Essays on the Macroeconomics of Housing Markets

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Contents

Preface

This dissertation consists of three chapters. Each chapter is self-contained.

Chapter 1 is joint work with Moritz Drechsel-Grau.[1] We evaluate the hypothesis that rising inequality was a causal source of the US household debt boom since 1980. The mechanism builds on the observation that households care about their social status. To keep up with the ever richer Joneses, the middle class substitutes status-enhancing houses for status-neutral consumption. These houses are mortgage-financed, creating a debt boom across the income distribution. Using a stylized model we show analytically that aggregate debt increases as top incomes rise. In a quantitative general equilibrium model we show that *Keeping up with the Joneses* and rising income inequality generate 60% of the observed boom in mortgage debt and 50% of the house price boom. We compare this channel to two competing mechanisms. The Global Saving Glut hypothesis gives rise to a similar debt boom, but does not generate a house prices boom. Loosening collateral constraints does not generate booms in either debt or house prices.

Chapter 2 is joint work with Moritz Drechsel-Grau as well. This chapter shows that the well-documented parallel surge in household debt and top income inequality in the United States has an important geographical component. First, we establish that rising incomes of the top 10% are tightly linked to rising debt of the non-rich after controlling for non-rich income as well as fixed state and year characteristics. Second, we show that this relationship is entirely driven by mortgage debt. While state-level changes in non-mortgage debt are not related to state-level top incomes, our estimates suggest that a 10% increase in a state's top incomes induces a persistent increase in mortgage debt of the state's non-rich households by up to 5% over the following years. The tight relationship between state-level top incomes and non-rich mortgage debt has important implications for our understanding of the drivers of the US household debt boom. In particular, our findings

[1] University of Mannheim.

are consistent with the theory that top income inequality drives non-rich debt through growing demand for debt as non-rich households attempt to keep up with the housing of the rich.

Chapter 3 is joint work with Frederick Zadow.[2] Housing wealth effects—the reaction of consumption to changes in house prices—were at the heart of the Great Recession. Empirical and quantitative macroeconomic studies have found that housing wealth effects are stronger for more indebted households. One important policy implication is that lowering debt limits for borrowers will dampen the consumption slump in a house price bust. Such conclusions might be premature. We build a simple life-cycle model with housing with closed form solutions for housing wealth effects. We show that the strength of housing wealth effects crucially depends on the underlying household characteristics which also determine the debt levels. In this framework imposing one-size-fits-all debt limits does not necessarily mitigate housing wealth effects. To be effective, policies have to be tailored to borrowers' characteristics. Aggregate housing wealth effects can be reduced in three ways: (*i*) if old homeowners reduce their housing wealth; (*ii*) if the home ownership rate decreases; (*iii*) if agents have smaller houses. We provide a simple empirical test of our model predictions. When explaining housing wealth effects, we find that the level of mortgages turns statistically insignificant once relevant household characteristics (age and a proxy for housing preferences) are added.

[2] University of Mannheim.

Chapter 1

Falling Behind: Has Rising Inequality Fueled the American Debt Boom?

Joint with Moritz Drechsel-Grau.

1.1 Introduction

Between 1980 and 2007, US household debt doubled relative to GDP. Mortgage debt was by far the most important driver of this household debt boom (see Figure 1.1.1a). In lockstep with mortgages, top income inequality has risen since 1980 and reached its peak in 2007 (see Figure 1.1.1b). While real incomes have stagnated for the bottom half of the population, the incomes of the top 10% have more than doubled over this time period (see Figure 1.1.2a). In the public debate, it was argued that rising top income inequality fueled the boom in household debt (e.g. Rajan, 2010; Stiglitz, 2009; Frank, 2013a), which in turn played an important role in the Global Financial Crisis of 2007 and the ensuing Great Recession.[1]

In this paper, we formally assess the hypothesis that rising top income inequality was a causal driver of the household debt boom. The underlying mechanism builds on the idea that households care about their social status. When top incomes rise and the rich upgrade their houses, the non-rich lose some of their social status. The non-rich substitute status-enhancing housing for status-neutral consumption to *keep up with the richer Joneses*. These

[1] See the survey by van Treeck (2014) on the hypothesis that inequality caused the financial crisis.

houses are mortgage-financed, causing a debt boom across the whole income distribution.

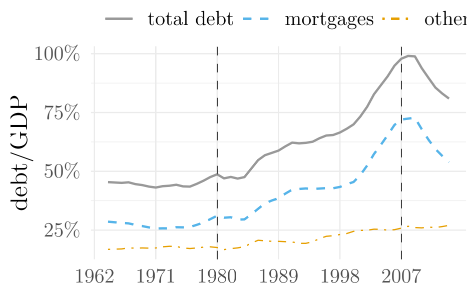
The idea that people care about how their belongings compare to those of their neighbors is certainly not new (among others Veblen, 1899; Duesenberry, 1949). Recently, there has been a growing empirical literature showing that social comparisons shape people's decision-making (e.g. Kuhn, Kooreman, Soetevent, and Kapteyn, 2011; Luttmer, 2005; Bursztyn, Ederer, Ferman, and Yuchtman, 2014; De Giorgi, Frederiksen, and Pistaferri, 2019).

We quantify the contribution of this mechanism to the observed mortgage and house price booms (Figure 1.1.3) between 1980 and 2007 and compare it to two alternative mechanisms in the literature. First, the Global Saving Glut hypothesis (e.g. Bernanke, 2005; Justiniano, Primiceri, and Tambalotti, 2014) according to which foreign capital inflow has driven down interest rates and hence enabled households to take out more debt. Second, financial liberalization (e.g. Favilukis, Ludvigson, and van Nieuwerburgh, 2017), which may increase borrowing due to a loosening of collateral constraints.[2]

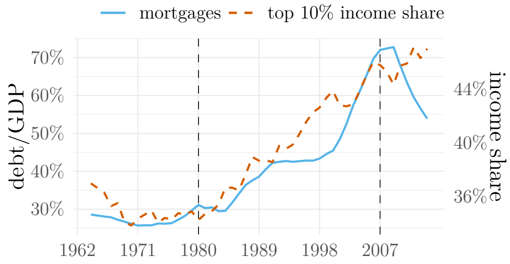
To that end, we build a heterogeneous agent general equilibrium model with housing and non-durable consumption goods, elastic housing supply, a collateral constraint, a state-of-the-art earnings process (Guvenen, Karahan, Ozkan, and Song, 2019) and a social comparison motive that we discipline using recent micro evidence on housing comparisons in the US (Bellet, 2019). We compare two steady states that differ only in the exogenous degree of income inequality. Based on evidence by Kopczuk, Saez, and Song (2010) and Guvenen, Kaplan, Song, and Weidner (2018) we scale the permanent component of income inequality to match the increase in cross-sectional income dispersion between 1980 and 2007.

We find that this rise in income inequality generates quantitatively significant mortgage and house price booms in the presence of *Keeping up with the Joneses*. Our model generates 60% of the observed increase in the mortgage-to-income ratio and 50% of the observed increase in house prices between 1980 and 2007. Even in the absence of *Keeping up with the Joneses* rising inequality drives houses prices through growing demand for housing at the top of the income distribution. Complementarities between housing and non-durable consumption increase the housing and mortgage demand of non-rich households. These general equilibrium effects are roughly doubled by social comparisons. Social comparisons directly raise the housing demand (and thus, demand for mortgages) for non-rich as a response to choices of the rich

[2]The expectations channel (Adam, Kuang, and Marcet, 2012; Kaplan, Mitman, and Violante, 2020) is another important channel, but it cannot be easily integrated into our model.



(a) The American household debt boom was mostly driven by mortgages.



(b) Inequality and mortgages have grown in lockstep since 1980.

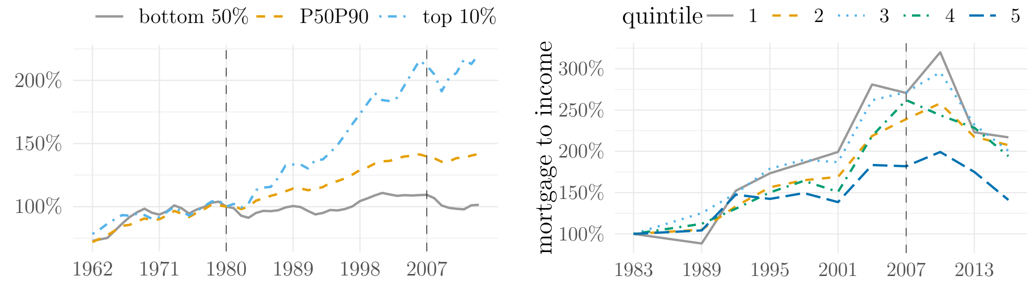
Figure 1.1.1: The American household debt boom and rising income inequality. Sources: US Flow of Funds and Alvaredo et al. (2016). Details see Appendix 1.A.

through the status externality.

In comparison, the Saving Glut generates a similarly strong debt boom through lower interest rates. However, it does not generate a strong house price increase. Both mechanisms together can explain 75% of the increase in the mortgage-to-income ratio and 60% of the house prices boom. Decomposing this total effect, we can attribute between one third and two thirds of the explained increase in debt and about 90% of the explained increase in house prices to rising inequality and social comparisons. Financial innovation, i.e., relaxed collateral constraints, raises neither debt nor house prices significantly.

Extensive robustness checks show that our quantitative findings are robust to perturbations in the internally and externally calibrated parameters. The generated effects stay quantitatively significant for deviations from the calibrated strength of the comparison motive.

In addition to the quantitative results, we show in closed form how top incomes can affect aggregate debt in a stylized version of the model without idiosyncratic earnings risk. In this infinite horizon network model (extending the one-period models in Ballester, Calvó-Armengol, and Zenou, 2006; Ghiglino and Goyal, 2010), we prove that an individual's debt is increasing in top incomes if the household cares about the rich (directly or indirectly). Moreover, we prove that if comparisons are upward looking (i.e., everybody cares about the rich directly or indirectly), aggregate debt is increasing in top incomes.



Real average pre-tax income growth from 1962 to 2014 in the US. Data are taken from Piketty, Saez, and Zucman (2018a). Growth rates are relative to the base year 1980.

(a) Since 1980 real incomes have stagnated for the bottom 50 %.

Growth of mean mortgage debt as a fraction of mean income by income quintiles. Use *OECD-modified equivalence scale* for income quintiles. Data from the Surveys of Consumer Finances.

(b) Mortgages rose across the whole income distribution.

Figure 1.1.2: Despite stagnating incomes, mortgage debt increased for the bottom 50 %.

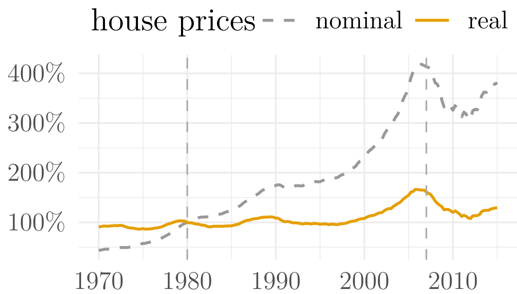


Figure 1.1.3: Nominal: Case-Shiller Home Price Index for the USA. Real: Deflated by the Consumer Price Index. Source: http://www.econ.yale.edu/~shiller/data.htm

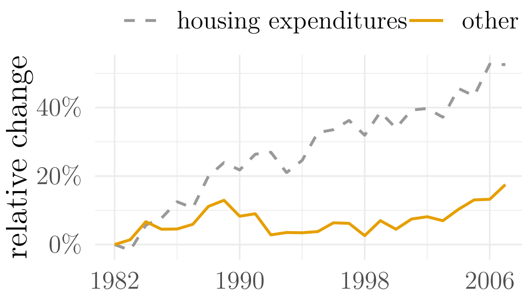


Figure 1.1.4: Relative change of housing expenditures and other expenditures over time. Data from Bertrand and Morse (2016, aggregated from the Survey of Consumer Expenditures) for the USA.

Contributions to the literature

Our findings contribute to the literature on distributional macroeconomics (e.g. Kaplan and Violante, 2014; Ahn, Kaplan, Moll, Winberry, and Wolf, 2017; Kaplan, Moll, and Violante, 2018), providing another reason why "inequality matters for macro". Rising income inequality has an effect on macroeconomic outcomes like house prices and aggregate mortgage debt as agents are linked not only through prices but also directly through social externalities of their consumption decisions.

Our main contributions concern the growing literature on the macroeconomics of the mortgage and house price booms. This literature builds on a variety of mechanisms: looser collateral constraints (e.g. Favilukis et al., 2017), lending limits (Justiniano, Primiceri, and Tambalotti, 2019), dynamics in foreign capital flows (Justiniano et al., 2014) and changes in house price expectations (Adam et al., 2012; Kaplan et al., 2020). Besides introducing a novel mechanism into this literature, we provide new insights and confirm findings on two other mechanisms. First, consistent with Kiyotaki, Michaelides, and Nikolov (2011) and others, we find that relaxation of collateral constraints does not generate sizable effects on debt and house prices.[3] Second, we confirm that foreign capital inflows can have sizable effects on household debt. In our model, the Saving Glut generates effects similar to those in Justiniano et al. (2014).

Kumhof, Rancière, and Winant (2015) formalize an alternative causal mechanism that links inequality and the debt boom in a model without housing. In their model, the debt boom is driven by the rich who derive utility from financial wealth, driving down interest rates. We provide an alternative causal mechanism that is consistent with micro-evidence and the fact that almost all of the debt boom was driven by mortgages (see Figure 1.1.1a). Livshits, MacGee, and Tertilt (2010) show that if cross-sectional inequality is driven by greater uncertainty (as opposed to variation in the permanent component) aggregate unsecured debt is decreasing. This quantitative result is driven by the precautionary savings motive. We complement their finding by showing that aggregate debt is increasing with higher permanent income inequality in an economy with durable goods.

In addition, a growing literature analyzes the consumption response to house price changes (Guren, McKay, Nakamura, and Steinsson, 2020; Garriga and Hedlund, 2020; Berger, Guerrieri, Lorenzoni, and Vavra, 2018). It finds that consumption reacts more when houses are bigger. Our model implies that house values become an ever bigger share of lifetime income

[3] This is in contrast to Favilukis et al. (2017) who generate sizable effects in their model with a large fraction of agents close to the collateral constraint.

when top incomes rise. Thus, rising top income inequality is amplifying the consumption response in financial crises.

A large empirical literature has established that social comparisons matter for well-being (e.g. Luttmer, 2005; Card, Mas, Moretti, and Saez, 2012; Perez-Truglia, 2019) and economic choices (Charles, Hurst, and Roussanov, 2009; Kuhn et al., 2011; Bursztyn et al., 2014; Bertrand and Morse, 2016; Bursztyn, Ferman, Fiorin, Kanz, and Rao, 2017; Bellet, 2019; De Giorgi et al., 2019). While the macroeconomic effects of keeping up with the Joneses have already been studied in the context of representative agent models (e.g. Abel, 1990; Campbell and Cochrane, 1999; Ljungqvist and Uhlig, 2000), we are the first to introduce social comparisons into a quantitative heterogeneous agents model.

We build on the macroeconomic literature on keeping up with the Joneses and bring it closer to the empirical evidence. First, we distinguish between conspicuous and non-conspicuous goods. In our model households compare themselves only in their houses, arguable the most important conspicuous good (e.g. Solnick and Hemenway, 2005; Bertrand and Morse, 2016). And second, agents compare themselves to the rich (e.g. Card et al., 2012; Bellet, 2019). Households only lose satisfaction with their own house, when a big house is built.

Our analytical results extend those by Ghiglino and Goyal (2010) and Ballester et al. (2006) who show that agents' choices depend on the strengths of social links in a one-period model. We extend their network models to infinite horizon and add a durable good (housing) to show that debt is increasing in the centrality of an agent. The centrality is reinterpreted as the weighted sum of incomes of the comparison group.

Structure of the paper The rest of the paper is structured as follows: In Section 1.2 we describe the model. In Section 1.3 we derive analytically how top incomes drive debt in a stylized version of the model. In Section 1.4 we describe the parameterization of the full model, followed by quantitative results in Section 1.5.

1.2 Model

We add social comparisons into an otherwise standard macroeconomic model of housing. Our model is a dynamic, incomplete markets general equilibrium model similar to the "canonical macroeconomic model with housing" in Piazzesi and Schneider (2016). We formulate our model in continuous time to take advantage of the fast solution methods of Achdou, Han, Lasry, Lions, and Moll (2017, in particular Section 4.3). We build our model with two aims in mind. First, we want to illustrate how rising top-incomes and social comparisons can lead to rising debt levels across the whole income distribution. And second, we want to quantify the effect of this channel on the increase in aggregate mortgage debt and house prices from 1980 to 2007.

1.2.1 Setup

Time is continuous and runs forever. There is a continuum of households that differ in their realizations of the earnings process. Households are indexed by their current portfolio holdings (a_t, h_t), where a_t denotes financial wealth and h_t denotes the housing stock, and their pre-tax earnings y_t. They supply labor inelastically to the non-durable consumption good and housing construction sectors. The financial intermediary collects households' savings and extends mortgages subject to a collateral constraint. The state of the economy is the joint distribution $\mu_t(a, h, y)$. There is no aggregate uncertainty.

1.2.2 Households

Households die at an exogenous mortality rate $m > 0$. The wealth of the deceased is redistributed to surviving individuals in proportion to their asset holdings (perfect annuity markets). Dead households are replaced by newborn households with zero initial wealth and earnings drawn from its ergodic distribution.[4] Households derive utility from a non-durable consumption good c and housing status s. They supply labor inelastically and receive earnings y. After-tax disposible earnings are given by

$$\tilde{y}_t = y_t - T(y_t),$$

where T is the tax function. Households choose streams of consumption $c_t > 0$, housing $h_t > 0$ and assets $a_t \in \mathbb{R}$ to maximize their expected discounted lifetime utility

$$\mathbb{E}_0 \int_0^\infty e^{-(\rho+m)t} \frac{\Big((1-\xi)c_t^\varepsilon + \xi s(h_t, \bar{h}_t)^\varepsilon\Big)^{\frac{1-\gamma}{\varepsilon}}}{1-\gamma},$$

where $\rho \geq 0$ is the discount rate and the expectation is taken over realizations of idiosyncratic earnings shocks. $1/\gamma > 0$ is the inter-temporal elasticity of substitution, $1/(1-\varepsilon) > 0$ is the intra-temporal elasticity of substitution

[4] This follows Kaplan et al. (2018).

between consumption and housing status and $\xi \in (0,1)$ is the relative utility-weight for housing status.

A household's utility from housing is a function of the *housing status* $s(h, \bar{h})$. Housing status increases in the household's housing stock h and decreases in reference housing $\bar{h}$ which is a function of the equilibrium distribution of housing as introduced in the next section.

Housing is both a consumption good and an asset. It is modeled as a homogenous, divisible good. As such, h represents a one-dimensional measure of housing quality (including size, location and amenities). An agent's housing stock depreciates at rate δ and can be adjusted frictionlessly.[5] Home improvements and maintenance expenditures x_t have the same price as housing (p) and go into the value of the housing stock one for one.

Households can save ($a > 0$) and borrow ($a < 0$) at the equilibrium interest rate r. Borrowers must post their house as collateral to satisfy an exogenous collateral constraint. The collateral constraint pins down the maximum possible loan-to-value ratio ω.

Households' assets evolve according to

$$\begin{aligned} \dot{a}_t &= \tilde{y}_t + r_t a_t - c_t - p_t x_t, \\ \dot{h}_t &= -\delta h_t + x_t, \end{aligned}$$

subject to the constraints

$$\begin{aligned} a_t &\geq -\omega p_t h_t, \\ h_t &> 0. \end{aligned} \tag{1.1}$$

1.2.3 Social comparisons

We build on the macroeconomic literature (e.g. Abel, 1990; Gali, 1994; Campbell and Cochrane, 1999; Ljungqvist and Uhlig, 2000) on keeping up with the Joneses and bring it closer to the empirical evidence. These papers feature representative agent models with one good and one asset. Agents compare themselves in the single consumption good, and their reference measure is the average consumption in the economy.[6]

We depart from this literature in two ways. First, we assume that households compare themselves only in their houses. This captures that people compare themselves only in conspicuous goods and that housing is one of the

[5] Frictionless adjustment is justified, because we will be comparing long-run changes (over a period of 27 years).

[6] In equilibrium the reference measure has to be equal to the optimal choice of the representative agent.

most important conspicuous goods—both in terms of visibility and expenditure share (e.g. Solnick and Hemenway, 2005; Bertrand and Morse, 2016).

Second, we allow the reference measure to be a function of the distribution of houses (and not necessarily its mean): $\bar{h}_i = \bar{h}_i(\mu_h)$. This reflects that the comparison motive is asymmetric, being strongest (and best documented) with respect to the rich (e.g. Clark and Senik, 2010; Ferrer-i-Carbonell, 2005; Card et al., 2012, on self-reported well-being). People buy bigger cars when their neighbors win in the lottery (Kuhn et al., 2011); non-rich move their expenditures to visible goods (such as housing) when top incomes rise in their state (Bertrand and Morse, 2016); and construction of very big houses leads to substantially lower levels of self-reported housing satisfaction for other residents in the same area—while the construction of small houses does not (Bellet, 2019).

