utility of forgone consumption for low income households is very high (due to the concavity of the utility function). Explicitly modeling lump-sum filing cost hence allows me to capture the fact that low income households are less likely to file for bankruptcy and rather stay delinquent. This observation is documented by Albanesi and Nosal (2015). Albanesi and Nosal find that increased monetary filing cost after the 2005 bankruptcy reform reduced Chapter 7 bankruptcies. This drop in bankruptcy filings leads to an increase in delinquencies. The effect is found to be most dominant for low income households.

**Delinquency**

If households choose to neither repay outstanding loans nor to officially file for bankruptcy, they become delinquent on their debt.\(^6\) In that case, creditors can restructure the outstanding loan and charge a late fee. Since I assume limited commitment, households cannot be forced to pay these fees, though. If households decide not to pay ($\nu(s, a) = 0$), households suffer collection efforts that create a utility loss. Payment of late fees arises endogenously, if households are better off paying the proposed fees than suffering collection efforts.

There are many legal ways for lenders to employ collection in order to inflict utility costs on borrowers: letters threatening legal consequences, calls to the debtors, the debtors’ employers or the debtors’ family members or in-person visits. Furthermore, payday lenders in particular threaten to cash the borrower’s post-dated check that was signed at origination. This would inflict further financial stress in the form of overdraft fees with the borrower’s bank, for example. Additionally, one might argue that some lenders also employ collection efforts outside the law.\(^7\)

---

\(^6\)To my knowledge, there is only one other paper that formally models delinquency. I depart from Athreya, Sánchez, Tam, and Young (2015)’s setup in allowing banks to recover as much as possible of any delinquent loan. In my setup, banks optimally levy late fees and restructure delinquent loans, subject to limited commitment on the household side. This is crucial when analyzing possible policy reforms since banks take advantage of the missing outside option of Chapter 7 bankruptcy for low income households. See Section 2.2.2 for more details.

\(^7\)See Hunt (2007) for an overview of debt collection practices in the US. Drozd and Serrano-Padial (2015, Appendix 1) provide detailed information on the influence of information technology on collection. The CFPB collects complaints about collection efforts that are perceived to exceed the legal boundaries set by the Fair Debt Collection Practices Act. For the full data, see https://data.consumerfinance.gov/dataset/Consumer-Complaints/s6ew-h6mp.
2.2 Model

$V^D$ defines the value of delinquency as

$$V^D(a, s) = \max_{c, \nu} [u(c) - (1 - \nu)\zeta_D + \beta \mathbb{E}_s V(\alpha(a', s'), s')]$$

s.t. $c = e - \nu L(s)$

$$\nu \in \{0, 1\}. \quad (2.3)$$

In delinquency, households choose how much to consume and whether to pay late fees ($\nu(a, s) = 1$) or not ($\nu(a, s) = 0$). If households do not pay late fees $L(s)$, they suffer collection efforts that induce a utility cost of $\zeta_D$.

In contrast to bankruptcy, delinquent debt is not written off by law. Since lenders are free to restructure delinquent debt contracts, next period’s debt holdings are set to $\alpha(a', s')$. The intermediaries’ problem of optimally setting $L(s)$ and $\alpha(a', s')$ is described in Section (2.2.2).

Complete Household Problem

The full household problem at the beginning of each period can be presented as

$$V(a, s) = \max_{p \in \{S, B, D\}} V^p(a, s). \quad (2.4)$$

After observing the period’s state $(a, s)$, households choose the payment mode $p(\cdot)$ that maximizes $V(a, s)$. Households choose between solvency ($p = S$), filing for bankruptcy ($p = B$) and delinquency ($p = D$). The corresponding Value Functions $V^S$, $V^B$ and $V^D$ are presented above, in Equations (2.1), (2.2) and (2.3). Note that, in certain states, some repayment modes might be infeasible. In particular, solvency is not feasible if income is sufficiently low or expenditure shocks are sufficiently high. Additionally, if income is lower than the lump-sum Chapter 7 filing costs, bankruptcy yields an empty budget set. I set the utility of empty budget sets to $u(c \leq 0) = -\infty$. In these cases, delinquency ($p(\cdot) = D$) is the only feasible alternative.

2.2.2 Banking Sector

The banking sector is assumed to be perfectly competitive with free entry and exit. Hence, banks price loans competitively, expecting zero profit for each contract offered. They can refinance externally, at the risk-free interest rate $r$. For each loan size and household state, banks form expectations over the probability of repayment and possible losses either because borrowers file for bankruptcy or because households become delinquent.
Debt Pricing Function

The pricing function is a function of the loan size and the household state when taking out a one-period loan. Both, \( a' \) and \( s \), govern the probability and size of repayment. Savings, on the other hand, are not risky and earn the risk free rate:

\[
q(a', s) = (1 + r)^{-1} \quad \forall a' \geq 0.
\]  

(2.5)

When creating loans, financial intermediaries face two types of transaction costs: fixed cost \( \delta \) and variable (proportional) cost \( \gamma \). These cost capture fixed per loan expenses such as labor cost of initiating a loan contract as well as variable operating expenses such as billing, payment processing and administration (c.f. Agarwal, Chomsisengphet, Mahoney, and Stroebel, 2015).

To my knowledge, this paper is the first paper in the consumer bankruptcy literature to introduce per loan fixed cost into debt pricing models. In the absence of market power, fixed cost are important to realistically capture the high prices observed for small loans. In standard models without fixed cost, interest rates only increase if default risk increases. In other words, credit spreads can only arise due to nonpayment risk. In order to capture the observed interest rates for small loans, nonpayment risk would thus have to be very large. In reality however, nonpayment risk only constitutes 30\% of credit spreads that are observed in the data (Skiba and Tobacman, 2007).

Let \( \bar{q} = (1 + r + \gamma)^{-1} \) denote the lender’s (constant) marginal cost of generating loans. Then, a risk-free loan of size \( \tilde{a}' \) would face the price

\[
q(\tilde{a}', s) = \bar{q} - \frac{\delta}{\tilde{a}'}.
\]

That means that households would receive a loan of size \( q(\tilde{a}', s)\tilde{a}' = \bar{q}\tilde{a}' - \delta \) and repay the face value of \( \tilde{a}' \) next period. Consequently, if \( q(\cdot) \to 0 \), the implicit interest rate on that loan approaches infinity. Vice versa, if \( q(\cdot) \to 1 \), the implicit interest rate on that loan approaches 0.

In equilibrium, bankruptcies and delinquencies occur and repayment is not certain. When banks take borrowers’ nonpayment decisions into account, risky loan prices evolve

\[\text{See Ernst and Young (2004) and Flannery and Samolyk (2005) for empirical evidence on the competitiveness of the small dollar lending market and further evidence on the importance of per loan fixed cost in the small dollar loan market.}\]
according to

\[ q(a', s) = \bar{q} \left( E[I_S(a', s') \mid s] \cdot 1 
+ E[I_B(a', s') \mid s] \cdot 0 
+ E[I_D(a', s') \frac{R(a', s')}{|a'|} \mid s] \right) - \frac{\delta}{|a'|} \]  

(2.6)

where \( E[\theta(a', s')^p \mid s] \) denotes the conditional probability of a household in state \( s \) that receives a loan of size \( a' \) to choose repayment mode \( p \) next period.

In the case of solvency (\( I_S(\cdot) = 1 \)), banks are repaid in full, hence the first line in Equation (2.6) is multiplied by 1. When filing for bankruptcy (\( I_B(\cdot) = 1 \)) though, banks retrieve nothing, hence these cases are multiplied by 0. Finally, when households become delinquent (\( I_D(\cdot) = 1 \)), banks recover \( R(a', s') \), as will be described in Problem (2.7). For a loan of size \( a' \), financial intermediaries thus expect to recover the fraction \( R(a', s')/|a'| \) in delinquency.

**Optimal Recovery in Delinquency**

If households become delinquent on a loan, banks can charge late fees \( L(s) \) and restructure the loan to hold a face value of \( \alpha(a, s) \) next period. When charging fees, banks take into account the limited commitment of households which can always choose not to pay. Hence, the utility cost of nonpayment restricts the amount of fees banks can charge in a given period.

When restructuring, banks are free to reduce the face value of the loan, subject to the households’ repayment behavior. That means banks can, on the one hand, decrease the payable amount to incentivize households to repay. On the other hand, banks can keep the payable amount as is in order to make repayment less likely and extract more resources from debtors through fees. Restructuring loans amounts to banks optimally adapting the size of outstanding debt, subject to the risky loan price schedule. Hence, banks set late fees (\( L \)) and reset the outstanding loan to size \( \alpha \) to maximize expected repayment. They solve the problem

\[ R(a, s) = \max_{L,\alpha} L + q \cdot (-\alpha) \]

s.t. \( u(e - L) \geq u(e) - \zeta_D \)

\[ q = q(\alpha, s') \]

\[ \alpha \geq a' \]  

(2.7)
Problem (2.7) states that, conditional on delinquency, banks maximize current and expected future returns. In choosing late fees and next period’s face value optimally, banks are subject to limited commitment on the household side in two ways: Firstly, households can choose to not pay any fees and rather suffer the utility cost $\zeta_D$ resulting from collection efforts. Hence, banks can only levy fees up to an amount that makes households indifferent between paying and not paying ($u(e - L) \geq u(e) - \zeta_D$). Secondly, setting next period’s face value, banks take into account that household can always choose between repayment, bankruptcy or delinquency. Banks thus discount next period’s face value $\alpha$ with the risky rate $q(\cdot)$. As described above, $q(\alpha, s')$ reflects the household’s expected repayment behavior in the following period, given the reset face value and the observed household state.

Finally, I assume that banks are not allowed to increase outstanding debts, hence $\alpha \geq a'$. Otherwise, for some incomes, it might be optimal to set $\alpha \to -\infty$ in order to ensure delinquency and thus late fee payment in all future states of the world (except when bankruptcy becomes available due to a steep increase in income). This is neither legal nor observable in reality, hence I restrict restructured debt $\alpha$ to be weakly less than the original amount owed.

Small Dollar Loans are issued employing the same technology as other unsecured loans. Since these loans are typically smaller than other unsecured credit, per loan fixed cost have a larger effect on their price. Loans carrying APRs in excess of 30% or loans not exceeding the amount of $1,000 are interpreted as Small Dollar Loans. This definition will be discussed in detail in Section 2.3.2.

### 2.2.3 Equilibrium

Given a risk-free rate $r$ and an income process $e$, a financial market equilibrium is the set of value functions $V$, $V^S$, $V^B$ and $V^D$, policy functions $c(\cdot)$, $a'(\cdot)$, $p(\cdot)$, conditional repayment probabilities $E[I_p(\cdot) | s]$, $p \in \{S, B, D\}$, a recovery function $R(\cdot)$ and a debt pricing function $q(\cdot)$ such that:

1. Households maximize $V$, $V^S$, $V^B$ and $V^D$, where $c(a, s)$, $a'(a, s)$, $p(a, s)$ are the resulting optimal policy functions.

2. The bond price $q(a', s)$ is determined in a competitive market with free entry, taking as given the expected default behavior $E[I_p(a', s') | s]$ and the expected recovery in delinquency $E[R(a', s') | a', s]$.

3. The measure of households over states $(a, s)$ is constant.
2.3 Calibration

The model has 16 free parameters of which nine are set exogenously. The remaining seven parameters are jointly determined in order to match important data moments. These moments describe the United States’ economy after the recent financial crisis.

2.3.1 Direct Specification

In order to capture the typical short duration of small dollar loans, the period length of the model is set to quarters. Households’ coefficient of relative risk aversion is set to a standard value of \( \sigma = 2 \).

The income process \( e = \hat{e} \cdot y \) represents idiosyncratic income that households earn. It has a constant component \( \hat{e} \) that represents the median endowment in the economy. Additionally, \( y \) represents idiosyncratic income risk that households face. It is defined as the residual of regressing household income on observables such as age and education. Following standard assumptions in the literature, it is composed of a persistent AR(1) process \( z_{i,t} \) and transitory white noise \( \varepsilon_{i,t} \). For household \( i \) at time \( t \) it is given by

\[
\log (y_{i,t}) = z_{i,t} + \varepsilon_{i,t}
\]

\[
z_{i,t} = \varrho z_{i,t-1} + \eta_{i,t},
\]

where \( \varrho \in [0,1], \varepsilon \sim N(0,\sigma^2_\varepsilon) \) and \( \eta \sim N(0,\sigma^2_\eta) \). Using data from the PSID, Storesletten, Telmer, and Yaron (2004) estimate the auto correlation coefficient \( \varrho \) and the standard deviations of \( \varepsilon \) and \( \eta \). I convert their values to quarterly values. To that end, I assume that Storesletten, Telmer, and Yaron’s annual process is the result of aggregating a quarterly income process. I report the quarterly values in Table 2.1. Median quarterly income is set to \( \hat{e} = \$7,200 \) as reported by the Social Security Administration (SSA).

The risk-free interest rate at which banks can refinance externally is set to \( r = 0.5\% \) quarterly. This roughly translates to an annual real interest rate of 2\%.\(^9\) Transaction cost for creating loans are set to \( \gamma = 0.7\% \), which corresponds to roughly 3\% p.a. and is in line with the operational cost of 3.4\% estimated by Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015). Lending fixed cost are set to \( \delta = \$25 \), as documented by Flannery and Samolyk (2005).

Out-of-pocket expenses when filing for bankruptcy \( (F) \) are set to \( \$2,500 \), following the analysis by Albanesi and Nosal (2015). These expenses comprise of court fees and lawyer fees, both of which are unavoidable if a household files for Chapter 7 bankruptcy.

\(^9\)See https://www.ssa.gov/oact/cola/central.html.
Table 2.1: Exogenous Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period Length</td>
<td>Quarters</td>
<td></td>
</tr>
<tr>
<td>CRRA Consumption ( \sigma )</td>
<td>2</td>
<td>standard</td>
</tr>
<tr>
<td>Median Wage ( \hat{e} )</td>
<td>$7,200</td>
<td>SSA (2014)</td>
</tr>
<tr>
<td>Wage Autocorrelation ( \varrho )</td>
<td>0.99</td>
<td>Storesletten, Telmer, and Yaron (2004)</td>
</tr>
<tr>
<td>Persistent Wage Var ( \sigma^2_\eta )</td>
<td>0.007/3.86</td>
<td>Storesletten, Telmer, and Yaron (2004)</td>
</tr>
<tr>
<td>Transitory Wage Var ( \sigma^2_\varepsilon )</td>
<td>0.04/4</td>
<td>Storesletten, Telmer, and Yaron (2004)</td>
</tr>
<tr>
<td>Risk Free Rate ( r )</td>
<td>0.5% &amp; approx. 2% p.a., World Bank (2015)</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost ( \delta )</td>
<td>$25</td>
<td>Flannery and Samolyk (2005)</td>
</tr>
<tr>
<td>Variable Cost ( \gamma )</td>
<td>0.7%</td>
<td>Agarwal et al. (2015b)</td>
</tr>
<tr>
<td>Bankruptcy Cost ( F )</td>
<td>$2,500</td>
<td>Albanesi and Nosal (2015)</td>
</tr>
</tbody>
</table>

2.3.2 Simulated Method of Moments

After setting nine parameters exogenously, seven parameters remain to be determined jointly. There are three utility parameters: the discount rate \( (\beta) \), the harassment cost in delinquency \( (\zeta_D) \) and stigma cost of bankruptcy \( (\zeta_B) \). Following Livshits, MacGee, and Tertilt (2010), I assume the expenditure shock to take two non-zero values. Hence, there are four additional parameters: two parameters governing the size \( (\kappa_1, \kappa_2) \) and two parameters governing the respective realization probabilities \( (P(\kappa_1), P(\kappa_2)) \). These parameters (summarized by \( \theta \)) are set to minimize the percentage deviations between model moments and data

\[
\min_\theta \sum_i \left( \frac{M_i(\theta)}{D_i} - 1 \right)^2. \tag{2.9}
\]

Here, \( M_i(\theta), i = \{1, \ldots, 7\} \) represent seven model moments as a function of the seven free parameters \( \theta \). \( D_i, i = \{1, \ldots, 7\} \) represent the corresponding data moments. The model and data moments are reported in Table 2.2; Table 2.3 presents the parameter estimates.

The debt to income ratio for unsecured debt relative to quarterly household income of roughly 35% is slightly overestimated by the model. On the other hand, the fraction of households holding debt is underestimated. The model captures equilibrium delinquencies and bankruptcies well, with 1.2% of households being delinquent in any given quarter and 0.3% of households declaring bankruptcy per quarter.

Following the empirical evidence presented in Ernst and Young (2004), Skiba and Tobacman (2007), and Stango (2012a), my model features one lending technology to generate unsecured credit. I use two definitions to define small dollar loans: Firstly, Small Dollar Loans are defined as all loans featuring APRs higher than 30%. Credit cards, which provide the main source of unsecured debt in the US, almost always have rates
## 2.3 Calibration

### Table 2.2: Data Fit

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Debt/Income</td>
<td>35.6%</td>
<td>28.9%</td>
<td>SCF (2013)</td>
</tr>
<tr>
<td>Fraction of HH in debt</td>
<td>19.1%</td>
<td>37.7%</td>
<td>SCF (2013)</td>
</tr>
<tr>
<td>Delinquencies</td>
<td>1.2%</td>
<td>1.7%</td>
<td>FRBNY/Equifax</td>
</tr>
<tr>
<td>Bankruptcies</td>
<td>0.3%</td>
<td>0.3%</td>
<td>SCF (2013)</td>
</tr>
</tbody>
</table>

| Small Dollar Loan APR (1) | 128% | 300% | Skiba et al., 2007     |
|                          | (2)  | 121% |                      |
| Credit Card APR (1)      | 12.7%| 17.5%| Stango et al., 2016   |
|                          | (2)  | 12.5%|                      |
| Default Premium         | 33.1%| 30%  | Skiba et al., 2007    |

Note: All moments represent quarterly values. SCF (2013) are the author’s calculation. Average debt to income is the average of each household’s ratio of unsecured debt holdings relative to quarterly household income. Fraction of households in debt is the fraction of households that hold any positive amount of unsecured debt. See Appendix 2.A for more details on the calculation of these moments. The default premium is defined as the credit price spread due to default risk (in contrast to the spread due to lending costs). FRBNY/Equifax is the Federal Reserve Bank of New York’s “Consumer Credit Panel / Equifax” data as estimated by Athreya, Sánchez, Tam, and Young (2015).

lower than 30%. Using a large administrative data set, Stango and Zinman (2016, Table 2) documents that 90% of revolving credit card balances feature APRs of less than 28%. Hence, a cut-off of 30% seems reasonable. Secondly, all loans smaller than $1,000 are defined as small dollar loans. According to Skiba and Tobacman (2008), payday loans generally do not exceed this size. When calibrating the model, I will show statistics using both definitions:

1. **APR**: All loans featuring APRs higher than 30% are defined as small dollar loans.

2. **Size**: All loans smaller than $1,000 are defined as small dollar loans.

In the calibration, I target the APR-based definition (1). I also report the APRs generated when using definition (2) in order to show that I am indeed capturing small loans that carry interest rates in excess of 30%.

Featuring realistic fixed cost of loan creation, the model is able to partially capture high APRs observed in the small dollar lending market according to both definitions presented above. Even though my framework falls short in terms of magnitude, it provides a much better fit than setups without any fixed cost in lending. These setups typically produce very low interest rates for the smallest loans.

Also due to fixed cost of loan creation, the model is able to match the default premium of credit prices very well (33% vs. 30% in the data). The default premium is defined as
the fraction of credit spreads that arises due to nonpayment risk rather than operating cost. In models without any fixed cost in lending, risk based credit spreads account for nearly 100% of credit spreads, since only variable lending costs (i.e. costs of funds) drive a small wedge between the risk free rate and the borrowing interest rate.

Finally, credit card interest rates are matched reasonably well according to both definitions. They remain slightly too low, though.

The endogenous parameters to generate these moments are shown in Table 2.3. The discount factor is $\beta = 0.975$. Utility cost in delinquency and bankruptcy are 7.2 and 7.4. In terms of dollar values, these correspond to the utility loss induced by reducing a median income household’s assets from $0 to $-30,000. Since households in the lowest income bin have much higher marginal utility, this amount is only $-2,800 for them.

The expenditure shocks estimated in the simulated method of moments are smaller but more likely than those estimated in Livshits, MacGee, and Tertilt (2010). Since these shocks are assumed to be i.i.d., the expected realization per period is simply $\sum_{i=1,2} P(\kappa_i)\kappa_i$. In my calibration, households are on average hit by unforeseen expenditures that amount to 13% of median income whereas households in Livshits, MacGee, and Tertilt on average have to cover 40% of median income. This is mainly because Livshits, MacGee, and Tertilt feature a very large health shock that exceeds the large shock in Table 2.3 by an order of magnitude.

