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A Simple Game-theoretic Framework for studying R&D expenditures and R&D Cooperation

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**Non-technical summary**

Under what conditions do firms increase or decrease their innovation expenditures? What is the effect of knowledge flows between firms on their innovation effort? What determines firms’ propensity to cooperatively conduct research? In times of rapid technological progress and increased governmental interest in innovation policy, these issues are high both on the political and on the industrial economist’s agenda.

Empirical evidence on the determinants of research joint venture formation and on the effects of these research cooperations on research efforts is scarce. Economic theory thus often guides economic policy to optimally allocate research and development subsidies. Economic theory traditionally assumes that a firm’s ability to absorb knowledge from competitors is independent from its own research expenditures. Such an assumption is unlikely to meet with reality well since a firm which does not invest in research at all does not have any capacity to absorb knowledge from other firms’ research programs. In this paper, the degree to which firms are able to absorb knowledge is made dependent upon their own research efforts: the more a firm invests in research, the more it can absorb from other firms’ stock of knowledge. It turns out that the effect of research joint ventures crucially depends upon the competitiveness of the market and upon the generality of firms’ research approach. The model derived in this paper hence leads to quite different conclusions than models ignoring the endogeneity of firms’ research approaches.
A Simple Game-theoretic Framework for studying R&D expenditures and R&D Cooperation

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Abstract: This paper derives a three stage Cournot duopoly game for research collaboration, research expenditures and product market competition. The amount of knowledge firms can absorb is made dependent on their own research efforts, e.g. firms’ absorptive capacity is treated as an endogenous variable. It is shown that cooperating firms invest more in R&D than non–cooperating firms if spillovers are sufficiently large. The degree of market competition is a key determinant of the effects of research cooperation on research efforts, implying that existing models which assume perfect competition might be too restrictive.

Keywords: research cooperation, research expenditures, knowledge spillovers
JEL classification: O31

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1 Introduction

In 1952, John Kenneth Galbraith noted that the ‘era of cheap innovation’ was over. He claimed that firms had exhausted low-cost R&D programs and were now forced to pool their R&D efforts in order to achieve scientific progress and to gain and to retain market power. Until the mid–eighties, however, antitrust law hampered firms’ collaboration in the R&D process. More than 30 years passed by since Galbraith’s statement before US and European governments considerably relaxed antitrust law to allow cooperative R&D.

Starting points of this relaxation were the positive results from some German and US research collaborations. Spencer and Grindley (1993) argue that the R&D consortium SEMATECH contributed significantly to the leading position of the US in semiconductor industries. Jorde and Teece (1990) trace the success of German mechanical engineering products in the seventies and eighties back to the partly industrially–financed research institutions.

For Germany, a strong increase in the number of research joint ventures (RJVs) can be observed. While only ten percent of all manufacturing firms in Germany were involved in R&D cooperations in 1971, 20 years later almost half of all the firms in manufacturing industries conducted cooperative research (König et al., 1994). Based on US Department of Justice data, Vonortas (1997) shows that a sharp increase in the number of RJVs is also present in the US. The interest of economic policy in RJVs is still unchanged since R&D subsidies are increasingly often bound to joint R&D efforts.

Microeconomists began to develop theoretical frameworks to describe R&D expenditure and R&D cooperation in the mid–eighties. Pioneering contributions on R&D investment with spillovers are Brander and Spencer (1983), Katz (1986) and Spence (1986). A large strand of the more recent literature is built on D’Aspremont and Jacquemin (1988, 1990), who develop a two–stage Cournot duopoly game for R&D expenditures and product market competition. Many subsequent papers adopted the structure of this model with modifications. A particular relevant extension is that of Kamien et al. (1992), who introduce oligopoly markets and also allow the degree of product substitution to vary between perfect complements and perfect substitutes. In fact, the contribution of Kamien et al. is more than just an extension of D’Aspremont and Jacquemin as
Amir (2000) has recently pointed out. He shows that the two models have some quite different implications, e.g. with respect to R&D levels under alternative co-operation scenarios. A survey of the existing literature on research cooperations can be omitted here since extensive reviews by De Bondt (1996), Cohen (1995) and Geroski (1995) already exist. While the first author is mainly concerned with theoretical contributions to the literature, the latter summarizes empirical findings.\footnote{Also see the special issue of ‘Annales d’Économie et de Statistique’, vol. 49/50 (1998), on ‘The Economics and Econometrics of Innovation’ and the references cited therein.}