For our analytical results we assume that $\bar{h}$ is a weighted mean of the housing distribution and use $s(h, \bar{h}) = h - \phi\bar{h}$ for tractability. For the quantitative results, we set $\bar{h}$ to the 90th percentile of the housing distribution and use $s(h, \bar{h}) = \frac{h}{\bar{h}^{\phi}}$ based on empirical evidence (see Section 1.4).

1.2.4 Pre-tax earnings process

In our main experiment, we want to adjust life-time (permanent) income inequality independently of income risk to capture the way income inequality has changed over time. We follow Guvenen et al. (2019), who estimate a pre-tax earnings process on administrative earnings data. The process consists of (i) individual fixed effects (α_i), a persistent jump-drift process (z_{it}), a transitory jump-drift process (ε_{it}), and heterogeneous non-employment shocks (ν_{it}).[7] We translate their estimated process to continuous time. Heterogeneity in α_i represents fixed ex-ante differences in earnings ability which is an important source of life-time inequality. The innovations of both the transitory and persistent process are drawn from mixture distributions to match higher order moments of income risk and impulse response functions. Finally, Guvenen et al. (2019) show that a non-employment shock with z-dependent shock probabilities greatly improves the model fit.[8]

[7] We use version (7), where we take out the deterministic life-cycle profile. The only component that this version does not have are differences in deterministic income growth rates.

[8] The only component that is missing compared to the Benchmark process is fixed heterogeneous income profiles, i.e. ex-ante permanent heterogeneity in lifecycle income growth rates.

If employed, individual pre-tax earnings are given by

$$y_{it}^{\text{pot}} = \exp(\tilde{\alpha}^i + z_{it} + \varepsilon_{it}).$$

We will refer to y^{pot} as *potential earnings*. The actual pre-tax earnings (taking into account unemployment) are

$$y_{it} = (1 - \nu_{it}) y_{it}^{\text{pot}},$$

where

$$\begin{aligned} \tilde{\alpha}_i &\sim \mathcal{N}(\mu_\alpha, \sigma_\alpha), \\ {}_{it} &= -\theta^z z_{it} + J_{it}^z, \\ \varepsilon_{it} &= -\theta^\varepsilon z_{it} + J_{it}^\varepsilon. \end{aligned}$$

J_{it}^z is a jump-process that arrives at rate λ^z. The size of the jump, η_{it}^z is drawn from a mixture of two normal distributions,

$$\eta_{it}^z = \begin{cases} \mathcal{N}\big(\mu^z(1-p^z), \sigma_1^z\big) & \text{with prob. } p^z \\ \mathcal{N}\big(-p^z\mu^z, \sigma_2^z\big) & \text{with prob. } 1-p^z. \end{cases}$$

Similarly, the jump process for the transitory process arrives at rate λ^ε and the jump size, η_{it}^z is drawn from a mixture of two normal distributions,

$$\eta_{it}^\varepsilon = \begin{cases} \mathcal{N}\big(-\varepsilon_{it} + \mu^\varepsilon(1-p^\varepsilon), \sigma_1^\varepsilon\big) & \text{with prob. } p^\varepsilon \\ \mathcal{N}\big(-\varepsilon_{it} - p^\varepsilon\mu^\varepsilon, \sigma_2^\varepsilon\big) & \text{with prob. } 1-p^\varepsilon. \end{cases}$$

The key difference between the persistent and the transitory process is that the jumps in the former are added to the current state whearas the jumps in the latter process reset the process such that the post-jump state is centered around zero.

The nonemployment shock arrives at rate $\lambda_0^\nu(z_{it})$ and has average duration $1/\lambda_1^\nu$. Specifically, the arrival probability as a function of the current state of the persistent process is modeled as

$$\lambda_0^\nu(z_{it}) = \frac{\exp\big(a + bz_{it}\big)}{1 + \exp\big(a + bz_{it}\big)}.$$

1.2.5 Production

There are two competitive production sectors producing the non-durable consumption good c and new housing investment I_h, respectively. Following Kaplan et al. (2020), there is no productive capital in this economy.

Non-Durable Consumption Sector The final consumption good is produced using a linear production function

$$Y_c = N_c$$

where N_c are units of labor working in the consumption good sector. As total labor supply is normalized to one, N_c is also the share of total labor working in this sector. The equilibrium wage per unit of labor is pinned down at $w = 1$.[9]

Construction Sector We model the housing sector following Kaplan et al. (2020) and Favilukis et al. (2017). Developers produce housing investment I_h from labor $N_h = 1 - N_c$ and buildable land $\bar{L}$, $I_h = (\Theta N_h)^\alpha (\bar{L})^{1-\alpha}$ with $\alpha \in (0, 1)$. Each period, the government issues new permits equivalent to $\bar{L}$ units of land, and these are sold at a competitive market price to developers. A developer solves

$$\max_{N_h} p_t I_h - w N_h \quad \text{s.t. } I_h = N_h^\alpha \bar{L}^{1-\alpha}$$

In equilibrium, this yields the following expression for optimal housing investment

$$I_h = (\alpha p)^{\frac{\alpha}{1-\alpha}} \bar{L}$$

which implies a price elasticity of aggregate housing supply of $\frac{\alpha}{1-\alpha}$.

1.2.6 Financial markets

The financial intermediary collects savings from households and issues mortgages to households. Lending is limited by the households' exogenous collateral constraint (1.1).

In addition, the intermediary has an exogenous net asset position with the rest of the world a_t^S. The equilibrium interest ensures that bank profits are zero and the asset market clears,

$$\int a_t(a, h, y) \mu_t = a_t^S. \tag{1.2}$$

[9] Neither labor supply nor the wage appear in the earnings process, because there is no aggregate risk, households inelastically supply one unit of labor, and the wage is equal to 1.

1.2.7 Stationary Equilibrium

A stationary equilibrium is a joint distribution $\mu(a, h, y)$, policy functions $c(a, h, y, \bar{h})$, $x(a, h, y, \bar{h})$, $h(a, h, y, \bar{h})$, $a(a, h, y, \bar{h})$, prices (p, r) and a reference measure $\bar{h}$ satisfying the following conditions

- Policy functions are consistent with agents' optimal choices $(c_t, h_t, a_t)_{t>0}$ given incomes $(y_t)_{t>0}$, prices p, r and the reference measure $\bar{h}$.
- Housing investment is such that the construction sector maximizes profits.
- $\mu(a, h, y)$ is stationary. That is, if the economy starts at μ, it will stay there.
- Asset market clears (1.2) and housing investment equals housing production $\int x(a, h, y) d\mu = I_h$.
- The reference measure is consistent with choices: $\bar{h} = \bar{h}(\mu)$.

1.3 Analytical Results

In this section we use a stylized version of the model described in section 1.2 to illustrate how rising top incomes can lead to rising mortgage levels across the whole income distribution via social comparisons. In this section we show analytically the following results.

In Proposition 1 we provide formulas for optimal housing and consumption, as functions of their permanent incomes, and the permanent incomes of the direct and indirect reference groups. In Proposition 2 we show that optimal debt is increasing in the incomes of the direct and indirect reference groups. In Proposition 3 we show that the impact of rising incomes $\tilde{y}_i$ on aggregate debt is increasing in type i's *popularity*. In Corollary 1 we show that total debt-to-income is increasing in top incomes if at least one person compares themselves to the rich. In Corollary 2 we show that under Cobb-Douglas aggregation ($\varepsilon = 0$), these results hold even under housing market clearing because they are independent of house prices p. In Corollary 3 we show that these results crucially depend on the fact the status good h is durable.

The assumptions needed to obtain tractability are that there is no idiosyncratic income risk; that the social status function is linear; and that the interest rate equals the discount rate (all of these assumptions are relaxed in the following sections).

Assumption 1. $r = \rho$.

Further, we assume that there is a finite number of types of households $i \in \{1, \ldots, N\}$. Agents vary by their initial endowments a_0 and flow disposable income $\tilde{y}$.

Assumption 2. Flow income $\tilde{y}_i$ is deterministic and constant over time, but varies across types i.

Without loss of generality, we assume that types are ordered by their permanent income $\mathcal{Y}_i = ra_0^i + \tilde{y}_i$,

$$\mathcal{Y}_1 \leq \mathcal{Y}_2 \leq \ldots \leq \mathcal{Y}_N.$$

We use bold variables to denote the vector variables for each type using the above ordering, e.g. $\boldsymbol{h} = (h_1, \ldots, h_N)^T$.

Assumption 3 (Tractable social comparisons)**.** The status function $s(h, \bar{h}) = h - \phi\bar{h}$ is linear and the reference measure $\bar{h}_i = \sum_{j \neq i} g_{ij} h_j$ is a weighted sum of other agent's housing stock (we assume $g_{ij} \geq 0$).

Note, that we can write the vector of reference measures as $\bar{\boldsymbol{h}} = (\bar{h}_1, \ldots, \bar{h}_N)^T = G \cdot \boldsymbol{h} := (g_{ij})(h_i)$. The matrix G can be interpreted as the adjacency matrix of the network of types capturing the comparison links between agents of each type. g_{ij} measures how strongly agent i cares about agent j.

We further require the comparisons to satisfy the following regularity condition.

Assumption 4. The Leontief inverse $(I - \phi G)^{-1}$ exists and is equal to $\sum_{i=0}^{\infty} \phi^i G^i$ for ϕ from Assumption 3.

This assumption is not very strong. This assumption is satisfied whenever the power of the matrix converges, $G^i \to G^\infty$. For example, if G represents a Markov chain with a stationary distribution or if G is nilpotent.

1.3.1 Characterizaton of the partial equilibrium

We solve for a simplified version of the equilibrium in Section 3.2.1. Agents solve their optimization problem given prices and the reference measure; the reference measure is consistent; but for now, we don't require market clearing. We use a lifetime budget constraint instead of the implicit transversality condition.

Households optimal decisions are given in the following proposition.

Proposition 1. *Under assumptions 1, 2, 3 and 4 the optimal choices* $\boldsymbol{h} = (h_1, \ldots, h_N)^T$ *and* $\boldsymbol{a} = (a_1, \ldots, a_N)^T$ *are given by*

$$\boldsymbol{h} = \Big(\sum_{i=0}^{\infty} (\kappa_1 \phi G)^i\Big)\kappa_2 \boldsymbol{\mathcal{Y}}.$$

$$-r\boldsymbol{a} = \tilde{\boldsymbol{y}} - \kappa_3 \boldsymbol{\mathcal{Y}} + (1-\kappa_3)\Big(\sum_{i=1}^{\infty} (\kappa_1 \phi G)^i\Big)\boldsymbol{\mathcal{Y}} \tag{1.3}$$

where $\kappa_1 = \frac{1}{\frac{p(r+\delta)}{\kappa_0}+1} \in (0,1)$, $\kappa_2 = \frac{\kappa_1}{\kappa_0}$, $\kappa_3 = \frac{1}{1+\frac{pr}{\delta p + \kappa_0}} \in (0,1)$ *and* $\kappa_0 = \Big((r+\delta)\frac{1-\xi}{\xi}p\Big)^{\frac{1}{1-\varepsilon}}$.

Proof. See appendix 1.B.2. □

Households' choices depend on a weighted average of the permanent incomes of their (direct and indirect) reference groups. The weights are positive, whenever there is a direct or indirect social link between those agents. This is captured by the *income-weighted Bonacich centrality*, $B = \sum_{i=0}^{\infty}(C_1\phi G)^i \boldsymbol{\mathcal{Y}}$. If the weight B_{ij} is positive, household j's lifetime income affects household i's choices. This is the case whenever j is in i's reference group (there is a direct link $g_{ij} > 0$), or if j is in the reference group of some agent k who is in the reference group of agent i (there is an indirect link of length two, $g_{ik}g_{kj} > 0$) or if there is any other indirect link ($\prod_{n=1}^{N-1} g_{\ell_n,\ell_{n+1}}$ where $\ell_1 = i$ and $\ell_{N-1} = j$).

These results are reminiscent of those in Ballester et al. (2006). They showed that the unique Nash equilibrium in a large class of network games is proportional to the (standard) Bonacich centrality.

1.3.2 Comparative statics

First, we show that optimal debt and optimal housing are increasing in incomes of the direct and indirect comparison groups.

Proposition 2. *For each type* j *in* i*'s reference group (that is,* $g_{ij} > 0$*) and for each* k *that is in the reference group of the reference group (etc.) of* i *(that is, there is* $j_1, j_2, \ldots, j_n$ *such that* $g_{ij_1}g_{j_1j_2}\cdots g_{j_{n-1}j_n}g_{j_nk} > 0$*), then* h_i *is increasing and* a_i *is decreasing in* $\mathcal{Y}_j$ *(or* $\mathcal{Y}_k$*).*

Proof. G is non-negative, so $\sum_i c^i G^i$ is non-negative for all $c \geq 0$. From the definition of the Leontief inverse, being the discounted sum of direct and indirect links it follows,

$$\frac{\partial h_i}{\partial \tilde{y}_j} > \kappa_2\kappa_1\phi g_{ij} > 0 \quad \text{and} \quad \frac{\partial h_i}{\partial \tilde{y}_k} > \kappa_2(\kappa_1\phi)^{n-1} g_{ij_1}g_{j_1j_2}\cdots g_{j_{n-1}j_n}g_{j_nk} > 0.$$

Similarly

$$-\frac{\partial a_i}{\partial \tilde{y}_j} > (1-\kappa_3)\kappa_1\phi g_{ij} > 0 \quad \text{and} \; -\frac{\partial a_i}{\partial \tilde{y}_k} > (1-\kappa_3)(\kappa_1\phi)^{n-1}\phi g_{ij_1}g_{j_1j_2}\cdots g_{j_{n-1}j_n}g_{j_nk} > 0.$$

□

Agent A's debt increases if agent B's lifetime income increases—as long as there is a direct or indirect link from A to B. That link exists, if agent A cares about agent B, or if agent A cares about some agent C who cares about agent B.

Second, we show how aggregage housing and debt react to changes in type j's income $\mathcal{Y}_j$. We first define the popularity of a type.

Definition 1 (Popularity)**.** We define the vector of popularities as

$$\boldsymbol{b}^T = \mathbf{1}^T \sum_{i=1}^{\infty} (\kappa_1 \phi G)^i,$$

and type i's popularity b_i as the ith component of $\boldsymbol{b}$.

The popularity is the sum of all paths that end at individual i. It measures how many agents compare themselves with i (directly and indirectly) and how strongly they do. The popularity of a type is crucial in determining how strongly their income will affect economic aggregates.

Proposition 3. *The impact of a change in type j's on aggregate housing and aggregate debt is proportional to its popularity.*

$$\frac{\partial}{\partial \tilde{y}_j} \sum_i h_i = \kappa_2(1 + b_j)$$
$$\frac{\partial}{\partial \tilde{y}_j} \sum_i r a_i = (1 - \kappa_3)(1 + b_j).$$

Proof. Take the expressions from proposition 1 and plug in the definitions for $\mathcal{Y}$ and b (Definition 1), aggregate housing can be written as $\sum_{i=1}^N h_i = \kappa_2 \sum_{i=1}^N (1 + b_i)(\tilde{y}_i + r a_0^i)$ and aggregate debt can be written as $-\sum_{i=1}^N r a_i = (1 - \kappa_3) \sum \tilde{y}_i - \kappa_3 \sum a_0^i + (1 - \kappa_3) \sum_{i=1}^N b_i(\tilde{y}_i + r a_0^i)$. The derivatives follow immediately. □

Corollary 1. *If all types $i \neq j$ are connected to agent j and $\tilde{y}_j$ increases, then debt-to-income increases for all types $i \neq j$.*

Proof. By Proposition 2 debt of types $i \neq j$ increases, while their income is unchanged. It follows that debt-to-income rises. □

Corollary 2. *Under Cobb-Douglas aggregation, the results for a in Propositions 1, 2 and 3 are independent of house prices.*

Proof. Under Cobb-Douglas κ_0 is divisible by p. This means that p cancels in κ_1 and κ_3. Thus, all p cancel in the expression for a in Proposition 1 and consequently doesn't show up in the respective expressions in Propositions 2 and 3. □

The results on optimal debt in Propositions 2 and 3 and Corollary 1 break down if houses are not durable. When houses are non-durable, for any small time interval Δ, the depreciation rate has to be $\delta = \frac{1}{\Delta}$, so that the housing stock depreciates immediately,

$$(1 - \Delta\delta)h_t = 0.$$

To analyze this case in continuous time, we thus let the depreciation rate δ go to infinity.

Corollary 3. *When $\delta \to \infty$, optimal debt does not depend on others' incomes.*

Proof. It can be easily seen that $\kappa_3 \to 1$ as $\delta \to \infty$, thus $(1 - \kappa_3) \to 0$. Since all other terms in expression (1.3) are bounded, the part containing the Leontief inverse vanishes and becomes $-r\boldsymbol{a} = \tilde{\boldsymbol{y}} - \boldsymbol{\mathcal{Y}} = -r\boldsymbol{a}_0$. □

1.3.3 How rising top incomes fuel the mortgage boom: Intuition

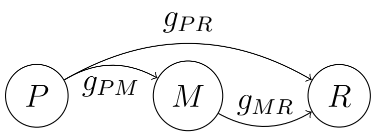
It is at the heart of the mechanism that there is a complementarity between a household's housing stock and their reference measure. When top incomes $\mathcal{Y}_N$ rise, households of type N will improve (or upsize) their housing stock h_N, increasing the reference measure $\bar{h}_i$ for all types i that care about type N directly or indirectly. Each of these agents will optimally substitute durable, status-enhancing housing for non-durable status neutral consumption.

For debt to be affected it is key that the status good is durable and the status-neutral good in non-durable. Agents want their stock of the durable good to be constant over time. They need to pay for the whole good ph upfront and only replace the depreciation δph in the future. Agents need to shift some of their lifetime income forward to finance their house. They use mortgages as an instrument to achieve that. The greater the value of the house, the bigger is the necessary mortgage.

Corollary 3 formalizes this intuition. It shows that if houses are non-durable ($\delta \to \infty$), the term containing the Leontief inverse of the adjacency matrix G vanishes.

1.3.4 Example: Upward comparisons with three types of agents

We now illustrate the results for the simple case of three types of agents, poor P, middle class M, and rich R. The poor type compares himself with both other types, the middle type compares himself only with the rich type, and the rich type not at all. Figure 1.3.1 shows the corresponding graph and its adjacency matrix.



(a) The graph.

$$G = \begin{array}{c c} & \begin{array}{ccc} P & M & R \end{array} \\ \begin{array}{c} P \\ M \\ R \end{array} & \begin{pmatrix} 0 & g_{PM} & g_{PR} \\ 0 & 0 & g_{MR} \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

(b) The adjacency matrix.

Figure 1.3.1: The social network structure with three types, assuming upward comparisons. The network can be represented as a graph and as its adjacency matrix.

Since G is a triangular matrix with only zeros on the diagonal, it is nilpotent ($G^3 = \mathbf{0}$), and thus the Leontief inverse exists.

$$G^2 = \begin{array}{c c} & \begin{array}{ccc} P & M & R \end{array} \\ \begin{array}{c} P \\ M \\ R \end{array} & \begin{pmatrix} 0 & 0 & g_{PM} g_{MR} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}, \quad G^3 = \mathbf{0}$$

The matrix G^2 counts the paths of length 2. In our example there is only one such path—from type P to type R. Defining $\tilde{\phi} = \kappa_1 \phi$, the vector of Bonacich centralities is given by

$$\sum_{i=0}^{\infty} \alpha^i G^i = I + \sum_{i=1}^{2} \alpha^i G^i = I + \begin{pmatrix} 0 & \alpha \cdot g_{PM} & \alpha \cdot g_{PR} + \alpha^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 0 & \alpha \cdot g_{MR} \\ 0 & 0 & 0 \end{pmatrix}$$

The partial equilibrium choices for housing and debt are now given by

$$\begin{pmatrix} h_P \\ h_M \\ h_R \end{pmatrix} = \kappa_2 \begin{pmatrix} 1 & \tilde{\phi} \cdot g_{PM} & \tilde{\phi} \cdot g_{PR} + \tilde{\phi}^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 1 & \tilde{\phi} \cdot g_{MR} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{Y}_P \\ \mathcal{Y}_M \\ \mathcal{Y}_R \end{pmatrix}$$

$$-r \begin{pmatrix} a_P \\ a_M \\ a_R \end{pmatrix} = \tilde{\boldsymbol{y}} - \kappa_3 \boldsymbol{\mathcal{Y}} + (1-\kappa_3) \begin{pmatrix} 0 & \tilde{\phi} \cdot g_{PM} & \tilde{\phi} \cdot g_{PR} + \tilde{\phi}^2 \cdot g_{PM} \cdot g_{MR} \\ 0 & 0 & \tilde{\phi} \cdot g_{MR} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{Y}_P \\ \mathcal{Y}_M \\ \mathcal{Y}_R \end{pmatrix}$$

An agent's housing choice increases linearly in own permanent income, $\mathcal{Y} = \tilde{y} + ra_0$, and on the permanent income of agents *in the reference group*. The poor agent's consumption increases through the direct links, but also indirect links (which are discounted more strongly). Agents' decisions to save or borrow depend on the ratio of initial wealth a_0 and income $\tilde{y}$. The higher the income relative to initial wealth, the greater the need to borrow.