### 2.3.3 Untargeted Moments

This section compares the model to data along non-targeted dimensions. I document cross-sections of credit prices with respect to loan size and borrowers’ income. Fixed cost of loan creation generate credit prices that match the cross-section of the data well. The model can thus be used to derive reliable implications when analyzing two potential policy reforms in the following sections.
2.3 Calibration

![Graph showing mean APR by loan size quintiles](image1)

(a) Data (SCF, 2013)

Figure 2.1: Average Credit Prices (APR), by loan size

Even though not directly targeted in the calibration, the model produces realistic price schedules for small dollar loans. Fixed cost of loan creation drive up interest rates for small loans, even in the absence of default risk. In standard models without fixed cost of loan creation, only default risk can increase interest rates on loans. This usually leads to small loans carrying counterfactually low interest rates. Since default risk is low for small loans, interest rates are low for small loans, too.\(^{11}\)

Using data from the SCF (2013), Figure 2.1a depicts a decreasing relationship between interest rates and unsecured loan size. I not only represent credit card debt in this figure but also all “other loans” reported in the SCF that are unsecured and not for business purposes. “Other loans” are any kind of loans a household hold besides credit card debt, mortgage debt or home equity lines of credit. Households may hold more than one loan, since each loan size–APR bundle is treated as one observation.\(^{12}\) Average interest rates only increase for very large loans. This is very closely replicated in equilibrium, as shown in Figure 2.1b. Despite not being targeted in the calibration, the model clearly exhibits decreasing interest rates over the four lowest quintiles with an increase for largest loans. Besides this qualitative feature, the model also does well in predicting the level of interest rates quantitatively, relative to loan size. Only for loans in the first and fourth quintile, the model underestimates interest rates.

Also when analyzing realized interest rates relative to income, the model replicates important facts. Figure 2.2a depicts average loan prices in the data, measured by the APR, plotted against income quintiles. Low income households face much higher average cost, with a peak at the second quintile. When income increases, costs of credit decrease.

\(^{11}\)For a theoretical derivation of increasing prices in the size of loans, see Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007, Theorem 6).

\(^{12}\)See Appendix 2.A.4 for more details on how these figures are created.
2 Regulating Small Dollar Loans: The Role of Delinquency

Figure 2.2: Average Credit Prices (APR), by income

Figure 2.2b depicts the model counterpart that exhibits the same features. Low income households tend to take smaller loans. Hence, fixed cost and higher nonpayment risk drive up prices for low income households.

The SCF oversamples high income households to precisely capture debt and asset holdings in the U.S. There is evidence that it does not contain complete information on the balance sheets of very low income households, though (Ratcliffe et al., 2007). The SCF recently introduced a question regarding the use of payday loans. When analyzing the balance sheet of households that report to use payday loans, these payday loans are rarely represented. Not being mentioned in the “other loans” category, small dollar loans are clearly underreported. Consequently, there are very few observations of very high interest rates in the SCF, even though small dollar loans are clearly used in reality.13

2.4 Delinquency as Insurance Device

This section highlights the trade-off of households that become delinquent or declare bankruptcy. Bankruptcy, on the one hand, might be prohibitively expensive in the period of filing for bankruptcy but offers total debt relief in the following period. If available, Chapter 7 bankruptcy thus insures households against adverse shocks. On the other hand, delinquency is more affordable in the short run (at least for low income households) but households enjoy no or only partial reduction of debt. I document that delinquent households might thus be “trapped” in debt long enough to repay more than they originally owed. Delinquency consequently provides low insurance or even anti-insurance. These findings will help to understand why the proposed repayment plan does not yield any

13See Appendix 2.A.5 for additional information on the marginal interest rates that households face.
2.4 Delinquency as Insurance Device

welfare gains (c.f. Section 2.5) whereas a bankruptcy advance makes households better off (c.f. Section 2.6).

2.4.1 Household Default Decisions

To understand the fundamental trade-off between delinquency and Chapter 7 bankruptcy, it is instructive to examine household default decisions more closely. Figure 2.3 depicts repayment decisions of low income, median income and high-income households that face a large expenditure shock. The policy functions \( p = \{S, D, B\} \) are plotted as a function of asset holdings minus the expenditure shock.\(^{14}\) High-income individuals can afford repayment for very high levels of debt. Consequently, these individuals file for bankruptcy \((p = B)\) only when debt is very high and otherwise stay solvent. Since they can easily pay out-of-pocket expenses from current income, these households avoid delinquency and directly file for Chapter 7 bankruptcy.

Median income households cannot afford to repay such high levels of debt. Hence, they choose bankruptcy for lower levels of debt. Since out-of-pocket expenses of declaring bankruptcy are significant for these households, they choose delinquency \((p = D)\) over bankruptcy when debt is moderate. Only when debt levels are sufficiently high, these individuals choose bankruptcy in order to forgo collection efforts by banks.

\(^{14}\)Negative asset levels correspond to debt holdings.
Low-income households, on the other hand, do not file for bankruptcy. When solvency becomes too expensive, these households become delinquent. Filing fees prove prohibitively high for these households, which de facto excludes them from Chapter 7 debt relief.\footnote{See Figure 2.B.1a in Appendix 2.B for the debt pricing function that results from this repayment behavior.}

The policy functions presented in Figure 2.3 clearly showed that low income households are effectively excluded from filing for Chapter 7 bankruptcy. This exclusion clearly materializes in the simulated equilibrium, too. Figure 2.4 presents repayment decisions as realized in equilibrium by income groups. Not surprisingly, higher income households are more likely to be solvent. While a significant fraction of low income households does not repay, they rarely file for Chapter 7 bankruptcy. Due to prohibitively high filing fees, these low income households rather become delinquent. As income increases, delinquency is substituted by filing for bankruptcy.

The cross-section of delinquency and bankruptcy filings with respect to income are not targeted in the calibration. The model nevertheless correctly predicts delinquencies to be more prevalent than bankruptcies for low income households. Table 2.4 compares the share of delinquencies and bankruptcy filings by households in the lowest 30% of the income distribution to the data. While fitting delinquencies quite well, the model underestimates bankruptcy filings by low income households.
Table 2.4: Non-Payment by 30% Lowest Income Households

<table>
<thead>
<tr>
<th>Non-Payment Decision</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delinquency</td>
<td>85%</td>
<td>75%</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>42%</td>
<td>64%</td>
</tr>
</tbody>
</table>

*Note:* This table depicts the share of delinquency and bankruptcy decisions by households with the lowest 30% of income relative to total delinquency and bankruptcy decisions. Calculated from the Federal Reserve Bank of New York’s “Consumer Credit Panel / Equifax” data set (as seen in Athreya, Sánchez, Tam, and Young, 2015).

### 2.4.2 Delinquency as a “Debt Trap”

In the absence of state-contingent contracts, households use bankruptcy and delinquency to insure against adverse shocks. When choosing bankruptcy, households gain full debt relief in the next period. Debt relief tomorrow comes at a cost today: Bankrupts have to pay out-of-pocket fees $F$ and they suffer utility cost $\zeta_B$.

When choosing delinquency, households do not necessarily receive full debt relief in the next period. But, in the current period, cost of delinquency do not contain lump-sum expenditures. Delinquent households repay late fees $L$ which depend on the household’s state. Since banks make households indifferent between suffering collection or paying late fees, the total utility cost (relative to consuming the endowment) are simply the utility loss due to collection, $\zeta_D$. Late fees can be directly backed out of the banks’ problem in Equation (2.7). $L = e - w^{-1}(u(e) - \zeta_D)$ makes the household indifferent between consuming $e - L$ without utility cost or consuming $e$ and facing a utility cost of $\zeta_D$.

The difference in instantaneous utilities between delinquency and bankruptcy is

$$\Delta_0 = u(e) - \zeta_D - (u(e - F) - \zeta_B) = U(e, F) + \zeta_B - \zeta_D,$$

(2.10)

where $U(e, F) = u(e) - u(e - F)$ denotes the utility loss when paying lump sum filing fees $F$ with income $e$. Hence, the difference in instantaneous utilities between delinquency and bankruptcy (first term in Equation (2.10)) can be interpreted as the difference between the utility cost of bankruptcy $U(e, F) + \zeta_B$ and the utility cost of delinquency $\zeta_D$ (second term in Equation (2.10)).

If $\Delta_0 > 0$, delinquency is preferable relative to bankruptcy in the current period. In a sense, it is a more affordable option to not repay outstanding debt. Thus, instantaneous utility from delinquency is higher than utility from bankruptcy, or equivalently, the utility cost of delinquency are lower than the cost of bankruptcy. Due to standard assumptions on the utility function (see the discussion following Equation (2.4)), utility cost of bankruptcy
2 Regulating Small Dollar Loans: The Role of Delinquency

become infinitely large if filing cost $F$ approach income:

$$U(e,F) \to \infty \text{ for } e-F \to 0, \ e>F.$$  

If $U(\cdot) \to \infty$, so does $\Delta_0 \to \infty$. In other words, delinquency becomes increasingly favorable as bankruptcy becomes infeasible (and instantaneous utility of bankruptcy diverges to $-\infty$). In these cases, bankruptcy is not available for households to insure against adverse shocks.\(^{16}\)

In the period after delinquency or bankruptcy, expected values differ, too. While bankruptcy offers complete debt relief ($a'=0$), households in delinquency get partial or no debt relief ($a'=\alpha \leq 0$). The difference between expected future utilities of delinquency and bankruptcy thus is

$$\Delta_1 = \mathbb{E}_s (V(\alpha, s') - V(0, s')).$$  \hspace{1cm} (2.11)$$

Since $V$ is a well-behaved value function, the expected future value of bankruptcy with debt relief is higher than that of delinquency: $-\infty < \Delta_1 \leq 0$. This is simply because $V$ is bounded and because $\partial V/\partial a > 0$.\(^{17}\)

To sum up, households trade-off current period expenditures with future debt relief. Better insurance in bankruptcy (i.e. higher debt relief tomorrow) comes at higher cost

\(^{16}\)On the other hand, if $F \to 0$ or $e \to \infty$, lump sum fees become unimportant i.e. $U(e,F) \to 0$. Then, the instantaneous utility difference converges to the difference in direct utility cost: $\Delta_0 \to \zeta_D - \zeta_B$.

\(^{17}\)The last inequality can be strict, if $\exists \alpha: q(\alpha, s')\alpha < 0$. 

Figure 2.5: Delinquency vs. Bankruptcy
(i.e., higher out-of-pocket expenditures today). Figure 2.5 depicts the trade-off between differences in filing fees in the current period \((\Delta_0)\) and differences in next period’s debt relief \((\Delta_1)\) as a function of income. As is apparent in the figure, low income households have very high (or infinite) utility cost of filing for bankruptcy in the current period and hence prefer delinquency \((\Delta_0 + \beta \Delta_1 > 0)\). This trade-off reverses for high income households.

Since low income households cannot afford to file for bankruptcy in order to insure against adverse events, they have to resort to delinquency. Households are “trapped” in debt in that they do not receive debt relief in delinquency, but rather end up paying late fees over multiple periods of time. Figure 2.6a depicts the distribution of realized repayment fractions in delinquency. That is the discounted sum of all late fee payments and possible repayments upon leaving delinquency. When leaving delinquency, households might repay zero if they file for bankruptcy or \(\alpha\) when they end up repaying their delinquent loan. Most households repay roughly 20% of the original loan, compared to nothing in bankruptcy. Note, though, that there are some households that end up repaying up to 160% of the original loan amount in late fees, as depicted in Figure 2.6b. Here, only repayment fractions above 100% are shown.

When comparing repayment across delinquency and bankruptcy directly, the “debt trap” becomes even more apparent. Table 2.5 describes the distribution of repayment fractions, both in delinquency and in bankruptcy. Here, the payment fraction in bankruptcy is defined as bankruptcy filing cost, relative to outstanding debt. Even though banks do not receive these payments, this measure describes how much households have to pay in the current period of bankruptcy to get all their debt forgiven. Thus, it makes the amount of insurance offered by bankruptcy and delinquency more comparable. Low repayment fractions...
Table 2.5: Realized Repayment Fractions, Delinquency and Bankruptcy

<table>
<thead>
<tr>
<th>Repayment Quantile</th>
<th>Delinquency</th>
<th>Bankruptcy</th>
</tr>
</thead>
<tbody>
<tr>
<td>25%</td>
<td>9.3%</td>
<td>3.9%</td>
</tr>
<tr>
<td>50%</td>
<td>15.1%</td>
<td>6.2%</td>
</tr>
<tr>
<td>75%</td>
<td>20.3%</td>
<td>8.2%</td>
</tr>
<tr>
<td>95%</td>
<td>48.1%</td>
<td>10.7%</td>
</tr>
<tr>
<td>99%</td>
<td>165.9%</td>
<td>18.6%</td>
</tr>
</tbody>
</table>

Note: In bankruptcy, repayment fractions are measured as fixed cost divided by total debt when declaring bankruptcy, i.e. $F/(-a + \kappa)$. Banks do not receive these payments, but households still pay $F$ in order to gain access to debt relief. These numbers are thus reported for comparability.

Fractions indicate a high amount of insurance, while high repayment fractions indicate a lower amount of insurance. Repayment fractions above one indicate anti-insurance: Households that cannot repay their debt through solvency and choose delinquency end up repaying more than was originally owed.

While the median household pays only 6.2% of his outstanding debt when filing for bankruptcy, the median delinquent households repays 15.1%. Households in the top 5% repay 10.7% in bankruptcy while nearly 50% in delinquency. Finally, the top 1% of bankrupts repays less than a fifth of the original debt, while delinquents repay 166%. These unlucky households can neither afford to repay outstanding debt nor afford to file for bankruptcy. Hence, being stuck in a “debt trap”, they remain delinquent for long periods of time.

For low-income households, delinquency is more affordable than bankruptcy. But on the flip side, delinquent households repay more of their debt than households that can afford to file for bankruptcy. Thus, while being more costly in the period of default, bankruptcy offers better insurance for households in bad states. Delinquent households might end up in a “debt trap”, unable to repay their debt or file for bankruptcy. After being hit by a bad income realization or expenditure shock, the most unlucky households repay more than 160% of their original debt.

2.5 Introduction of a Repayment Plan

This section evaluates a key component of the Consumer Financial Protection Bureau’s (CFPB) “Proposal to End Debt Traps.” The proposal aims to reduce the cost that households face when not being able to repay the full principal and interest in one balloon payment. Instead of keeping households in debt and letting fees accumulate, the CFPB
proposes that lender are mandated to offer a repayment plan for short-term small dollar loans. According to this plan, households are able to roll over outstanding debt for three periods at no additional cost. By decreasing loan size consecutively, borrowers opting into the repayment plan repay the full principal over three periods. Additionally, they are protected from any collection efforts. By effectively extending the loan duration, this proposal lowers the APR and is supposed to provide a cheap “off ramp” for indebted households. To ensure that households do indeed repay all outstanding small dollar loans, they are excluded from any additional borrowing while repaying under this plan.

According to the proposed regulation, the repayment plan is to be added to any small dollar loan contract. There is only one way to circumvent this requirement. Instead of offering the repayment plan, lenders could ensure a borrower’s ability to repay. However, evidence suggests that lenders already do screen loan applicants. Lenders receive information on income and expected repayment behavior by screening paychecks and retrieving credit scores from Teletrack.18 Additionally, the CFPB’s proposed process of determining ability to repay is fairly extensive. In addition to income and repayment behavior, lenders would be required to assess basic living cost, recurring expenses and other financial obligations. It is not clear if these requirements could be met without significantly increasing operational cost.

I focus on the introduction of a repayment plan due to three reasons. (1) Higher screening standards increase fixed cost of loan creation further. The resulting prices are likely to prove noncompetitive relative to prices of loans that would just include a repayment plan. (2) Borrowers in the small dollar loans market often are financially strained. They use small dollar loans exactly because they do not have access to other lending with more stringent screening. These borrowers would without much doubt choose loans that feature a repayment plan rather than more extensive screening.19 (3) The proposed repayment plan extends loan contracts to multiple periods. Multiple effects may arise: On the one hand, payments are delayed, freeing current resources to increase consumption. On the other hand, households lose the flexibility of adapting their borrowing or saving behavior to newly realized shocks in the future. The current framework allows to analyze which effect dominates.

In the model, I represent the CFPB’s policy proposal by offering an opt-in repayment plan that fully commits households to repay the outstanding amount over three periods. Households using the repayment plan cannot save or borrow but only consume whatever

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18Teletrack provides credit scores that are specifically targeted at the small dollar loans market. See Skiba and Tobacman (2008) for more details.

19Stango (2012b) finds that borrowers prefer traditional payday loans to short-term credit provided by Credit Unions because payday lenders are less restrictive in the application process.
is left of their income after repaying $\psi = 1/3$ of the outstanding loan amount. Since this proposal is aimed at small dollar loans, I only introduce the repayment plan for loans smaller than $1,000$.

In order to keep the state space constant, exiting the repayment plan is modeled stochastically. Exit thus occurs with the probability $\psi$. The expected duration is equal to the inverse of the exit probability $\psi$ and hence equal to three periods. The value of choosing the repayment plan can be written as

$$V^R(a, s) = u(e - \psi(|a| + \kappa)) + \beta\mathbb{E}[\psi V(0, s') + (1 - \psi)V^R(a, s')]$$

using the fact that households consume their endowment after repaying $\psi|a|$. If $\psi(|a| + \kappa) \geq e$, choosing the repayment plan is not feasible. In these cases, repayment would exceed the available income. Instantaneous utility is set to minus infinity.

With probability $\psi$, households in the repayment plan expect to leave the repayment plan and start over with zero debt: $\psi V(0, s')$. If exit does not occur, households stay in the repayment plan for the next period: $(1 - \psi)V^R(a, s')$.

With the repayment plan available, the full household problem in Equation (2.4) is expanded by $p(\cdot) = R$ and the corresponding value function of Equation (2.12).

### 2.5.1 Debt Pricing Function

Since households are now free to choose between solvency ($p = S$), bankruptcy ($p = B$), delinquency ($p = D$) or entering the repayment plan ($p = R$), credit prices adapt. The new price schedule is

$$q_R(a', s) = \hat{q} \left( \mathbb{E} [\mathbb{I}_S(a', s') | s] \cdot 1 + \mathbb{E} [\mathbb{I}_B(a', s') | s] \cdot 0 + \mathbb{E} [\mathbb{I}_D(a', s') \frac{R(a', s')}{|a'|} | s] + \mathbb{E} [\mathbb{I}_R(a', s') \frac{\Psi(a', s')}{|a'|} | s] \right) - \frac{\delta}{|a'|},$$

where $\Psi(\cdot)$ represents the interest rate losses that banks realize for loans that are repaid through the repayment plan. These losses can be written recursively, taking into account the exit probability $\psi$. Next period’s repayment is thus not only discounted at the risk free rate $(1 + r)$, but also multiplied with the probability of the household remaining in
2.5 Introduction of a Repayment Plan

When introducing this repayment plan into the economy, it is never chosen in equilibrium (see Figure (2.7)). Consequently, there is no effect on welfare.

Households do not choose the repayment plan in good times (i.e. in the absence of adverse income or expenditure shocks). Rather than spreading repayment over multiple periods they prefer to repay directly with the option to take out new loans or to save, depending on the future state. The option value of flexibly choosing the asset position next period exceeds the effect of increasing current consumption and repaying parts of the debt later.

Households do not choose the repayment plan when hit by an adverse shock that triggered delinquency in the benchmark, either. Large, negative income shocks are persistent and spreading repayment over three periods is not sufficient to make repayment affordable. Additionally, expenditure shocks are simply too large to repay them over three periods. As a result, the proposed repayment plan does not offer a viable alternative in states where borrowers previously chose delinquency. Since outstanding debt has to be repaid over three periods, the plan only offers minimal insurance and is hence dominated by delinquency.

\[ \Psi(a, s) = \psi(a, s) + (1 + r)^{-1}(1 - \psi(a, s))E[\Psi(a', s') \mid s]. \] (2.14)
To sum up, the repayment plan is neither chosen in states where households choose solvency in the benchmark, nor in states where households resort to delinquency in the benchmark. This finding is robust to extending the period in which households can repay to up to eight quarters. Longer repayment horizons are not very realistic, given the short-term nature of the original market.\footnote{In a future extension, I plan to evaluate income-related repayment schemes. Dye (1986) uses a three period model to theoretically argue that optimal repayment schemes should be based on income. These regimes are prone to moral hazard, though. Allowing for endogenous labor supply, Exler (2016) finds positive garnishment rates to maximize welfare, too.}

### 2.6 Introduction of Bankruptcy Advances

As shown in Section 2.4, low income households mainly use delinquency when hit with a bad shock. Compared to bankruptcy, delinquency offers less insurance and households end up paying late fees for multiple periods. The most unlucky households are “trapped” in debt long enough to repay significantly more than originally owed. In the previous section, I showed that a repayment plan does not offer a viable alternative to delinquency.

A more direct policy measure to improve consumption smoothing for low income households is to offer a more affordable option of debt relief. As argued in Section 2.4.2, the future discounted utility of bankruptcy exceeds that of delinquency because of complete debt relief. Low income households choose delinquency, though, because bankruptcy might be prohibitively expensive in the current period. Many low income households consequently stay delinquent until they experience a steep (and unlikely) increase in wages so that out-of-pocket filing cost can be covered.