A main question of the literature on RJV formation is: ‘Does cooperative R&D increase or decrease R&D efforts?’ The common answer is that it depends on the relation of the level of spillovers to a term usually consisting of product substitutability and market demand. Research spillovers arise whenever knowledge produced by firm \(i\) is voluntarily or involuntarily given to some other firm \(j\) without firm \(j\) having paid for it. If spillovers are sufficiently large, R&D investment under RJV exceeds that of competition. Intuitively, there are two opposing effects of research joint ventures on research efforts. Due to the internalization of spillover — it is assumed that knowledge is fully exchanged in an RJV —, R&D investment is stimulated. Business-stealing counteracts this positive effect on R&D spending and may dominate the positive effect attributable to the internalization of technical spillovers. The competition effect depends upon the degree of product substitution so that this factor is a main determinant of the effect of joint research on research spending. This point is detailed in Section 2 below.

The theoretical part of this paper shares the essential features of the D’Aspremont and Jacquemin (1988, 1990) model. As in Kamien et al. (1992), the D’Aspremont and Jacquemin framework is extended to explicitly model the R&D cooperation decision. Firms’ R&D expenditure level, their R&D decision and their competition on the output market is modelled in a three-stage duopoly game. In the first stage, firms decide whether or not to conduct R&D in cooperation. In the second stage, they decide upon their R&D expenditures. Lastly, they compete in a Cournot-duopoly product market.

While in most existing studies — with the exception of Kamien and Zang (2000) — the extend to which firms can absorb knowledge is assumed to be exogenously determined, it is treated as a function of own innovation efforts this paper. The model developed here is closely related to that of Kamien and Zang (2000) but
takes into account more complex and interesting market demand function since it does not restrict products to be perfect substitutes as in Kamien and Zang. As it shall turn out later on the answer to the question whether or not research effort is larger under cooperation than under competition hinges upon the degree of product substitution.

While existing studies usually do not take into account the endogeneity of absorptive capacity, it in fact appears to be unlikely that firms can gain from each other’s knowledge independently of their own research effort. Cohen and Levinthal (1989) have empirically shown and theoretically described that firms’ absorptive capacity critically depends on own research efforts. In traditional models, it is assumed that even a firm which does not invest in R&D at all gains from the stock of knowledge to an identical extent as another firm which spends a large amount of money on research.

Important and empirically testable findings of the theoretical model are (i) that research efforts are larger under RJV than under competition if exogenous spillovers are sufficiently large, (ii) that an increase in R&D productivity positively affects both R&D efforts and RJV formation and (iii) that an increase in market demand leads to an increase in R&D efforts. Under the condition that the direct effect of changes in market demand, in the elasticity of product substitution and in the generality of the R&D approach is larger than the effect of these changes in innovation efforts, the following additional conclusions can be drawn: (i) increasing market demand and (ii) an increasing generality of the R&D approach provide incentives to form RJVs while (iii) an increase in product substitutability has negative effects on RJV formation.

## 2 Model

### 2.1 Market demand

In order to keep things tractable and interpretable, this paper deals with process innovation only. In Kaiser and Licht (1998), we consider both process and product R&D in a Cournot oligopoly framework with exogenous spillovers. We show that the optimality conditions for product and process R&D have virtually the same structure and that results obtained for product R&D are qualitatively also valid for process R&D.
Let the market be characterized by two one–product firms producing products $i$ and $j$. Market demand is linear and given by:

$$p_i = 1 - b\sigma q_j - bq_i,$$

where $p_i$ denotes the price of firm $i$’s product and $q_i$ ($q_j$) denotes the quantity of product $i$ ($j$). The parameter $\sigma$ is a measure of substitutability of the two goods with $\sigma \in [0, 1]$. If $\sigma = 1$, the two goods are perfect substitutes and if $\sigma = 0$, the two goods are perfect complements (this is the monopoly case). The term $b$ denotes inverse market demand, the ratio of the number of firms over the number of customers.