1.4 Parameterization

Now we return to the full model. We parameterize the model to be consistent with the aggregate relationships of mortgage debt, house value and income in the US at the beginning of the 1980s. We use the estimated income process from Guvenen et al. (2019) and assign eight other parameters externally. The remaining two parameters (the discount rate ρ and the utility weight of housing status ξ) are calibrated internally so that in general equilibrium the aggregate net-worth-to-income ratio and aggregate loan-to-value ratio match these aggregate moments in the 1983 Survey of Consumer Finances.

Income Process We translate the estimated income process from Guvenen et al. (2019) to continuous time. It has a permanent, a persistent and a transitory component and state-dependent unemployment risk. Guvenen et al. (2019) estimate it to data from the time period 1994–2013. In order to construct the income process for the baseline economy $\mathcal{E}$ (corresponding to the year 1980) we rescale the permanent component following evidence on the changes in the income distribution from Kopczuk et al. (2010), Guvenen, Ozkan, and Song (2014) and Guvenen et al. (2018).

The cross-sectional dispersion of incomes has increased substantially between 1980 and 2007. Figure 1.4.1 (taken from Guvenen et al., 2018, Figure 12) shows the variation of three common measures over time: the P90/P50 ratio, the P90/P10 ratio and the standard deviation of log-earnings. These changes in the variation of incomes can come from either component of the income process, or even a combination of them.

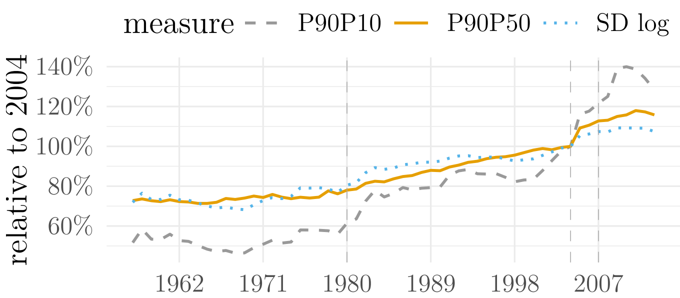


Figure 1.4.1: Change in the cross-sectional distribution of male earnings in the US. Vertical bars in 1980, 2004 and 2007. Source: Guvenen et al. (2018)

While there is no consensus yet,[10] as to which of those factors contributed how much, there is evidence that rising permanent inequality explains a substantial share in increased cross-sectional variation. Kopczuk et al. (2010, Figure V) find that almost all of the change in earnings variation came from increases in permanent inequality. This finding is supported by Guvenen et al. (2014, Figure 5) who show that the variances of earnings shocks have had a slight downward trend since 1980.

Given this evidence, we attribute all change in inequality to changes in permanent inequality (σ_α). In our income process, permanent income inequality is represented by the permanent component $\tilde{\alpha}$. So, given the discretized version of the process, we stretch the upper half of the $\tilde{\alpha}$-grid to match the changes in the cross-sectional P90/P50 ratio.

When translating the process to continuous time, we assume that shocks arrive on average once a year (instead of every year). Moreover, we replace the discrete time *iid* process by jump-drift process (ε_{it}) that is re-centered around zero whenever a shock hits so that shocks do not accumulate. The mean reversion rate of the persistent process (z_{it}) is the negative log of the discrete time persistence parameter which preserves the same annual autocorrelation. The exit rate out of nonemployment is chosen to match the average duration of nonemployment stays in the discrete time process. As households in our infinite horizen model die at a constant rate, we remove all age-dependence by setting the age profile constant (to the value at the mean age $\bar{t}$).[11] Table 1.1 shows all parameters of our continuous time earnings

[10] Carr and Wiemers (2016, 2018) show that depending on data source, sample selection, and statistical model one can find substantial differences in the decomposition into risk and permanent inequality.

[11] This affects the mean of log earnings as well as the arrival rate of nonemployment shocks.

Table 1.1: Earnings Process Parameters

Parameter		Value
Fixed Effects		
μ_α	mean	$2.7408 + 0.4989\bar{t} - 0.1137\bar{t}^2$
σ_α	standard deviation	0.467
Persistent Process		
λ^z	arrival rate	1.0
θ^z	mean reversion rate	$-\log(0.983)$
p^z	mixture probability	0.267
μ^z	location parameter	-0.194
σ_1^z	std. dev. of first Normal	0.444
σ_2^z	std. dev. of second Normal	0.076
σ_0^z	std. dev. of z_{i0}	0.495
Transitory Shocks		
λ^ε	arrival rate	1.0
θ^ε	mean reversion rate	0.0
p^ε	mixture probability	0.092
μ^ε	location parameter	0.352
σ_1^ε	std. dev. of first Normal	0.294
σ_2^ε	std. dev. of second Normal	0.065
Nonemployment Shocks		
a	constant	$-3.2740 - 0.8935\bar{t}$
b	slope	$-4.5692 - 2.9203\bar{t}$
λ_1^ν	exit rate	$1/0.9784$

process.

We put the process on a discrete state space, using the approach of Kaplan et al. (2018). We discretize each component separately, obtaining continuous-time Markov chains[12] for the persistent and transitory components and combining them afterwards. Finally, we add the state-dependent non-employment risk.

Income Taxation We use the progressive income tax function from Heathcote, Storesletten, and Violante (2017),

$$T(y) = y - \tau_0 y^{1-\tau_1}.$$

[12]Mostly called Poisson processes in the literature.

Table 1.2: Baseline Parameters

	Parameter description	Source	Value
Preferences			
ϕ	strength of keeping up motive	Bellet (2017)	0.7
ρ	discount rate	internal	0.02
ξ	utility weight of housing	internal	0.277
$\frac{1}{1-\varepsilon}$	intra-temporal elasticity of substitution	Flavin and Nakagawa (2008, AER)	0.15
γ	inverse intertemporal elasticity of substitution	standard	1.5
$\frac{1}{m}$	constant mortality rate	45 years worklife	45.0
Housing and financial technogy			
$\frac{\alpha}{1-\alpha}$	price elasticity of housing supply	Saiz (2010, QJE)	1.5
δ	depreciation rate of housing	Bureau of Economic Analysis	0.021
ω	maximum loan-to-value ratio	P95 of LTV	0.85
$a^S/\bar{y}$	exogenous net asst supply	cum. current account	-0.01
Taxation and Unemployment Insurance			
τ_0	level of taxes	internal	0.932
τ_1	progressivity	Heathcote et al. (2017)	0.15
b	replacement rate	Dept of Labor	0.32

If non-employed, households receive a fraction b of their potential earnings from unemployment insurance. Thus, the post-tax disposable income is given by

$$\tilde{y}_t = \begin{cases} y_{it}^{\text{pot}} - T(y_{it}^{\text{pot}}) & \text{if employed} \\ b y_{it}^{\text{pot}} & \text{otherwise.} \end{cases}$$

We follow Kaplan et al. (2020) in our choice of the parameters τ_0, τ_1. The progressivity parameter τ_1 is an estimate from Heathcote et al. (2017) and the scale parameter τ_0 is set to match the tax revenue from personal income tax and social security contribution as a share of GDP in 1980 (14.4%).[13] We set the replacement rate to 32%, matching average unimployment insurance benefits, as a fraction of average wage, as reported by the US Department of Labor.[14]

Preferences and demographics The discount rate ρ and the utility weight of housing status ξ are internally calibrated to match the economy-wide mortgage-debt-to-income and loan-to-value ratios from the 1983 SCF. The interpretation of the utility weight ξ differs from other models, because ξ is the utility weight of housing status (not housing stock).

The literature has not yet converged to a common value for the intratemporal elasticity of substitution $\frac{1}{1-\varepsilon}$. Estimates range from 0.13–0.24 (from structural models; e.g. Flavin and Nakagawa, 2008; Bajari, Chan, Krueger,

[13] Retrieved from https://taxfoundation.org/federal-tax-revenue-source-1934-2018/.
[14] Retrieved from https://oui.doleta.gov/unemploy/DataDashboard.asp.

and Miller, 2013) up to 1.25 (Ogaki and Reinhart, 1998; Piazzesi, Schneider, and Tuzel, 2007, using estimates from aggregate data). Many papers have picked parameters out of this range.[15] We follow the evidence from structurally estimated models and set the elasticity to 0.15.

The inverse intertemporal elasticity of substitution γ is set to the standard value 1.5. The constant annual mortality rate $m = 1/45$ is set to get an expected (working) lifetime of 45 years.

Social comparisons For the status function we use a ratio-specification $s(h, \bar{h}) = \frac{h}{\bar{h}^{\phi}}$ as in Abel (1990). Bellet (2019) shows that this functional form captures the empirical finding that the utility loss from a big houses decreases with own house size. Households with a medium sized house are more affected by top housing than households living in a small house.[16]

We define the reference measure as the 90^{th} percentile of the (endogenous) housing distribution, $\bar{h} = h_{\text{P90}}$. This follows Bellet (2019) who shows that households are only sensitive to changes in the top quintile of the house (size) distribution and strongest when the reference measure is defined as the 90^{th} percentile.[17]

The parameter ϕ pins down the strength of the comparison motive. It is the ratio of two utility elasticities

$$\phi = -\frac{\text{elasticity of utility w.r.t. } \bar{h}}{\text{elasticity of utility w.r.t. } h}.$$

If the reference houses improves by 1%, then agents would have to improve their own house ϕ% to keep utility constant. Bellet (2019) estimates ϕ to be between 0.6 and 0.8 when setting $\bar{h}$ equal to the 90^{th} percentile of the housing distribution. We thus choose $\phi = 0.7$.[18] Note that Bellet (2019) estimates exactly this sensitivity allowing us to take his estimates without an intermediate indirect inference procedure.

Technology and Financial Markets The construction technology parameter α is set to 0.6 so that the price elasticity of housing supply $(\frac{\alpha}{1-\alpha})$ equals 1.5, which is the median value across MSAs estimated by Saiz (2010).

[15] Garriga and Hedlund (2020) use 0.13, Garriga, Manuelli, and Peralta-Alva (2019) use 0.5, many papers use Cobb-Douglas (that is, an elasticity of 1.0, e.g. Berger et al., 2018; Landvoigt, 2017) and Kaplan et al. (2020) use 1.25.

[16] Note that the more tractable linear specification $(h - \phi\bar{h})$ as used in Campbell and Cochrane (1999), Ljungqvist and Uhlig (2000) and Section 1.3 would imply the opposite relationship between own house size and comparison strength.

[17] See Figure 6 in Bellet (2019).

[18] See Table 2 in Bellet (2019)

Table 1.3: Targeted moments

moment	model	data (80/83)
aggregate loan-to-value	0.24	0.24
aggregate networth-to-income	4.63	4.6
tax-revenue-to-income	0.14	0.14

The maximum admissible loan-to-value ratio (ω) is set to 0.85, to match the 95th percentile of the LTV distribution in the SCF (Kaplan et al., 2020, use a similar approach for setting the debt-service-to-income constraint). Finally, we specfiy the exogenous net supply of assets a^S to match the net foreign debt position of the US. The net foreign debt position can be well approximated by the cumulative current account deficit of the US (Gourinchas, Rey, and Govillot, 2017), which was 1% of GDP in 1980 (see also Figure 1.5.3).

1.4.1 Internal calibration and model fit

For the internal calibration we target the aggregate networth-to-income ratio (4.6) and the aggregate loan-to-value ratio (0.24) from the first wave of the Survey of Consumer Finances in 1983. We pick the utility weight of housing ξ and the the discount rate ρ so that simulated moments match their counterparts in the data. Table 1.3 shows that the model fits the data very well.

1.5 Quantitative Results

In this section we study how the model economy reacts to changes in the environment in the long-run. We compare the initial stationary equilibrium $\mathcal{E}$ (corresponding to 1980) with alternative stationary equilibria $\mathcal{E}^x$ where we adjust income inequality I, capital inflow (Saving Glut) S and and the collateral constraints ω to reflect the observed changes in the data.

In the first experiment we compare $\mathcal{E}$ to $\mathcal{E}^I$ where only income inequality rises. Afterwards we set these results into perspective, comparing them to equilibria that reflect other (combinations) of mechanisms like $\mathcal{E}^S$ (Saving Glut), $\mathcal{E}^\omega$ (relaxation of borrowing limit) and $\mathcal{E}^{IS\omega}$ (all three mechanisms).

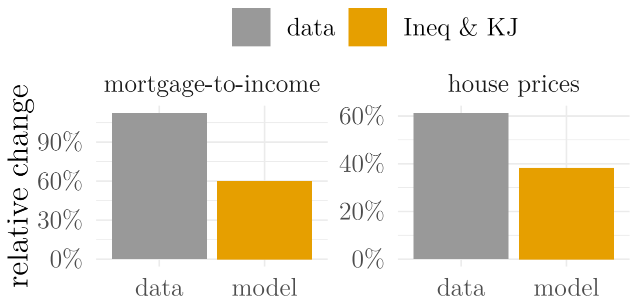


Figure 1.5.1: Comparison simulated changes in aggregate variables between the steady states in 1980 and 2007.

1.5.1 Rising inequality, mortgages and house prices

We now move to the main experiment of the paper. We start from the steady-state calibrated to the U.S. economy in 1980. Then we raise income inequality to match the level in 2007 and compare the mortgage debt, house prices and housing production between 1980 and 2007. Before getting to the results, we describe how we model the increase in income inequality.

Modelling rising inequality

As we have discussed in Section 1.4, the cross-sectional dispersion of income has increased substantially between 1980 and 2007. Given the evidence in Kopczuk et al. (2010) and Guvenen et al. (2014) we attribute the whole change in cross-sectional inequality to changes in permanent inequality. In our model permanent inequality is reflected by the standard deviation of the distribution of the permanent component σ_α. Thus, we increase σ_α to match the increase in the cross-sectional P90/P50 ratio.

Results

Rising inequality and keeping up with the Joneses creates both a mortgage boom and a house price boom in our model. Figure 1.5.1 shows that our mechanism generates an increase in the mortgage-to-income ratio of about 60%—about half of the increase that is observed in the data. Similarly, we generate a house price boom (+38%) that generates 62% of the increase in the data.

Keeping up with the Joneses are a quantitatively important to generate the results. Figure 1.5.2 shows how much of the simulated debt and mortgage

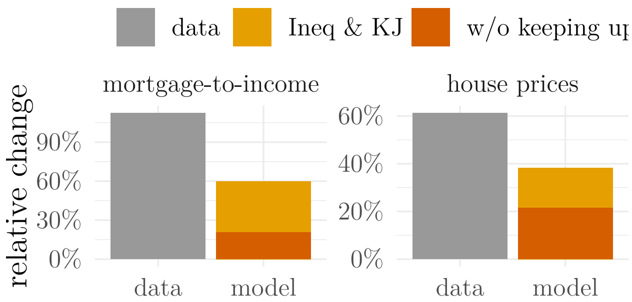


Figure 1.5.2: Comparison simulated changes in aggregate variables between the steady states in 1980 and 2007. "w/o keeping up" shows the changes when the reference measure $\bar{h}$ is kept fixed at level $\bar{h}_{1980}$ from the initial stationary equilibrium.

booms can be obtained with rising inequality, but without status concerns.[19] Without keeping up with the Joneses, the debt boom would be 71% weaker and the house price boom would be 44% weaker. The sizable effect of rising income inequality comes from general equilibrium effects. Rising inequality raises house prices and thus housing expenditures across the distribution. Since houses are financed by mortages, demand for credit increases. The interest rate rises to clear asset markets.

There are four channels at play. (i) Rising top incomes raise the demand for housing and house prices because the richer households want to live in bigger houses. (ii) Agents react to the new reference measure. They substitute houses for consumption to keep up with the Joneses. (iii) All households react to the higher house prices. Agents will spend a larger fraction of their income on houses, because houses and consumption are not perfects substitutes. (iv) The three channels above raise the demand for housing, and thus the demand for mortgages. Interest rates rise until demand for savings (i.e. credit supply) meets credit demand.

1.5.2 Horse race against alternative mechanisms

Rising inequality together with a "keeping up with the Joneses" motive is not the only possible explanation for the rise in mortgages and house prices. The

[19]Instead of recalibrating the model with $s(h, \bar{h}) = h$ one can use that for a given reference measure $\bar{h}$ that is constant across the population, the initial equilibrium $\mathcal{E}$ is equivalent to a parameterization with $s(h, \bar{h}) = h$ and housing weight $\tilde{\xi}$ such that $\frac{\tilde{\xi}}{1-\tilde{\xi}} = \frac{\xi}{1-\xi}\frac{1}{\bar{h}^{\phi}}$. This holds because our specification of social comparisons, just reweights the utility of housing and consumption.

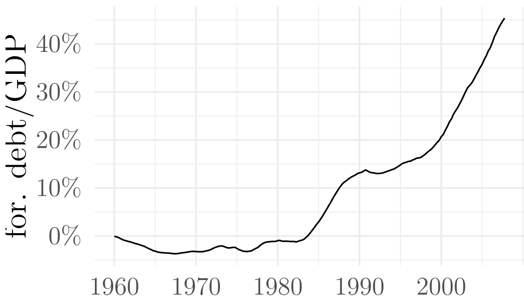


Figure 1.5.3: Cumulative current account deficit (which is approximately US external debt) as a fraction of GDP. Source: BEA and FRED. Details see Appendix 1.A

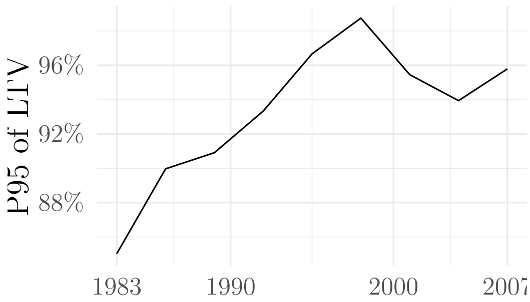


Figure 1.5.4: The 95th percentile of the loan-to-value (LTV) distribution, Source: Surveys of Consumer Finances 1983–2007. Details see Appendix 1.A

main complementary mechanisms are the Global Saving Glut (capital inflow from emerging markets; Bernanke, 2005), financial innovation (securitization allows banks to lend more liberally to less credit-worthy households; e.g. Favilukis et al., 2017) and a bubble in the housing market (house prices rose in expecation of rising house prices; e.g. Kaplan et al., 2020).

In this section compare the magnitudes of two competing channels (Saving Glut and relaxation of borrowing limits) with our main mechanism. Analyzing the role of expectations for the housing boom is beyond the scope of our model.

Global Saving Glut

Just like the US mortgage boom, growing international imbalances have been discussed as a source of instability leading to the Global Financial Crisis. Bernanke (2005, then Fed govenor) was one of the first to interpret these imbalances not in terms of trade imbalances, but as an accumulation of external debt: The cumulative current account deficit is approximately equal to the net foreign asset position. As seen in Figure 1.5.3 the cumulative current account reached −40% of GDP in 2006. That is, the US was a net debtor with net debt amounting to 40% of GDP[20]

Bernanke (2005) also provides a potential source for this rise in foreign debt: the steep increase in the global demand for savings, especially from

[20] Gourinchas et al. (2017) estimate that the precise net foreign asset position was less negative due to valuation effects.

China and India. He argues that these savings flowed into the US economy, building up the US debt position.

Through the lens of our model, the global saving glut changes the market clearing condition (1.2) of the asset and mortgage market. Exogenous asset supply is given by a_t^S, where $a_t^S/\bar{y}_t$ is the cumulative current account from Figure 1.5.3 ($\bar{y}_t$ is average pre-tax earnings, our measure of GDP).

Comparing the Saving Glut to our main mechanism, Figure 1.5.5 shows that the Saving Glut indeed causes a substantial increase in the mortgage-to-income ratio (at the same order of magnitued as inequality and keeping up with the Joneses) and only a very weak increase in house prices if inequality is held fixed at the 1980 level.

Note that the way we model the Saving Glut potentially biases the effects upwards. We assume that the capital inflow is purely driven by foreign demand for assets. If, on the other hand, part of the capital inflow is driven by increased supply of assets (from higher demand for mortgages), the part of the external debt position might just be a symptom of a demand-side mechanism like ours. Assuming a small open economy (constant interest rate), our main mechanism generates a mortgage boom that is large enough to explain the build-up of external debt. Kumhof, Ranciere, Richter, Throckmorton, Winant, and Ozsögüt (2017) indeed find that rising top incomes are an important predictor of a current account deficits (and thus, foreign debt). In this case, the Saving Glut (the increased demand for assets) would be less powerful.

Financial liberalization and innovation

Another promiment explanation for debt boom is a relaxation of constraints in the financial sector. These might come from regulatory changes or financial innovation. In 2007, banks could give out more mortgages than in 1980 because law required lower collateral requirements on the banks' and the households' balance sheets. Moreover, banks' technology might have improved, so that they are no able to lend on worse terms (e.g. higher loan-to-value ratios). Favilukis et al. (2017) and Justiniano et al. (2019) have shown that under certain condition the relaxation of constraints can have sizable effects on total lending. We show, that in our model this is not the case because interest rates rise in general equilibrium.