In order to increase the amount of insurance that low income households can afford, I introduce a bankruptcy advance to cover bankruptcy filing cost. Using this advance, households gain access to debt relief through Chapter 7 bankruptcy even when income is very low. Under the policy reform, every borrower with below-median income can take out a loan covering all monetary bankruptcy filing cost. The loan is sheltered from bankruptcy and due once the household has left bankruptcy protection. Since the loan does not carry any nonpayment risk, only the risk-free interest rate applies. Households consequently have to repay \((1 + r)F\) after filing for bankruptcy using the bankruptcy advance.\footnote{This setup is modeled similar to the legal status of student loans which are not written off during Chapter 7 bankruptcy, either. See Ionescu and Simpson (2016) for an example with unsecured credit and student loans.} Additionally, households repay the risk-free interest that accrues. This policy is consequently budget neutral for the regulator.
2.6 Introduction of Bankruptcy Advances

The value of using a bankruptcy advance is

\[ V^{BA}(a, s) = \max_c \left[ u(c) - \zeta_B + \beta \mathbb{E}_s V(-(1 + r)F, s') \right] \]

s.t. \( c = e \)

and can be simplified to

\[ V^{BA}(a, s) = u(e) - \zeta_B + \beta \mathbb{E}_s V(-(1 + r)F, s'). \] \hspace{1cm} (2.15)

The policy is means-tested and only households with income that does not exceed median income benefit from the bankruptcy advance. Equation (2.15) is thus only true for all households with \( e \leq \hat{e} \), where \( \hat{e} \) is median income in the economy. If income exceeds median income, the policy is not available: I set \( V^{BA}(a, s) = -\infty \), if \( e > \hat{e} \).

Filing cost are to be payed back in full (including interest). Households choosing a bankruptcy advance enter the following period with debt of \((1 + r)F\). Full repayment means that bankrupts cannot directly default on the bankruptcy advance. This ensures that the policy is budget neutral.\(^{22}\)

The beginning-of-period household problem in Equation (2.4) is expanded by adding the choice of a bankruptcy advance \( p(\cdot) = BA \) and the corresponding value function of Equation (2.15).

2.6.1 Debt Pricing Function

Since households are now free to choose between solvency \( (p = S) \), bankruptcy \( (p = B) \), delinquency \( (p = D) \) or using a bankruptcy advance \( (p = BA) \), credit prices adapt. The new price schedule is

\[ q_B(a', s) = \bar{q} \left( \mathbb{E} \left[ I_S (a', s') \mid s \right] \cdot 1 + \mathbb{E} \left[ I_B (a', s') \mid s \right] \cdot 0 + \mathbb{E} \left[ \frac{R(a', s')}{|a'|} I_D (a', s') \mid s \right] + \mathbb{E} \left[ I_{BA} (a', s') \mid s \right] \cdot 0 \right) - \frac{\delta}{|a'|}. \] \hspace{1cm} (2.16)

\(^{22}\)When I solve the model, I do not enforce this restriction directly. Rather, I check that households stay solvent after using the bankruptcy advance for at least one period. 100% of households choosing a bankruptcy advance do so. This corresponds to repaying the government funded advance, even if households refinance parts of the advance with private loans in subsequent periods.
where recovery during Chapter 7 bankruptcy is zero, no matter if households pay the filing fees or use a bankruptcy advance. See Figure 2.B.1b in Appendix 2.B for more details and a plot of the debt pricing function.

### 2.6.2 Effects

#### Household Behavior

When entering each period, households now have an additional choice: Instead of traditional Chapter 7 bankruptcy or delinquency, below-median households can choose to get a loan to cover Chapter 7 filing cost. Figure 2.8a plots the adapted household decisions when a Chapter 7 advance is available. For better readability, Figure 2.8b presents the equilibrium repayment choices in the benchmark again (c.f. Figure 2.4).

Households with below-median income enter delinquency significantly less than in the benchmark. In order to get debt relief, they prefer a bankruptcy advance over standard Chapter 7, since filing fees can be repaid after filing for bankruptcy. All low income bankrupts choose to make use of the introduced bankruptcy advance of paying the filing fees up front.

Higher income households, on the other hand, are excluded from the policy reform; their repayment choices resemble the benchmark case more closely. Standard Chapter 7 bankruptcy is slightly reduced because lower income households do not stay delinquent and wait for a higher income realization to be able to file for bankruptcy.
2.6 Introduction of Bankruptcy Advances

Average Realized Credit Prices

Figure (2.9) presents credit prices realized in the new equilibrium, relative to the benchmark without bankruptcy advances. It depicts average APRs for quintiles of the wage distribution. Since low income individuals have a more affordable option to file for bankruptcy now, banks consider these households to be more risky. Compared to the benchmark, banks recover less through late fees in delinquency and credit for low income borrowers become more costly. Households choose bankruptcy in more states of the world, yielding complete debt relief and consequently zero recovery for banks. This effect is most pronounced for the lowest income quintile.

Abolishing “Debt Traps”

Expected recovery in delinquency is reduced along the extensive and intensive margin: Making bankruptcy available to low income households reduces the number of households in delinquency along the extensive margin. As a result, banks expect zero recovery due to bankruptcy to occur more often. On the intensive margin, bankruptcy advances also change banks’ expected recovery conditionally on borrowers being delinquent. Banks cannot keep low income households in debt for long periods and thus cannot charge high late fees to recover as much of the loan as in the benchmark. Since filing fees do not bar low income households from filing anymore, banks have to offer better terms on delinquent
balances in order to incentivize at least some repayment. The outside option of filing for Chapter 7 bankruptcy and owe debt of \((1 + r)F\) next period thus reduces repayment in delinquency along the intensive margin, too.

Along the extensive margin, delinquencies drop significantly in this policy experiment. Table 2.6 presents the effects on equilibrium repayment choices of the new policy relative to the benchmark. Bankruptcy filings are divided into standard filings and filings financed by a bankruptcy advance. While solvency is roughly constant, the fraction of households choosing delinquency is reduced by two thirds. These households file for bankruptcy instead and aggregate bankruptcy filings quadruple (+300%).

The overwhelming fraction of bankruptcy filings are financed by a bankruptcy advance (96% = 1.15/1.2). While aggregate bankruptcy filings increase, standard bankruptcy cases actually drop to very low levels. Standard Chapter 7 bankruptcy was mainly used by delinquent formerly low income households that received a positive permanent wage shock. Now, these households directly have access to bankruptcy through a government sponsored advance and do not wait to file for standard Chapter 7 bankruptcy later.

Also along the intensive margin, delinquencies become less severe, as depicted in Table 2.7. Conditional on being delinquent, households repay significantly less. Especially, households that were formerly “trapped” in debt ended up repaying more than they
originally owed. Now that households have the outside option of bankruptcy, though, the unluckiest delinquent households only repay less than 30% of original debt through late fees (compared to 165% in the benchmark).

### 2.6.3 Welfare Effects

Upon the introduction of bankruptcy advances, low income households’ welfare improves significantly. When delinquency is used as an insurance against bad income realizations or expenditure shocks, households suffer significant utility cost due to the late fees they pay to avoid collection efforts by banks. Also, there is no way for low income households to receive full debt relief. Hence, households readily use subsidized bankruptcy when it is introduced.

While average welfare increases by 0.1%, low income households gain up to 1% in welfare. These households could not formally declare bankruptcy before, and hence value the policy reform the most. The welfare gains are computed using consumption equivalence variation (CEV). For each household, I calculate the necessary consumption increase in the benchmark to make them indifferent between this benchmark and the policy experiment. Thus, for every household with state \((a, s)\) in the stationary distribution, I solve for \(\xi\) such that

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23I do not account for the transition period. Transition effects should be small, though, since there is no aggregate capital stock in my model. Even in the presence of an aggregate capital stock, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) find very small transition effects.
\[ V_\xi(a, s) = \mathbb{E}\left( \sum_{i=0}^{\infty} \beta^i u(c_i(1 + \xi)) \mid a, s \right) \equiv \tilde{V}(a, s), \]  

where \( V_\xi \) denotes benchmark welfare, if consumption is increased by \( \xi \). \( \tilde{V} \) denotes welfare when a bankruptcy advance is available.

Figure 2.10 reports the population average of the CEV (i.e. \( \mathbb{E}(\xi) \)) and the average by income (i.e. \( \mathbb{E}(\xi \mid e) \)). Since high income households are very unlikely to ever become eligible to use a bankruptcy advance, welfare effects for those households are roughly zero. Low income households, on the other hand, face steeper pricing, but gain better access to insurance against bad states. The latter effects dominates and produces up to 1\% in welfare gains.

These welfare gains can be realized at no cost to the regulator. All bankruptcy advances are repaid after unsecured debt is written off. Compared to introducing a repayment plan, this policy has positive effects on the economy. Furthermore, there is unanimous support for the reform: Nobody loses when the reform is introduced. While high income individuals are unlikely to be affected, low income households prefer better access to insurance with the reform over lower prices without it. Introducing this reform should be politically feasible since it offers a Pareto Improvement.

### 2.7 Conclusion

Fixed bankruptcy filing costs may “trap” low income households in debt. In the face of bad shocks, these households become delinquent on their loans because bankruptcy is unaffordable. Banks have no incentive to offer a discount in order to incentivize repayment. Rather, they maximize expected repayment by repeatedly charging late fees and keeping households in delinquency for long periods of time. For high income households on the other hand, bankruptcy is available as an outside option. Consequently, the most unlucky households can insure the least.

A policy proposal by the Consumer Financial Protection Bureau (CFPB) aims at ending these “debt traps” in the small dollar loan market. The proposed repayment plan is introduced in Section 2.5. It allows delinquent households to repay outstanding debt over three periods. Without suffering bad income or expenditure shocks, households do not value this option, though. They rather repay directly and reserve the flexibility to adapt future asset or debt holdings to changes in income or unforeseen expenditures. In the face of bad shocks, the repayment plan is not affordable because low income realizations are persistent and expenditure shocks are too large. Consequently, the CFPB proposal is
never chosen in equilibrium and has no welfare effect.

Introducing a bankruptcy advance in Section 2.6 yields significant welfare gains. Under this policy, low income households can file for bankruptcy and repay the filing cost after their unsecured debt is forgiven. With this option available, the burden of delinquency decreases both along the extensive and intensive margin: Households use a bankruptcy advance instead of being delinquent and delinquent households repay less since Chapter 7 becomes a viable outside option.

For the lowest incomes, welfare increases by up to 1% when subsidized bankruptcy is available. The average welfare gain is 0.1% and all households are weakly better off. These effects arise because households have access to better insurance against adverse shocks. The value of insurance dominates the negative effects of increased credit prices.
Appendix

2.A Unsecured Debt in the Survey of Consumer Finance

2.A.1 Unsecured Debt

The Survey of Consumer Finances (SCF) contains very detailed information on household balance sheets. In order to construct total unsecured debt holdings of a household, I sum up non-business credit card debt (variables X413, X421, X427, X430, X7575), debt owed on lines of credit that is neither secured by housing or vehicles nor a business loan (variables X1108, X1119, X1130) and debt owed on other unsecured loans (variables X2714, X2731, X2814, X2831, X2914, X2931).

Let $X_{i,j}$ define the set of measures of unsecured credit for each household $i$ as described above. Hence, total unsecured debt for each household $i$ in sample $I$ is

$$D_i = \sum_{X_{i,j} \in X_i} X_{i,j}. \quad (2.18)$$

2.A.2 Debt to Income

The debt to income ratio of each household is constructed using the total unsecured debt measure described above and dividing it by total household income (variable X5729). I follow Livshits, MacGee, and Tertilt (2010) in constructing debt to income ratios using actual outstanding debt instead of net debt positions. Livshits, MacGee, and Tertilt argue that outstanding unsecured debt is more relevant to a households bankruptcy decision than the net position. Many assets (such as housing) are illiquid and cannot be used to repay outstanding debt. Additionally, exemptions shield large parts of household assets.

Let $Y_i$ denote household income $X5729$. Then, the debt to income ration of households is defined as

$$d_i = \frac{D_i}{Y_i}. \quad (2.19)$$
In the calibration, I target the population average of this ratio:

\[ d = \frac{\sum_{i \in I} d_i}{N} \]  

where \( N \) is population size.

### 2.A.3 Fraction in Debt

In order to calculate the fraction of households in debt \( (f) \), I calculate the population average of an indicator that is one if \( D_i \) is strictly larger than zero and zero otherwise:

\[ f = \frac{\sum_{i \in I} \mathbb{1}(D_i > 0)}{N} \]  

### 2.A.4 Cross-Sectional APRs

In order to generate Figures 2.1a and 2.2a, I treat any (loan size, APR) pair in the data as one observation. Each household \( i \) thus might hold several loans summarized in \( X_i \). In order to generate credit card debt, I sum up all credit card debt and apply the interest rate reported for the highest balance (X7132). More detailed interest rate information is not available for credit cards. All other loans come with individually reported interest rates (X2724, X2741, X2824, X2841, X2924, X2941). Let \( R_i \) denote an individual \( i \)'s set of interest rates \( \{R_{i,1}, R_{i,2}, \ldots, R_{i,j}, \ldots, R_{i,J}\} \).

I then take the conditional mean of the observed APRs by loan size quintiles (Figure 2.1a) and by household income quintiles (Figure 2.2a):

\[ \text{APR}_n = \sum_{i,j} \frac{R_{i,j} \mathbb{1}(Y_i \in Q_n(Y))}{\mathbb{1}(Y_i \in Q_n(Y))} \]

\[ X_n = \sum_{i,j} \frac{R_{i,j} \mathbb{1}(Y_i \in Q_n(Y))}{\mathbb{1}(Y_i \in Q_n(X))} \]  

where \( Q_n \) denotes the set of incomes and loan sizes in the \( n \)th quintile and \( j \) indicates each loan held by individual \( i \).

### 2.A.5 Marginal Borrowing Cost

To get a further impression on the distribution of interest rates in the SCF, I construct a measure of effective borrowing cost of a household. Here, I assume that households use the cheapest available credit first. As a consequence, the highest observed interest rate for all unsecured loans \( X_i \) (i.e. credit cards and other unsecured loans) measures the interest...
rate of the marginal loan. In Figure 2.A.1, I plot these interest rates for each household, relative to income quintiles.

2.B Debt Pricing Function

Resulting from the household behavior described in Section 2.4, Figure 2.B.1 depicts the equilibrium credit pricing function that is employed by banks for three different wage realizations as a function of debt. Prices are infinitely large (i.e. $q = 0$) for loans smaller than the fixed cost. The impact of these fixed cost vanishes with increasing loan size. Since low income households stop repaying earlier than high-income households, credit prices for the former deteriorate at lower levels of debt.

In the benchmark however (c.f. Figure 2.B.1a), low income households do not file for bankruptcy but rather become delinquent. Consequently, banks can recover some resources through levying late fees once these households are not solvent any more. The drop in $q$ is less pronounced than for median income households who can afford Chapter 7 bankruptcy. Banks expect median income households to file for bankruptcy and correspondingly expect zero repayment for high debt levels.

When bankruptcy funding is available to low income households, low income households do not stay in delinquency and accordingly do not repay late fees. Figure 2.B.1b clearly
shows the drop in delinquencies and the corresponding drop in expected repayment described in Section 2.6.2. Low income households not only use bankruptcy advances at lower levels of debt to file for Chapter 7 bankruptcy, but, once they file for bankruptcy, expected repayment is much lower than in delinquency. Thus, $q$ drops at lower levels of debt and $q$ drops more steeply.

In both setups, high-income individuals still repay very high levels of debt. Hence, credit remains cheap (corresponding to $q$ close to one) and only deteriorates for even higher levels of debt.

### 2.C Computational Approach

The numerical solution is computed in MATLAB 2016b on the “MLS & WISO bwForCluster.” The solution algorithm is parallelized and solved on multiple sixteen core Intel Xeon nodes. The model is solved numerically by value function iteration on a discretized state space $A \times Z \times E \times K$, denoting the set of asset holdings, persistent income realizations, transitory income realizations and expenditure shocks. The income shock processes are discretized using Rouwenhorst’s method (Kopecky and Suen, 2010). Total income thus is $e \in Z \times E$.

#### 2.C.1 Algorithm

1. Pick structural parameters $\theta_0$ of length $J$. 
2. Computational Approach

2. Let $\tilde{\theta}_j = \theta_0 + \hat{j} d$ for each $j = \{1, 2, \ldots, J\}$, where $\hat{j}$ is the standard unit vector with $j$th element equal to one and $d$ is the stepsize.

I. Do Value Function Iteration

i. Guess $V_0$ and $q_0$.

ii. Compute $\mathbb{E}[V_0(a', s') | s]$ for all $z \in Z$.

iii. Compute the new value functions $V_1^S, V_1^D, V_1^B$ by maximizing over $c, a'$ for each $p = S, D, B$.

iv. Compute $V_1$ by solving for optimal $p^*$.

v. Compute expected repayment in delinquency $R(a, s)$, taking household choices $c^*, a^*, p^*$ as given.

vi. Compute new implied credit price $q_1$ taking repayment as given.

vii. If $\|V_1^S - V_0^S\|, \|V_1^D - V_0^D\|, \|V_1^B - V_0^B\| < \epsilon_V$ and $\|q_1 - q_0\| < \epsilon_q$, end.

Else, $V_0 = V_1$ and $q_0 = \mu q_1 + (1 - \mu)q_0$ and go to ii.

II. Calculate model moments

i. Simulate the invariant distribution over $A \times Z \times \mathcal{E} \times \mathcal{K}$.

ii. Calculate model moments $M(\tilde{\theta}_j)$.

3. Update structural parameters

I. Pick $\theta^* = \arg\min_j \sum_i w_i (M_i(\theta_j) - D_i)^2$ where $w_i = D_i^{-2}$ to minimize sum of squared relative deviations to data $D_i$.

II. If $j \in \{1, \ldots, J\}$, update $\theta_0 = \theta^*$ and increase step size $d$.

Else, $j = 0$. Keep initial $\theta_0$ and decrease step size $d$.

III. If $d > \epsilon_d$, go to 2. Else, end.
3 Personal Bankruptcy and Wage Garnishment

3.1 Introduction

Policy makers face a fundamental trade-off when designing personal bankruptcy laws: when bankruptcy laws are very lax, consumers have a powerful tool to insure against adverse events such as job loss, illness or divorce by not repaying their debts. However, higher default will translate to increasing interest rates in equilibrium because lenders face higher non-payment risk. While countries such as the U.S. and the U.K. grant debt relief rather easily, countries such as Germany have much harsher regimes.

While in “Fresh Start” regimes such as in the U.S. lenders do not have claims towards the future income of bankrupts, the German bankruptcy system feature harsh wage garnishment rules for six years. Only after fulfilling these repayment requirements can households receive debt relief. In garnishment, 70% of annual net income exceeding 12,600€ is garnished. Income in excess of 38,500€ is fully garnished. Garnishment might reduce risk premia because lenders recover more resources upon default. However, Dobbie and Song (2014) find that besides suffering other negative consequences households subject to wage garnishment significantly reduce labor supply. This adverse labor supply effect is ignored in most of the literature dealing with bankruptcy and wage garnishment. There, labor supply is modeled exogenously. However, when evaluating bankruptcy regimes that feature income garnishment, endogenous labor supply plays an important role when determining how much is repaid through garnishment, default premia, and welfare.

Allowing for endogenous labor supply, I investigate the optimal garnishment policy for the German economy. For this purpose, I explore the properties of a bankruptcy regime with labor income garnishment. I focus on the trade-off between insurance against adverse shocks and access to unsecured credit if loan prices incorporate the risk of default. While wage garnishment effectively reduces moral hazard, it may also reduce the amount of insurance that bankruptcy offers to individuals and it reduces the incentive to work.

\footnote{See §850c ZPO (civil process order).}
3 Personal Bankruptcy and Wage Garnishment

in subsequent periods. On the one hand, garnishment makes bankruptcy more costly to individuals. This reduces the value of using bankruptcy as insurance against adverse income or expenditure shocks. On the other hand, banks expect some repayment even in default and price loans more favorably under garnishment. Cheaper loans mean households gain greater access to credit.

To quantify these effects and assess welfare implications, I set up a limited commitment model with equilibrium bankruptcy and endogenous labor supply. I explicitly model the time in bankruptcy during which households are subject to harsh wage garnishment. The quantitative model is calibrated to match important facts of household income, debt, and bankruptcy filings in Germany. I answer the following questions: What are the individual labor supply effects of wage garnishment? How does garnishment affect loan prices and access to credit? Finally, which garnishment regime is optimal for the German economy?

In order to answer the first two questions, I compare the current German garnishment regime with a “Fresh Start” bankruptcy regime without any wage garnishment. I find that labor supply distortions due to bankruptcy are reduced. However, abolishing wage garnishment makes bankruptcy more favorable for households and lenders expect higher write-offs. Thus, interest rates significantly increase and the amount of debt supported in equilibrium drops by 20%. Lower total debt actually leads to a small decrease in bankruptcy filings. Resulting from more restricted access to credit, German households would suffer a 0.8% welfare drop if garnishment were abolished. No income group would gain from the reform.

The third question is answered in line with typical prescriptions in public economics: the optimal garnishment regime should feature lower marginal effective tax rates (i.e. lower garnishment rates) and the tax base should be widened (i.e. longer time in bankruptcy). More specifically, in the optimal garnishment regime, garnishment rates are lowered by 22 percentage points and the duration of garnishment is increased from six to ten years. This yields significantly higher repayment because distortions are reduced and the duration of garnishment is increased. In response, lenders reduce interest rates and households borrow significantly more. Due to higher equilibrium debt, bankruptcies double despite a longer garnishment period. On average, households enjoy a 3.3% welfare improvement. Comparing ex-ante welfare, no income class loses from this policy and low-income households gain up to 7%.

\[2\] See, e.g. Rea (1984). In a three-period setting that ignores negative work incentives, Dye (1986) shows that optimal bankruptcy regimes garnish future income.

\[3\] Indeed, Lin and White (2001) find evidence of this mechanism. They show that in U.S. states where banks expect higher repayment (through lower exemption levels), consumers are more likely to gain access to loans.

\[4\] See, for example, OECD (2010).
Most quantitative research has focused on models representing the “Fresh Start” bankruptcy system without claims towards future labor income (i.e. bankruptcy under Chapter 7 in the U.S.). Hence, papers in the tradition of Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007) do not consider labor supply distortions. Despite these papers including a garnishment component, labor supply is exogenously fixed, abstracting from potential distortions.