2.2 R&D production function

Following the tradition of R&D cooperation models (c.f. Kamien et al., 1992), market structure is modelled as a Cournot game in which firms can decrease production costs by conducting R&D. R&D efforts do not only contribute to a reduction of own production costs but also spill over to competitors, customers or suppliers. R&D–performing firms, however, have the possibility of conducting R&D in cooperation with other firms. In this case, results of R&D are assumed to be fully exchanged. By performing cooperative R&D, firms can internalize the externalities related to the R&D process. The deterministic R&D model suggested here falls short of real innovation processes which are driven by risk and irreversibilities.\(^2\)

The main assumptions on production techniques, R&D spillovers and R&D production functions are briefly introduced below. The production conditions are captured by a cost function $k_i$. By conducting R&D, firms can decrease marginal costs. Denoting $X_i$ the effective level of R&D — own R&D plus R&D received from other firms — of firm $i$, the unit cost function of firm $i$ is assumed to be given by:

$$k_i = c_i - f(X_i),$$

where $f(X_i)$ denotes the R&D production function of process innovation and $c_i$ denotes fixed costs. The cost function (2) represents per–unit production costs

\(^2\)Beaudreau (1996) discusses a model that takes into account the uncertainty and multidimensionality without, however, finding markedly different results compared to contributions based on the D’Aspremont and Jacquemin (1988, 1990) framework.
which are measured in monetary units. It is required that

$$\begin{align*}
  f(0) &= 0, \quad f(X_i) < c_i, \quad f'(X_i) > 0, \quad f''(X_i) < 0, \\
  \lim_{X_i \to -\infty} f'(X_i) &\to 0 \quad \text{and} \quad (1 - k_i)f''(X_i) + f'(X_i)^2 < 0.
\end{align*}$$

These assumptions assure that no process innovation is achieved if it is not invested in R&D, production costs are positive, the R&D production function is increasing and concave in effective R&D, marginal productivity of R&D goes to zero as effective R&D approaches infinity and that R&D costs show a steeper increase than the returns of R&D so that it is prevented that firms boundlessly invest in R&D.

Following Kamien and Zang (2000), firm $i$’s effective R&D, $X_i$, depends upon own R&D, $x_i$ and the spillovers firm $i$ receives from the other firm. Both effective and own R&D are measured in monetary units. Effective R&D is assumed to be given by

$$X_i = x_i + (1 - \delta) \beta x_j^{\delta} x_i^{1-\delta}$$

with $\delta, \beta \in [0, 1].$ Equations (4) implies that if firm $i$ does not invest in R&D at all, it cannot receive any spillovers from other firms’ research efforts.

The parameter $\beta$ denotes the exogenously–given intensity of R&D spillovers. It can, e.g. be interpreted as a parameter reflecting the degree of patent protection. For $\beta = 0$, patents perfectly protect research results, for $\beta = 1$, patents are completely unable to protect research results; $\beta$ reflects the restricted possibility to protect research results.

The parameter $\delta$ denotes firm $i$’s “R&D approach” (Kamien and Zang, 2000, p. 998). That is, if $\delta = 0$, firms are both universal recipients from and universal donors of other firms’ R&D efforts (‘general R&D approach’). Firm $i$’s effective R&D function then reduces to the standard formulation of effective R&D (e.g., Beath et al., 1998; D’Aspremont and Jacquemin, 1988, 1990; Kamien et al., 1992) for duopolies, $X_i = x_i + \beta x_j$.

At the other extreme, with $\delta = 1$, effective R&D is equal to own R&D. Then, firms are neither able to internalize any of the other firms’ knowledge nor do they contribute to other firms’ effective R&D (‘specific R&D approach’). If $\delta$ lies in between the two extreme cases, effective R&D is homogeneous of degree one in $x_i$.