We model financial liberalization and financial innovation in a reduced form way. We assume that the exogenous LTV limit (ω_t) increases over time. As a proxy for this borrowing limit, we use the 95th percentile of the LTV distribution in the Surveys of Consumer Finances, which is shown in

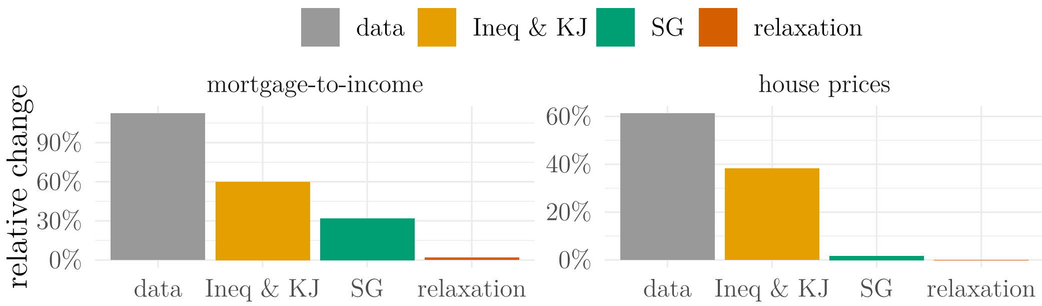


Figure 1.5.5: Compare simulated changes in aggregate variables for different scenarios. *Saving Glut*: Constant inequality and reference measure $\bar{h}$, varying a^S to match net foreign debt position (see Figure 1.5.3). *relaxation*: Constant inequality and reference measure $\bar{h}$, varying ω to match P95 of the LTV distribution (see Figure 1.5.4)

Figure 1.5.6: Decomposition of the three mechanisms

Figure 1.5.4. In line with the data we assume that the LTV limit increases from 0.85 in 1980 to $\omega_{2007} = 0.96$.

Figure 1.5.5 shows that in general equilibrium, this mechanism doesn't contribute to the debt and house price booms. These results differ from Favilukis et al. (2017) because there are not many constrained agents in our equilibrium. Their equilibrium is constructed in a way that a big part of the population is at or close to the constraint. Moreover, they use an exogenous inflow of capital to keep the interest rate down.

Decomposition of the three mechanisms

Instead of looking at the channels individually, we will now analyze their marginal effects. We add three mechanisms to the baseline economy one by one and compute their marginal effects. In a first step we compare the baseline economy $\mathcal{E}$ with the Saving Glut economy $\mathcal{E}^S$. Then we compute the marginal effect of adding rising inequality and keeping up with the Joneses in $\mathcal{E}^{IS}$ and finally we compute the marginal effect of a relaxation of the collateral constraint in $\mathcal{E}^{IS\omega}$.

All three mechanisms together generate an increase in the mortgage-to-income ratio of 83% and an increase in house prices of 38%. In Figure 1.5.6 and Table 1.D.2 we provide a decomposition. The contributions of each channel depends on the ordering in which they are added. Rising inequality and social comparison contributes between 39% and 72% to the total generated increase in mortgage-to-income and more than 95% of the total generated house price boom. The Saving Glut contributes 21–55% to the debt boom and has only negligible effects on house prices. Relaxation of the collateral constraint has only a minor contribution to both.

Thus, rising inequality and keeping up with the Joneses are an important amplifier of the Saving Glut when it comes to mortgage debt. Moreover, among the three mechanisms, rising inequality and and keeping up with the Joneses is the only channel that generates a substantial increase in house prices.

Mortgages and houses across the income distribution

The mechanisms have different predictions on how housing and mortgage holdings change across the income distribution. Figure 1.5.7 shows the percentage change of house value (ph) and mortgage holdings as a fraction of income across the income distribution. In the data, there is in inverse-U shape in housing growth. The middle income quintiles had the strongest growth in house-value-to-income. Rising inequality and keeping up with the Joneses generates a very similar pattern, where the second and the third quintiles react strongest. The Saving Glut predicts only negligible effects on housing across the income distribution, and it does counterfactually predict that the effect is increasing over the income distribution.

1.6 Conclusion

Rising inequality and keeping up with the Joneses were an important driver of mortgage debt and house prices in the decades prior to the Great Recession. In our calibrated heterogenous agent macroeconomic model, rising inequality and keeping up with the Joneses generate an increase in the mortage-to-income ratio of around 60%, about half as much as observed in the data between 1980 and 2007.

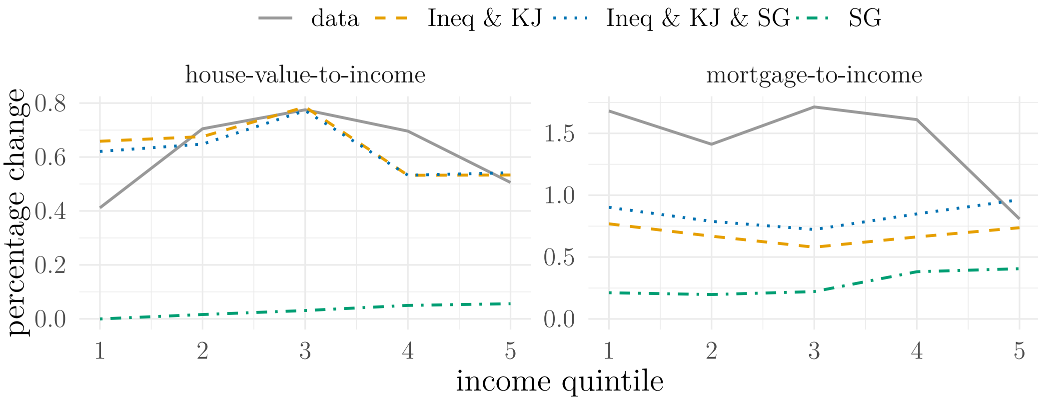


Figure 1.5.7: Compare simulated changes in housing and mortgages across the income distribution to the data (from the Survey of Consumer Finances).

Is also an important amplifier of alternative mechanisms that generate a debt boom. In a model with exogenous capital inflow (to capture the Global Saving Glut) and relaxing borrowing constraints (to capture financial liberalization) adding Keeping up with the Joneses and rising inequality boosts the debt boom by a factor of two (generating 83% instead of 38%). Among these three mechanisms, rising inequality and keeping up with the Joneses is the only mechanism that generates a sizable house price boom (generating 62% of that observed in the data).

Both of these results are robust to perturbations of the parameters.

We show analytically that under social comparisons households' optimal debt level is increasing in incomes of the direct and indirect reference group. When everybody is directly or indirectly connected to the rich, aggregate debt rises in response to rising top incomes. Our tractable framework exposes how this mechanism works. Households substitute durable houses for non-durable consumption when top incomes rise because houses are a status good. Since houses are durable, they are optimally debt-finance. So an increase in the housing share also increase debt levels.

Avenues for future research With our mechanisms, rising inequality can be an important amplifier of financial crises. First, it amplifies the aggregate consumption response to house price shocks, because these *housing wealth effects* are increasing in the house share (Berger et al., 2018; Chapter 3). Second, trends in top income inequality can lead to expectations of future house price growth, and thus serve as a trigger for the expectations channel of the housing boom and bust (Kaplan et al., 2020).

The insights from this paper can also lead to interesting research in international finance. It provides a different angle on the growing current account imbalances of the US. Rising demand for credit can attract foreign capital leading to a current account deficit.

Appendix

1.A Data sources

Figure 1.1.1: Aggregate debt and inequality We data on outstanding household debt from the US Flow of Funds, retrieved from FRED: total debt (`TLBSHNO`) and mortgages (`HMLBSHNO`). *Other debt* is constructed as the difference between total debt and mortgages. Debt is displayed as a share of nominal GDP (Bureau of Economic Analysis, BEA, via FRED: `GDP`).

The top 10% income share is from the World Wealth and Income Database (Alvaredo et al., 2016).

Figure 1.1.2b We use the micro data from the Survey of Consumer Finances (SCF). The SCF uses multiple imputation to overcome problems of missing data. We join all five imputations and treat them as one data set. This is valid because we do not do inference.

1.A.1 Horse race

Figure 1.5.3: Net foreign debt position of the US We use the current account and GDP series from the BEA, retrieved via FRED (`BOPBCA`, `GDP`). Following Gourinchas et al. (2017) we compute the cumulative sum of the current account

$$\text{cum CA}_t = \sum_{i=1960}^{t} \text{CA}_t$$

and show it as a fraction of GDP in that given year $\frac{\text{cum CA}_t}{\text{GDP}_t}$.

Figure 1.5.4: P95 of LTV distribution (proxy for ω) We use the micro data from the Survey of Consumer Finances (SCF). We join all five imputations and treat them as one data set. This is fine since we don't do inference. We use the definition of mortgages and house value from above.

We calculate individual $\text{LTV}_{i,t} = \frac{\text{outstanding mortgages}_{i,t}}{\text{house}_{i,t}}$. For each year we report the 95% percentile of the LTV distribution.

1.B Proofs

1.B.1 Lemmas

Lemma 1. *The necessary conditions for an optimum of the households' problem are*

$$u_c\big(c_t, s(h_t, \bar{h}_t)\big) = \lambda_t \tag{1.4}$$

$$u_s\big(c_t, s(h_t, \bar{h}_t)\big) s_h(h_t, \bar{h}_t) = \lambda_t (r + \delta) p \tag{1.5}$$

$$\dot{\lambda}_t - \rho \lambda_t = -r \lambda_t \tag{1.6}$$

where λ is the co-state in the continuous time optimization problem.

Proof. Without adjustment costs, the two endogenous state variables a_t and h_t collapse into one state variable net worth w_t.

$$\dot{w}_t = r w_t + y_t - (r + \delta) p h_t - c_t$$

The present-value Hamiltonian is

$$H(w, h, c, \lambda) = u\big(c, s(h, \bar{h})\big) + \lambda\big(r w_t + y_t - (r + \delta) p h_t - c_t\big),$$

where w is the state, c and h are the controls and λ is the co-state. The necessary conditions are

$$\begin{aligned}
\frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial c} &= u_c\big(c_t, s(h_t, \bar{h}_t)\big) - \lambda_t = 0 \\
\frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial h} &= u_s\big(c_t, s(h_t, \bar{h}_t)\big) s_h(h_t, \bar{h}_t) - \lambda_t (r + \delta) p = 0 \\
\dot{\lambda}_t - \rho \lambda_t = \frac{\partial H(w_t, h_t, c_t, \lambda_t)}{\partial w} &= -r \lambda_t. \qquad \square
\end{aligned}$$

Lemma 2. *Under our assumption of CRRA-CES preferences, the optimal relation of c_t and h_t is given by*

$$\frac{\xi}{1 - \xi} \Big(\frac{s(h_t, \bar{h}_t)}{c_t} \Big)^{\varepsilon - 1} s_h(h_t, \bar{h}_t) = (r + \delta) p. \tag{1.7}$$

Further assuming Assumption 3 yields

$$c_t = \kappa_0 h_t - \kappa_0 \phi \bar{h}_t, \quad \textit{where } \kappa_0 = \Big((r + \delta) p \frac{1 - \xi}{\xi} \Big)^{\frac{1}{1 - \varepsilon}}. \tag{1.8}$$

Proof. Combining conditions (1.4) and (1.5) yields

$$\frac{u_s(c_t, s_t)}{u_c(c_t, s_t)} s_h(h_t, \bar{h}_t) \stackrel{!}{=} (r + \delta)p.$$

For the given CRRA-CES preferences the marginal utilites are given by

$$\begin{aligned} u_c(c_t, s_t) &= ((1-\xi)c_t^\varepsilon + \xi s_t^\varepsilon)^{\frac{1-\gamma}{\varepsilon}-1}(1-\xi)c_t^{\varepsilon-1} \\ u_s(c_t, s_t) &= ((1-\xi)c_t^\varepsilon + \xi s_t^\varepsilon)^{\frac{1-\gamma}{\varepsilon}-1}\xi s_t^{\varepsilon-1}. \end{aligned} \tag{1.9}$$

Thus,

$$\frac{u_s(c_t, s_t)}{u_c(c_t, s_t)} = \frac{\xi}{1-\xi}\left(\frac{s_t}{c_t}\right)^{\varepsilon-1}.$$

Plugging in above yields the first statement. Using Assumption 3 we get

$$\begin{aligned} \frac{\xi}{1-\xi}\left(\frac{h_t - \phi\bar{h}}{c_t}\right)^{\varepsilon-1} &= (r+\delta)p. \\ \left(\frac{c_t}{h_t - \phi\bar{h}}\right) &= \left((r+\delta)p\frac{1-\xi}{\xi}\right)^{\frac{1}{1-\varepsilon}} = \kappa_0 \\ c_t &= \kappa_0 h_t - \kappa_0\phi\bar{h}_t \end{aligned}$$

□

Lemma 3. *Under the assumption of time-constant house prices p, and all previous assumptions of this section, individual choices c_t, h_t are constant over time.*

Proof. The costate λ is constant over time. This follows from using Assumption 1 in condition (1.6), which gives $\dot{\lambda}_t = 0$.

Plugging in (3.9) in condition (1.5) one gets that an decreasing function of h is constant over time, thus h_t is constant over time. Knowing that h_t constant over time, and a similar argument for condition (1.4) it follows that c_t is constant over time. □

1.B.2 Proof of Proposition 1

From the lemmas above we get that

$$c = \kappa_0 s(h, \bar{h}) = \kappa_0 h - \kappa_0\phi\bar{h}.$$

Using the lifetime budget constraint we get

$$\begin{aligned}
\mathcal{Y} &:= ra_0 + y = ph(r+\delta) + c \\
&= h\Big(p(r+\delta) + \kappa_0\Big) - \kappa_0\phi\bar{h} \\
\implies h &= \frac{\mathcal{Y} + \kappa_0\phi\bar{h}}{p(r+\delta)+\kappa_0} = \underbrace{\frac{1}{p(r+\delta)+\kappa_0}}_{\kappa_2}\mathcal{Y} + \underbrace{\frac{\kappa_0}{p(r+\delta)+\kappa_0}}_{\kappa_1}\phi\bar{h} = \kappa_2\mathcal{Y} + \kappa_1\phi\bar{h}
\end{aligned} \tag{1.10}$$

where

$$\kappa_1 := \frac{\kappa_0}{p(r+\delta)+\kappa_0} = \frac{1}{\frac{p(r+\delta)}{\kappa_0}+1} \in (0,1)$$

since

$$\frac{p(r+\delta)}{\kappa_0} = \left(\frac{1}{(r+\delta)p}\right)^{\frac{1}{1-\varepsilon}-1}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{1-\varepsilon}} > 0.$$

Stacking equations (1.10) for and using $\bar{\boldsymbol{h}} = G\boldsymbol{h}$

$$\begin{aligned}
\boldsymbol{h} &= \kappa_2\boldsymbol{\mathcal{Y}} + \kappa_1\phi G\boldsymbol{h} \\
\boldsymbol{h} &= (I - \kappa_1\phi G)^{-1}\kappa_2\boldsymbol{\mathcal{Y}} = \Big(\sum_{i=0}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}}.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\bar{\boldsymbol{h}} = G\boldsymbol{h} &= \frac{\kappa_1\phi}{\kappa_1\phi}G\Big(\sum_{i=0}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}} \\
&= \frac{1}{\kappa_1\phi}\Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\kappa_2\boldsymbol{\mathcal{Y}} \\
&= \frac{1}{\kappa_0\phi}\Big(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\Big)\boldsymbol{\mathcal{Y}}
\end{aligned}$$

$(I - \kappa_1\phi G)^{-1}$ is a Leontief inverse. It exists if the matrix power series $\sum_{i=0}^{\infty}(\kappa_1\phi G)^i$ converges[21]. In that case

$$(I - \kappa_1\phi G)^{-1} = \sum_{i=0}^{\infty}(\kappa_1\phi G)^i.$$

[21] This is the case for all nilpotent matrices (there exists a power p such that $G^p = 0I$) (there are no infintely-long paths in the network) or if all eigenvalues of $\kappa_1\phi G$ are between 0 and 1. This holds whenever G can be interpreted as a Markov Chain.

Now, we calculate debt.

$$-ra = y - \delta ph - c$$

using 1.B.2,

$$\begin{aligned}
&= y - \delta ph - \kappa_0 h + \kappa_0 \phi \bar{h} \\
&= y - (\delta p + \kappa_0) h + \kappa_0 \phi \bar{h}
\end{aligned}$$

$$\begin{aligned}
-r\boldsymbol{a} &= \boldsymbol{y} - (\delta p + \kappa_0) \underbrace{\left(\sum_{i=0}^{\infty} (\kappa_1 \phi G)^i\right)}_{=I+\left(\sum_{i=1}^{\infty}(\kappa_1\phi G)^i\right)} \kappa_2 \boldsymbol{\mathcal{Y}} + \left(\sum_{i=1}^{\infty} (\kappa_1 \phi G)^i\right) \boldsymbol{\mathcal{Y}} \\
&= \boldsymbol{y} - \kappa_3 \boldsymbol{\mathcal{Y}} + (1 - \kappa_3) \left(\sum_{i=1}^{\infty} (\kappa_1 \phi G)^i\right) \boldsymbol{\mathcal{Y}}
\end{aligned}$$

where

$$\kappa_3 = (\delta p + \kappa_0)\kappa_2 = \frac{\delta p + \kappa_0}{p(r+\delta) + \kappa_0} = \frac{1}{1 + \frac{pr}{\delta p + \kappa_0}} \in (0, 1).$$

1.C Numerical solution for a stationary equilibrium

We first describe how we discretize the complex income process, then we show how to solve the partial equilibrium using a finite difference method from Achdou et al. (2017). Finally we present the algorithm used to compute equilibrium prices and reference measure.

The model was solved using version 1.2 of the `Julia` language. For a given parameterizaton, 200 endogenous grid points and 2000 exogenous gridpoints solving for a general equilibrium takes about 30 minutes on standard laptop using just one core.

For the calibration we ran the code in parallel (using 30 nodes with 16 cores) for 12 hours on a high performance cluster.

1.C.1 Discretizing the income process

Pre-tax earnings depend on four exogenous states $\theta = (\tilde{\alpha}, z, \varepsilon, \nu)$,

$$y(\theta) = (1 - \nu) \exp(\tilde{\alpha} + z + \varepsilon).$$

We first discretize the two jump-drift processes z and ε following the procedure of Kaplan et al. (2018). We discretize them separately, creating two continuous time Markov chains and combining them. The statespace of the combined continous time Markov Chain is given by

$$\{z_1, \dots, z_{N_z}\} \times \{\varepsilon_1 \dots \varepsilon_{N_\varepsilon}\}.$$

Then we add non-employment states for each state, where the transition probabilities into the non-employment state are state-dependent. The statespace of the CTMC with non-employment becomes

$$\{z_1, \dots, z_{N_z}\} \times \{\varepsilon_1 \dots \varepsilon_{N_\varepsilon}\} \times \{0, 1\}.$$

Finally we add the discretize the permanent component $\tilde{\alpha}$. We choose $N_\alpha = 10$ gridpoints, where each of those gridpoints represents a decile of $\tilde{\alpha}$'s distribution. Conditional on drawing $\tilde{\alpha}_i$, the other three components follow the same CTMC with $N_z \cdot N_\varepsilon \cdot 2$ states. Denote the changing states by $\tilde{\theta} = (z, \varepsilon, \nu)$

The transition between states $\tilde{\theta}$ is given by the intensities q_{ij}. For an agent at state $\tilde{\theta}_i$ the probability of jumping to a new state $\tilde{\theta}_j$ within the time short time period Δ is approximately given by $p_{ij}(\Delta) \approx q_{ij}\Delta$. More precisely, given the intensity matrix $Q = (q_{ij})$ where $q_{ij} \geq 0$ for $i \neq j$ and $q_{ii} = -\sum_{k \neq i} q_{ik}$, the matrix of transition probabilities is given by

$$P(\Delta) = \exp(-\Delta Q),$$

where exp is the matrix exponential. $P(\Delta)$ is a stochastic matrix.

1.C.2 Partial equilbrium given p, r, $\bar{h}$

Given prices (p, r) and reference measure $\bar{h}$ the households' problem can be characterized by a coupled system of partial differential equations: the Hamilton-Jacobi-Bellman (HJB) equation and the Kolmogorov forward (KF) equation. The HJB equation describes the optimization problem of the households and the KF equation describes the evolution of the cross-sectional distribution $\mu(a, h, y)$.