In the absence of labor supply effects, there is some evidence that increasing commitment through garnishment might be welfare-improving. However, it is unclear whether these findings hold true in a framework with endogenous labor supply. Livshits, MacGee, and Tertilt (2007) argue that the benefits of insurance versus access to credit critically depend on the nature of income and expenditure risk. In a setup without unexpected expenditures, Chatterjee and Gordon (2012) find that the positive effects of increased access to credit outweigh the negative effects of reduced insurance. Under the authors’ garnishment regime, households are forced to repay outstanding debt in full, which leads to cheaper and larger lines of credit. This is an extreme view, given that most European systems do not force households to fully repay and they lie somewhere between “Fresh Start” regimes and full commitment regimes. Besides wage garnishment, European bankruptcy regimes generally feature tighter rules concerning the amount of debt that households can discard and the generosity of exemptions (Gerhardt, 2009).

Besides potentially increasing commitment to repay, garnishment reduces the incentive to default and – once in bankruptcy – the incentive to supply labor. Indeed, Fay, Hurst, and White (2002) find that – controlling for adverse events – households are more likely to default if their financial gains are higher. Vice versa, stricter garnishment should reduce bankruptcy filings. Lower non-payment risk lowers credit prices and increases access to credit (Lin and White, 2001). Dobbie and Song (2014) find strong evidence for labor supply reactions. They show significant negative effects of income seizure on future annual gross earnings, estimating the elasticity of earnings with respect to income seizure to be 0.94. Chen (2013) also finds negative employment effects of garnishment in a search model of the labor market. W. Li and Sarte (2006) model garnishment under U.S. Chapter 13 as an income tax and find negative labor supply effects. Debt relief or risk-dependent loan prices are not considered.

The remainder of the paper is structured as follows. Section 3.2 provides some background information on the details of the German bankruptcy law, before the model is presented in Section 3.3. I present the calibration in Section 3.4. Section 3.5 discusses the main features of the benchmark economy. The effects of abolishing the current garnishment regime are explored in Section 3.6, before Section 3.7 discusses the features of the optimal garnishment regime. Finally, Section 3.8 concludes.
3.2 German Bankruptcy Code

Germany introduced its personal bankruptcy law in 1999. Contrary to the U.S. system – which grants debt forgiveness and a “Fresh Start” – the German bankruptcy code tries to deter consumers from defaulting and promotes repayment (Gerhardt, 2009; Niemi, 2009). Before insolvent households can file for personal bankruptcy, they must exhibit substantial effort to settle their debts directly with their creditors. Only if an out-of-court settlement fails (which it does in more than 98% of all cases) can consumers proceed to seek bankruptcy protection.

Being granted bankruptcy protection typically entails asset seizures and a six-year repayment period. There are some exemptions to asset and housing seizures to allow for basic needs. During the repayment period, a significant part of income is garnished and distributed among creditors. As discussed in further detail in Section 3.4.1, garnishment rates for a typical household are 70% and they apply to all net income in excess of ca. 13,000€ per year. Above 38,500€, the garnishment rate is 100%.

If households show good conduct during six years by sticking to the repayment plan and not taking on new debt, remaining debt is forgiven at the end of the garnishment period (Gerhardt, 2009). Households are free to repay more than required by law. If they end up repaying all outstanding debt before the end of the period of good conduct, households can exit bankruptcy earlier than six years.

Figure 3.1 shows the bankruptcy filing rate in Germany. After the introduction of
personal bankruptcy legislation in 1999, German bankruptcy rates rose quickly and remained stable since. From 2006 to 2012, on average 0.25% of German households filed for bankruptcy each year.

### 3.3 Model

I set up a limited commitment model with equilibrium bankruptcy. In line with most of the literature, the model abstracts from secured lending (e.g. mortgages or car loans) and focuses on unsecured credit such as credit card debt or overdraft loans. The economy is populated by a continuum of heterogeneous households in an overlapping generations framework, similar to Livshits, MacGee, and Tertilt (2007). Each agent faces idiosyncratic risk in labor productivity and expenditure shocks. Households can consume, save (or borrow), file for bankruptcy and decide how much to work. Financial intermediaries operate competitively and offer loans that are priced depending on household characteristics and loan size.

Wages and the risk-free interest rate are set exogenously. Since unsecured borrowing and lending only account for a small fraction of capital in the economy, this is arguably not a strong assumption. Changes in borrowing and lending behavior do not significantly influence the marginal product of labor or the aggregate capital stock.\(^5\)

#### 3.3.1 Households

Households derive utility from consumption \(c\) and disutility from hours worked \(h\). Their life-time utility is the expected discounted sum of one-period CRRA utility functions. For a household of type \(i\) and age \(j = 1\) it can be written as

\[
U \left( \{c^i_j, h^i_j\}_{j=1}^{J} \right) = \mathbb{E} \sum_{j=1}^{J} \beta^{j-1} u(c^i_j, h^i_j) = \mathbb{E} \sum_{j=1}^{J} \beta^{j-1} \left( \frac{(c^i_j)^{1-\sigma}}{1-\sigma} - \psi \frac{(h^i_j)^{1+\phi}}{1+\phi} \right), \quad \psi > 0 \quad (3.1)
\]

In each period, solvent households (i.e. those not in bankruptcy):

1. observe their idiosyncratic labor productivity \(p\), expenditure shock \(\kappa\) and assets \(a\);
2. optimally choose whether to default \(d(a', s') = 1\) or not \(d(a', s') = 0\); and
3. choose consumption, savings (i.e. next period’s asset holdings) and labor supply optimally.

---

\(^5\)Indeed, when running their policy experiment, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) conclude that general equilibrium effects do not create noticeable dynamics.
The solvent household’s state is thus fully represented by \((a, s)\), with \(s = \{j, p, \kappa\}^6\). Households enter the model at age 21 \((j = 1)\) and die with certainty at the age of 80 \((j = J = 60)\).

For solvent households, the recursive formulation of the life-time maximization problem given state \((a, s)\) and conditional on not declaring bankruptcy \((d = 0)\) is \(V^S(a, s, 0)\):

\[
V^S(a, s, 0) = \max_{c, h, a'} \left[ u(c, h) + \beta \mathbb{E} \max \left\{ V^S(a', s', 0), V^D(a', s', 1) \right\} \right]
\]

s.t. \[ c + q(a', s)a' = y + a - \kappa \]
\[ h \in (0, 1), \ c > 0. \]  

(3.2)

The budget constraint in solvency simply states that expenditures cannot exceed labor income \((y)\) plus initial wealth \((a)\) minus the expenditure shock \((\kappa)\). \(q(\cdot)\) denotes the bond price that households are offered for saving/borrowing. This will be discussed in detail in Section 3.3.2.

An important part of the policy analysis in this paper will center around wage garnishment rules and how households adjust their labor supply to it. In order to create realistic effective marginal tax rates under garnishment, labor income taxes have to be taken into account. Thus, a household’s gross income is subject to a progressive income tax. Gross labor income comprises the wage rate \(w\), which is multiplied by the household’s labor supply \(h\). I apply the labor income tax function proposed by Benabou (2002) such that net labor income is

\[
y = \lambda_0 (wh)^{1-\lambda_1}. \]

(3.3)

The wage rate \(w\) comprises \(w = p \cdot x(j)\), where labor productivity \(p\) is multiplied by an age-dependent experience premium \(x(j)\). The productivity component \(p\) of (log) wages represents the idiosyncratic wage risk that a household faces. It is modeled as a persistent AR(1) process \(z_{i,t}\) and transitory white noise \(\varepsilon_{i,t}\). For household \(i\) at time \(t\), it reads:

\[
\log (p_{i,t}) = z_{i,t} + \varepsilon_{i,t}
\]
\[ z_{i,t} = \varrho z_{i,t-1} + \eta_{i,t}, \]

(3.4)

where \(\varrho \in [0, 1], \varepsilon \sim N(0, \sigma^2_\varepsilon)\) and \(\eta \sim N(0, \sigma^2_\eta)\).

Upon default, households enter the phase of good conduct and labor income is subject to garnishment for \(T\) periods. Upon deciding to file for bankruptcy \((d = 1)\), \(t = \{1, 2, ..., T\}\) keeps track of the household’s time in bankruptcy.

---

\(^6\)Note that due to the OLG structure, age \(j\) enters the state space.
3.3 Model

The recursive formulation in default is

\[ V^D(a, s, t) = \max_{c, h, a'} \left[ u(c, h) - d\zeta + \beta \mathbb{E} \left( V^D(a', s', t') \right) \right] \]

s.t. \[ c = [1 - g(y)] y - g^* \]
\[ (1 + \bar{r})^{-1} a' = a + g(y) y + g^* \]
\[ g^* \geq 0 \]
\[ t' = t + 1. \]

(3.5)

When defaulting, agents incur a utility cost of \( \zeta \). Once in bankruptcy, households are not free to borrow but have to comply with wage garnishment. Hence, consumption can only be as large as labor income less the part that is seized. Additional repayment (i.e. \( g^* > 0 \)) further reduces consumption. The total repayment \( g(y) + g^* \) is used to pay down debt. The remainder is rolled over at rate \( \bar{r}. \)

Allowing the two aforementioned ways of exiting bankruptcy, all outstanding debt is forgiven upon completion of the period of good conduct

\[ V^D(a, s, T + 1) \equiv V^S(0, s, 0) \]

or early exit is allowed if all outstanding debt is repaid in full. Hence

\[ V^D(a, s, t) \equiv V^S(a, s, 0) \quad \text{if} \quad a \geq 0 \]

With the value functions from equations (3.2) and (3.5) at hand, the value function for solvent households in the beginning of each period – after observing the household state \((a, s)\) but before deciding whether to default or not – can be expressed as

\[ V(a, s) = \max_{d(a, s) \in \{0, 1\}} (1 - d(a, s)) V^S(a, s, 0) + d(a, s) V^D(a, s, 1) \]

(3.6)

For households in their t-th year of default, the value function is given in equation (3.5).

3.3.2 Financial Intermediaries

Banks operate in a perfectly competitive market with free entry. Each bank can refinance or invest at the exogenous risk-free rate \( r \) outside the model economy. Upon emitting loans, banks face proportional transaction costs of \( \gamma \). At each point in time, a schedule

\[ \text{Note that it is assumed that households are protected from expenditure shocks during bankruptcy. Since these are very rare and bankruptcy rates are around 0.25\% per year, this assumption has no measurable implications.} \]
of one-period contracts is offered. Each contract is defined as a quantity-price bundle \((a', q(a', s))\). Since current household states are observed by the financial intermediary, prices vary not only by loan size but also by household type.

Due to perfect competition, the expected profits of offering any loan contract are zero, given any type of household. This condition is used to pin down the loan price as a function of loan size and household type. The expectations of next period’s repayment rate \(\tilde{\rho}\) are a function of the size of the loan \(a'\) and next period’s state of the household \(s'\), given state \(s\) today. It is denoted by \(\mathbb{E}[\tilde{\rho}(a', s') \mid s]\). Accordingly, the expectations of profits \(\pi(a', q(\cdot))\) can be written as

\[
\mathbb{E}[\pi(a', q(a', s)) \mid s] = q(a', s)a' - (1 + r + \gamma \cdot I_{a' < 0})^{-1} \mathbb{E}[\tilde{\rho}(a', s') \mid s]a' = 0 \quad \forall s, \forall a'.
\]

(3.7)

Expected profits are (expected) revenue minus (expected) cost. In case of offering a savings contract \((a' \geq 0)\), revenues are defined by the first term: \(q(\cdot)a'\). Costs are derived from the second term and amount to \(a'/ (1 + r)\). Here, the indicator function is equal to zero and \(\mathbb{E}[\tilde{\rho}(\cdot) \mid s] = 1\) since banks do not face default risk for savings contracts. Savings are therefore secure and paid the risk-free interest rate. Hence, \(q(a', s) = (1 + r)^{-1}\) if \(a' \geq 0\).

If banks provide loans, repayment might be lower: \(\mathbb{E}[\tilde{\rho}(a', s') \mid s] \in [0, 1]\). Hence, revenues are uncertain and read \(- (\mathbb{E}[\tilde{\rho}(a', s') \mid s]a') / (1 + r + \gamma)\). The costs of offering a loan contract \((a', q(a', s))\) are simply the face value \(-q(a', s)a'\).

Denote the risk-free loan price where loans are fully repaid (i.e. \(\mathbb{E}[\tilde{\rho}(\cdot) \mid s] = 1\)) as \(\bar{q} = (1 + r + \gamma)^{-1}\). Solving equation (3.7) for \(q(\cdot)\), one can then write

\[
q(a', s) = \bar{q} \cdot \mathbb{E}[\tilde{\rho}(a', s') \mid s]
\]

\[
= \bar{q} \cdot \mathbb{E}[\tilde{\rho}(a', s') (1 - d(a', s')) \mid s] + \bar{q} \cdot \mathbb{E}[\rho(a', s')d(a', s') \mid s]
\]

\[
= \bar{q} (1 - \mathbb{E}[d(a', s') \mid s]) + \bar{q} \cdot \mathbb{E}[\rho(a', s')d(a', s') \mid s] \quad \forall s, \forall a' < 0,
\]

(3.8)

where the last step in equation (3.8) uses the fact that repayment is full given no default occurring:

\[
\mathbb{E}[\tilde{\rho}(a', s') (1 - d(a', s')) \mid s] = \begin{cases} 1, & \text{if } d(\cdot) = 0, \\ 0, & \text{if } d(\cdot) = 1. \end{cases}
\]

(3.9)

Finally, denoting the fraction that is repaid conditional on defaulting \(\rho(a', s') = \tilde{\rho}(a', s')d(a', s')\), the full price schedule can be written as

\[
q(a', s) = \begin{cases} (1 + r)^{-1}, & \text{if } a' \geq 0, \\ \bar{q} (1 - \mathbb{E}[d(a', s') \mid s]) + \bar{q} \cdot \mathbb{E}[\rho(a', s') \mid s], & \text{if } a' < 0. \end{cases}
\]

(3.10)
3.4 Calibration

The fraction of recovered loans is the discounted sum of garnished incomes (and voluntary repayment $g^*$), normalized by the original loan size, denoted here by $a'_{0}$. $^8$

$$E [\rho(a'_{0}, s_{0}) | s_{0}] = \frac{\sum_{i=1}^{\tilde{T}} E [g(y_{i})y_{i} + g_{i}^{*} | s_{0}] \cdot \bar{q}^{i}}{|a'_{0}|}$$

with $\tilde{T} = \min\{T, J - j_{0}\}$. (3.11)

Garnishment ends after $\tilde{T}$ periods if either garnishment has been completed after $T$ periods or if the household dies before.

3.3.3 Equilibrium

Given a bankruptcy code, a risk-free rate $r$ and a wage process $w \cdot x(j)$, a financial market equilibrium is the set of value functions $V^S$ and $V^D$, policy functions $c(\cdot)$, $a'(\cdot)$, $d(\cdot)$, $h(\cdot)$, a set of default probabilities $E [d(\cdot) | s]$ and expected repayment rates $E [\rho(\cdot) | s]$ and an asset pricing function $q(\cdot)$ such that:

1. Households maximize $V$, $V^S$ and $V^D$, where $c(a, s)$, $a'(a, s)$, $d(a, s)$, $h(a, s)$ are the resulting optimal policy functions.

2. The bond price $q(a', s)$ is determined in a competitive market with free entry, taking as given the expected default and repayment rates $E [d(a', s') | s]$ and $E [\rho(a', s') | s]$.

3. The measure of households over states $(a, s, t)$ is constant.

3.4 Calibration

In order to provide a useful tool for policy recommendations, I calibrate the model to the German economy between 2012 and 2014. The model reproduces important facts on income, debt and bankruptcy filings. Some parameters are directly specified (see Table 3.1), while others are jointly chosen to match the data on income, debt and bankruptcy filings (see Tables 3.2 and 3.3).

3.4.1 Direct Specification

Preferences

The period utility function is assumed to be additively separable in consumption and hours worked. Discounting and the CRRA parameter of consumption are set to standard

---

$^8$In a slight abuse of notation, I introduce time indices into recursive formulation.
Table 3.1: Direct Specification

<table>
<thead>
<tr>
<th>Var</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount</td>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>CRRA conspt</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Frisch elasticity</td>
<td>$\phi$</td>
<td>$0.7^{-1}$</td>
</tr>
<tr>
<td>Wage autocorrelation</td>
<td>$\varrho$</td>
<td>0.92</td>
</tr>
<tr>
<td>Persistent wage std. dev.</td>
<td>$\sigma_\eta$</td>
<td>0.12</td>
</tr>
<tr>
<td>Transitory wage std. dev.</td>
<td>$\sigma_\epsilon$</td>
<td>0.17</td>
</tr>
<tr>
<td>Expenditure shock (€)</td>
<td>$\kappa_0$</td>
<td>0 98.6%</td>
</tr>
<tr>
<td></td>
<td>$\kappa_1$</td>
<td>7,800 0.9%</td>
</tr>
<tr>
<td></td>
<td>$\kappa_2$</td>
<td>24,500 0.5%</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r$</td>
<td>2%</td>
</tr>
<tr>
<td>Income tax level</td>
<td>$\lambda_0$</td>
<td>0.78</td>
</tr>
<tr>
<td>Income tax progressivity</td>
<td>$\lambda_1$</td>
<td>0.20</td>
</tr>
<tr>
<td>Income exemption (€)</td>
<td>$y$</td>
<td>12,600</td>
</tr>
<tr>
<td>Income cap (€)</td>
<td>$\overline{y}$</td>
<td>38,500</td>
</tr>
<tr>
<td>Garnishment rate</td>
<td>$\tau$</td>
<td>70%</td>
</tr>
</tbody>
</table>

values. To obtain the coefficient of labor supply, I use the Frisch elasticity of labor supply estimated in Hall (2009). One can argue that this is a conservative estimate, since Hall accounts for the external margin, which I abstract from.\(^9\)

**Income Process**

The idiosyncratic productivity component $p$ of household (log) wages corresponds to the residual of regressing wages on observables such as age and education. For most purposes, it is assumed to follow a combination of a persistent AR(1) process $z_{i,t}$ and transitory white noise $\varepsilon_{i,t}$, as described in equation (3.4). Using GSOEP data (German Socioeconomic Panel), Bayer and Juessen (2012) estimate the AR(1) coefficient ($\varrho$) and the standard deviations of $\varepsilon$ and $\eta$ for the 1984-2006 period. I report the results in Table 3.1.

Both components of the idiosyncratic wage process are discretized. The persistent component $z_{i,t}$ is approximated by a five-state Markov chain using the Rouwenhorst method for highly correlated processes described in Kopecky and Suen (2010). The white

\(^9\)Households can decide whether to participate in the market or not. The Frisch elasticity of labor supply is then calculated conditional on participating and captures labor supply responses to shifts in the wage rate.
noise term $\varepsilon_{i,t}$ is discretized to take three possible values.

I estimate the age-dependent experience component $x(j)$ from data on monthly gross wages in Germany by age in 2006 and 2010. The data bins are interpolated to yield yearly values using cubic splines. The mean $1/J \sum_{j \in \{1, \ldots, J\}} x(j) = 1$ is normalized to one. The experience profile is plotted in Figure 3.B.1 in the appendix.

### Expenditure Shocks

Following Livshits, MacGee, and Tertilt (2007), I assume that the expenditure shock is independently and identically distributed and discretize it into three realizations: $\kappa \in \{\kappa_0, \kappa_1, \kappa_2\}$. Besides no expense shock ($\kappa_0 = 0$), the authors calculate a moderate and a large realization. The moderate realization comprises family disruptions (such as unwanted children or divorce), whereas the large realization mainly covers medical bills (see Livshits, MacGee, and Tertilt, 2007, Table 1).

Using data from the statistic on over-indebtedness (German Federal Statistical Office, 2014b), I follow the author’s categorization and sort the reasons for over-indebtedness into reasons causing moderate and large debt, respectively. Households cite reasons such as family disruptions and poor financial planning, which create debts of around $\kappa_1 = 7,800\,€$. Larger expenses of around $\kappa_2 = 24,500\,€$ are mainly related to health reasons (addiction and disabilities) and failed housing investments.\(^{11}\)

In order to calculate the probabilities of each of these shocks, I calculate the share of over-indebtedness caused by either of the two kinds of reasons discussed above. Subsequently, I multiply these shares with the unconditional probability of filing for bankruptcy. I obtain probabilities of $\pi(\kappa_1) = 0.9\%$ and $\pi(\kappa_2) = 0.5\%$, respectively.

### Wage Garnishment

Upon default, agents enter the period of good conduct for $T = 6$ years. During this period, a household faces wage garnishment of $\tau = 70\%$ for all net labor income in excess of $y = 12,600\,€$. Above $\overline{y} = 38,500\,€$, all additional income is garnished. Figure 3.2 depicts disposable labor income with and without garnishment and marginal garnishment rates as a function of gross labor income. As is apparent from the figure, total effective marginal tax rates in garnishment (i.e. income taxes plus income garnishment) jump to 80% at a gross income of 15,000€ and reach 100% at a gross income of 59,000€.