$^3$In the original paper by Kamien and Zang (2000), firms decide upon $\delta$ in an additional stage of a Cournot oligopoly game.
Hence, the parameter $\delta$ reflects how applied, as opposed to how specific, how orientated towards science, the research program is. For large values of $\delta$, the research program is focused on basic research whereas it aims at applied research for small values of $\delta$.

2.3 Stage III: Product market competition with R&D expenditures given

The R&D oligopoly game is solved by backwards induction. In stage III of the game, the two firms choose the optimal level of output given sunk costs. Collusive agreements concerning the level of output are ruled out. Firms maximize their profits, $\Pi$, independently by choosing the optimal level of output $q_i$:

$$\max_{q_i} \Pi_i = (p_i - k_i)q_i - x_i. \quad (5)$$

Optimal output is derived by using the Cournot assumption and is given by

$$q_i^* = \frac{(1 - k_i) + \frac{\sigma}{2-\sigma}((1 - k_i) - (1 - k_j))}{b(2 + \sigma)}. \quad (6)$$

Comparative-static analysis shows, see Appendix A, that an increase in the degree of substitutability leads to a decrease in own output, that own output increases with market size and that it decreases if more specific R&D approaches are chosen.

2.4 Stage II: Determination of the R&D level

In the second stage of the game, firms maximize profits by optimally choosing R&D efforts. If firms decide not to cooperate in R&D in the first stage of the game, firm $i$’s profit function is given by:

$$\max_{x_i} \Pi_i = b q_i^*(x_i, x_j)^2 - x_i. \quad (7)$$

In a symmetric equilibrium, where firm subscripts can be omitted, optimal R&D expenditures follow from the first order condition:

$$\Psi^c = \frac{f'(X^c)(1 - c + f(X^c))}{b(2 - \sigma)(2 + \sigma)^2} \left(2 + \beta(1 - \delta)(\delta(2 + \sigma) - \sigma)\right) - 1 = 0 \quad (8)$$
where \( X^c \) denotes effective R&D of firm \( i \) under separate profit maximization (Cournot). The impact of spillovers on R&D expenditures under R&D competition is ambiguous. It is positive if

\[
\frac{f'(X^c)(\delta(2 + \sigma) - \sigma)(1 - k)}{x^c(2 + \beta(1 - \delta)(\delta(2 + \sigma) - \sigma)(f'(X^c)^2 + (1 - k)f''(X^c)))} > 0
\]

and negative otherwise.\(^4\) This condition simply states that there are two effects working against one another in a RJV: there are positive technological spillovers which arise from the joint use of research results and there are negative competitive spillovers which are due to the fact that firm \( i \) can use firm \( j \)'s research results to improve its relative competitive position.

If firms decide to cooperate in R&D in the first stage of the game, they maximize joint profits over their R&D efforts. The optimal R&D equations of Kamien et al. (1992) are obtained by neglecting the endogeneity of absorptive capacity by setting \( \delta = 0 \). Under RJV — as, e.g. in Kamien et al. (1992) — full information sharing is assumed, \( \beta \) takes on the value 1:

\[
\max_{x_i, x_j} \Pi = b q^*_i(x_i, x_j)^2 - x_i + b q^*_j(x_i, x_j)^2 - x_j,
\]  

which leads to the following first–order–condition:

\[
\Psi_j = \frac{f'(X^j)(1 - c + f(X^j))}{b(2 + \sigma)^2} 2 (1 - \delta) - 1 = 0
\]

where \( X^j \) denotes effective R&D expenditures under joint profit maximization.