We solve these two equations using the finite difference method from Achdou et al. (2017). The discretized system can be written as

$$\begin{aligned} \rho \boldsymbol{v} &= \boldsymbol{u}(\boldsymbol{v}) + A(\boldsymbol{v}; r, p, \bar{h})\boldsymbol{v} \\ \boldsymbol{0} &= \big(A(\boldsymbol{v}; r, p, \bar{h}) + M\big)^T \boldsymbol{g}, \end{aligned}$$

where $\boldsymbol{v}$ is the discretized value function, $\boldsymbol{g}$ is the discretized cross-sectional distribution, $\boldsymbol{u}(\boldsymbol{v})$ is the discretized flow utility, $A(\boldsymbol{v}; r, p, \bar{h})$ is the discretized infinitesimal generator of the HJB equation (a very sparse matrix) and M is a matrix that corrects the intensities for births and deaths. The discretized system reveals how tightly coupled the HJB and KF equations are. The matrix $A(\boldsymbol{v}; r, p, \bar{h})$ shows up in both equation. Once it is known from the solution of the HJB equation, it can be directly used to get the distribution $\boldsymbol{g}$ from the KF equation.

Solving the Hamilton-Jacobi-Bellman equation

We assume that housing h can be adjusted frictionlessly. So the two states h and a collapse into one, "net worth"

$$w_t = a_t + ph_t,$$

with its law of motion

$$\dot{w}_t = rw_t + y_t - (r+\delta)ph_t - c_t.$$

The collateral constraint can be rewritten in terms of w

$$\begin{aligned} w_t &= ph_t + a_t \geq ph_t - \omega ph_t \\ \implies ph_t &\leq \frac{w_t}{1-\omega}. \end{aligned}$$

The households' HJB equation is

$$\begin{aligned} (\rho+m)v(w,\theta_i) = \max_{c,h\leq\frac{w}{1-\omega}} \; & u(c, s(h,\bar{h})) \\ & + v_w(w,\theta_i)(rw + \theta_i - (r+\delta)ph - c) \\ & + \sum_{k\neq i} q_{ik}(v(w,\theta_k) - v(w,\theta_i)). \end{aligned}$$

The intensities q_{ij} are the intensities of the continuous time Markov chain from Section 1.C.1. In order to solve this equation, we need to replace the maximum operator with the maximized Hamiltonian. That is, we need to plug in the optimal policy functions $c^*(w,y)$, $h^*(w,y)$ which are given in Corollary 4 below. The result depends on the following lemma.

Lemma 4. *When the collateral constraint is slack, we get the optimality conditions*

$$\begin{aligned} h(w,y) &= \left(\frac{1}{\tau_2}\left(\bar{h}^{\phi}(\rho+\delta)pv_w(w,y)\right)\right)^{-\frac{1}{\gamma}} \bar{h}^{\phi} \\ c(w,y) &= s(h(w,y),\bar{h})\tau_1, \end{aligned}$$

where $\tau_1 = \left((r+\delta)p\frac{1-\xi}{\xi}\bar{h}^{\phi}\right)^{\frac{1}{1-\varepsilon}}$ *and* $\tau_2 = ((1-\xi)\tau_1^{\varepsilon}+\xi)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi$.

Proof. Using the optimality conditions (1.7) and (1.5) with (1.9) we get

$$(r+\delta)p = \frac{u_s(c,s)}{u_c(c,s)}s_h(h,\bar{h}) = \frac{\xi}{1-\xi}\Big(\frac{s(h,\bar{h})}{c}\Big)^{\varepsilon-1}s_h(h,\bar{h}) \quad (1.22)$$

$$(\rho+\delta)pv_w(w,y) = u_s\big(c,s\big)s_h = ((1-\xi)c^{\varepsilon}+\xi s^{\varepsilon})^{\frac{1-\gamma}{\varepsilon}-1}\xi s^{\varepsilon-1}s_h. \quad (1.23)$$

Using (1.22) we express optimal c as a function of optimal s

$$c(h,\bar{h}) = s(h,\bar{h})\Big((r+\delta)p\frac{1-\xi}{\xi}\frac{1}{s_h(h,\bar{h})}\Big)^{\frac{1}{1-\varepsilon}}$$

using the ratio specification for s

$$= s(h,\bar{h})\Big((r+\delta)p\frac{1-\xi}{\xi}\bar{h}^{\phi}\Big)^{\frac{1}{1-\varepsilon}} =: s(h,\bar{h})\tau_1.$$

Then we can plug this expression into (1.23) and get

$$\begin{aligned}(\rho+\delta)pv_w(w,y) &= ((1-\xi)(\tau_1 s)^{\varepsilon}+\xi s^{\varepsilon})^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi s^{\varepsilon-1}s_h \\ &= \underbrace{((1-\xi)\tau_1^{\varepsilon}+\xi)^{\frac{1-\gamma-\varepsilon}{\varepsilon}}\xi}_{=:\tau_2} s^{1-\gamma-\varepsilon}s^{\varepsilon-1}s_h \\ &= \tau_2 s^{-\gamma}s_h\end{aligned}$$

Thus we get

$$s(h,\bar{h}) = \left(\frac{(\rho+\delta)pv_w(w,y)}{\tau_2 s_h}\right)^{-\frac{1}{\gamma}},$$

and using ratio-specification for s,

$$h = \left(\frac{1}{\tau_2}\big((\rho+\delta)pv_w(w,y)\bar{h}^{\phi}\big)\right)^{-\frac{1}{\gamma}}\bar{h}^{\phi}. \qquad \square$$

Corollary 4. *The optimal policies are given by*

$$h^*(w,y) = \begin{cases} h(w,y) & \text{if } h(w,y) < \frac{w}{p(1-\omega)} \\ \frac{w}{p(1-\omega)} & \text{otherwise}\end{cases}, \quad c^*(w,y) = \begin{cases} c(w,y) & \text{if } h(w,y) < \frac{w}{p(1-\omega)} \\ \tilde{c}(w,y) & \text{otherwise}\end{cases}$$

where $h(w,y)$ *and* $c(w,y)$ *are from Lemma 4 and* $\tilde{c}(w,y)$ *is the solution to the optimality condtion for c, given* $h = \frac{w}{p(1-\omega)}$,

$$v_w(w,y) = ((1-\xi)c^{\varepsilon}+\xi s^{\varepsilon})^{\frac{1-\gamma-\varepsilon}{\varepsilon}}(1-\xi)c^{\varepsilon-1},$$

which is solved numerically.

Given the optimal policies, it is straight-forward to solve the HJB using the implicit upwind scheme in Achdou et al. (2017).

Solving the Kolmogorov forward equation

We construct the birth and death matrix M as in Kaplan et al. (2018) and solve for the distribution using the implicit scheme from Achdou et al. (2017).

1.C.3 General equilibrium: Solving for r, p and $\bar{h}$

We use the following algorithm to compute general equilibria.

0. Guess r_0, p_0 and $\bar{h}_0$
1. Clear housing markets given r_{n-1} and $\bar{h}_{n-1}$
   (a) Use Newton steps until the sign of the excess demand for housing changes
   (b) Use Bisection to find the market clearing price p_n
2. Compute the excess demand on the asset market
3. Use a Newton step to update the interest rate r_n
4. Compute the implied reference measure $\bar{h}_x$ and update $\bar{h}_n = \bar{h}_{n-1} + a(\bar{h}_x - \bar{h}_{n-1})$
5. If $r_n \approx r_{n-1}$ and $\bar{h}_n \approx \bar{h}_{n-1}$, an equilibrium has been found. If not, go back to step 1.

1.D Additional tables

In this appendix we show the tables corresponding to the figures in the main text.

Table 1.D.1: Disentengling the effects of rising inequality and keeping up with the Joneses

	mortgage-to-income			house prices		
	growth	share	% of data	growth	share	% of data
Rising inequality	0.17	0.29	0.15	0.22	0.56	0.35
& Keeping up	+0.43	0.71	0.38	+0.17	0.44	0.27
total	0.60	1.0	0.53	0.38	1.0	0.62

Table 1.D.2: Decomposition of the constributions of the three channels on the mortgage and house price booms.

(a) Starting from Keeping up with the Joneses

	mortgage-to-income			house prices		
	growth	share	% of data	growth	share	% of data
Inequality and keeping up	0.60	0.72	0.53	0.38	1.0	0.62
& Saving Glut	+0.17	0.21	0.15	+0.00	0.0	0.0
& Relaxed collateral constraint	+0.06	0.07	0.05	+0.00	0.0	0.0
total	0.83	1.0	0.74	0.38	1.0	0.63

(b) Starting from the Saving Glut

	mortgage-to-income			house prices		
	growth	share	% of data	growth	share	% of data
Saving Glut	0.32	0.39	0.28	0.02	0.04	0.03
& Inequality and keeping up	+0.45	0.55	0.4	+0.37	0.95	0.6
& Relaxed collateral constraint	+0.06	0.07	0.05	+0.00	0.0	0.0
total	0.83	1.0	0.74	0.38	1.0	0.63

Table 1.D.3: The effects of each channels on mortgages and house prices

	mortgage-to-income		house prices	
	growth	% of data	growth	% of data
Inequality and keeping up	0.6	0.53	0.38	0.62
Saving Glut	0.32	0.28	0.02	0.03
Relaxed collateral constraint	0.02	0.02	0.0	0.0

Chapter 2

Top Incomes and Mortgage Debt Across the United States

Joint with Moritz Drechsel-Grau.

2.1 Introduction

Income inequality and household debt have risen in parallel between 1980 and 2007. In theory, there are at least two causal mechanisms that generate this relationship. We refer to them as the *savings channel* and the *comparisons channel*. This chapter uses distributional national accounts (DINA) data compiled by Piketty, Saez, and Zucman (2018b) to provide some new empirical findings on the relationship between household debt and income inequality. These findings are consistent only with the *comparisons channel*.

The *comparisons channel* was proposed in Chapter 1.[1] Debt growth is driven by the appetite for housing of the non-rich. Non-rich households upgrade their houses to keep up with the houses of the ever richer rich. Debt rises because these houses are usually mortgage-financed.

The *savings channel* was proposed by Kumhof et al. (2015). Debt growth is driven by the appetite for savings of the rich. Asset market clearing dictates that the non-rich have to take out debt to match the savings of the rich. This hypothesis is echoed by Mian, Straub, and Sufi (2020) who show that most of the debt of non-rich is held by the rich.

[1] In Chapter 1 we formalize ideas put forward by Stiglitz (2009), Rajan (2011) and Frank (2013b) and builds on complementary empirical evidence by Bellet (2019), De Giorgi, Frederiksen, and Pistaferri (2020) and Bertrand and Morse (2016).

We construct a state-year-income group panel for the period from 1980 to 2007 from US DINA data. These data cover the joint distribution of income, wealth and outstanding debt by year and US state. The data set allows us to track outstanding debt by debt category, income group and state over time.

We provide evidence from three different methods. First, we compare the long-run changes of top incomes and household debt between 1980 and 2007. Second, we run panel regressions using state and year fixed effects. And third, we study the dynamic response of debt to changes in top incomes using local projections.

We document three new empirical findings. First, there is a positive relationship between incomes of the rich and mortgage debt of the non-rich at the US state level. Mortgage debt of the non-rich grows faster in states that experience a stronger increase in top incomes. Second, we show that the state-level relationship between top incomes and debt of the non-rich is solely driven by mortgage debt. When top incomes rise, mortgage debt increases, while non-mortgage debt does not react. Third, we find that rising top incomes affect the *level* of outstanding mortgages. When top incomes rise by 10%, the level of mortgages will persistently go up by 5%. It takes some years for the new level to be reached.

Our findings show that the well-established *savings channel* cannot account for all aspects of the debt boom. The *comparisons channel* from Chapter 1 is an important complementary explanation. It is necessary to explain geographical variation and the differential effects across debt categories.

We do not expect the *savings channel* to generate our empirical findings because it works through interest rates. First, if financial markets are sufficiently integrated then local demand for savings need not be compensated by local debt. Instead, abitrage should lead to a uniform increase in debt of the non-rich across all states. Second, there is no reason to expect mortgage debt to react differently than non-mortgage debt.

By contrast, the *comparisons channel* is consistent with the geographic variation in responses and the fact that only mortgage debt reacts. As noted in Chapter 1 top incomes drive debt because housholds substitute durable status goods for non-durable non-status goods. This channel should be strongest for housing and mortgage debt, because housing is the most important visible good—both in terms of value and visibility.

We confirm that the relationship between mortgage debt and top incomes is not specific to the DINA data set. We build a county-year panel of mortgage originations from the Home Mortgage Disclosure Act (HMDA) database. The data include information on the purpose of the mortgage (new purchase, home improvement or refinancing) and the income of the mortgage applicant. This allows us to construct distinct mortgage panels per loan purpose and per income group. We complement these data with state-level income deciles from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). Consistent with our findings from the DINA data, there is a significant positive relationship between county-level mortgage debt and state-level top incomes.

A note on causality In this chapter we do not exploit exogenous variation in income inequality. All estimates in this paper should be interpreted as correlations rather than causal effects. That said, the correlations are consistent with the causal theoretical mechanism based on social comparisons from Chapter 1. Chapter 1, in turn, builds on a big empirical literature showing that social comparisons causally shape economic desicion making.

Relation to the literature This chapter is most closely related to Coibion, Gorodnichenko, Kudlyak, and Mondragon (2020), who find that, on average, borrowing of the middle class goes up (while borrowing of the poor does not increase) when inequality goes up. Mian et al. (2020) have shown that most of the debt of non-rich is held by the rich, taking this as evidence for the *Saving Glut of the Rich* hypothesis. Neither of these papers considers differences between mortgages and non-mortgage debt.

We provide indirect evidence that individual economic decisions causally depend on the consumption of others because of social preferences. This adds to a growing literature on consumption externalities. Bellet (2019) has shown that homeowners increase their spending on home improvements when a big house is built in their neighborhood and Bertrand and Morse (2016) show that consumption of the non-rich shift from non-conspicuous to conspicuous goods when top incomes rise.

We borrow our empirical strategy from Bertrand and Morse (2016). We analyze how the debt of the bottom 90% of the income distribution reacts to income changes at the top of the income distribution. For our state-level analysis we use the data set from Piketty et al. (2018b), who construct distributional national accounts (DINA) for the United States.

Roadmap The remainder of this chapter is structured as follows. In Section 2.2, we describe the datasets that we use and how we aggregate them. We present the empirical results in Section 2.3. Section 2.4 concludes.

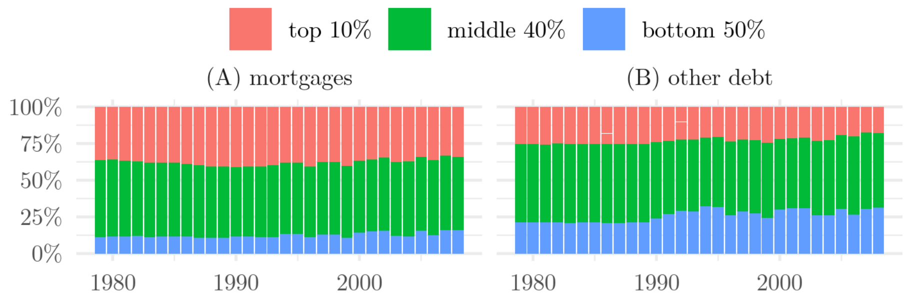


Figure 2.2.1: Outstanding debt by income group. Source: Authors' calculations from the DINA database.

2.2 Data

We compile two different datasets. The main data set is aggregated from US distributional national accounts (DINA) data (from Piketty et al., 2018b). We compile a state-year-income-group panel for the period 1980–2007 covering income, outstanding mortgages and outstanding non-mortgage debt.

For our some additional analyses, we build a county-year-income-group panel from two data sources. We combine individual-level mortgage originations data from the Home Mortgage Disclosure Act (HMDA) database and state-level inequality measures that we construct from the Current Population Survey (CPS) Annual Social and Economic Supplement (CPS-ASEC).

2.2.1 US-DINA Data

The US-DINA data were constructed by Piketty et al. (2018b) to study the evolution of economic inequality over time. They were constructed from tax, survey, and national accounts data and cover the joint distribution of income and wealth, including household debt, also across U.S. states. The DINA data allow us split the data by state, debt category and income group.

The DINA data allow us to split outstanding debt by income group. Figure 2.2.1 shows that the bottom 50% percent of income earners hold only around 10% of outstanding mortgages, while the top 10% account for around 30% of all mortgages.

2.2.2 HMDA Mortgage Data

The HMDA was enacted by the Congress of the United States in 1975. It requires most mortgage lenders to report individual mortgage applications with the lender's decision (originated or declined), loan amount, purpose

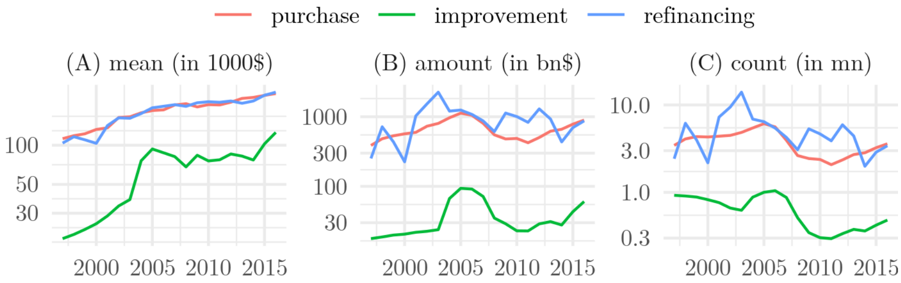


Figure 2.2.2: Mortgage originations by purpose over time. Y-axis in $\log_{10}$-scale. Source: Authors' calculations from the HMDA database.

(purchase, home improvement or refinancing), borrower characteristics (gross annual income, race, sex, census tract). Dell'Ariccia, Igan, and Laeven (2008) report that the HMDA data base covers a substantial fraction of total mortgage originations: 78% for 2000 and between 88% and 96% for the years 2001 to 2006. The FED estimates coverage of more than 90% for 2016.[2]

Figure 2.2.2 shows some descriptive statistics from the HMDA database. We see that the mean mortgage for purchase and refinancing was about $100,000 in 1997 and grew quite montonously to about 250,000$ in 2016. The total loan amount and the number of originated mortgages have seen some cyclical behavior. Originations for purchases and home improvement peaked around 2005, and fell until 2011 before starting to rise again. While the number of originated loans did not change much between 1997 and 2016, the aggregate loan amount tripled in all three categories (purchases and refinancing were at about $300bn in 1997 and at almost $900bn in 2016).

We construct a county-year panel, aggregating the mortgage originations by loan purpose (purchase, home improvement, refinancing) and income group of the borrower. We use the state-level income distribution of a given year as reference, and split the population into the bottom 50% (P0P50), P50P70, P70P90 and the top 10% (P90P100).

Each of these four income groups accounts for a substantial share of mortgage originations. As shown in Figure 2.2.3, the P90P100 and P70P90 groups each account for more than a quarter of all originated mortgages. The bottom 50% account for about 15% of mortgage originations.

[2]Fed Bulletin November 2016: https://www.federalreserve.gov/publications/2017-november-residential-mortgage-lending-in-2016.htm

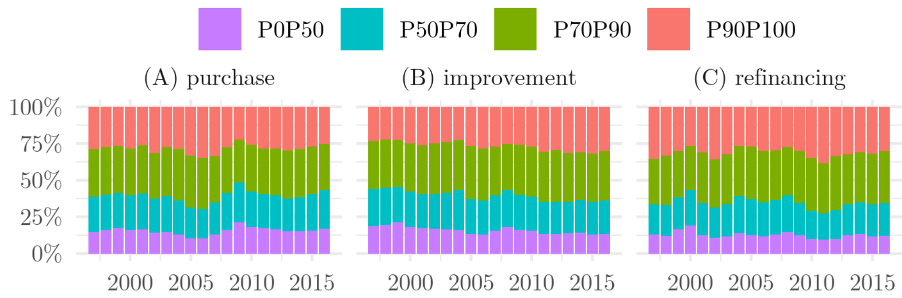


Figure 2.2.3: Mortgage originations by income group. Source: Authors' calculations from the HMDA database.

2.2.3 Alternative Inequality Data

For our complementary analysis of county-level debt we calculate income and income inequality measures on a state level from the CPS-ASEC. We use household income and household weights.[3]

2.2.4 A Note on the Differences Between DINA and HMDA Data

Other than being more granular, the HMDA data differ from to the DINA data in that they contain mortgage originations, and not outstanding mortgages. In order to make the analysis as comparable as possible we use the level of originations form HMDA as an equivalent to the change in outstanding mortgages from DINA. (This neglects debt repayment.)

Moreover, while the income in DINA covers the income of the whole population, HMDA only contains information on a person's income if this person has taken out a mortgage in a given year.

2.2.5 The Distribution of US Counties

US counties are quite unevenly distributed across states, and they have dispersion in terms of populations size. This is shown in Figure 2.2.4. The right panel shows that most counties have a population between 10,000 and

[3]We need to take an income measure (household income or individual income) that is comparable to the HMDA data. The HMDA data contain the gross annual income relevant for the lender. So this should be the sum of incomes of all applicants. Assuming (but not having checked) that household usually apply for mortgages jointly, we use household income. Incidentally, Bertrand and Morse (2016), who compare consumption from the Consumer Expenditure Survey (CEX) to income groups, also need household income.