\(^{10}\)Source: German Federal Statistical Office (2013). Figure 3.B.1 plots the age profile.

\(^{11}\)I exclude reasons that are related to labor market outcomes such as job loss or a failed business. Job loss is a reason associated to the income process in my model, whereas I abstract from business filings.
Financial Intermediaries

Banks are assumed to have access to outside financing at the risk-free interest rate, which is set to the yields of German government bonds around the 2012 value of 2%.

3.4.2 Jointly-Targeted Moments

The model is able to very well match bankruptcy filings, average debt in bankruptcy (i.e. bad debt) and average income in Germany (see Table 3.2). In order to compute aggregate statistics, the model is solved by backward iteration over the life-cycle. The model economy is then simulated in a Monte-Carlo fashion with \( N = 100,000 \) random life-cycle draws of the wage process and expenditure shocks. In a last step, the model moments are aggregated from this sample. In order to match the model to the data, the following objective function is solved

\[
\min_{\theta} \sum_i \omega_i (M_i(\theta) - D_i)^2
\]  

(3.12)

Hence, optimal parameter values \( (\theta) \) are chosen such that the sum of squared differences between the model moments \( M_i(\theta) \) and data targets \( (D_i) \) is minimized. \( \theta = \{\zeta, \gamma, \psi\} \) and deviations are weighted equally (i.e. \( \omega_i = 1 \ \forall i \)). These parameters are chosen to match three data targets: bankruptcy filings per thousand, debt when filing for bankruptcy and labor income.
3.4 Calibration

### Table 3.2: Jointly-Targeted Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy filings per 1,000 HH</td>
<td>2.5</td>
<td>2.44</td>
</tr>
<tr>
<td>Average bad debt (€)</td>
<td>55,000</td>
<td>55,150</td>
</tr>
<tr>
<td>Average Labor Income (€)</td>
<td>37,300</td>
<td>32,000</td>
</tr>
</tbody>
</table>


### Table 3.3: Internally-Determined Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stigma cost</td>
<td>ζ</td>
</tr>
<tr>
<td>Transaction cost</td>
<td>γ</td>
</tr>
<tr>
<td>Utility weight labor</td>
<td>ψ</td>
</tr>
</tbody>
</table>

**Bankruptcy Statistics**

As discussed in Section 3.2, German bankruptcy rates were very stable between 2006 and 2012. On average, 2.5 per one thousand German households filed for bankruptcy per year (see Figure 3.1). The model hits this target very closely. The resulting stigma cost are equivalent to the utility loss induced by taking 1,000 € from a 21-year-old median income individual.

**Bad Debt**

The model does a very good job in matching average debt when declaring bankruptcy (i.e. “bad debt”). Defaulted debt is any unsecured debt that households hold \(a < 0\) plus potential unforeseen expenditures \(κ\).

Despite only targeting the average value, the model captures the shape the distribution extremely well. Figure 3.3 shows the distribution of bad debt in the model and the data. The model slightly over-estimates debts below 100,000 €, while very high amounts of debt (above 300,000 €) are not captured. Since only unsecured lending is modeled, very high debt holdings are difficult to obtain. Nonetheless, one might argue that debt from other sources might be reported in the statistic. In practice, it might be difficult to identify liabilities from failed businesses or liquidated mortgages that are carried over into private bankruptcy. Hence, the data might over-state very high debt realizations.

To induce sufficiently high amounts of borrowing, the proportional transaction cost between saving and (secure) lending is \(γ = 0\). However, this does not mean that all loans
come at the safe rate of 2%. The default premium drives a wedge between save interest rate and actual interest rates quoted on loans (see Section 3.5.2).

**Labor Income**

In the benchmark case, households earn around 32,000€ per year. This is below the data equivalent of 37,300€. The resulting utility weight of labor is roughly $\psi = 2$. Reducing this parameter would increase labor supply, albeit at the cost of increasing both bankruptcy filings and average bad debt. This link cannot be broken by higher default cost because while they directly reduce bankruptcy filings, they increase average bad debt even further.

**3.5 Benchmark**

In the benchmark economy described above, about 2.5 per thousand households file for bankruptcy. More than 80% of filers have outstanding loans of around 30,000€ and about 10% have debts in excess of 80,000€. The average debt that is carried into bankruptcy amounts to around 55,000€.
3.5 Benchmark

Figure 3.4: Labor Supply Decisions Before, During, and After Bankruptcy.

3.5.1 The Effect of Garnishment on Endogenous Labor

Garnishment rates in Germany are very high. Figure 3.2b depicts the evolution of effective marginal tax rates under garnishment as a function of gross labor income. An effective marginal tax rate in excess of 70% is levied for gross incomes above 15,000€ and the marginal rate increases to 100% for net incomes above 59,000€.

Endogenous labor supply emerges as a key margin along which households adapt in response to (planning to file for) bankruptcy. Figure 3.4 shows the average realized labor supply of households before, during, and after filing for bankruptcy and being subject to wage garnishment. The gray area depicts the six years of wage garnishment that follow the declaration of bankruptcy in period 0. Left (right) of the gray area, the six years before (after) being subject to garnishment are depicted. Labor supply is normalized to 1 in the year prior to filing for bankruptcy (i.e. year −1).

In the years prior to bankruptcy, households of any income level increase their labor supply. By increasing labor earnings, households try to repay their outstanding debt and avoid bankruptcy. Once repayment becomes undesirable (or impossible) households declare bankruptcy in period 0. Labor supply drops by up to 10% in response to wage garnishment in the first period of bankruptcy (indicated by the beginning of the gray area).

During the garnishment spell, labor supply drops by another 10 percentage points to roughly 80% of pre-bankruptcy levels. This effect is mainly a composition effect: since
households are allowed to leave garnishment early if they fully repay their debts, some households work hard to achieve this early repayment. The remaining bankrupts are not able (or willing) to repay their debts early and thus wait for debt relief after six years of garnishment. As a result, average labor supply drops with the duration of the garnishment spell because the pool of bankrupts includes a decreasing amount of bankrupts working hard in order to repay early.

Since households that become subject to wage garnishment significantly reduce their labor supply one can conclude that the current garnishment regime creates substitution effects (labor becomes less desirable relative to leisure) that outweigh the income effect (poorer households tend to work more).

Finally, Figure 3.4 documents that garnishment mainly discourages labor supply by higher-income individuals. Low-income individuals are close to the income exemption level and consequently exhibit weaker reactions. Once their labor income drops below the exemption level, garnishment does not distort their labor supply decision further.

### 3.5.2 Equilibrium Loan Price

Evading garnishment has a stark impact on loan prices, as documented in Figure 3.5. Using the notation from equation (3.2), one can interpret the loan price as $q = 1/(1 + r)$.

---

12I do not report labor supply responses by top income earners because the number of observed bankruptcy filings by high-income individuals is low. Consequently, average labor supply of these bankrupts is quite noisy.
Hence, \( q \to 1 \) means a low interest rate \( r \to 0 \), while \( q \to 0 \) means increasing interest rates \( r \to \infty \).

Loan prices are (weakly) decreasing in loan size. However, there is a sharp drop at the threshold of sustainable debt. This comes from the fact that banks understand the incentives for households to default and reduce working hours, avoiding high amounts of garnishment. Hence, the expected repayment for these loans is very low.

Comparing high and low wage earners, unsurprisingly, the former have considerably higher access to credit. This manifest along two dimensions: first, credit prices only deteriorate at higher levels of debt; and second, the plunge in loan prices is less pronounced for high-income individuals. Regarding the former, higher disposable income simply allows a higher repayment of loans without the necessity to default. Furthermore, filing for bankruptcy is more costly for high-wage households relative to not filing, owing to the distortions to labor income. As a result, default becomes optimal at much higher levels of debt. Second, if highly-productive individuals file for bankruptcy, banks can recoup a larger fraction of the outstanding loans. This means that expected losses are smaller, which leads to a less pronounced drop in credit prices.

### 3.6 Abolishing Garnishment

Before discussing the welfare-maximizing garnishment regime in Section 3.7, it is instructive to study an alternative reform to the bankruptcy code. The proposed “no garnishment” regime resembles “Fresh Start” regimes such as Ch. 7 in the U.S. These regimes do not feature wage garnishment and hence the marginal garnishment rate is set to zero. This allows us to study the effects of garnishment on labor supply, as well as how claims towards future income influence credit prices.

The “no garnishment” regime corresponds to the following parameter setup: under the reformed bankruptcy code, labor income is not subject to garnishment. During the period of good conduct, individuals are only prevented from increasing debt. Effectively, bankrupts are thus excluded from borrowing. After \( T = 6 \) years, all outstanding debt is forgiven and bankrupts have a “Fresh Start”. Technically, I set \( y = \bar{y} = \infty, \tau = 0 \).

Abolishing garnishment has a strong impact on effective (marginal) tax rates during bankruptcy. Since there is no garnishment, net income and marginal tax rates in bankruptcy are equal to those in solvency. In Figure 3.2, income and marginal taxes under “no garnishment” correspond to the “net income” plots.\(^\text{13}\)

\(^{13}\text{Appendix 3.A discusses two additional garnishment regimes: “lenient garnishment” and “mean garnishment.”}\)
3.6.1 Labor Supply and Interest Rates

The effects of abolishing wage garnishment on labor supply and the loan price schedule are shown in Figure 3.6. Removing wage garnishment results in little distortions of labor supply during bankruptcy (see Figure 3.6a) compared to the benchmark with wage garnishment (see Figure 3.4). There is a slight increase in labor supply when declaring bankruptcy because bankrupts cannot borrow during the garnishment period. Towards the end of the garnishment period, labor supply declines back to the levels prior of filing for bankruptcy. On average, labor supply remains relatively stable.

When debts are relieved in period +1 after six years of good conduct, labor supply drops significantly. Households experience a positive income effect (because their debt is forgiven) and thus reduce labor supply.

Credit prices react in two ways (see Figure 3.6b): first, interest rates start increasing (i.e. the loan price schedule $q$ drops) at lower levels of debt; and second, interest rates rise more steeply.

Relating to the first point, abolishing garnishment increases the incentives to default at any given debt level. This leads households to prefer default at lower levels of debt. Due to higher expected losses, lenders react by increasing interest rates at lower levels of debt.

Relating to the second point, without garnishment, households have no way to pledge future labor income as securities for banks. Credit prices deteriorate quickly as soon as default is optimal since banks lose all of the outstanding loans and do not recover anything through garnishment. This effect is more pronounced for high-income households. These are strongly affected by garnishment and hence change default behavior more radically.
3.6 Abolishing Garnishment

3.6.2 Aggregate Effects

The aggregate effects of moving from the current German bankruptcy law to a regime without any garnishment are summarized in Table 3.4. Since default is less painful without garnishment, financial intermediaries fear that households might default earlier and interest rates might rise (see before). Consequently, less debt can be sustained in the “no garnishment” economy, where default is easier: the fraction of borrowers drops by 10% and average debt in the economy drops by more than 20%.

Interestingly, the effect of removing wage garnishment on bankruptcies is rather small. Both, the number of bankruptcies and the amount of bad debt remain rather constant. There are two opposing effects that cancel each other out: on the one hand, less painful bankruptcy requirements make households more likely to default; and on the other hand, financial intermediaries respond by increasing interest rates, which in turn makes households less likely to borrow. If households borrow less, they are less likely to default.

3.6.3 Welfare Effects

All possible reforms face the trade-off between higher punishment (e.g. through wage garnishment), allowing greater and cheaper access to credit versus greater leniency and thus better insurance against adverse events at the cost of more expensive credit. In the case of Germany, the current system is particularly harsh on high-income individuals and – as previously discussed – creates serious labor supply distortions.

I employ two different kinds of welfare measures to assess the desirability of the “no garnishment” reform. First, I evaluate welfare effects if the reform was introduced into the equilibrium of the current German economy. I assume that all new credit contracts are signed under the new law, i.e. taking into account that wage garnishment is abolished.\textsuperscript{14}

\textsuperscript{14}Since only one-period loans exist in the model, the simulated economy adapts the new rules immediately. Transition dynamics might be over-stated.
Second, I evaluate ex-ante welfare. I calculate the consumption equivalence variation (CEV) for a newborn household between being born into the “no garnishment” economy and the German benchmark economy.

Figure 3.7 presents two ways to evaluate the welfare effects of introducing the reform into the benchmark economy. Panel 3.7a shows – by age – the fraction of individuals who are in favor of the reform. A solvent household with assets $a$ and household state $s$ prefers the policy shift if

$$\tilde{V}(a, s) > V(a, s),$$

where $\tilde{V}(a, s)$ indicates the value function under the new regime for given asset holdings $a$ and household state $s$, which includes age.

40% of the population prefer a bankruptcy system without garnishment when introduced in the steady state of the benchmark economy. However, very young and very old households are clearly against this policy.

Panel 3.7b depicts the average CEV by age. In order to compute the CEV, I determine the factor $\xi$ by which consumption in the benchmark case needs to be increased annually to make a household indifferent between the benchmark and introducing the reform into the benchmark economy:

$$V_\xi(a, s) = E \sum_{i=j}^{j} \beta^{i-j} u(c_i(1 + \xi), h_i) = \tilde{V}(a, s).$$

This means that any $\xi > 0$ implies that households prefer the reform, given their current state $(a, s)$. All households are equally weighted to construct this measure and I report the economy-wide average (i.e. $E(\xi)$) and the average by age (i.e. $E(\xi \mid j), j \in \{1, 60\}$).
3.6 Abolishing Garnishment

Young households are worse off – and consequently not in favor of the reform – since “no garnishment” forces them to forgo current consumption to start building up assets. Credit is more expensive after the reform. However, under the benchmark garnishment regime, young households had much larger credit lines since garnishment made them less likely to exercise default. Additionally, they could pledge future income as collateral. It was thus easier to smooth consumption over the life-cycle and start repaying loans once wages increased with age.

Prime-age individuals overwhelmingly prefer abolishing wage garnishment. These households are net-savers and credit prices do not impact their welfare directly. Debt becomes relevant when very bad shocks realize. Under the new regime, they have an improved option of insurance since bankruptcy is much less painful. Hence, “no garnishment” reduces the negative effect of adverse shocks considerably and households prefer it over the current garnishment regime.

Finally, retirees suffer more from losing access to credit than they gain from easier bankruptcy. Since their income is low and they face significantly smaller risks, the insurance option of bankruptcy is not very relevant to them. Thus, a deterioration of credit conditions outweighs the positive effects of the reform. As depicted in Panel 3.7b, the average welfare effect of abolishing garnishment in the old benchmark economy would be -1.7%.

Besides evaluating the introduction of “no garnishment” into the current German
3 Personal Bankruptcy and Wage Garnishment

economy, the second relevant welfare measure is an ex-ante measure: How would newborns fare when being born into the “no garnishment” economy relative to being born into an economy with the current German bankruptcy law? Figure 3.8 presents the ex-ante CEV, both controlling for persistent income and behind the veil of ignorance. It is calculated similar to equation (3.14), but only for newborns of age 21 \((j = 1)\) who enter the economy with zero assets \((a = 0)\). The average ex-ante effect is roughly -0.8% and low-income households suffer the most. They experience very sharp increases in interest rates and gain relatively little since garnishment was not very hard on them. No income group is better off without garnishment.

Even when explicitly taking into account endogenous labor supply effects, I find that “no garnishment” is not a favorable regime. This is in line with Livshits, MacGee, and Tertilt (2007), who suggest that if income risk was lower than in the U.S., garnishment might be welfare-enhancing.

3.7 Optimal Garnishment Regime

After investigating the effects of completely repealing wage garnishment, this section describes the optimal garnishment regime within the benchmark’s class of policies. The social planner’s welfare function equally weights each newborn’s life-time utility. This welfare measure is equivalent to the expected ex-ante welfare of an individual born into the economy.

In order to maximize social welfare, the planner optimally chooses all four parameters of the garnishment regime: duration of garnishment \(T\), income exemption \(y\), income cap \(\bar{y}\), and the fraction garnished above the income exemption \(\tau\).\(^{15}\)

The maximization problem of the social planner is

\[
\max_{\{T,y,\bar{y},\tau\}} \sum_{z \in Z, \varepsilon \in E, \kappa \in K} V(a = 0, j = 1, z, \varepsilon, \kappa) \times \mu(z, \varepsilon, \kappa). \quad (3.15)
\]

Here, \(\mu(z, \varepsilon, \kappa)\) denotes the probability at birth of receiving persistent wage \((z \in Z)\), transitory wage \((\varepsilon \in E)\), and expenditure shock \((\kappa \in K)\).

When designing the optimal garnishment regime, the social planner faces a fundamental trade-off between readily offering insurance to unlucky households through a cheap bankruptcy option (i.e. low levels of garnishment) and ensuring low interest rates by

\(^{15}\)This amounts to assuming that the social planner cannot change the garnishment regime itself but only optimizes within the current class of garnishment functions. However, setting an exemption level, a garnishment rate and an income cap provides sufficient flexibility to define a linear or progressive garnishment regime.
3.7 Optimal Garnishment Regime

Table 3.5: Planner Solution

<table>
<thead>
<tr>
<th></th>
<th>Current Law</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in Bankruptcy ($T$)</td>
<td>6 years</td>
<td>10 years</td>
</tr>
<tr>
<td>Garnishment Rate ($\tau$)</td>
<td>70%</td>
<td>52%</td>
</tr>
<tr>
<td>Exempt Income ($y$)</td>
<td>12,600</td>
<td>50</td>
</tr>
<tr>
<td>Income Cap ($\bar{y}$)</td>
<td>38,500</td>
<td>29,200</td>
</tr>
</tbody>
</table>

discouraging default through a tough bankruptcy option (i.e. high levels of garnishment). Additionally, garnishment rates not only influence the trade-off between insurance and interest rates by making default more or less painful but also influence the write-offs that lenders experience conditional on default.

Table 3.5 presents the optimal garnishment regime, solving the planner’s problem in equation (3.15).\textsuperscript{16} The optimal regime features a garnishment period of ten years, a garnishment rate of 52%, basically no income exemption and an income cap of roughly 30,000€. While the income exemption is basically set to zero and the income cap is lowered, the time in bankruptcy rises by more than half. At the same time, the garnishment rate drops by 26% to a rate of 52%.

3.7.1 Labor Supply and Interest Rates

Lower marginal garnishment rates, ceteris paribus, lead to less recovery and higher write-offs. However, lower garnishment rates also reduce the distortions to labor supply. The total effect of lower marginal garnishment rates and longer garnishment duration actually drives up labor supply. The substitution effect of garnishment is reduced due to lower marginal rates and the income effect becomes stronger due to a longer garnishment duration. Both effects work to increase labor supply, even relative to no garnishment as discussed in the previous section.

The positive effects on labor supply are depicted in Figure 3.9a. Individuals of all income classes increase hours worked as a response to being subject to garnishment under the optimal regime. Under the optimal regime, there is less bunching around the income exemption (because it is basically zero) such that even low-income households increase labor supply and repay some of their debt.

Despite lower marginal garnishment rates, repayment increases under the optimal garnishment regime. In other words, lenders face lower write-offs due to default. Consequently, interest rates decrease and access to credit increases under the optimal garnishment regime.

\textsuperscript{16}See Appendix 3.C.2 for a detailed description of my computational approach.
(i.e. \( q \) increases). Figure 3.9b depicts the increase of \( q \). The reduction in interest rates is visible along two dimensions. First, bankruptcy becomes less attractive for households. Both low-income and high-income households only declare bankruptcy for higher levels of debt, below which households always repay. Compared to the benchmark, credit is cheaper for much larger amounts of debt. In other words, \( q \) only drops for higher amounts of debt.

Second, in case of bankruptcy, repayment under the optimal law is higher than repayment under the current law. Hence, even if households file for bankruptcy, banks expect lower write-offs. Higher expected repayment is evident with a less pronounced drop in \( q \) for debt levels above which households start filing for bankruptcy.

Reducing garnishment rates and increasing garnishment duration resembles findings on optimal taxation in the public finance literature. Optimal tax codes generally feature low marginal tax rates to reduce distortions but a wider tax base (c.f. OECD, 2010). By reducing the garnishment rate, distortions are reduced. A longer time in bankruptcy effectively lets the planner widen the base for repayment of debt by including more future household income.

### 3.7.2 Aggregate Outcomes

In reaction to cheaper access to credit, households hold significantly more debt. Table 3.6 presents key statistics in the benchmark and under optimal garnishment. While aggregate labor income slightly increases, average debt holdings increase by a factor of 3. The fraction of households in debt increases by 15 percentage points. As a result of higher indebtedness, more households are at risk of filing for bankruptcy in response to adverse
3.7 Optimal Garnishment Regime

### Table 3.6: Equilibrium Outcomes, Benchmark vs. Optimal Regime

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy filings per 1,000 HH</td>
<td>2.44</td>
<td>5.71</td>
</tr>
<tr>
<td>Fraction of HH in debt (in %)</td>
<td>56</td>
<td>71</td>
</tr>
<tr>
<td>Average debt (€)</td>
<td>30,550</td>
<td>93,009</td>
</tr>
<tr>
<td>Average bad debt (€)</td>
<td>55,150</td>
<td>209,518</td>
</tr>
<tr>
<td>Average savings (€)</td>
<td>42,184</td>
<td>40,180</td>
</tr>
<tr>
<td>Average Labor Income (€)</td>
<td>32,000</td>
<td>32,890</td>
</tr>
</tbody>
</table>

Expenditure or wage shocks. Consequently, bankruptcies more than double in equilibrium. Furthermore, the amount of bad debt in bankruptcy increases nearly three-fold. These sharp increases in default do not yield to higher interest rates, as discussed above. Since garnishment recovers more resources than in the benchmark, interest rates are lowered.