The consequences of research collaboration for the level of R&D expenditures in the case of R&D cooperation can be drawn from comparing equations (11) and (8) and using the set of assumptions (3). For sufficiently large spillovers, e.g.,

\[
\beta \geq \frac{(2 - \sigma)(2 - \delta) - 2}{(1 - \delta)(\delta(2 + \sigma) - \sigma)},
\]  

R&D efforts are larger under RJV than under Cournot competition. Condition (12) is always satisfied for specific R&D approaches, \( \delta > 2 - \left( \frac{2}{(2 - \sigma)} \right) \). The difference to the Kamien et al. (1992) special case of truly exogeneous spillovers

\(^4\)Note the difference for \( \delta = 0 \): under exogenous spillovers, the impact of an increase in exogenous spillovers on R&D expenditures is unambiguously negative if goods are substitutes.
(\(\delta = 0\)) are striking since in their model research efforts are always larger under RJV than in research competition. In the Kamien and Zang (2000) model with \(\sigma = 1\), firms choose R&D approaches which ensure that the cooperative R&D exceeds noncooperative R&D if \(\beta \geq 0.5\).

The extreme cases of perfect competition and perfect substitution are also contained in the model presented here. If goods are perfect substitutes in the model presented here, the cooperative level of R&D exceeds the noncooperative level if the R&D approach is sufficiently specific: \(\delta > 1/3\), implying that in the case of tough competition, business-stealing effects due to cooperation overweigh the internalization effects only if the R&D approach is not to general. In the other extreme case, the monopoly case with \(\sigma = 0\), the cooperative R&D levels never exceed the noncooperative ones. This is quite unsurprising since there are no incentives to cooperate in R&D in monopoly, at least in models which focus on the in terms of policy relevance more interesting horizontal spillovers.\(^5\)

The contour plot of equation (12) displayed in Figure 1 visualizes the effects of the alternative combinations of the degree of product substitution, \(\sigma\) and the generality of the R&D approach, \(\delta\). It is restricted to be in the \([0,1]\) range since \(\beta\) also is in \([0,1]\) so that condition (12) is always met in the black area while it is never met in the white area in Figure 1, for example if \(\sigma = 0\) as pointed out above. A decrease in the generality of the R&D approach has two opposing effects on the incentive too form an RJV. On the one hand, involuntary knowledge leakage is reduced. On the other hand, firms gain from one another’s knowledge to a lesser extend if they pursue narrow R&D approaches. Which effect actually overweighs — i.e. if competition effects exceed internalization effects — depends upon the market structure, implying that it is worthwhile to consider more complex market structures instead of focussing attention to the extreme case of perfect substitution.

Other results from comparative–static analysis, see Appendix B, of equations (8) and (11) are that (i) an increase in R&D generality leads to an increase in research efforts under research competition if the research approach already is sufficiently general, e.g. \(\delta < \sigma/(2 + \sigma)\), while the effect in the case of research cooperation is not determined a priori,\(^6\) (ii) an increase in the degree of substitutability has


\(^{6}\)Research expenditures increase with a decrease in R&D generality if 
\[f'(1-k) + x(1-\delta)(f" + (1-k)f") < 0.\]
Note: The darker the shaded areas are, the more likely it is that cooperative R&D is larger than noncooperative R&D, and vice versa. The completely white area indicates that under the corresponding $\sigma/\delta$-combinations cooperative R&D always is smaller than noncooperative R&D.

...a disincentive effect on research efforts under competition if $\sigma < 2/3$; the effect of product substitution on optimal R&D under cooperation is not determined, (iii) an increase in market demand leads to an increase in research efforts both under RJV and competition, and (iv) an increase in R&D productivity positively affects research efforts in both cases as well.\footnote{The last result is not shown in Appendix B since it is obvious from equations (8) and (11).}

2.5 Stage I: R&D cooperation

Incentives for firms to cooperatively conduct R&D become apparent from comparing the level of profits firms earn with and without cooperation. An RJV is started if:

$$\Pi_{ij}^c - \Pi_i^c = b (q_{ij}^c)^2 - x_i^c - b (q_i^c)^2 + x_i^c > 0.$$  \hspace{1cm} (13)

Both profit functions are globally concave in $x_i$ as long as conditions (3) hold. Incentives to start a research joint venture increase with increasing differences in profits.
Incentives to start an RJV increase with increasing exogenous spillovers, $\beta$, if $\varepsilon_{x^e,\beta} > f'[X^c] \varepsilon_{x^e,\beta}$ with $\varepsilon_{x^e,\beta}$ denoting the elasticity of research expenditures with respect to spillovers.