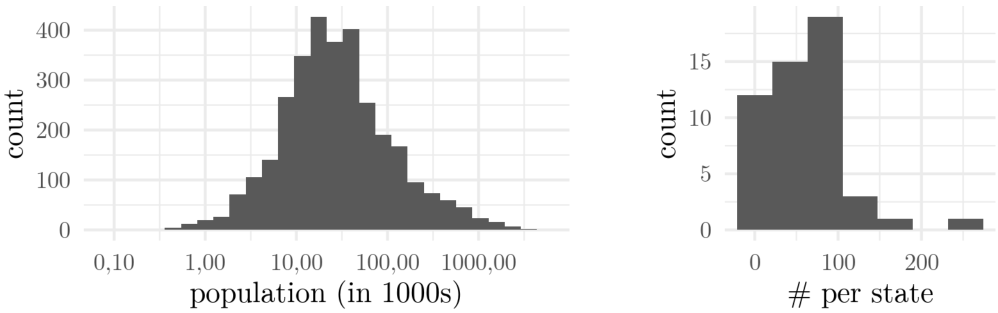


Figure 2.2.4: The distribution of county population (as of 2000) and the distribution of counties across states.

100,000. Most states have less than but close to 100 counties. This is why we use population weights for our county-level analysis.

2.3 Empirical Analysis

On an aggregate level, mortgage debt and top income inequality have risen in lockstep between 1980 and 2007 in the United States (see, Chapter 1). In this section we use geographical variation across US states and counties to show that mortgage debt has grown faster in regions that have experienced stronger growth in income inequality over this time period.

We use average top incomes of the top 10% as our measure of income inequality. This has two reasons. First, Piketty et al. (2018b) have shown that average incomes of the bottom 50% have stagnated between 1980 and 2007 while incomes of the top 10% have doubled in the US. So, top incomes capture an important dimension of the change in income inequality over this time period. And second, we have shown in Chapter 1 that this exact nature of rising inequality drives debt under the *comparisons channel*. Rising top incomes drives the debt of the bottom 90%.

We will use the following notation. Let $\text{incomes}^g_{s,t}$ be the average income and debt^g_{st} be the average debt for the income group g in state s and year t. We consider the income groups

$$g \in \{\text{bottom } 90\%, \text{bottom } 50\%, \text{middle } 40\%\}.$$

Let further $\text{top incomes}_{s,t}$ be the average incomes of the top 10% in a given state s and year t. Unless noted otherwise all data come from the DINA data set.

2.3.1 Debt and Inequality Over the Long-run (1980–2007)

First, we look at simple long-run correlations between income inequality and debt. We show that there is a positive relationship between top incomes and household debt. This relationship is driven by mortgages only.

For a state s and income group g we look at the following relationship,

$$\Delta \log(\text{debt}_s^g) = \alpha + \beta \Delta \log(\text{top incomes}_s) + \varepsilon_s^g, \tag{2.1}$$

where we look at changes between 1980 and 2007. We provide additional results using the change in income of group g as a control variable. (This is not possible for the top 10%, for whom own income and top incomes are identical.) Figure 2.3.1a shows that there is a positive relationship between household debt growth and growth in top incomes. This positive relationship is present in household debt across the income distribution.

Figure 2.3.1b reveals that this relationship between debt and inequality is only driven by mortgages. We split total household debt in mortgage debt and non-mortgage debt. While the positive relationship between debt and top incomes is preserved for mortgages, non-mortgage debt growth even seems to be slightly negatively related to changes in top incomes.

Table 2.3.1 shows that, absent any control variables, top incomes are a statistically significant predictor for mortgage debt across all income groups, while being insignificant in predicting growth of other debt. When adding income growth of own incomes as a first control variable, the picture doesn't change much. Top incomes stay significant for bottom 50% mortgages, becoming just insignifcant for middle 40% mortages. The coefficients for non-mortgage debt turn negative.

2.3.2 Fixed Effect Regressions

The regression model (2.1) uses only the information of the years 1980 and 2007. We now estimate a fixed effect regression using the full state-year-income group panel. We estimate a set of regression models of the form

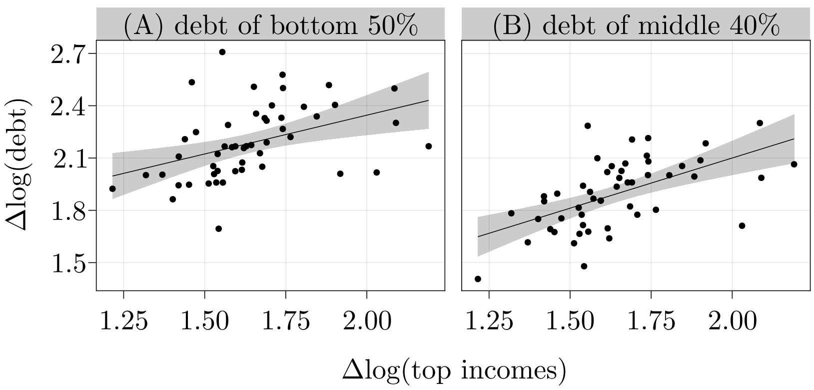
$$\log(\text{debt}_{st}^g) = \alpha + \beta \log(\text{top incomes}_{st}) + \gamma \log(\text{incomes}_{st}^g) + \delta_s + \delta_t + \varepsilon_{st} \tag{2.2}$$

for the two different debt categories (mortgage debt, non-mortgage debt). If β is positive, higher top income levels are associated with higher current levels of non-rich debt when non-rich incomes are held constant and state and year effects (δ_s, δ_t) are controlled for.

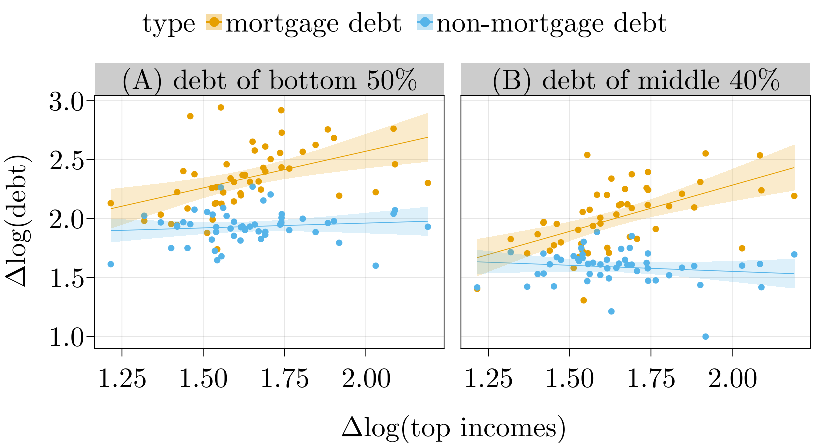
Table 2.3.2 shows that this coefficients is indeed positive. A 10% increase in top incomes is associated with a 3% increase in total debt of the

Figure 2.3.1: Change in top 10% incomes and household debt between 1980 and 2007 across US states

(a) Total household debt by income group



(b) Mortgage and non-mortgage debt by income group



Note: Authors' calculations based on data from DINA (Piketty et al., 2018b)

Table 2.3.1: Growth of debt by income group and growth of top 10% incomes between 1980 and 2007 for US states

	mortgages				
	bottom 50%		middle 40%		top 10%
	(1)	(2)	(3)	(4)	(5)
(Intercept)	1.327***	0.402	0.715**	-0.101	0.958***
	(0.290)	(0.430)	(0.272)	(0.315)	(0.202)
top incomes	0.622***	0.450**	0.784***	0.260	0.615***
	(0.175)	(0.176)	(0.165)	(0.196)	(0.122)
own income		0.951***		1.488***	
		(0.342)		(0.376)	
Estimator	OLS	OLS	OLS	OLS	OLS
N	51	51	51	51	51
R^2	0.204	0.314	0.316	0.485	0.342

	non-mortgage debt				
	bottom 50%		middle 40%		top 10%
	(1)	(2)	(3)	(4)	(5)
(Intercept)	1.797***	1.156***	1.759***	1.434***	1.259***
	(0.171)	(0.245)	(0.174)	(0.221)	(0.154)
top incomes	0.082	-0.037	-0.104	-0.312**	-0.017
	(0.103)	(0.100)	(0.105)	(0.137)	(0.093)
own income		0.659***		0.592**	
		(0.195)		(0.263)	
Estimator	OLS	OLS	OLS	OLS	OLS
N	51	51	51	51	51
R^2	0.013	0.202	0.019	0.113	0.001

Notes: Standard errors in parentheses. Authors' calculations based on data from DINA (Piketty et al., 2018b).

bottom 90%. Hence there is both economically and statistically significant co-variation of local top incomes and non-rich debt. Like in the previous specification this relationship is entirely driven by mortgage debt. While mortgage debt is affected by top incomes, non-mortgage debt is not.

Figure 2.3.2 visualizes the estimation results by showing residualized binned scatter plots of non-rich debt and top incomes. The slope of the fit is equal to β in equation (2.2). The regression model is able to capture a substantial amount of the relationship of non-rich debt and top incomes after accounting for fixed effects and non-rich income.

2.3.3 Impulse Responses from Local Projections

We now analyze the dynamic response of non-rich debt to changes in top incomes. In particular, we estimate the impulse response function of non-rich debt using local projections of the form

$$\begin{aligned}\Delta^{h+1}\log(\text{debt}_{s,t+h}) &= \alpha^h + \beta^h \Delta\log(\text{top incomes}_{st}) + \delta_t^h + \\ &\sum_{k=1}^{3}\Big(\gamma_k^h \log(\text{debt}^g_{s,t-k}) + \phi_k^h \log(\text{top incomes}_{s,t-k})\Big) + \varepsilon_{st}^h \quad (2.3)\end{aligned}$$

for each $h \in \{0, \dots, 10\}$, where

$$\Delta^{h+1}\log(\text{debt}_{s,t+h}) = \log(\text{debt}^g_{s,t+h}) - \log(\text{debt}^g_{s,t-1}).$$

The coefficients β^h give us the cumulative %-change in non-rich debt that is induced by a one-time change in top-incomes by 1%. Figure 2.3.3 plots the estimated impulse response function for mortgage and non-mortgage debt for the middle 40% and bottom 50% of the income distribution. By adding past debt and inequality measures as controls, specification (2.3) essentially compares states with the some pretrends in debt and inequality, but where one state experiences a stronger increase in inequality in t.

Consistent with the previous specifications, top incomes drive up mortgage debt substantially over the following ten years while non-mortgage debt remains roughly constant. The elasticity of mortgage debt with respect to top incomes is somewhat higher for the middle 40% than for the bottom 50% and more precisely estimated.[4] For the middle 40%, a 10% increase in top incomes from $t-1$ to t translates into persistent increase in mortgage debt of roughly 5% after five to ten years. The same change in top incomes is associated with a 2% increase in mortgage debt of the bottom 50%. For non-mortgage debt, there are no (persistent) effects in both income groups.

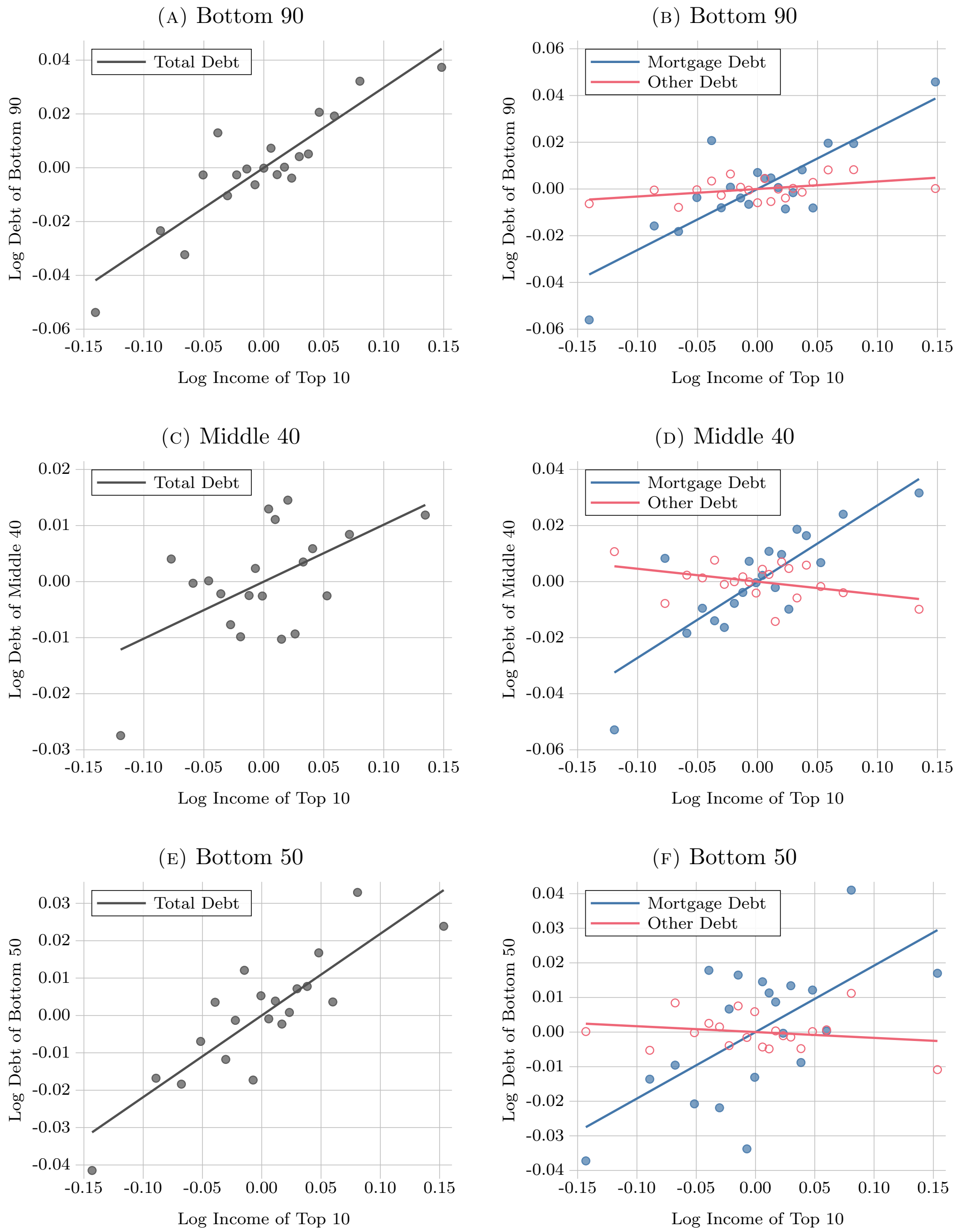
[4]Controlling for lags of non-rich income does not change the estimated IRFs.

Table 2.3.2: Relationship Between Non-Rich Debt and Top Incomes

	Bottom 90%			Middle 40%			Bottom 50%		
	total	mortgage	other	total	mortgage	other	total	mortgage	other
log(top incomes$_{st}$)	0.296**	0.252**	0.0386	0.115	0.267**	-0.0382	0.206**	0.180**	-0.0147
	(0.0688)	(0.105)	(0.0578)	(0.0751)	(0.108)	(0.0689)	(0.0676)	(0.0775)	(0.0452)
log(own incomes$_{st}$)	0.277**	0.236**	0.214**	0.964**	0.541**	0.555**	0.200**	0.214**	0.195**
	(0.0319)	(0.0600)	(0.0359)	(0.0652)	(0.125)	(0.0524)	(0.0206)	(0.0638)	(0.0347)
State FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	1326	1326	1326	1326	1326	1326	1326	1326	1326
R^2	0.987	0.990	0.994	0.975	0.965	0.986	0.985	0.989	0.993

Note: This table shows the estimation results of the regression model in equation 2.2 for $k = 0$. Data: DINA.

Figure 2.3.2: Relationship Between Non-Rich Debt and Top Incomes



Note: This figure shows the relationship between debt (total, mortgage, other) of non-rich households (bottom 90, middle 40, bottom 50 % of income distribution) conditional on state and year fixed effects. The All variables are demeaned. The slope of the regression line is equal to the conditional elasticity. Each dot represents the mean log top income and mean log non-rich-debt within 20 equal-sized bins of the x-variable (log top income).

Figure 2.3.3: Response of Non-Rich Debt to Increase in Top Incomes

Note: This figure shows the cumulative effect of a 1% change in top-10 incomes on mortgage and non-mortgage debt of the middle 40 and bottom 50. The confidence bands are constructed using a significance level of 10%.

2.3.4 County-level Analysis

In the previous subsections we have shown that there is a clear positive relationship between mortgages and inequality in the DINA data. In this subsection we show that one can find the same relationship using independent data. We compile a county-year-income group panel of mortgage originations from the HMDA database and combine it with annual state-level income inequality measures from the CPS-ASEC. Since coverage of the HMDA data was much worse in the early 1990s we analyze changes between 1997 and 2007.

These datasets force us to use alternative measures of debt and inequality. Since the HMDA database tracks mortgage originations (as opposed to outstanding mortgages), we use the sum of originations over the time period as a dependent variable. This is a close as we we can get to the actual change in outstanding debt over this time horizon.

$$\Delta \text{debt}_{r,g,p} := \sum_{t=1996}^{2007} \text{originations}_{r,t,g,p}.$$

We normalize this variable by average income of the full time period. Let $I_{c,g,t,p} = \{1, \ldots, N_{c,g,t,p}\}$ be the set of all observed mortgage originations for

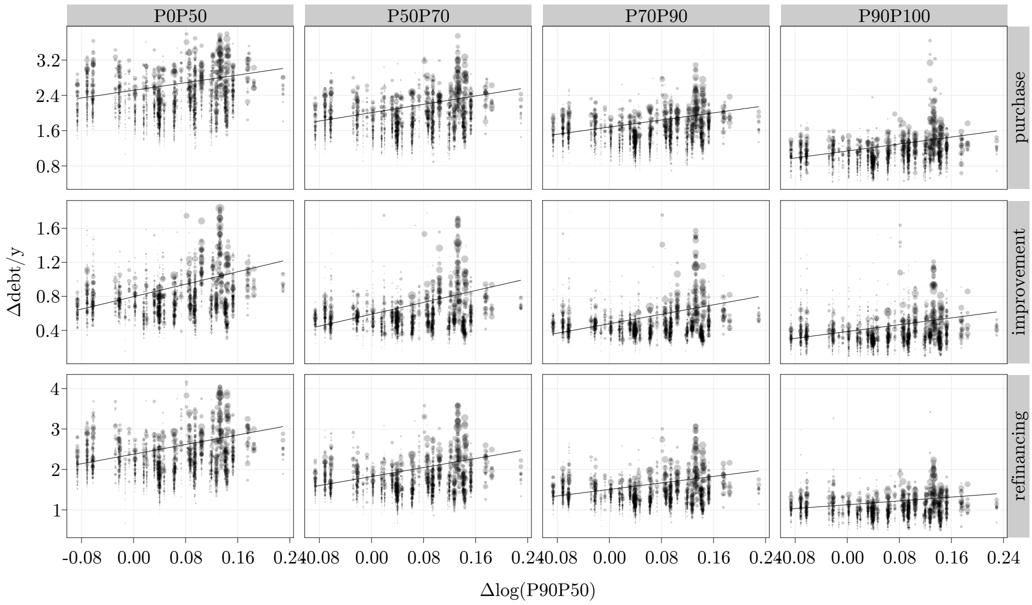


Figure 2.3.4: County-level mortgage-originations and change in state-level income inequality between 1997 and 2007. The regression line is computed with population weights. Source: Authors calculations based on HMDA and CPS-ASEC data.

income group g in county c during year t with purpose p. Then

$$y_{c,g,p} := \frac{1}{\sum_t N_{c,g,t,p}} \sum_t \sum_{i \in I_{c,g,t,p}} \text{income}_i.$$

As for the inequality measure, we follow Bertrand and Morse (2016) and use percentiles instead of group averages. We use the P90/P50 ratio as a measure for income inequality.

We run regression of the following kind,

$$\frac{\Delta \text{debt}_{c,g,p}}{y_{c,g,p}} = \alpha + \beta \Delta \log(\text{inequality}_{s(c)}) + \varepsilon_{c,g,p},$$

where c is the county, $s(c)$ the corresponding state, g the income group and p is the purpose of the mortgage (purchase, refinancing or home improvement). Regressions are run separately for each income group g and mortgage purpose p. Since U.S. counties are quite hetergeneous in terms of their size we use weighted statistics whenever appropriate.

Figure 2.3.4 shows the relationship between the change in income inequality (measured by the change of log of the P90/P50 ratio) on the state level and new originations on the county level. The figure shows that there is a positive correlation between debt growth and income inequality for all income groups and mortgage purpose categories. Table 2.3.3 shows that this correlation is statistically significant across all income groups for purchases, and for some income groups for the other two debt purpose categories.

This findings are consistent with the previous sections. Regions that experience stronger increases in income inequality also experience stronger growth in mortgage debt.

2.4 Conclusion

This chapter provides further evidence that household debt and income inequality are fundamentally linked. We showed that US states that experienced a stronger growth in top incomes, also have a stronger growth in mortgage debt. This finding is confirmed by the results of a fixed effects regression, local projections and complementary analysis using different data. We find that the effect on mortgage balances builds up over time and it seems that higher inequality affects the level of mortgages in the long-run. Importantly, this relationship between debt of the non-rich and top incomes only holds for mortgages. Non-mortgage debt does not seem to be affected by rising top incomes.