#### 3.7.3 Welfare Effects

Figure 3.10 shows that once households have made borrowing and savings choices under the current German garnishment system, a shift to the optimal regime does not receive a majority (see Panel 3.10a). While literally every newborn would vote for such a reform, many middle-aged households are not in favor of the reform. When examining Panel 3.10b, it is striking how much welfare of young households increases. However, those opposing the reform are only slightly worse off. These households have already paid back their initial debts and only need to borrow in the face of adverse shocks. When suffering adverse shocks, the option value of bankruptcy is lower in the optimal regime: repayment takes much longer. Thus, without taking advantage of cheaper credit early in life, middle-aged households do not prefer the policy shift.

Consistent with the very strong increase in the welfare of newborns, ex-ante welfare effects are large and positive. As depicted in Figure 3.11, being born into an economy with the optimal garnishment regime increases aggregate welfare and no income group suffers from the reform. In terms of CEV, individuals born into an economy with the optimal garnishment law gain 3.3% in expectations relative to those born into an economy with the benchmark garnishment law. When controlling for persistent wage at the age of 21, low-income household gain up to 7% while the highest wage individuals still gain 0.8%.

High-income households gain the least from this garnishment reform, because their credit prices only significantly improve for high amounts of debts. These households already faced favorable credit prices prior to the reform. Additionally, very productive
households rarely file for bankruptcy. Hence, changes in garnishment law do not have a large impact on their welfare.\textsuperscript{17}

Low-income households strongly benefit from cheaper and more credit. Taking advantage of upside wage risk, they are better able to smooth consumption over the life-cycle. Despite being likely to file for bankruptcy, tough garnishment rules do not strongly affect them since labor income is low. The positive effect of cheap credit significantly outweighs this drawback.

### 3.8 Conclusion

This paper sets up a quantitative model of consumer bankruptcy and endogenous labor supply in a regime with wage garnishment. It is able to match key statistics concerning bankruptcy and debt and it also fits the distribution of bad debt, which is not directly targeted. It clearly shows the negative effects of the German garnishment regime on labor supply, especially of highly-productive agents with high wages. Since households evade garnishment by strongly reducing labor supply, banks only recoup a small fraction of defaulted loans. Hence, banks expect low repayment upon default. This leads to a steep drop in credit prices around debt levels that make households file for bankruptcy.

Under the optimal garnishment law, garnishment rates are reduced by more than 26\% while at the same time bankruptcy duration is increased from six to ten years. This shift reduces labor supply distortions while at the same time widening the “tax base” for debt

\textsuperscript{17}Note that the income cap $\bar{y}$ serves as a strong punishment for high incomes. The induced upper bound of labor income prevents bankruptcy filings by highly productive individuals. Despite not raising funds for repayment, keeping the income cap actually proves to be welfare-superior.
3.8 Conclusion

Figure 3.11: % CEV: Optimal Regime vs. Benchmark, by Income.

repayment through garnishment. As a result, labor supply under the optimal garnishment regime increases, total recovery through garnishment increases and interest rates drop, especially for low-income households.

The optimal garnishment regime increases aggregate welfare by 3.3%. Under the new law, low-income households’ welfare increases by up to 7% by granting access to larger and cheaper lines of credit. High-income individuals gain 0.8% since these households already face favorable credit prices prior to the reform. By lowering interest rates, the optimal law leads to higher outstanding debt in the economy, which actually produces more equilibrium default.

By contrast, shifting to a “Fresh Start” regime without any income garnishment reduces welfare by 0.8% on average. The amount of debt in the economy declines and default rates slightly decline, which results from a strong increase in credit prices since banks expect lower repayment in equilibrium. Households react by borrowing less and saving more, making them less likely to default.
Appendix

3.A Two Additional Policy Experiments

Here, I analyze two additional policy experiments. Case 1 represents an intermediary case between the current German legislation and U.S. legislation. In this policy experiment, only income in excess of average income is subject to garnishment. Case 2 resembles the legal limits on income seizure in the U.S. for households that are not protected under Ch. 7 bankruptcy. For those households, 30% of income is seized by their creditors to repay outstanding debt. Figure 3.A.1 depicts disposable income and marginal tax rates under both regimes.

In both experiments, I remove the income cap during garnishment to reduce the distortionary effects on labor supply by highly-productive households. The policy regimes to be analyzed correspond to the following parameter setup:

1. “Mean income exemption”: The income exemption in garnishment is increased to mean labor income, hence \( y = 29,800 \) EUR, \( \bar{y} = \infty \), \( \tau = 70\% \).

2. “Lenient garnishment”: Exempt income is kept constant, but upon entering garnishment, only 30% of net income is subject to garnishment. Thus, \( y = 12,600 \) EUR, \( \bar{y} = \infty \), \( \tau = 30\% \).

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>Mean Exemption</th>
<th>Lenient Garnishment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bankruptcy filings per 1,000 HH</td>
<td>2.44</td>
<td>2.48</td>
<td>2.20</td>
</tr>
<tr>
<td>Fraction of HH in debt (in %)</td>
<td>56</td>
<td>52</td>
<td>55</td>
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<tr>
<td>Average debt (€)</td>
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<td>24,001</td>
<td>27,366</td>
</tr>
<tr>
<td>Average bad debt (€)</td>
<td>55,150</td>
<td>56,963</td>
<td>58,527</td>
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<tr>
<td>Average savings (€)</td>
<td>42,184</td>
<td>44,889</td>
<td>45,016</td>
</tr>
<tr>
<td>Average Labor Income (€)</td>
<td>32,000</td>
<td>31,884</td>
<td>31,951</td>
</tr>
</tbody>
</table>
3 Personal Bankruptcy and Wage Garnishment

Figure 3.A.1: Garnishment under Alternative Policy Experiments.

(a) Net Income. (b) Effective Marginal Tax Rates.

Figure 3.A.2: Effects of Introducing “Mean Income Exemption.”

(a) Labor Supply Decisions. (b) Equilibrium Loan Prices, Age 50.
3.A Two Additional Policy Experiments

3.A.1 Mean Income Exemption

Figure 3.A.2 depicts the labor supply effects and equilibrium loan price schedules of Case 1, “mean income exemption.” Since most low-income households are exempt and high-income households no longer face an income cap, the negative labor supply effects of garnishment become weaker. Panel 3.A.2a shows that, on average over six years of garnishment, labor supply remains relatively constant around pre-filing levels. The slight labor supply increase upon filing for bankruptcy is compensated by lower labor supply during the garnishment period. On the lender side, recovery through garnishment declines. Consequently, lenders increase interest rates (i.e. $q$ drops), as displayed in Panel 3.A.2b.

In equilibrium, bankruptcies slightly increase (see Table 3.A.1). Since default is less costly, lenders expect more bankruptcies and raise interest rates. Hence, the fraction of borrowers and the amount borrowed are both reduced. Average bad debt slightly increases because high-income households no longer face an income cap.

Figure 3.A.3 shows the aggregate welfare effects: ex-ante, households are 0.75% worse off. When controlling for income, especially low-income households suffer from the reform. These households are exempt from garnishment, face the strongest increase in interest rates and consequently face the steepest welfare decrease.

3.A.2 Lenient Garnishment

Figure 3.A.4 presents the labor supply effects and interest rate changes induced by Case 2, “lenient garnishment.” The effects are similar to Case 1, “mean income exemption.” However, lenient garnishment distorts labor supply downward during the garnishment period. Due to lower repayment in bankruptcy, interest rates rise. The drop in $q$ is not as
pronounced for low incomes as in Case 1, as under the current policy experiment these households are forced to repay some of their bad debt through garnishment.

As presented in Table 3.A.1, the introduction of “lenient garnishment” slightly reduces bankruptcies. Since default is less costly, lenders increase loan prices and the fraction of borrowers and the amount borrowed are both reduced. As a result, fewer households are pushed into bankruptcy by negative shocks.

Figure 3.A.5 shows that the “lenient garnishment” regime also makes households worse off, measured by CEV at birth. Again, low-income households suffer the most from worsening credit conditions.
3.B Life-Cycle Profile of Wages

![Experience Profile in Monthly Wages](image)

Figure 3.B.1: Experience Profile in Monthly Wages.

3.C Computational Approach

The numerical solution is computed in MATLAB 2014b on the “MLS & WISO bwFor-Cluster.” The solution algorithm is parallelized and solved on multiple sixteen core Intel Xeon nodes.

3.C.1 Model Solution and Calibration

The model is solved numerically by iterating backwards on the value function. The state space is discrete: \( A \times Z \times \mathcal{E} \times \mathcal{K} \times J \times T \), denoting the set of asset holdings, persistent income realizations, transitory income realizations, expenditure shocks, age, and time in bankruptcy. The income shock processes are discretized using Rouwenhorst’s method (Kopecky and Suen, 2010).

A. Pick structural parameters \( \theta_0 \) of length \( I \).
3 Personal Bankruptcy and Wage Garnishment

B. Let $\tilde{\theta}_i = \theta_0 + \hat{i}d$ for each $i = \{1, 2, \ldots, I\}$, where $\hat{i}$ is the standard unit vector with ith element equal to one and $d$ is the stepsize. For all $\tilde{\theta}_i$:

I. Do Value Function Iteration

1. Set $j = J$. Let $E[V(j + 1, \cdot) | z] = 0$ and $q(j, \cdot) = 0$ for all $z \in Z$.

2. Compute current period value functions $V^S(j, \cdot), V^D(j, \cdot)$ by maximizing over $c, a', h$ for both $d = \{0, 1\}$.

3. Compute $V(j, \cdot)$ by solving for optimal $d^*$.

4. Compute expected repayment in garnishment $E[p(j, \cdot)]$, taking household choices $c^*, a'^*, h^*, d^*$ as given.

5. Compute previous period credit price $q(j - 1, \cdot)$, taking repayment as given.

6. Compute $E[V(j, \cdot) | z]$ for all $z \in Z$.

7. If $j = 0$, end.

Else, set $j = j - 1$ and go to 2.

II. Calculate model moments

1. Simulate the invariant distribution over $A \times Z \times E \times K \times J \times T$.

2. Calculate model moments $M(\tilde{\theta}_i)$.

C. Update structural parameters

I. Pick $\theta^* = \arg \min_i \sum_n w_n \left( M_n(\tilde{\theta}_i) - D_n \right)^2$ to minimize the sum of squared residuals to the data targets $D_n$.

II. If $i \in \{1, \ldots, I\}$, update $\theta_0 = \theta^*$ and increase step size $d$.

Else, $i = 0$. Keep initial $\theta_0$ and decrease step size $d$.

III. If $d > \epsilon_d$, go to B. Else, end.

3.C.2 Optimal Garnishment Regime

In order to solve equation (3.15), I take the structural parameters found in Section 3.C.1 as given. Let $T = \{0, 1, 2, \ldots, 20, \infty\}$ be the set of possible garnishment durations.

Note that $T = 0$ is the no commitment case. Upon filing for bankruptcy, no monetary cost or utility cost are suffered and all debts are forgiven. Households remain solvent. Consequently, no debt can be sustained in equilibrium. At the other extreme, $T = \infty$ is defined as the full repayment case in which households can only exit garnishment, if they either repay their debts in full ($a \geq 0$) or die ($j = J + 1$).
3.C Computational Approach

- For each time in bankruptcy $T \in \mathcal{T}$
  
  A. Pick garnishment parameters $p_0 = [y, \Delta y, \tau]$ and define $\bar{y} = y + \Delta y$, $\Delta y \geq 0$.
  
  B. Let $\tilde{p}_i = p_0 + \hat{i}d$ for $i = \{1, 2, 3\}$, where $\hat{i}$ is the standard unit vector with $i$th element equal to one and $d$ is the stepsize. For all $\tilde{p}_i$:

  I. Do Value Function Iteration
     
     1. Set $j = J$. Let $\mathbb{E}[V(j + 1, \cdot) \mid z] = 0$ and $q(j, \cdot) = 0$ for all $z \in Z$.
     
     2. Compute current period value functions $V^S(j, \cdot), V^D(j, \cdot)$ by maximizing over $c, a', h$ for both $d = \{0, 1\}$.
     
     3. Compute $V(j, \cdot)$ by solving for optimal $d^*$.
     
     4. Compute expected repayment in garnishment $\mathbb{E}[\rho(j, \cdot)]$, taking household choices $c^*, a'^*, h^*, d^*$ as given.
     
     5. Compute previous period credit price $q(j - 1, \cdot)$, taking repayment as given.
     
     6. Compute $\mathbb{E}[V(j, \cdot) \mid z]$ for all $z \in Z$.
     
     7. If $j = 0$, end.
        
        Else, set $j = j - 1$ and go to 2.

  II. Calculate newborns expected life-time utility

     1. Using stationary distribution $\mu$, let $V(\tilde{p}_i, T) = \sum_{z \in Z, \varepsilon \in \mathcal{E}, \kappa \in \mathcal{K}} V(a = 0, j = 1, z, \varepsilon, \kappa) \times \mu(z, \varepsilon, \kappa)$.

  C. Update garnishment parameters

     I. Pick $p^*_T = \arg \max_i V(\tilde{p}_i, T)$ to maximize expected life-time utility $V$ given garnishment duration $T$.
     
     II. If $i \in \{1, \ldots, I\}$, update $p_0 = p^*_T$ and increase step size $d$.
        
        Else, $i = 0$. Keep initial $p_0$ and decrease step size $d$.
        
     III. If $d > \epsilon_d$, go to B. Else, end.

- Choose optimal bankruptcy duration

  A. Pick $T^* = \arg \max_T V(p^*_T, T)$ to maximize expected life-time utility $V$.
  
  B. Optimal set of policy parameters is $\{T^*, p^*_T, \}$. 

I use a multi-start approach when picking initial garnishment parameters $p_0$. I also verify that ex-ante welfare – given optimal garnishment parameters $p^*$ – decreases constantly
3 Personal Bankruptcy and Wage Garnishment

for all persistent income classes when the bankruptcy duration exceeds $T = 15$ years. While I am not able to prove the optimality of my result, I am reasonable certain that the reported optima are actually the solution to the planner’s problem.
4 Regulation of Consumer Credit with Over-Optimistic Borrowers

Joint with Igor Livshits, James MacGee, and Michèle Tertilt.

4.1 Introduction

In the wake of the Subprime Crisis, national controversy regarding regulating consumer credit has sparked. Following the Dodd–Frank Wall Street Reform and Consumer Protection Act in 2010, the Consumer Financial Protection Bureau was established to regulate credit products. The ensuing policy debate mostly centered around whether borrowers understand all relevant information when making financial decisions, how borrowers process this information and whether cognitive biases can induce potentially harmful borrowing decisions.¹

The debate is growing over whether some households are over-borrowing and being “trapped in debt”² and whether additional regulation is required to protect behavioral consumers from strategical pricing by lenders. Campbell (2016) argues in the 2016 Richard T. Ely Lecture that the absence of financial regulation overly hurts behavioral households. According to Campbell, regulation plays an important role in mitigating the effects of “financial mistakes.” Despite the growing policy debate, surprisingly little effort has been made to understand the implications of the presence of behavioral borrowers in a quantitative model. In this paper, we develop a framework to address this void.

We quantitatively evaluate the effect of introducing over-optimistic borrowers into an economy with unsecured debt and equilibrium default. Lenders price credit endogenously and potential spill-overs between rational and over-optimistic borrowers may arise.³ We

¹C.f. Bar-Gill and Warren (2008): “We harness both theory and data to demonstrate that sellers of credit products have learned to exploit the lack of information and cognitive limitations of consumers in ways that put consumers’ economic security at risk, turning them into far more dangerous products than they need to be”.

²Senator Chris Dodd, U.S. Senate, Congressional Record, 155, S5314 (2009).

³Over-optimists form expectations that over-(under)-estimate the probability of good (bad) future events. “Realists” form rational expectations and use the true fundamental probabilities.
study how lenders form beliefs about borrower types and how these beliefs influence the interest rates quoted. We establish that over-optimists are generally cross-subsidized by realists and that over-optimists declare bankruptcy more often than they would if not pooled with realists. If the fraction of over-optimists rises in our economy, both types borrow less and default less at an individual level. However, due to a composition effect, aggregate debt and aggregate bankruptcies increase. We also show that introducing harsher repayment requirements for bankrupts would harm over-optimistic borrowers more than realistic borrowers. Over-optimists suffer more from harsher bankruptcy requirements because they face structurally higher expense risks and consequently prefer cheaper options to default on their debt.

The equilibrium bankruptcy model that we employ is based on Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007). As in the latter framework, our model is an incomplete markets model populated by multiple generations of heterogeneous agents. Households are subject to idiosyncratic uncertainty about earnings and unforeseen expenditures (which we term “expense shocks”). When households learn the realization of this uncertainty, they can choose to file for bankruptcy or not, subject to the rules governing Chapter 7 bankruptcy in the U.S. Besides rational households as in those standard models, we introduce over-optimistic households that think of themselves as realists (and – conditional on their state – behave as realists) but actually face systematically higher expense risk. If households do not default, they can borrow or save in a one-period bond that is priced in a perfectly competitive debt market. While financial intermediaries observe household earnings, age and current debt or asset positions, they do not know with certainty whether a household is overly optimistic or not. However, financial intermediaries observe income and expense shock realizations and form beliefs about the probability of a household being a realist. We refer to these beliefs as type scores. In equilibrium, credit prices will depend on current income, age and the level of borrowing, as well as this type score.

There are two key differences between our paper and previous work on models of equilibrium bankruptcy: first, we model over-optimistic borrowers who are not conscious about the fact they face structurally higher expense risk; and second, these over-optimists are endogenously pooled with realists (with correct beliefs about the future) dependent on the type score that lenders attach to them. To obtain these type scores, lenders continuously learn about a borrower’s type by observing her income and expense realizations. This setup allows us to not only study the borrowing behavior of over-optimists in a market with equilibrium default, but also to analyze the degree to which cross-subsidization is important for our welfare implications.
4.1 Introduction

Over-optimism is not well defined in the literature.\textsuperscript{4} In the popular debate, over-optimists are usually assumed to be unable to fully grasp the negative consequences of not serving outstanding loans. Thus, over-optimism is usually assumed to lead to over-borrowing. However, Hynes (2004) argues that over-optimism could also imply insufficient debt by under-estimating default risk in the future. In our framework with endogenous default, we find exactly that over-optimists take out less debt than they would if they were to understand their fundamental risk because they do not expect to default on this debt in the future. Even when over-optimists are pooled with realists and thus face more favorable credit conditions, they borrow less than if they understood their fundamental risk. Despite “under-borrowing,” welfare losses of over-optimists are small in equilibrium because of cross-subsidization from realists.

“Under-borrowing” is supported by Rozsypal and Schlafmann (2017). The authors use the Michigan Surveys of Consumers to document that households over-estimate the persistence of their income. For lower income individuals, a forecast error that leads to over-estimating the persistence of income implies less borrowing relative to the rational benchmark.

Although our model features lenders who are better informed than borrowers about the risk of default, our structure differs from one common definition of predatory lending. Bond, Musto, and Yilmaz (2009) define a predatory loan as one that a borrower would decline if they had the same information as the lender. Depending on each household’s type score, borrowing in our model pools borrowers with correct beliefs about future default risk with borrowers who incorrectly share the same beliefs. However, contrary to Bond, Musto, and Yilmaz (2009), over-optimists are aware of and agree with their type score as it is simply a function of realized past shocks. They are ignorant about their fundamentally higher risk and simply think of themselves as being unlucky and thus they are pooled with worse risks. As a result, they agree to the loan contract offered to them. Even more strikingly, if one was to resolve their ignorance, over-optimists would understand that their loan contracts have been subsidized by realists and would be more than happy to accept those contracts.

There is ample work on behavioral traits in consumers’ choices finding heterogeneity across consumers in their consistency with rationality. Based on experimental evidence, Bruhin, Fehr-Duda, and Epper (2010) argue that many individual choices deviate from standard expected utility preferences.

Gathergood (2012) designed an add-on questionnaire to the DebtTrack survey conducted in the U.K. He finds that consumers reporting financial difficulties are also more likely

\textsuperscript{4}This point was already made by Hynes (2004).
to report that they have self-control issues. He documents that behavioral households are more likely to endure unforeseen expenditures. In line with these findings, our model setup will use unexpected expense shocks to separate over-optimists from realists.

In applied theory work, Heidhues and Kőszegi (2015) argue that lenders can take advantage of borrowers who under-estimate their future impatience. These borrowers back-load repayments and thus incur penalties that they did not anticipate ex-ante.

Despite broad evidence on behavioral traits in consumers, there is little work taking these traits into account when analyzing models of consumer bankruptcy.\footnote{Moreover, to our knowledge, we are the first to explicitly model over-optimism in a consumer bankruptcy framework.} One exemption is Laibson, Repetto, and Tobacman (2000). Their carefully calibrated life-cycle model can only predict observed credit card borrowing rates and observed levels of illiquid wealth if households are hyperbolic discounters.