Increases in R&D productivity create incentives to form an RJV. Provided that the direct effects of changes in the generality of the R&D approach, in market demand and in product substitutability are larger than their indirect effects via research efforts, increases in (i) the generality of the R&D approach and (ii) in product substitutability create incentive effects to RJV formation. (iii) An increase in market demand leads to an increased likelihood of RJV formation.

### 2.6 Testable model implications

The hypotheses derived from the theoretical model can be summarized as follows:

1. An increase in research productivity has a positive effect on RJV formation.
2. An increase in the generality of a firm's R&D approach creates incentive to form an RJV.
3. An increase in market demand has a positive effect on RJV formation.
4. Research efforts are larger under RJV than under research competition provided that spillovers are sufficiently large.
5. An increase in research productivity has a positive effect on R&D expenditures.
6. An increase in the generality of a firm's R&D approach leads to an increase in R&D expenditures under research competition provided that the R&D approach already is sufficiently general.
7. An increase in market demand has a positive effect on R&D expenditures.

These hypotheses are empirically tested on the basis of German service sector data by Kaiser (2001), who finds that his estimation results are broadly consistent with the predictions of the model derived here. The empirical evidence provided by Kaiser and Licht (1998) for German manufacturing industries, who

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8Note that if firms pursue a very specific R&D approach ($\delta = 1$), there are no incentives at all to collaborate in R&D since absorptive capacity is zero in this case.
do not consider endogenous absorptive capacity, also is in accordance to the hypotheses listed above. An insignificant impact of both vertical cooperations (cooperations between a firm and its suppliers or/and customers) and horizontal cooperations (cooperations among competitors) on the R&D intensity of German firms is found by Inkmann (2000). He also finds significant negative effects of intra–industry spillovers on R&D intensity and a significantly positive effect of inter–industry spillovers, while horizontal spillovers increase the tendency to cooperate with customers. Cassiman and Veugelers (1999) analyze Belgian firms to uncover the differential effects of incoming and outgoing spillovers and find that firms with large incoming spillovers and lower outgoing spillovers (better appropriation) have a higher probability of cooperating in R&D.

Other empirical work on RJVs has focused on the anatomy of the research partners. Kleinknecht and Reijnen (1992) study the determinants of research cooperation in Dutch manufacturing industries. They come to the quite surprising conclusion that firm size does not have a significant effect on the propensity to cooperate. By contrast, the existence of an R&D department, granted patents, licensing and sectoral affiliation significantly affect firms’ propensity to cooperate. The results by Kleinknecht and Reijnen (1992) may suffer from simultaneous equation bias. Röller et al. (1998) use a simultaneous equation setup. In their analysis of U.S. firms that participate in RJVs they find a tendency towards cooperation among firms of similar size and that RJV formation is dependent on a number of industry–specific effects. Veugelers (1993) describes the profile of 668 international research alliances and finds that improved market access, monitoring and control as well as complementarities in assets drive cooperative research.

3 Conclusions

In this paper a three stage Cournot–oligopoly game for product market competition, optimal R&D effort and research cooperation is derived. In contrast to most existing studies, the degree to which firms can absorb knowledge from other firms’ R&D efforts is made dependent on their own research expenditures: firms which do not invest in R&D are unable to retrieve any knowledge from competitors.