These findings shed some light on potential causal mechanisms that link mortgage balances and top income inequality. Among the two theoretical mechanisms that have been proposed in the literature, only one mechanism is consistent with two of these facts. The *comparisons channel* (proposed in Chapter 1) is consistent with a state-level variation in mortgage growth and with the fact that non-mortgage debt is unaffected. By contrast, the *savings channel* (proposed by Kumhof et al., 2015) is consistent with neither of these facts if financial markets are sufficiently integrated. Local demand for savings need neither be compensated by local debt, nor by mortgages alone. Instead, abitrage should lead to a uniform increase in debt of the non-rich across all states and across both debt categories. The *savings channel* alone cannot account for all of the variation in debt growth during the debt boom. The *comparisons channel* should be regarded as an important complementary explanation.

We show that the relationship between debt and top incomes is also present in a different dataset based on HMDA data. The analysis of these county-level data has some issues that need to be dealt with in future work. First, we only observe mortgage originations, but not the outstanding balances of mortgage debt. Second, and more importantly, we do not observe

	Δdebt/y											
	purchase				improvement				refinancing			
	P0P50	P50P70	P70P90	P90P100	P0P50	P50P70	P70P90	P90P100	P0P50	P50P70	P70P90	P90P100
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
(Intercept)	2.303***	1.824***	1.542***	1.025***	0.754***	0.556***	0.473***	0.367***	2.204***	1.687***	1.396***	1.016***
	(0.068)	(0.051)	(0.045)	(0.035)	(0.032)	(0.021)	(0.026)	(0.023)	(0.065)	(0.049)	(0.039)	(0.028)
Δlog(P90P50)	1.400*	1.399**	1.081**	1.218***	0.843	0.735*	0.363	0.250	1.607**	1.184*	0.747	0.429
	(0.785)	(0.587)	(0.467)	(0.364)	(0.514)	(0.386)	(0.365)	(0.291)	(0.786)	(0.663)	(0.534)	(0.350)
Estimator	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
N	3,136	3,136	3,131	3,115	3,131	3,128	3,116	3,029	3,135	3,135	3,135	3,126
R^2	0.031	0.050	0.039	0.054	0.034	0.047	0.010	0.005	0.045	0.036	0.022	0.011

Table 2.3.3: County-level mortgage-originations and change in state-level income inequality between 1997 and 2007. The regression line is computed with population weights. Source: Authors calculations based on HMDA and CPS-ASEC data.

the full income distribution at the county-level. All we have is the income data for new borrowers. Better income data is important to correctly control for income growth of the non-rich, and also to get within-state variation of top income growth.

Chapter 3

Understanding Housing Wealth Effects: Debt, Home-Ownership and the Lifecycle

Joint with Frederick Zadow.

3.1 Introduction

During the Great Recession, the US saw a pronounced drop in house prices along with a stark reduction in consumption expenditures. The large reduction in spending has been attributed to housing wealth effects: Households reduced their non-durable consumption as a reaction to the depreciation of their housing wealth. Mian, Rao, and Sufi (2013) emphasize the role of debt for housing wealth effects. They find that aggregate housing wealth effects are stronger in more indebted regions.[1] This finding suggests that imposing low enough debt limits is a potent policy to dampen consumption response to a house price bust. In response, as of 2018, 60% of advanced countries have introduced maximum loan-to-value ratios.[2]

In this paper we show that this conclusion might be premature. A one-size-fits-all approach to regulating debt limits might not be the best measure

[1] Aladangady (2017) provide further empirical evidence on the relationship between debt and housing wealth effects. Macroeconomic models of housing generate a similar correlation (e.g. Berger et al., 2018).

[2] For more details see Cerutti, Claessens, and Laeven (2017) and https://voxeu.org/article/increasing-faith-macroprudential-policies.

to increase resilience to a house price bust. Instead, measures should be taylored to household characteristics and take into account the aggregate taste for housing.

To this end, we build a simple life-cycle permanent-income model that allows closed form solutions for housing wealth effects. Housing wealth effects do not directly depend on debt. They do depend on household characteristics (age and the utility weight for housing) which also determine the level of indebtedness. Thus, there is a reduced form correlation between aggregate debt and the aggregate consumption reaction on a regional level. The sign of this correlation depends on the composition of agents.

Consider a baseline region A with a continuum of agents and compare it to three other regions. Region B has identical distributions of age and incomes, but agents have a larger utility weight of housing. Region B will be more indebted than region A and will have *stronger* housing wealth effects.

Region C has identical distributions of incomes and housing preferences, but agents are younger than in region A. Region C will be more indebted than region A and will have *weaker* housing wealth effects.

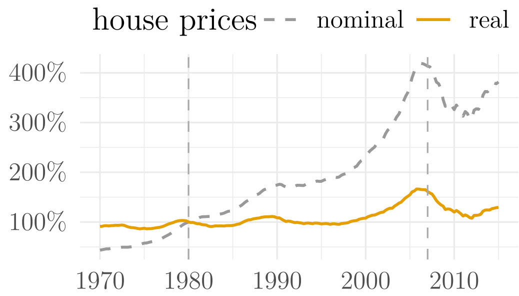
Region D has identical distributions of age and housing preferences, but agents have more front-loaded incomes than in region A. Region D will be more indebted than region A and will have *as strong* housing wealth effects as region A.

Thus, the underlying reason for being indebted changes the sign of the effect on housing wealth effects. This finding is not inconsistent with Mian et al. (2013), who find that more indebted regions had a stronger consumption response in the crisis. If the age distribution is similar across regions, their estimate is simply picking up the effect of differences in the taste for housing.

We test the predictions of our model empirically. We use data on consumption and housing from the CEX and MSA-level house price indices from `Zillow.com`. We construct a proxy for housing preferences from the residual of an auxiliary regression and show that mortgages are a statistically significant predictor of the size of housing wealth effects only if age and housing prefences are excluded from the regression. If, on the other hand, mortgages are excluded, age and housing preferences both have the predicted statistically significant positive effect on housing wealth effects.

From these findings we derive two main policy implications. First, debt limits should vary with age. For young households home ownership provides an opportunity to build up wealth. At the same time, this group does not exhibit strong housing wealth effects. Hence, it does not appear reasonable to further constrain their choices with respect LTV limits. Instead, the ability of older households to take out debt should be curtailed more strongly. It is these people that display stronger consumption reactions and thereby

Figure 3.1.1: Real and nominal house prices in the USA.



Notes: Nominal: Case-Shiller Home Price Index. Real: Deflated by the Consumer Price Index. Source: http://www.econ.yale.edu/~shiller/data.htm

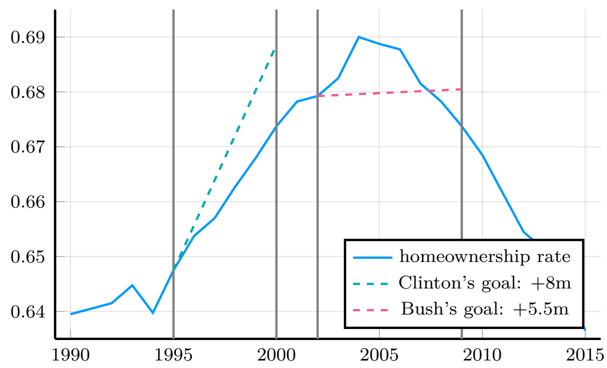
impose more of the negative externalities of housing wealth effects on the economy. The reason is that housing makes up a larger share of older households' remaining lifetime wealth, therefore they react stronger to price busts. Hence, from the policy maker's perspective it makes sense to encourage diversification of senior people's portfolios into other assets such as stocks and downsizing of houses. The importance of this conclusion will only increase during the coming years when the baby boomer generation, which holds a larger share of national wealth (in housing) is about to enter retirement. According to our model, a given housing bust in the future will lead to a more severe reduction in consumption expenditures compared to today due to the changing demographics.

Second, politicians should reconsider policies which promote home ownership. In the US such policies often convey the message that owning a big house is still part of the American Dream. Through the lense of our model, a high home ownership rate can be considered as an expression of strong household preferences for housing. However, stronger preferences for housing are associated with bigger housing wealth effects. Hence, policies that raise home ownership contribute to financial instability.

Contributions to the literature This paper connects to the empirical and the quantitative macroeconomic literatures on housing wealth effects. We provide a tractable model that rationalizes both empirical and quantitative findings about the heterogeneity of housing wealth effects.

Empirical studies have shown that the reaction of consumption to changes in housing wealth varies with household characteristics. Mian et al. (2013) show that counties that are more leveraged and poorer react more strongly to

Figure 3.1.2: Homeownership in the US and policy goals.



house price changes. Campbell and Cocco (2007) find heterogeneous housing wealth effects with respect to housing status and age. They find strongest reactions for older homeowners and the weakest reaction for young renters. Aladangady (2017) shows that the response to house prices is neglibile for renters and large for homeowners. His specifications implies that the consumption response is proportional to initial house values.

Our model generates the findings that homeowners that are older or have bigger houses react stronger while renters don't react. We show that the role of indebtedness is ambiguous on an individual level, but likely negative (consistent with previous findings) on an aggregate level.

Similar to the empirical studies mentioned above, quantitative macroeconomic studies have found that housing wealth effects vary across individuals. Guren et al. (2020) show that the consumption response, as a function of loan-to-value (LTV), is hump shaped. Berger et al. (2018) show numerically that consumption elasticities vary with income, age, housing, liquid assets and renting decision. With the exception of liquid assets, we can investigate all of these dimensions analytically in our model.

Moreover, Berger et al. (2018) derive a rule-of-thumb for housing wealth effects, which are given by the initial value of the house times the marginal propensity to consume. We complement their finding by providing a formula that is solely based on primitives of the model, without relying on an endogenous object like the MPC.

Structure of the paper The paper is organized as follows. In Section 3.2 we present our tractable life-cyle model with housing and mortgages and its solution. Subsequently, we derive closed forms for housing wealth effects in our model (this is our main result) and discuss comparative statics in Section 3.3. In Section 3.4 we provide an empirical test of our model's predictions.

3.2 A Simple Lifecycle Model with Housing

Time is discrete and runs forever. Households are born with an initial endowment of assets $a_0 \geq 0$ and live for $J \in \mathbb{N}$ periods. There are two types of households: homeowners and renters. These two types differ in their access to technology: homeowners are not allowed to rent, renters are not allowed to buy.[3] Households derive utility from a non-durable consumption good c and their durable housing stock h (rented or owned). They supply labor inelastically and receive earnings y. Households choose streams of consumption $c_t > 0$, housing stock $h_t > 0$ and assets $a_t \in \mathbb{R}$ to maximize their discounted lifetime utility.

3.2.1 Homeowners

Homeowners' discounted lifetime utility is

$$\sum_{t=0}^{J-1} \beta^t \frac{\left(c_t^{1-\xi} h_t^{\xi}\right)^{1-\gamma}}{1-\gamma} + \beta^{J-1}\psi(h_{J-1}),$$

where $\beta > 0$ is the discount factor and $\psi : \mathbb{R}_+ \to \mathbb{R}$ represents a warm-glow bequest motive. Households consume a Cobb-Douglas-aggregated composite good from which they derive utility according to a standard constant relative risk aversion (CRRA) utility function.

Housing is both a consumption good and an asset. It is modelled as a homogeneous, divisible good. As such, h represents a one-dimensional measure of housing quality (including size, location and amenities). An agent's housing stock depreciates at rate δ and can be adjusted frictionlessly. Home improvements and maintenance expenditures x_t have the same price as housing (p) and go into the value of the housing stock one for one. The law of motion for the housing stock is

$$h_t = (1-\delta)h_{t-1} + x_t,$$

where $h_{-1} = 0$. The asset a serves both as a savings device and short-term mortgage. Saving and borrowing can be done at the equilibrium interest rate r. The law of motion for end-of-period assets is

$$a_{t+1} = (1+r)a_t + y_t - c_t - px_t,$$

[3] This assumption is a short-cut to explicitly modelling the renting-vs-owning decision of households.

where a_0 is the given initial endowment. Agents are not allowed to die in debt,

$$a_{J+1} \geq 0.$$

Otherwise, there are no borrowing constraints in our model. This is justified, because there is no uncertainty, and thus, no reason to default in our model.

In order to obtain closed-form results for optimal choices, we make the following assumptions. First, we assume that incomes are deterministic and constant over time.

Assumption 5 (Constant incomes)**.** $y_t = y$ for all t.

Second, we assume that the bequest function takes the following parametric form.

Assumption 6. The bequest function is $\psi(h, p) = \kappa_1 \frac{(\kappa_2 h)^{1-\gamma}}{1-\gamma}$ where $\kappa_1 = \xi \frac{\beta(1-\delta)}{1-\beta(1-\delta)}$ and $\kappa_2 = \left((1-\beta(1-\delta))\frac{1-\xi}{\xi}p\right)^{1-\xi}$.

Assumption 6 will ensure that optimal choices for consumption c and housing stock h are constant over time. It requires the marginal utility of bequeathing a house being equal to the marginal utility of selling the house during one's lifetime.

Optimal choices

We first show that the optimal consumption and housing stock for a household is constant over time.

Lemma 5. *Under Assumptions 5 and 6 the optimal choices for housing stock and consumption are constant over time, $c_t = c$ and $h_t = h$, for all $0 < t < J-1$ and in optimum $c(h) = \kappa_3 ph$, where $\kappa_3 = (1-\beta(1-\delta))\frac{1-\xi}{\xi}$.*

Proof. See Appendix 3.A.1. □

Then we derive optimal choices for consumption c and housing h.

Proposition 4. *Under Assumptions 5 and 6 optimal choices are*

$$\begin{aligned} ph &= \mathcal{Y}\frac{1}{1-\delta+\theta(J,r)\cdot(\delta+\kappa_3)}, \\ c &= \kappa_3 ph, \end{aligned}$$

where $\theta(J,r) := \sum_{j=0}^{J-1}\left(\frac{1}{1+r}\right)^j$ and $\mathcal{Y}$ is life-time income.

Proof. The life-time budget constraint is given by

$$(1-\delta)ph + \sum_{j=0}^{J-1}\Big(\frac{1}{1+r}\Big)^j(\delta ph + c) = a_0 + \sum_{j=0}^{J-1}\Big(\frac{1}{1+r}\Big)^j y =: \mathcal{Y}$$

Using the definition of θ, and the assumption on c we get

$$\begin{aligned}\mathcal{Y} &= (1-\delta)ph + \theta(J,r)\cdot(\delta ph + \kappa_3 ph)\\ &= ph\big((1-\delta) + \theta(J,r)\cdot(\delta+\kappa_3)\big)\end{aligned}$$

Rearranging yields the desired result. □

Lemma 5 and Proposition 4 show that optimal choices are constant over time and proportional to discounted lifetime income. An agent buys her optimal house in the first period, irrespective of how low the initial endowment is. To cover the gap between initial resources and the downpayment, the agent takes out a mortgage m_0 and repays it over time. Let m_t be the level of outstanding mortgages at the beginning of period t and π_t the debt service in period t. The law of motion is given by

$$m_t = (1+r)(m_{t-1} - \pi_t). \tag{3.1}$$

Proposition 5. *For homeowners, initial outstanding mortgages are*

$$m_0 = (\theta - 1)\left(\frac{y}{1+\theta\frac{\kappa_3+\delta}{1-\delta}} - \frac{a_0}{\theta + \frac{1-\delta}{\kappa_3+\delta}}\right), \tag{3.2}$$

The debt service payment is constant overtime,

$$\pi_t = \pi = y - c - \delta ph,$$

and the beginning-of-period outstanding mortgage at age j is

$$m_t = \sum_{i=0}^{J-t-1}\Big(\frac{1}{1+r}\Big)^i(y - c - \delta ph). \tag{3.3}$$

Proof. See Appendix 3.A.2. □

From Proposition 5 it follows immediately that mortgages are positive, as long as the income is sufficiently high or initial assets are sufficiently low.

Corollary 5. *Initial mortgages are positive, $m_0 > 0$, iff*

$$\frac{y}{a_0} > \frac{\delta+\kappa_3}{(1-\delta)}.$$

(a) Infinite lifetime ($J \to \infty$)

(b) Finite lifetime (here $J = 4$)

Figure 3.2.1: Optimal choices for consumption c, housing stock h and assets a.

Proof. Follows immediately from the intermediate equation (3.10) in the proof of Proposition 5. □

If an agent inherits a sufficiently large initial endowment, she can finance the downpayment of the house, without the need for a mortgage. If the initial endowment exceeds the downpayment, the agent will be a saver. If, on the other hand, an agent does not inherit any initial endowment, the initial income will not be sufficient to cover the downpayment. She will need to take shift part of her lifetime income to the present using a mortgage.

Moreover, we can see the determinants of indebtedness.

Corollary 6. *For homeowners, initial debt is increasing in the taste for housing ξ and flow income y and decreasing in initial endowments a_0. Outstanding debt is decreasing with age.*

Proof. Follows immediately from (3.2) and (3.3) in Proposition 5 because $\kappa_3 \propto 1/\xi - 1$ and $\frac{\partial \kappa_3}{\partial \xi} < 0$. □

There are three reasons, why households are more indebted than others: being young, having a stronger taste for housing, having low initial endowments relative to lifetime income. Households that are younger are more indebted, because they have had less time to repay their mortgage. Households that have a stronger taste for housing are more indebted because they need to finance a bigger house. Finally, for a given lifetime income, households with low initial endowments earn a larger share of their incomes later in life. That is, they need to shift a larger amount of their lifetime income to the present to finance the downpayment of the house.

3.2.2 Renters

Renters have no bequest motive. Their problem is then given by

$$\max_{\{c_t,h_t\}_{t=0}^{J-1}} \sum_{t=0}^{J-1} \beta^t u(c_t^{1-\xi} h_t^{\xi})$$
$$\text{s.t. } c_t + \rho h_t + a_{t+1} = (1+r)a_t + y_t$$
$$a_J \geq 0$$

where ρ denotes the price of renting one unit of the housing good.

Under our given assumptions, agents' consumption choice will not depend on the rental price (which is a function of the house price).

Proposition 6. *Optimal policies of renters are constant across time. Furthermore, the level of consumption is independent of the cost of renting,*

$$c^* = (1-\xi)\frac{\mathcal{Y}}{\theta}, \qquad \rho h^* = \xi \frac{\mathcal{Y}}{\theta}.$$

Proof. See Appendix 3.A.3. □

In this framework renters' optimal consumption is independent of the cost of rent. Now suppose, that rent increases with rising house prices and vice versa. A decrease in house prices then, which reduces consumption of home owners, has no effect on renters. Their wealth is unaffected and therefore also spending on consumption. This is in line with e.g. Berger et al. (2018); Aladangady (2017) who find very small reactions of renters' consumption expenditure to changes in house prices.

3.3 Housing wealth effects with closed forms

We can now derive the main result of this paper: closed forms for the consumption response to house price shocks. We have already shown that there are no housing wealth effects for renters, so the remaining work to do is to derive results for homeowners.

We assume that house price shocks are unexpected and permanent. In our thought experiment, an agent wakes up at age j and observes that the house price has fallen from p to q. She reconsiders her optimal choices given her net worth,

$$\tilde{a}_j = q(1-\delta)h - (1+r)m_{j-1},$$

her unchanged flow income y and her remaining lifetime $J-j$. Due to exponential discounting, her optimal choices are time consistent (Strotz, 1955) and will be as if she was a $(J-j)$-period-lived agent with initial endowment $\tilde{a}_j$ and flow income y given house price q.

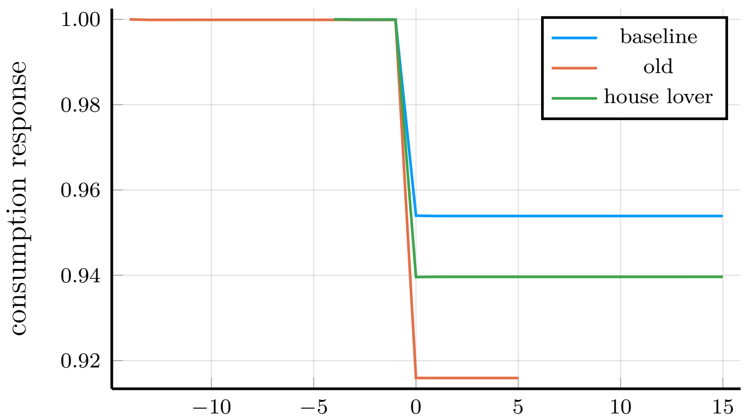


Figure 3.3.1: Housing wealth effects—the consumption response to a drop in house prices—by age and housing preferences. Shock happens in $t = 0$.

Proposition 7. *After an unexpected price change from p to q at the beginning of a period, a homeowner of age j will adjust their consumption*

$$\frac{c_j^*}{c_0^*} = \frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta + \Omega)}{(1-\delta) + \theta^{J-j}(\delta + \Omega)}$$

Proof. See Appendix 3.A.4. □

Given this closed form result, it is easy to analyze the heterogeneity in housing wealth effects along different household characteristics.