Closest in spirit to this paper is Nakajima (2012, 2017). In these papers, the author introduces “temptation” preferences into an equilibrium bankruptcy model. In Nakajima’s model, behavioral consumers suffer utility losses from exerting self-control. Consequently, these consumers value any means of commitment to withstand their temptation. Contrary to our findings, behavioral consumers in this context are better off when higher repayment requirements in bankruptcy are introduced because this lowers their temptation.

A growing body of literature examines the costs of behavioral decision-making in the realm of financial markets. Although Calvet, Campbell, and Sodini (2007) find modest losses from the investment portfolio decisions of Swedish households, they also find that less financially-sophisticated households tend to under-invest in higher return (but riskier) assets. In a large administrative data set obtained from a large American bank Agarwal, Chomsisengphet, Liu, and Souleles (2015) show that 40% of consumers did not choose the cheapest credit card contract available to them.

The remainder of the paper is organized as follows. We present our model in Section 4.2 and calibrate it to U.S. data in Section 4.3. In Section 4.4, we present the quantitative results and discuss the evolution of type scores and equilibrium effects. Section 4.5 discusses how a stricter repayment requirement in bankruptcy affects realists and over-optimists. Finally, Section 4.6 concludes.

### 4.2 Model Environment

This section presents the model used to explore the impact of introducing over-optimistic consumers into a standard model of unsecured debt and consumer bankruptcies. We build on Livshits, MacGee, and Tertilt (2010). The key novel feature is to include a
positive mass of consumers with incorrect beliefs in this “Fresh Start” model of consumer bankruptcy. Regarding fundamental shocks, these consumers have distorted beliefs that are overly optimistic relative to rational expectations. Consumers can be mistaken about their persistent or transitory income risk as well as the risk of unforeseen expenditures. While consumers are ignorant about the possibility of distorted beliefs, lenders learn about the probability of a given household to have incorrect beliefs. We call these probabilities type scores. Lenders attach a type score to each household and update these type scores once new information becomes available.

These extensions allow us to analyze the effects of (partially) pooling households with correct beliefs and households with incorrect beliefs. Relating to the current policy debate, we plan to answer whether behavioral households are indeed paying too much relative to actuarially fair interest rates. In other words, behavioral households might cross-subsidize rational households due to endogenous pooling in equilibrium. Furthermore, since banks update their type score over time, we can also analyze whether and how lenders target specific pricing schemes to households with distorted beliefs.

The economy is inhabited by overlapping generations that each live for \( J \)-periods. Each generation is of measure 1. While we abstract from aggregate uncertainty, households face idiosyncratic risk. A fraction \( \lambda \in (0, 1) \) of households are over-optimistic about the idiosyncratic uncertainty that they face, while \((1 - \lambda)\) have correct beliefs. These over-optimistic households face lower fundamental income processes and higher fundamental expense risks. However, we assume that over-optimists expect the same fundamental risks as “realists.” Despite persistently under-estimating (over-estimating) downside (upside) risk, over-optimists think of themselves as fully-rational agents facing the same fundamental uncertainty as the “realists.”

Markets are incomplete. Households can borrow in non-contingent one-period bonds only. They can save at the exogenous risk-free interest rate. Debt is priced endogenously. It is state-contingent because households are able to declare Chapter 7 bankruptcy.

\( ^6\)Since lenders are assumed to make zero profits in our framework, we cannot directly speak to “exploitation” of behavioral consumers. However, behavioral consumers in our model (i.e. over-optimists) might pay too high interest rates in equilibrium to the benefit of fully rational consumers (i.e. realists). 

\( ^7\)This paper focuses on unsecured debt. Following standard assumptions, the aggregate capital stock is taken as constant. This assumption seems reasonable when taking into account the small share of unsecured debt in total debt in the U.S. and it significantly reduces computational burden. The latter is needed for us to track type scores over the life-cycle.
4 Regulation of Consumer Credit with Over-Optimistic Borrowers

4.2.1 Households

Household maximize their expected discounted life-time utility,

$$\mathbb{E}^B \sum_{j=1}^{J} \beta^{j-1} u \left( \frac{c_j}{n_j} \right),$$

(4.1)
discounting at $\beta$. $c_j/n_j$ is household consumption adjusted to household size at age $j$. $B \in \{R, O\}$ denotes the whether a household forms realistic ($B = R$) or over-optimistic ($B = O$) beliefs.

Over-optimistic beliefs are persistently distorted and we assume that households never learn. In other words, over-optimists remain overly optimistic throughout their entire life without ever learning that their beliefs are biased. They think of themselves as perfectly rational and interpret bad realizations simply as continued bad luck.

Labor income at age $j$ is the product of age-dependent labor productivity and productivity shocks. An individual forming beliefs $B$ thus earns

$$y^B_j = \tau^B_j z^B_j \eta^B_j,$$

(4.2)

where, for any type of consumer $B$, $\tau^B_j$ is deterministic labor productivity, $z^B_j$ is a persistent auto-regressive earnings shock, and $\eta^B_j$ is a transitory earnings shock. These income measures carry a superscript $B$ to indicate the possibility that over-optimists face systematically different income risks.

Households face additional uncertainty as they may need to cover unforeseen expenses. Expense shocks are modeled as discrete and independently and identically distributed. $\kappa^B \geq 0$ is drawn from a finite set $K = \{0, \kappa_1, ..., \kappa_N\}$ with probabilities $\{\pi^B_0, ..., \pi^B_N\}$. An expense shocks alters a household’s net asset position. Expense shocks are not correlated with income shocks.

**Fundamental and Perceived Risk**

Realistic households perfectly understand future income and expenditure shocks:

$$\mathbb{E}^R(x'|x) = \mathbb{E}(x'|x),$$

(4.3)

where $x$ is any random variable, $x'$ is next period’s realization and $\mathbb{E}$ is the true mean.

However, over-optimistic households expect income and expense shocks to persistently
differ from their fundamental values:

\[ E^O y_j^O > E^O y_j \quad \text{and} \quad E^O \pi^O < E^O \pi. \]  

(4.4)

Most importantly, over-optimistic households are assumed to think of themselves as rational. In other words, they expect exactly the same future income and future expenditure shocks as realists:

\[ E^O y_j^O < E^O y_j^R = E^R y_j^R \]
\[ E^O \pi^O > E^O \pi^R = E^R \pi^R, \]

(4.5)

where the first inequality in each line uses equation (4.4).

This means that when controlling for the household state, over-optimists behave exactly like realists because they are ignorant about the fundamentally different risk that they face. It also follows directly from equation (4.5) that over-optimists face systematically worse shocks than realists. They face lower income shocks and higher expenditure shocks:

\[ E^O y_j^O < E^R y_j^R \quad \text{and} \quad E^O \pi^O > E^R \pi^R. \]  

(4.6)

**Bankruptcy**

Households in the U.S. can choose between Chapter 7 and Chapter 13 when filing for bankruptcy protection.\(^8\) When a household’s bankruptcy filing is accepted, it is completely relieved of any unsecured debt. Creditors do not have any claims towards the bankrupt’s future income. However, assets above a certain exemption level are seized. Due to the 2015 Bankruptcy Abuse and Consumer Protection Act, Chapter 7 is now means-tested.\(^9\)

After declaring Chapter 7 bankruptcy, consumers are exempt from re-filing for six years. Total filing cost comprise court fees and legal fees and range from roughly $1,000 to $1,700 (Sullivan, Warren, and Westbrook, 2000). The court also demands a full list of creditors, outstanding debt, available assets, regular cost of living and the details on a debtor’s income. Typical Chapter 7 bankruptcies rulings take four months until completion.

In our model, households hold zero debt when entering the period following bankruptcy.\(^10\)

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\(^9\)Roughly 70% of bankrupts file under Chapter 7. Chapter 13 – the other option – is not present in our model.

\(^10\)We do not allow bankrupts to save or borrow. This represents asset seizures during Chapter 7. Livshits, MacGee, and Tertilt (2010) contains a full argument supporting the appropriateness of this assumption.
Regulation of Consumer Credit with Over-Optimistic Borrowers

Bankruptcy entails two kinds of consequences to proxy for the legal properties of Chapter 7. First, to proxy for six years of exclusion from bankruptcy, we do not allow bankrupts to re-file for an additional three-year period after the bankruptcy period. Second, to proxy for monetary cost of bankruptcy, filers are forced to repay a fraction $\gamma$ of their income. This captures the good faith effort required from borrowers to repay their debt as well as filing fees and legal fees.

Finally, households suffer a utility cost of filing, $\chi$. It is supposed to cover the stigma cost associated with filing for bankruptcy.

Timing works as follows:

1. When entering each period, households learn their productivity and expense shocks.
2. Subsequently, households decide to file for bankruptcy or repay outstanding debt.
3. a) When declaring bankruptcy, creditors garnish earnings. Households consume the remainder $(1 - \gamma)y_j^B$. Bankrupts are barred from saving or borrowing.
   b) When not declaring bankruptcy, consumers face a standard consumption savings problem. Negative savings are unsecured debt.

4.2.2 Financial Intermediaries

Financial markets feature free entry and financial intermediaries face perfect competition. Intermediaries take deposits at the exogenous risk-free rate $r_s$. They offer loans that are priced to reflect the expected rate of repayment, taking into account the fact that households can default through Chapter 7 bankruptcy. Loans are one-period contracts. Due to bankruptcy, repayment is (partially) state-contingent. We denote the face value of each loan as $d'$. $d'$ is to be repaid next period. When creating loans, intermediaries face a proportional transaction cost, $\tau$.

When pricing debt, intermediaries form expectations about future default. When forecasting default, intermediaries have incomplete information about borrowers: while they observe debt $d'$ and the household’s persistent productivity level $z$ as well as age $j$, they cannot distinguish between realistic and over-optimistic households.\(^{12}\)

While intermediaries do not know how households form their beliefs with certainty, they infer some information on the type by observing a household’s shock realizations. By

\(^{11}\)In our setting, savings are simply denoted as negative debts, i.e. $d' < 0$.

\(^{12}\)Since the transitory shock $\eta$ and the expense shock $\kappa$ are idiosyncratic, lenders do not obtain any additional information on default risk in subsequent periods. Loan prices are thus independent of those shocks. Only $z$ carries information about future income and thus the ability to repay in the next period.
assuming completely ignorant households – over-optimistic households do not learn that they face systematically worse risk and believe that they are realist types – household behavior does not convey any information on the household type. Over-optimistic decision rules – conditional on a household state and bond price – are exactly the same as those of realists. The only observable difference between realists and over-optimists is that over-optimists consistently draw their income and expenditure shock realizations from a worse distribution. Hence, it is only by observing income and expenditure realizations that lenders can learn about the underlying household types.\footnote{Note that because both types of agents are ignorant about their fundamental differences, they do not choose to signal their type by separating in equilibrium. Hence, there can only be a (partial) pooling equilibrium.}

**Type Scores**

are the probabilities that intermediaries attach to a household being a realist. Intermediaries update their type scores using Bayes’ rule. A household of age \( j + 1 \) receiving shocks \( z', \eta', \kappa' \) has the type score

\[
s'(z', \eta', \kappa', j + 1, s) = \frac{Pr_R(z', \eta', \kappa' | z) s}{Pr_R(z', \eta', \kappa' | z) s + Pr_O(z', \eta', \kappa' | z) (1 - s)},
\]

(4.7)

where \( s \in [0, 1] \) and we set the prior to \( s(z, \eta, \kappa, 1) = \lambda \).

In case over-optimistic households have biased expectations about only one shock, \( Pr_B(z', \eta', \kappa' | z) \) simplifies to

\[
Pr_B(z' | z), \\
Pr_B(\eta'), \quad \text{or} \\
Pr_B(\kappa'),
\]

(4.8)

where the last two lines use the fact that neither transitory income (\( \eta \)) nor the expense shock (\( \kappa \)) are auto-correlated. Hence, conditional probabilities equal unconditional probabilities.

Conditional on the probability of any household being a realist (\( s \)), intermediaries accurately forecast the default probability of a borrower, \( \theta(d', z, j, s) \), and price the loan accordingly.

**4.2.3 Equilibrium**

Due to perfect competition, lenders earn zero expected profits on each loan. Conditional on observable characteristics (persistent labor income \( z \) and age \( j \)) as well as a household’s
type score \((s)\), bond prices are determined by the default probability of such a household and the risk-free rate. Free entry implies that there can be no cross-subsidization of interest rates between contracts for different types of consumers. If households default, banks receive a fraction \(\gamma y/(d + \kappa)\) of the original loan from the required repayment. We assume that any debt recovery is proportionally allocated to outstanding loans and unpaid expenses.

The zero-profit condition then implies a bond price schedule of

\[
q^{ab}(d', z, j, s) = (1 - \theta(d', z, j, s))\bar{q}^b + \theta(d', z, j, s)E(\frac{\gamma y'}{d' + \kappa'})\bar{q}^b, \tag{4.9}
\]

where \(\bar{q}^b = \frac{1}{1+r_{s}+\tau}\) is the hypothetical price of a safe bond.

In the numerical solution of the model, the interest rate is restricted by a ceiling \(\bar{r}\), which yields the equilibrium bond price

\[
q^b(d', z, j, s) = \begin{cases} q^{ab}(d', z, j, s) & \text{if } q^{ab}(d', z, j, s) \geq \frac{1}{1+\bar{r}} \\ 0 & \text{otherwise.} \end{cases} \tag{4.10}
\]

When choosing their debt position, households take the equilibrium bond price schedule as given. Setting up the households’ optimization problem, we employ three distinct value functions. \(V\) is the value of being solvent, while \(\overline{V}\) is the value of filing for bankruptcy. Since we assumed that bankruptcy cannot be declared twice in a row, households can informally default when not eligible for bankruptcy.\(^{14}\) If a household defaults informally, it is forced to repay the same fraction of its income as in bankruptcy. However, contrary to bankruptcy, debt is not discharged in informal default. Households in default do not have access to borrow from the market. We thus assume a fixed interest rate \(r^r\) at which the debt is rolled over. Since households only default informally if they declared bankruptcy in the previous period, they only hold debt stemming from an expense shock. Next period’s debt is equal to \((\kappa - \gamma y)(1 + r^r)\): the original (expense shock) debt reduced by forced repayments, rolled over at \(r^r\). A household defaulting right after bankruptcy receives value \(W\).

Additionally, all value functions depend on whether individuals form realistic or over-

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\(^{14}\)Informal default is introduced to avoid the issue of empty budget sets for households that are ineligible for bankruptcy and face an expense shock. Introducing default does not alter our results since it is very rarely chosen.
4.3 Benchmark Calibration

optimistic beliefs $B \in \{R, O\}$:

$$V_j^B(d, z, \eta, \kappa, s) = \max_{c, d'} \left[ u \left( \frac{c}{n_j} \right) + \beta \mathbb{E}^B \max \left\{ V_{j+1}^B(d', z', \eta', \kappa', s'), \nabla_{j+1}^B(z', \eta', s') \right\} \right]$$

s.t. $c + d + \kappa \leq y_j^B + q^b(d', z, j, s)d'$

(4.11)

$$\nabla_j^B(z, \eta, s) = u \left( \frac{c}{n_j} \right) - \chi + \beta \mathbb{E}^B \max \left\{ V_{j+1}^B(0, z', \eta', \kappa', s'), W_{j+1}^B(z', \eta', \kappa', s') \right\}$$

s.t. $c = (1 - \gamma)y_j^B$

(4.12)

$$W_j^B(z, \eta, \kappa, s) = u \left( \frac{c}{n_j} \right) - \chi + \beta \mathbb{E}^B \max \left\{ V_{j+1}^B(d', z', \eta', \kappa', s'), \nabla_{j+1}^B(z', \eta', s') \right\}$$

s.t. $c = (1 - \gamma)y_j^B$, $d' = (\kappa - \gamma y_j^B)(1 + r^r)$

(4.13)

An equilibrium is a set of value functions, optimal decision rules for consumption $c^B(\cdot)$ and default $n^B(\cdot)$ for the consumer, default probabilities $\theta(\cdot)$, and bond prices $q^b(\cdot)$, such that households optimize (equations (4.11)-(4.13)), and bond prices are the solution to intermediaries’ problem (zero-profit condition in equation (4.10)), taking the default probabilities as given. In order to solve the model numerically, we iterate backwards on the value functions.

4.3 Benchmark Calibration

In calibrating the model, we assume a benchmark without any over-optimistic households; we set $\lambda = 0$. As discussed in the introduction, there is ample evidence on non-sophisticated consumers in the economy. However, there is no consensus as to which bias is most important or what exact fraction of consumers in the US are behaving not rational. We thus understand the following analysis as an exploration into the effects of introducing over-optimistic households without taking a stand on how large the actual fraction might be. Consequently, we introduce a range of plausible measures of over-optimists, c.f. Section 4.3.3, and present the results for each of those measures in Section 4.4.

Without over-optimism and type scoring our model collapses to that of Livshits, MacGee, and Tertilt (2007). In the following numerical example we consequently follow their calibration closely. The authors document that their model matches important facts of the US economy in 1995-1999. In order for this chapter to be self-contained we briefly
present the parameter values used to solve our model. For a more detailed exposition please refer to Livshits, MacGee, and Tertilt (2007).

### 4.3.1 Household Parameters

Households live for 54 years, which are modeled in 18 three year periods. Households enter the economy at age 20 and – for 15 model periods – receive stochastic (labor) income until they are 65 years old. During the last three model periods, households are retired and receive non-stochastic retirement benefits. The felicity function is $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$. Households discount future consumption at $\beta = 0.94$. We set the coefficient of relative risk aversion to $\sigma = 2$. Livshits, MacGee, and Tertilt (2007) use equivalence units $n_j$ to proxy for household size over the life-cycle.

Expense shocks are parameterized to represent unexpected expenses that – according to bankrupts – trigger bankruptcy. There are three sources of shocks: medical expenses, divorces and unplanned parenthood. The support of expense shocks $K$ has three elements: $\kappa \in K = \{0, \kappa_1, \kappa_2\}$. Livshits, MacGee, and Tertilt (2007) calibrate the large shock using medical expenses in the 1996 and 1997 Medical Expenditure Panel Survey (MEPS) and from the US Health Care Financing Administration (HCFA). To calibrate the smaller shock, family disruptions (i.e. divorces and unplanned and unwanted children) are used. This smaller shock is 26.4% of (three year) average income. The large medical shock corresponds to 82.18% of average endowment. The probabilities $[\pi^R_1, \pi^R_2]$ of these shocks realizing are 7.1% and 0.46%, respectively.\footnote{Expense shocks are assumed to only hit working-age households.} Livshits, MacGee, and Tertilt (2003) contains a more detailed account of the expense shock parameterization.

Following a large body of literature, Livshits, MacGee, and Tertilt assume log earnings to follow $\log y^t = z^t + \eta^t + g(X^i)$, where $g(X)$ is the deterministic component of income, $z$ is a persistent income shock and $\eta \sim N(0, \sigma^2_\eta)$ is a transitory income shock. Persistent idiosyncratic income shocks are modeled as an AR(1) process, $z^t_j = \rho z^t_{j-1} + \epsilon^t_j$, where $\epsilon^t_j \sim N(0, \sigma^2_\epsilon)$. The persistence is set to $\rho = 0.99$ annually, the annual variances are $\sigma^2_\epsilon = 0.007$ and $\sigma^2_\eta = 0.043$.\footnote{These parameters are consistent with what Storesletten, Telmer, and Yaron (2004), Hubbard, Skinner, and Zeldes (1994), and Carroll and Samwick (1997) report.} These annual figures are translated into triennial values. Then, we employ the Tauchen method to discretize the income shocks.\footnote{C.f. Adda and Cooper, 2003.} We represent the persistent shock as a five state Markov process. For the transitory shock, it is assumed that 10% of households are hit with a positive or negative realization each period. The support of the shock is set to match the variance.
Income of retirees comprises two parts: Each retiree receives a pension of 20% of average income in the economy. Additionally, retirees receive 35% of their last persistent income realization before retirement.\textsuperscript{18} Besides public pensions, retirees also run down their savings to increase consumption above pension payments.

4.3.2 Financial Market Parameters

The risk-free rate is set to 4%. The rate at which informally defaulted debt is rolled over ($r^r$) is fixed at 20% per year. Utility cost of declaring bankruptcy are set to $\chi = 0$.

There are three remaining parameters that Livshits, MacGee, and Tertilt (2007) determine jointly: Transaction cost of creating loans are $\tau = 4\%$ annually. Adding the risk-free savings rate of 4%, risk-free loans carry an annual interest rate of 8%. The interest rate cap is set to $\bar{r} = 100\%$ annually.\textsuperscript{19} The garnishment rate that forces some debt repayment in bankruptcy is $\gamma = 0.355$. This parameter does not directly represent legal garnishment rates that are in effect after households declare bankruptcy. Rather, besides garnishment, it also represents the good faith requirements under which borrowers are to repay some of their unsecured debt before defaulting and monetary cost of filing for bankruptcy.\textsuperscript{20}

Livshits, MacGee, and Tertilt (2010) document that – for lack of a direct empirical counterpart – setting the garnishment rate $\gamma$ to roughly 35% replicates the data on unsecured debt, interest rates and default very well. Additionally, it is noteworthy that Nakajima (2017) uses a very similar value of 34%.

4.3.3 Introducing Over-Optimistic Households

We introduce a measure $\lambda > 0$ of over-optimistic households into the economy. While expecting exactly the same income and expense risk, these over-optimists actually face fundamentally worse income and expense shocks. We define the degree of over-optimism by $\psi > 1$.