The theoretical framework is closely related to Kamien and Zang (2000) but
considers a more complex and more interesting market demand function since it does not restrict products to be perfect substitutes. It turns out that the crucial economic question if research expenditures are larger under research cooperation than under research competition crucially depends on the degree of product substitution. Intuitively, two opposing effects determine if research expenditures are larger in an RJV than in research competition: a positive internalization effect and a negative competition, or business-stealing, effect. The degree of product substitution of course is a main determinant of the degree related to the business-stealing effect so that it in fact appears worthwhile not to restrict goods to be perfect substitutes as in Kamien and Zang (2000).
Appendix A: Comparative-static properties of the product quantity equation (6)

Degree of product substitution, $\sigma$:

$$\frac{\partial q_i}{\partial \sigma} = - \frac{x \beta f'}{b (2 + \sigma)} < 0 \text{ in symmetric equilibrium}$$  (14)

Inverse market size, $b$:

$$\frac{\partial q_i}{\partial b} = - \frac{1 - c + f}{b^2 (2 + \sigma)} < 0 \text{ in symmetric equilibrium}$$  (15)

Generality of R&D approach, $\delta$ (i.e. change in output if R&D approach becomes more specific):

$$\frac{\partial q_i}{\partial \delta} = - \frac{x \beta f'}{b (2 + \sigma)} < 0 \text{ in symmetric equilibrium}$$  (16)
Appendix B: Comparative–static properties of optimal process innovation spending, equations (8) and (11)

Since the partial derivatives of \( \Psi^c \) and \( \Psi^{jv} \) with respect to process innovation spending, \( x \), are negative under both regimes, the partial derivatives of \( \Psi^c \) and \( \Psi^{jv} \) with respect to \( \sigma, b \) and \( \delta \) directly determine the effect of the respective variables on process innovation expenditures.

**Cournot competition:**
\[
\frac{\partial \Psi^c}{\partial x} = \frac{2(1 + \beta(1 - \delta)) (2 + \beta(1 - \delta)(\delta(2 + \sigma) - \sigma)) \left( f'^2 + (1 - k) \ g'' \right)}{b(2 - \sigma)(2 + \sigma)^2} < 0 \tag{17}
\]

**RJV:**
\[
\frac{\partial \Psi^{jv}}{\partial x} = \frac{2(2 - \delta)(1 - \delta)(f'^2 + (1 - k)f'')}{b(2 + \sigma)^2} < 0 \tag{18}
\]

**Effect of \( \sigma \)**

**Cournot competition:**
\[
\frac{\partial \Psi^c}{\partial \sigma} = \frac{4(2 - 3\sigma + \beta(\delta - 1)(\sigma + 2\delta - 2 + (1 - \delta)\sigma^2))(1 - k)f'}{b(\sigma - 2)^2(2 + \sigma)^3} < 0 \tag{19}
\]
if \( \sigma < 2/3 \)

**RJV:**
\[
\frac{\partial \Psi^{jv}}{\partial \sigma} = \frac{f'(1 - k) + x(1 - \delta)(f'^2 + (1 - k)f'')}{b(2 + \sigma)^3} \rightarrow \text{undetermined} \tag{20}
\]

**Effect of \( b \)**

**Cournot competition:**
\[
\frac{\partial \Psi^c}{\partial b} = -\frac{2(2 + \beta(1 - \delta)(\delta(2 + \sigma) - \sigma))(1 - k)g'}{b^2(2 - \sigma)(2 + \sigma)^2} < 0 \tag{21}
\]

**RJV:**
\[
\frac{\partial \Psi^{jv}}{\partial b} = -\frac{2(1 - \delta)(1 - k)g'}{b^2(2 + \sigma^2)} < 0 \tag{22}
\]

**Effect of \( \delta \)**

**Cournot competition:**
\[
\frac{\partial \Psi^c}{\partial \delta} = \frac{2\beta(2 + (1 + \sigma - \delta(2 + \sigma))(1 - k)g' - x(2 + \beta(1 - \delta)(2\delta - (1 - \delta)\sigma))(f'^2 + (1 - k)f''))}{b(2 - \sigma)(2 + \sigma)^2} < 0 \tag{23}
\]
if \( \delta < \sigma/(2 + \sigma) \)

**RJV:**
\[
\frac{\partial \Psi^{jv}}{\partial \delta} = -\frac{2(f'(1 - k) + x(1 - \delta)(f'^2 + (1 - k)f''))}{b(2 + \sigma)^2} \rightarrow \text{undetermined} \tag{24}
\]
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