Proposition 8. *The consumption response to an unexpected negative house price shock is (i) zero for renters and (ii) negative for homeowners. Moreover, the absolute response for homeowners is*

1. *increasing in age j and*

2. *increasing in the utility weight for housing ξ.*

Proof. See Appendix 3.A.5. □

Homeowners that are older or have stronger preferences for housing are hit harder by house price shocks. This is illustrated in Figure 3.3.1. Agents have to reduce their consumption to compensate their losses in housing wealth. Intuitively, older agents react more strongly, because they have less time to smooth out their losses. Agents with stronger preferences for housing own a larger house, so they are facing larger losses that have to be compensated.

Housing wealth effects and indebtedness

Proposition 8 is silent about the role of debt on housing wealth effects. This is because indebtedness endogenous, rather than a primitive of the model. Rather than looking at the role of debt directly, we can analyze how the *drivers* of debt (see Corollary 6) affect the strength of housing wealth effects. Each driver—income profile, age and taste for housing—acts on housing wealth effects differently.

Income profile (y vs a_0) Varying the ratio of initial endowment and flow incomes will change the debt holdings for a given lifetime income $\mathcal{Y}$. If more of the lifetime income is earned through flow income, optimal debt will be higher. For the choices of c and h however, the composition of lifetime income is irrelevant in our complete markets setup. So, more indebted agents react *equally strongly.*

Age j Households repay their debt over their lifetime. Older agents are less indebted than poor agents. As shown above, older agents have a stronger consumption response. When comparing agents of different ages, more indebted agents react *less strongly.*

Taste for housing ξ We have shown that agents with stronger preferences for houses, are more indebted. They also react stronger to house price changes. When comparing agents of different housing preferences, more indebted agents react *more strongly.*

From the perspective of our model, the effect of debt on individual housing wealth effects is ambiguous. The reason for being indebted determines the strength of the consumption response to a change in house prices.

3.3.1 Rationalizing the findings in the empirical literature

Campbell and Cocco (2007) use survey data from the UK to find heterogeneous housing wealth effects with respect to housing status and age. They find the strongest reactions for older homeowners and the weakest reaction for young renters. To do so they look at changes in house prices across three regions (North, Center, South).[4] Due to data limitations, Campbell

[4]This coarse distinction masks a lot of heterogeneity within these regions. For example, the region "North" contains all of Scotland with more densely populated, urban areas around Edinburgh or Glasgow and very sparsely populated in the north of Scotland.

and Cocco (2007) cannot condition their findings on individual house size. Our model is consistent with their findings: older people react more strongly, renters do not react.

Aladangady (2017) links the individual expenditure data from the CEX with house price information on the MSA-level, using restricted-use geographical information from the CEX. He shows that the response to house prices is neglibile for renters and large for homeowners. His specifications implies that the consumption response is proportional to initial house values. Additionally, he finds that households with low LTV ratio react more strongly than households with high LTV ratio. Our model is consistent with the finding that effects are stronger for homeowners with bigger houses. On the other hand, according to our model, his estimated effect of the LTV ratio must pick up the underlying effect of the taste for housing.

Mian et al. (2013) use aggregate data (county and ZIP code level) on expenditures and household balance sheets to show that there the elasticity of consumption out of housing wealth is higher more leveraged and poorer households. While we cannot (yet) make a statement about the reaction of poorer households, we can rationalize the stronger effects of more indebted regions. If the age distribution is similar across regions, their estimate is simply picking up the effect of differences in the taste for housing. Regions with a stronger taste for housing will react more strongly according to our model.

3.3.2 Aggregate housing wealth effects

From Corollary 7 and Proposition 6 we know the housing wealth effects for owners and renters. We showed that under Cobb-Douglas aggregation, renters do not react at all.

Thus we can write the aggregate response as

$$\text{Homeownership rate} \times \text{mean response of owners.}$$

Thus, bigger homeownership rate will lead to a stronger aggregate consumption response to a house price change.

Hence, people may very well be assigned to a housing wealth effect when, in reality, they do not experience one (e.g. when there is strong price appreciation in London, affecting the whole region). We are able to avoid this problem by only considering narrowly defined MSAs.

3.4 Testing the predictions on housing wealth effects

Now we take the model predictions to the data. We combine the CEX, a quarterly panel with MSA-level geographic identifiers, with regional house price data from Zillow.com. We construct a simple measure of housing preferences from the residual of a regression explaining house sizes. We show that, in line with our model, age and housing preferences are significant predictors of the size of housing wealth effects. When these two explanatory variables are ommitted, their effect is picked up by the level of mortgage debt.

3.4.1 Data

For the empirical exercise we employ data from two sources. First, we obtain publicly available house price indices on the MSA level from Zillow.com, a real estate listing site. In particular, we use the Zillow Home Value Index, which is a "smoothed, seasonally adjusted measure of the median estimated home value across a given region and housing type" according to the website.[5] This data has been used in several other papers such as Graham (2018). The data set covers the period from 1996 until 2017 on a monthly frequency, which includes both the sharp decline following the financial crisis as well the strong recovery in house prices that followed. This is an advantage over many other papers in the literature that often only look at the sharp increase in prices prior to the crisis.

Secondly, we use publicly available data from the Consumer Expenditure Survey (CEX) from the Bureau of Labor Statistics. The CEX is a household survey which includes detailed information on expenditures (such as durable and non-durable goods). Additionally, the survey contains information on the housing status of the household (i.e., if the household is a renter, homeowner etc.), mortgage information and some other, more general household characteristics. Households are observed at most four times within in 12 months (the time period does not have to correspond to a calendar year). Between observations, there are always three months. This structure gives the data set a panel dimension which allows us to identify the effect across time.[6]

To match both data sets we use the fact that the publicly available CEX

[5]More information and the detailed methodology can be found under https://www.zillow.com/research/data/

[6]For more details on the CEX data see https://www.bls.gov/cex/pumd.htm.

data includes geographical information on the MSA level for a subset of all households (23 MSAs). Hence, we allocate the house price level (measured by the index) in a given month to every household which was residing in that particular MSA. Our final data set consists of around 37.000 unique households observed between 2006 and 2017.

3.4.2 Constructing a measure of housing preferences

The model predicts that the utility weight of housing ξ is an important determinant of the magnitude of housing wealth effects. Since preferences for housing are not directly observable, we need to construct a proxy from the available data. We use information on *rent equivalents*, which is the imputed market rent for a household's house or apartment. The basic idea behind this approach is that, given a household's observable characteristics (such as income and size), a higher implicit price one would pay for housing suggests stronger preferences for housing (consistent with our model). To operationalize this idea, we run a regression explaining the rent equivalents using a set of household characteristics $\boldsymbol{x}_i$ (such as location, education, age, family size and income),

$$\text{rent}_i = \alpha + \boldsymbol{\beta}\boldsymbol{x}_i + u_i.$$

Then we define the measure of housing preferences as the residual of the regression (as a percentage deviation),

$$\text{pref}_i = \frac{\text{rent}_i - \widehat{\text{rent}}_i}{\widehat{\text{rent}}_i}.$$

A household with a larger residual has a larger (or more expensive) home than households with similar characteristics. We interpret that as the household having larger than average utility weight of housing ξ.

3.4.3 Results

We run a regression of the form

$$\frac{\Delta c_{i,a,t}}{\Delta p_{a,t}} = \boldsymbol{x}_{i,a,t}\boldsymbol{\gamma} + \varepsilon_{i,a,t},$$

where a is the index for the MSA, $\Delta c_{i,a,t}$ is household-level change in non-durable consumption and $\Delta p_{a,t}$ in the change in the MSA-level house price index and $\boldsymbol{x}_{i,a,t}$ contains the a subset variables of interest (mortgages, age,

Table 3.4.1: Regression output

	$\Delta c/\Delta p$		
	(1)	(2)	(3)
own × log(mortgage)	0.727**		0.746$^{\circ\circ}$
	(0.362)		(0.523)
own × h-pref-proxy		2.444**	1.722°
		(1.229)	(1.371)
own × age		0.323*	0.247
		(0.177)	(0.269)
own × age^2		-0.003*	-0.002
		(0.002)	(0.003)
MSA FE	Yes	Yes	Yes
year FE	Yes	Yes	Yes
Estimator	OLS	OLS	OLS
N	36,350	52,307	33,796
R^2	0.002	0.001	0.002

$^{***}p \leq 0.01$, $^{**}p \leq 0.05$, $^{*}p \leq 0.1$, $^{\circ\circ}p \leq 0.2$, $^{\circ}p \leq 0.3$

the preference proxy, home ownership) and MSA and year fixed effects as control variables.

We find that this simple test supports the predictions of our model. The log of outstanding mortgages is a significant predictor of the size of housing wealth effects—but only as long as the determinants of mortgages are excluded from the regression. Indeed, regression (2) in Table 3.4.1 shows that the housing preference measure and age are statistically significant predictors of housing wealth effects, when mortgages are excluded. When including all three variables in regression (3) of Table 3.4.1, they all turn insignificant. This result reflects the fact that there is a high correlation between these variables.

3.5 Conclusion

Empirical and quantitative macroeconomic studies have found that housing wealth effects are stronger for more indebted households. One important policy implication is that lowering debt limits for borrowers will dampen the consumption slump in a house price bust. In this chapter we show that such conclusions might be premature.

We build a simple life-cycle model with housing with closed form solutions for housing wealth effects. We show that the strength of housing wealth effects crucially depends on the underlying household characteristics which also determine the debt levels. In this framework imposing one-size-fits-all debt limits does not necessarily mitigate housing wealth effects. To be effective, policies have to be tailored to borrowers' characteristics. Aggregate housing wealth effects can be reduced in three ways: (i) if old homeowners reduce their housing wealth; (ii) if the home ownership rate decreases; (iii) if agents have smaller houses. We provide a simple empirical test of our model predictions. When explaining housing wealth effects, we find that the level of mortgages turns statistically insignificant once relevant household characteristics (age and a proxy for housing preferences) are added.

Appendix

3.A Proofs

3.A.1 Proof of Lemma 5

Proof. The Lagrangian is given by

$$\sum_{t=0}^{J-1} \beta^t \Bigg(u(c_t, h_t) - \lambda_t \Big(a_{t+1} - (1+r)a_t - y_t + c_t + p(h_t - (1-\delta)h_{t-1}) \Big) \Bigg) + \beta^{J-1}\psi(h_{J-1})$$

where $a_J = 0$ is given. The first order conditions are as follows. For a_t, $t \leq J-1$,

$$\lambda_{t-1} = \beta(1+r)\lambda_t \implies \lambda_0 = \cdots = \lambda_{J-1} = \lambda.$$

For c_t for $t \leq J-1$,

$$u_c(c_t, h_t) = \lambda_t = \lambda$$

For h_t for $t < J-1$,

$$\begin{aligned} u_h(c_t, h_t) &= \lambda_t p - (1-\delta)p\beta\lambda_{t+1} = \lambda p(1 - \beta(1-\delta)) \\ \implies \frac{u_h}{u_c} &= p(1 - \beta(1-\delta)) \end{aligned} \tag{3.4}$$

and for $t = J-1$

$$u_h(c_{J-1}, h_{J-1}) = \lambda_{J-1}p - \psi_h(h_{J-1}). \tag{3.5}$$

Using the CRRA-Cobb-Douglas functional form assumption we get

$$u_c(c, h) = (1-\xi)\frac{(c^{1-\xi}h^{\xi})^{1-\gamma}}{c} \tag{3.6}$$

$$u_h(c, h) = \xi\frac{(c^{1-\xi}h^{\xi})^{1-\gamma}}{h} \tag{3.7}$$

$$\frac{u_h}{u_c} = \frac{\xi}{1-\xi}\frac{c}{h}. \tag{3.8}$$

Combining (3.4) and (3.8) yields

$$\frac{\xi}{1-\xi}\frac{c}{h} = p(1-\beta(1-\delta))$$

which gives an optimal relationship of c and h,

$$c^*(h) = \underbrace{(1-\beta(1-\delta))\frac{1-\xi}{\xi}}_{\kappa_3} ph = \kappa_3 ph. \tag{3.9}$$

Using this relationship, (3.6) and (3.7) simplify to

$$\begin{aligned}
u_c(c^*(h),h) &= (1-\xi)\frac{((\kappa_3 ph)^{1-\xi}h^{\xi})^{1-\gamma}}{\kappa_3 ph} = (1-\xi)h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)-1} \\
u_h(c^*(h),h) &= \xi\frac{((\kappa_3 ph)^{1-\xi}h^{\xi})^{1-\gamma}}{h} = \xi h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)}.
\end{aligned}$$

We choose ψ to ensure that (3.9) also holds at age $J-1$. So we plug the previous expressions into (3.5),

$$u_h - p\lambda = u_h - pu_c = h^{-\gamma}(\kappa_3 p)^{(1-\xi)(1-\gamma)}\left(\xi - p\frac{1-\xi}{\kappa_3 p}\right) = \psi'(h).$$

Finally, define

$$\begin{aligned}
\tilde{\kappa}_1 &:= \xi - p\frac{1-\xi}{\kappa_3 p} = \xi\frac{\beta(1-\delta)}{1-\beta(1-\delta)}, \\
\tilde{\kappa}_2 &:= (\kappa_3 p)^{1-\xi},
\end{aligned}$$

and use the guess for ψ from Assumption 6 to determine the undetermined coefficients,

$$\begin{aligned}
&\psi'(h) = \kappa_1\kappa_2^{1-\gamma}h^{-\gamma} = h^{-\gamma}\tilde{\kappa}_2^{1-\gamma}\tilde{\kappa}_1. \\
\implies &\kappa_1 = \tilde{\kappa}_1, \quad \kappa_2 = \tilde{\kappa}_2.
\end{aligned}$$

□

3.A.2 Proof of Proposition 5

From Proposition 4 and the flow budget constraint XX we know that the initial mortgage is

$$m_0 = ph + c - y - a_0.$$

Plugging in optimal choices we get

$$
\begin{aligned}
&= (1+\kappa_3)\frac{\theta y + a_0}{1-\delta+(\delta+\kappa_3)\theta} - (y + a_0) \\
&= \frac{(1+\kappa_3)(\theta y + a_0) - (y+a_0)(1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= y\frac{(1+\kappa_3)\theta - (1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} + a_0\frac{(1+\kappa_3) - (1-\delta+(\delta+\kappa_3)\theta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= y\frac{(\theta-1)(1-\delta)}{1-\delta+(\delta+\kappa_3)\theta} - a_0\frac{(\theta-1)(\kappa_3+\delta)}{1-\delta+(\delta+\kappa_3)\theta} \\
&= (\theta-1)\frac{y(1-\delta) - a_0(\kappa_3+\delta)}{1-\delta+(\delta+\kappa_3)\theta} \qquad (3.10) \\
&= (\theta-1)\left(\frac{y}{1+\theta\frac{\kappa_3+\delta}{1-\delta}} - \frac{a_0}{\theta+\frac{1-\delta}{\kappa_3+\delta}}\right),
\end{aligned}
$$

which is the first claim of the proposition. The second claim follows from the flow budget constraint and the fact that y, h and c are constant over time (Proposition 4).

For the third claim, use the fact that $m_J = 0$, and solve the difference equation (3.1) forward,

$$
\begin{aligned}
m_t &= \frac{1}{1+r}m_{t+1} + \pi_t \\
&= \frac{1}{1+r}\left(\frac{1}{1+r}m_{t+2} + \pi_{t+1}\right) + \pi_t \\
&= \left(\frac{1}{1+r}\right)^s m_{t+s} + \sum_{i=1}^{s}\left(\frac{1}{1+r}\right)^{i-1}\pi_{t+s-1}
\end{aligned}
$$

let $J = t + s$

$$
\begin{aligned}
&= \left(\frac{1}{1+r}\right)^{J-t}\underbrace{m_J}_{0} + \sum_{i=1}^{J-t}\left(\frac{1}{1+r}\right)^{i-1}\pi \\
&= \sum_{i=1}^{J-t}\left(\frac{1}{1+r}\right)^{i-1}\pi_t. \qquad \square
\end{aligned}
$$

3.A.3 Proof of Proposition 6

As before the budget constraints can be combined into one lifetime budget constraint. Let $\mathcal{Y}$ denote lifetime income. Let λ denote the Lagrange multiplier of the constraint maximization problem. The FOCs of the new problem are given by

$$(1-\xi)u'(c_t^{1-\xi}h_t^{\xi})\Big(\frac{h_t}{c_t}\Big)^{\xi} = \lambda$$
$$\xi u'(c_t^{1-\xi}h_t^{\xi})\Big(\frac{c_t}{h_t}\Big)^{(1-\xi)} = \lambda\rho$$

From here it follows that policies are constant over time. Furthermore, rearranging the FOCs and plugging them back into the lifetime budget constraint yields the optimal policies:

$$c^* = (1-\xi)\frac{\mathcal{Y}}{\theta}$$
$$\rho h^* = \xi\frac{\mathcal{Y}}{\theta}$$

where θ is as defined before. The desired result follows.

3.A.4 Proof of Proposition 7

$$c^*(\mathcal{Y}_0, J) = c^*(\mathcal{Y}_j, J-j)$$
$$h^*(\mathcal{Y}_0, J) = h^*(\mathcal{Y}_j, J-j)$$

where

$$\mathcal{Y}_0 = \tilde{a}_0 + \theta^J y$$
$$\mathcal{Y}_j = \tilde{a}_j + \theta^{J-j} y$$

and

$$\tilde{a}_j = (1-\delta)ph_{j-1} - m_j.$$

If, however, the environment changes, the agent will want to reallocate their expenditures.

After the price change, agent's optimal choices are given by

$$h_j^* = \mathcal{Y}_j \frac{1}{q\big((1-\delta) + \theta^{J-j}(\delta+\Omega)\big)} \tag{3.11}$$
$$c_j^* = q\Omega h_j^*$$

where the new lifetime income at age j is given by a combination of current wealth (h and m) and future income

$$\mathcal{Y}_j = \tilde{a}_j + \theta^{J-j} y$$

with

$$\begin{aligned}\tilde{a}_j &= (1-\delta)qh - m_j \\ &= (1-\delta)qh - m_{J-(J-j))}.\end{aligned}$$

Plugging in our formulas for m_{J-t} (lemma XXX) and c (assumption XXX) we get

$$\begin{aligned}\tilde{a}_j &= (1-\delta)qh - \theta^{J-j}\pi \\ &= (1-\delta)qh - \theta^{J-j}(y-c-\delta ph) \\ &= \big((1-\delta)q + p\theta^{J-j}(\Omega+\delta)\big)h - \theta^{J-j}y.\end{aligned}$$

Thus, agents lifetime income at age j is

$$\mathcal{Y}_j = \big((1-\delta)q + p\theta^{J-j}(\Omega+\delta)\big)h_0^* \tag{3.12}$$

Combining equations (3.11) and (3.12) we get new optimal house at age j,

$$h_j^* = \frac{p}{q}\frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta+\Omega)}{(1-\delta)+\theta^{J-j}(\delta+\Omega)}h_0^*$$

The new optimal consumption level at age j is given by

$$c_j^* = q\Omega h_j^* = \frac{p}{q}\frac{(1-\delta)\frac{q}{p} + \theta^{J-j}(\delta+\Omega)}{(1-\delta)+\theta^{J-j}(\delta+\Omega)}\frac{q}{p}\underbrace{p\Omega h_0^*}_{c_0^*}$$

The optimal consumption response to an unexpected house price shock follow directly from the previous equation.

3.A.5 Proof of Proposition 8

The consumption response has the following structure,

$$\frac{c_j^*}{c_0^*} = \frac{a+f(x)}{b+f(x)}$$

where $a = (1-\delta)\frac{q}{p}$, $b = (1-\delta)$ and $f(J,j,\delta,\Omega) = \theta^{J-j}(\delta+\Omega)$. The derivative is given by

$$\begin{aligned}\frac{\partial}{\partial x}\frac{c_j^*}{c_0^*} &= \frac{f'(x)(b+f(x)) - f'(x)(a+f(x))}{\big(b+f(x)\big)^2} \\ &= \frac{f'(x)(b-a)}{\big(b+f(x)\big)^2} \\ &= f'(x)\frac{(1-\delta)(p-q)}{p\big(1-\delta+f(x)\big)^2} \\ &\propto f'(x)\end{aligned}$$

for a ngetive shock to prices. That is we can look at the partial derivative of f only.

$$\frac{\partial f}{\partial \Omega} = \theta^{J-j} > 0$$

Higher Ω means a bigger weight on consumpion, that is agents *hate houses more*. Thus,

$$0 < CR(\Omega_L) < CR(\Omega_H) < 1,$$

or

$$-100\% < \%CR(\Omega_L) < \%CR(\Omega_H) < 0\%.$$

That is, agents who love house more (lower Ψ), react stronger. For the following result consider the extension of θ to the real numbers,

$$\theta(t,r) = \frac{1 - \left(\frac{1}{1+r}\right)^t}{1 - \frac{1}{1+r}}$$

$$\frac{\partial f}{\partial j} = (\delta + \Omega)\underbrace{\theta'(J-j,r)}_{>0}(-1) < 0$$

That is, agents with higher age react stronger (Using the same reasoning as above).

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