In the model setup, we introduce three kinds of over-optimistic households that face either lower expected persistent income ($z$), lower expected transitory income ($\eta$) or a more likely expense shock ($\kappa$). After presenting the technical implementation, Section

\textsuperscript{18}As documented by Livshits, MacGee, and Tertilt (2007), the pension system is progressive and produces an average replacement rate of 55%. These facts are well in the range of Butrica, Iams, and Smith (2004).

\textsuperscript{19}This value is substantially larger than implied by current usury laws. However, official legal ceilings can be avoided. See Livshits, MacGee, and Tertilt (2010) for a more detailed discussion. Numerically, the ceiling does not have strong effects as it rarely binds for borrowers.

\textsuperscript{20}For a more detailed discussion, see Livshits, MacGee, and Tertilt (2010).
4 Regulation of Consumer Credit with Over-Optimistic Borrowers

4.4 discusses the quantitative outcomes of introducing over-optimistic households with distorted beliefs about unforeseen expenditures into the economy.

**Persistent Income:**

Over-optimists with respect to persistent income face lower upside risk (i.e. the chance of experiencing a persistent improvement of their income) and higher downside risk than “realists.” We scale the conditional transition probabilities in the upper triangle of the Markov transition matrix by \( \frac{1}{\psi} < 1 \) and the probabilities in the lower triangle by \( \psi > 1 \). Formally, let \( Pr(z_j|z_i) \) denote the true transition probabilities of realists with income \( z_i \) in a given period receiving income \( z_j \) in the following period. Furthermore, let \( Pr^O(z_j|z_i) = \frac{1}{\psi} Pr(z_j|z_i) \) \( \forall j > i \) and \( Pr^O(z_j|z_i) = \psi Pr(z_j|z_i) \) \( \forall j < i \). Finally, we adapt the probabilities of keeping the current income realization (i.e. the diagonal elements of the Markov matrix) such that \( \sum_j Pr^O(z_j|z_i) = 1 \forall i \).

**Transitory Income:**

Over-optimistic households are \( \psi \)-times more likely to face the lowest transitory income shock compared to realistic households. The probability of receiving the highest transitory income shock is adapted accordingly and the median transitory income shock still occurs with a probability of 80%.

**Expenditure shock:**

If households are over-optimistic, they face a \( \psi \)-fold higher probability of incurring the moderate expense shock. The probability of the high expense shock is kept constant. This implies that the probability of not receiving any expense shock is reduced one-for-one with the increase in the moderate shock’s probability.

### 4.4 Quantitative Evaluation of Over-Optimism

This section evaluates the effects of over-optimistic households in our quantitative model. We focus on agents that are systematically wrong about their expense risk.\(^{21}\) Section 4.4.1 presents simulations of the evolution of type scores that intermediaries use to price credit. Section 4.4.2 presents the equilibrium effects of simulating different fractions of over-optimistic agents in our economy. It describes composition as well as interaction effects.

\(^{21}\)Over-optimism about transitory and persistent income risk are left for future work.
4.4 Quantitative Evaluation of Over-Optimism

Finally, Section 4.5 documents the effects of changing repayment requirements in bankruptcy when there are realistic and over-optimistic agents in the economy.

The benchmark model without any over-optimistic households is consistent with data on debt and bankruptcy filings as well as the life-cycle profile of bankruptcies and consumption as presented in Livshits, MacGee, and Tertilt (2007). To evaluate the equilibrium effects of including over-optimistic agents, we run several experiments steadily increasing the measure of over-optimists in the economy. These over-optimists have wrong beliefs about their fundamental expense shock risk. We evaluate the effects of introducing a fraction \( \lambda = \{0, 0.1, 0.25, 0.35, 0.5, 0.75\} \) of over-optimistic households. The degree of over-optimism is set to \( \psi = 3 \) and we assume that over-optimists only face a higher probability of incurring the smaller expense shock. Hence, \( \pi_1^O = 3\pi_1^R = 21.3\% \), and \( \pi_2^O = \pi_2^R = 0.46\% \). We select this rather high value to produce experiments that clearly illustrate the effects of over-optimism on the equilibrium. We ran several experiments with lower \( \psi \) and found the same basic forces to be at work. Despite being quantitatively smaller, the effects of lower degrees of over-optimism exhibit the same qualitative features.

Since over-optimists have distorted beliefs about the fundamental risks that they face, their expected value at birth does not correspond to the value that a potential planner would attach to their life. Over-optimistic beliefs weigh positive outcomes too heavily and vice versa. Consequently, over-optimists’ expectations do not correspond to the average outcomes of over-optimistic individuals or – since over-optimists are not aware of their own presence – to average outcomes of all types in the economy. In order to compare the welfare of over-optimists being born into one of our experiments, we introduce a welfare measure that is not distorted by biased expectations.

We thus define realized welfare \( V^O \) as the welfare that over-optimists would expect if they were to use the correct rational expectations but still behaved ignorantly over their whole life:

\[
V_1^O(d, z, \eta, \kappa, s) = u\left(\frac{c^O}{n_j}\right) + \beta E^R \max^O \left\{ V_2^O(d', z', \eta', \kappa', s'), V_2^O(z', \eta', s') \right\}, \tag{4.14}
\]

where \( c^O \) represents the optimal over-optimistic consumption policy and \( \max^O \) represents the default choice induced by the optimal over-optimistic default policy \( n^O \). These policies solve the household problem in equations (4.11)-(4.13) for over-optimistic beliefs \( B = O \). Note that while behavior is unchanged, expected values are formed using rational expectations \( E^R = E \).

\[\text{When solving the model numerically, we calculate each household's discounted utility flow that is derived from realized consumption over the life-cycle. We obtain these realized welfare measures by}\]
The expected realized welfare $V$ of being born into a certain economy is simply the average of realized welfares $V^O_1$ weighted by the ergodic distribution of newborns, $\mu$:

$$V = \sum_{z,\eta,\kappa} V^O_1(0, z, \eta, \kappa, \lambda) \mu(z, \eta, \kappa). \quad (4.15)$$

### 4.4.1 Evolution of Type Scores

As discussed in Section 4.2.2, intermediaries update type scores by observing the shocks that households face. Type scores correspond to the probability that lenders attach to a certain household being a realist. Conditional on these scores, lenders quote their credit prices.

In order to illustrate how new information is incorporated into individual type scores, Figure 4.1 plots four simulated type scores over the working life of a consumer. Each panel represents type scores created from one sequence of draws from the underlying sample space. By applying the realistic or over-optimistic probability measure, the draw is mapped to a sequence of expense shock realizations over the life-cycle of a realist and “over-optimist.”

---

simulating $N = 100,000$ overlapping generations over 30 generations. Since $N$ is large, this measure is a very close proxy to the theoretical measure in equation (4.15).

Since $\kappa$ is a random variable, it is a function $\Omega \to \mathbb{R}$, with an appropriately defined probability space $(\Omega, \mathcal{A}, P^B)$. For each panel, we draw a new sequence of $\{\omega_1, \ldots, \omega_J\}$ with $\omega_j \in \Omega$. The sequence of expense shocks is obtained by applying the probability measure $P^B$, where $P^R(\kappa_1) = \pi_1^R$ for the “realist” and $P^O(\kappa_1) = \pi_1^O$ for the “over-optimist.” $P^R(\kappa_2) = P^O(\kappa_2) = \pi_2^R = \pi_2^O$ for both.
Panel 1 shows how credit scores increase monotonically if not hit by any expense shocks, as is the case for the “realist.” The over-optimist is hit with two consecutive expense shocks $\kappa_1$ and the type score sharply decreases. His type score only recovers towards the end of the life-cycle, when he is lucky not to suffer any expense shocks.

The realistic and over-optimistic households in Panel 2 receive the same type scores as they both suffer identical expense shocks in the beginning of their life. The type scores diverge when the over-optimist is hit by a sequence of shocks that the realist does not suffer. The same pattern arises in Panel 4 after the over-optimistic household suffered an initial expense shock.

In Panel 3, the realist is rather unlucky. She faces a series of expense shocks, making her look very similar to an over-optimistic borrower. Her type score only recovers after the age of 35, when she rarely experiences any unforeseen expenses.

Figure 4.2 presents the evolution of the distribution of type scores over the life-cycle of a given generation. This generation features a fraction of $\lambda = 0.5$ “over-optimists.” Over-optimistic agents face a three-fold probability of incurring the moderate expense shock $\kappa_1$. As in our current numerical solution, we allow for nine distinct type score bins.

At the age of 22, intermediaries only observed households for one model period. Accordingly, most households did not incur an expense shock. Their type score slightly increases to above 0.5. Those households hit by an expense shock are more likely to be over-optimistic, hence the small mass around a type score of 0.25. When not hit by an
expense shock in the first period, all households are pooled and face the same interest rate schedule.

At the age of 37, fifteen years into the life-cycle, more households have faced expense shocks. Nonetheless, five model periods do not allow determining fundamentally higher expense risk with certainty. As a result, the distribution of type scores is widely dispersed.

Only towards the end of the working life can intermediaries clearly distinguish between over-optimists and “realists.” Observing the distribution at age 51 and 66, there is a large mass of type scores at the extremes of $s = 0$ (over-optimistic) and $s = 1$ (realistic). Consequently, later in the life-cycle, there is barely any cross-subsidization in interest rates and households with and without distorted beliefs each face type-specific lending contracts.

4.4.2 Equilibrium Effects

When assessing the implications of over-optimists in our framework, different effects arise: firstly, over-optimists face substantially higher expense risk; second, over-optimists do not form rational expectations about their fundamental risk and make distorted decisions; and third, they are pooled with realists and thereby influence realists’ interest rates.

In order to disentangle those effects, Table 4.1 presents two relevant intermediate scenarios that allow identifying the effects of (1) higher expense risk, (2) over-optimism and (3) partial pooling separately:

1. This economy is populated by realists only. However, the fraction $\lambda$ faces fundamentally higher expense risk corresponding to the risk of over-optimists, $\pi^O_i$, in the policy experiment. Households and financial intermediaries can identify the different types. Thus, there are no behavioral biases, cross-subsidization does not occur and borrowers are perfectly separated in the credit market. Relative to the benchmark, the effect of fundamentally higher expense risk can be identified.

2. This case features realists and a fraction $\lambda$ of over-optimists. Over-optimists are subject to higher expense risk but are perfectly ignorant about it. However, financial intermediaries can identify the different types. Thus, cross-subsidization does not occur and borrowers are perfectly separated in the credit market. Relative to Case 1, the effect of over-optimism on borrowing can be identified.

3. Finally, Table 4.2 presents the effects of increasing the share $\lambda$ of over-optimists in the full model with pooling. Over-optimists face fundamentally higher expense risk, cannot be identified by intermediaries and consequently are (partially) pooled with
realists, depending on their type scores. Relative to Case 2, the effects of pooling both types of consumers become apparent.

**Case 1: Higher fundamental expense risk**

As apparent from the first block of Table 4.1, relative to the benchmark replicated in the \( \lambda = 0 \) column, welfare declines with increasing expense risk. This was to be expected, since expense shocks are purely wasteful. As more individuals are subject to more expense shocks, the fraction of borrowers as well as the amount borrowed (measured by the debt-to-income ratio) increases.

Unsurprisingly, the higher the expense risk in the economy, the higher the bankruptcies. The amount of debt written off in bankruptcy (measured both as total debt written off – i.e. “bad debt” – or when examining the debt-to-income ratio of bankrupts) also increases with the expense risk.

Higher write-offs directly drive up credit prices. Since there is no pooling, high-risk realists and standard realists are perfectly separated in the credit market and face interest rates of 59.9% and 32.6%, respectively, regardless of the population shares. The reported interest rate is an economy-wide average, thus weighting these rates with \( \lambda \) and \((1 - \lambda)\).

Perfect separation also explains the constant welfare measures by type: regardless how many high-risk realists populate the economy, all of them face the same (higher) interest rates and can borrow accordingly. This is also true for all other measures: the fraction of borrowers, debt-to-income ratios, and bankruptcies show no interaction effects and are simply derived by appropriately weighting the \( \lambda = 0 \) and \( \lambda = 1 \) cases. In other words, there are only composition effects.

**Case 2: Higher fundamental expense risk and over-optimism**

The second block of Table 4.1 presents the outcomes when over-optimists with higher expense risk populate the economy, without being pooled with realists. Hence, there are also no interaction effects. In line with standard economic theory, realized welfare is lower than if only agents with realistic expectations populate the economy. While realists are exactly as well off as in Case 1, households facing higher expense risk (i.e. over-optimists) fare worse since they are not aware of their higher risk and thus behave sub-optimally.

The effects of increasing the fraction of over-optimists are in line with Case 1: welfare decreases, debt increases, bankruptcies and interest rates rise.

The most striking difference between Cases 2 and 1 is the interest rate that high-risk over-optimists face at 75.2%, relative to high-risk realists, which only pay 59.9%. Since banks perfectly observe the type of beliefs that households hold, over-optimists pay a
Table 4.1: Intermediate Cases

<table>
<thead>
<tr>
<th>Case 1: higher risk, no over-optimism, no pooling</th>
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<tbody>
<tr>
<td>Share $\lambda$</td>
</tr>
<tr>
<td>% Borrowers</td>
</tr>
<tr>
<td>D/I Borrowers</td>
</tr>
<tr>
<td>Bankruptcies</td>
</tr>
<tr>
<td>Bad Debt</td>
</tr>
<tr>
<td>D/I Bankrupts</td>
</tr>
<tr>
<td>Loan Interest Rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: higher risk, over-optimism, no pooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share $\lambda$</td>
</tr>
<tr>
<td>Over-optimists</td>
</tr>
<tr>
<td>% Borrowers</td>
</tr>
<tr>
<td>D/I Borrowers</td>
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<tr>
<td>Bankruptcies</td>
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<td>Bad Debt</td>
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<tr>
<td>D/I Bankrupts</td>
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<tr>
<td>Loan Interest Rate</td>
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</tbody>
</table>
4.4 Quantitative Evaluation of Over-Optimism

high interest rate premium. In equilibrium, high premia prompt them to borrow less often and borrow lower amounts than their realist counterparts in Case 1. In other words, over-optimism prompts households to “under-borrow.” Over-optimistic households actually end up defaulting less than their realist counterparts.

Case 3: Full model with over-optimists and pooling

In the full model without assuming that lenders can perfectly identify over-optimists, households are (partially) pooled across types. Hence, Table 4.2 presents equilibrium effects that not only exhibit composition effects as in Cases 1 and 2 but also interaction effects that depend on the pool of borrowers.

Note that because the measure of over-optimists affects the pool of borrowers that face the same lending contract, outcomes such as debt-to-income ratios, interest rates, defaults or welfare now depend on the share of over-optimists. Besides composition effects, interaction effects arise.24

Cross-subsidization runs from realists to over-optimists, since the latter are worse risks to lenders. The smaller the fraction of over-optimists, the better off that the remaining over-optimists are (-10.34 if \( \lambda = 0.1 \) vs. -10.41 if \( \lambda = 0.75 \)). Over-optimists face significantly better lending conditions: while in Case 2 they paid 75.2%, they only have to pay between 51.4% (when \( \lambda = 0.1 \)) to 69.3% (when \( \lambda = 0.75 \)) when being pooled with realists. On the other hand, realists face the other end of the bargain: their interest rates increase from 32.6% to between 37.4% (when \( \lambda = 0.1 \)) to 51.6% (when \( \lambda = 0.75 \)).

As a result, the welfare of over-optimistic households is higher than in Case 2, where they were not pooled with better risks. This is especially true for smaller fractions of over-optimists. Realists also do better when fewer over-optimists populate the economy, but are always worse off than without them.

In the full model with pooling, it is still true that increasing the fraction \( \lambda \) of over-optimists leads to lower welfare, higher average debt, higher average defaults, and higher average interest rates. However, due to the interaction effect of being pooled with an increasing number of bad risks, each type borrows less (in terms of debt-to-income ratio and fraction of borrowers) if \( \lambda \) increases. This leads to fewer bankruptcies for each type.

Even though both realists and over-optimists borrow less and default less often at an individual level, the composition effect of more over-optimists in the economy as a whole

\[ \text{Due to these interaction effects, borrowing behavior, default choices and prices now depend on the composition of the economy. Thus, we now report most measures for over-optimists and realists separately. In Table 4.1, without pooling, the outcomes reported in the } \lambda = 0 \text{ and } \lambda = 1 \text{ represent realists' and over-optimists' outcomes, respectively. When comparing Table 4.2 to Case 1 or 2, these columns are used as a reference.} \]
4 Regulation of Consumer Credit with Over-Optimistic Borrowers

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<thead>
<tr>
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over-compensates these individual-level effects and leads to an increase of interest rates in the fraction of over-optimists.

4.5 Policy Experiment

This section presents the results of changing the repayment requirement $\gamma$ upon filing for bankruptcy. In the current policy debate, many advocates of tougher regulation of credit products quote “over-borrowing” by consumers who over-estimate their ability to repay as a rationale for regulation. As a result of over-optimism about their future ability to repay, they supposedly end up “trapped in debt.” However, as discussed in the introduction, when taking consumer bankruptcy into account, this conclusion might not hold up. Whether or not over-borrowing is an issue critically depends on how painful consumer bankruptcy is. If behavioral agents under-estimate the likelihood of default but bankruptcy laws are rather lenient, over-borrowing might not be very harmful. However, if bankruptcy legislation is very tough, over-borrow might be detrimental to welfare.

In order to shed light on this mechanism, Table 4.3 presents the results of changing the repayment requirement $\gamma$ upon default. We compare an economy with $\lambda = 0.1$ of
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over-optimists to the benchmark with only realists populating the economy. Increasing the required repayment can be interpreted as any regulation that increases the lender’s expected recovery for defaulted loans: harsher requirements for bankrupts to show good faith or stricter garnishment rules before the bankruptcy case goes to trial are just two examples. We present the effects of significantly increasing forced repayment to the fraction $\gamma = \{0.53, 0.84\}$ of income and lowering it to the fraction $\gamma = \{0, 0.24\}$ of income.

On average, as well as for both types of agents, welfare increases if the required repayment is lowered. In both economies – with and without over-optimists – lower repayment requirements yield significantly more bankruptcies and thus much higher interest rates. While the debt-to-income-ratio of borrowers does not strongly respond to higher interest rates, the number of borrowers sharply declines. As bankruptcies become less painful, less debt can be sustained in equilibrium. The effects of increasing $\gamma$ are exactly opposite to that.

Figure 4.3 plots the welfare effects of changing the repayment requirement ($\gamma$) in terms of consumption equivalence variation (CEV). Over-optimistic households are over-proportionally affected by shifts in forced repayment: when reducing $\gamma$, realists’ welfare increases by only 1% in terms of CEV while over-optimists’ welfare increases by up to 5%. When increasing $\gamma$, realists’ welfare decreases by only 2% while over-optimists’ welfare decreases by more than 5%.

Clearly, both types of agents are better off with laxer repayment requirements. Additionally, the results imply that the welfare gap between realists and over-optimists is smaller.
if repayment requirement are low. This observations suggests that “over-borrowing” is a more severe problem if the consequences of default (proxied by the repayment requirements) are tough. In the case of lenient bankruptcy rules, over-optimists still borrow more than their realist counterparts. However, the welfare consequences are rather small if $\gamma$ is low and over-optimists are roughly as well off as realists.

When comparing the welfare results to Nakajima (2017), who conducts a similar analysis, it is most striking that increasing garnishment in our model never leads to higher welfare. Especially biased consumers are made worse off by harsher repayment requirements whereas this is not true in Nakajima, ’s model. Most of this difference is most likely explained by the fact that Nakajima models behavioral consumers as being subject to temptation. Consequently, they value commitment devices such as a very painful bankruptcy option. Higher repayment requirements achieve exactly this.

### 4.6 Conclusion

In this paper, we quantitatively assess the effects of allowing for over-optimistic households in a standard model of bankruptcy. We assume that over-optimistic households think of themselves as fully rational. This implies that they face higher fundamental expense risk than realists. Lenders do not directly observe the type of a household and hence lenders learn about the type score, i.e. the probability of a household being a realist. Loans are priced competitively, pooling households with the same type score.

Relative to a model without pooling, over-optimists are better off when facing borrowing interest rates that are cross-subsidized by realist types (who are worse off in turn). However, this slightly increases the number of bankruptcy filings observed in the economy. When introducing harsher repayment requirements upon bankruptcy, everyone is made worse off, although this is specifically true for over-optimists. Lowering the garnishment rate substantially increases welfare. Additionally, the welfare gap between over-optimists and realists narrows.

In future work, we plan to define direct data moments for the amount and the kind of over-optimism observed in the U.S. In a second step, the current model is to be calibrated to match these targets. Finally, we plan to use the carefully calibrated framework to evaluate a battery of policy proposals aimed at non-sophisticated borrowers. Examples include interest rate caps, restrictions on late fees and minimal repayment requirements for delinquent loans.
Bibliography


Bibliography


Han, Song and Geng Li (2011). “Household Borrowing after Personal Bankruptcy.” *Journal of Money, Credit and Banking* 43.2-3, pp. 491–517.


Bibliography


Curriculum Vitae

2011 – 2017 University of Mannheim (Germany)
   PhD in Economics

2015 Federal Reserve Bank of Minneapolis (USA)
   Visiting Scholar

2006 – 2011 University of Freiburg (Germany)
   Diploma (M.Sc. equivalent) in Economics

2009 – 2010 University of Wisconsin, Madison (USA)
   Visiting Graduate Student