Double Bertrand Tax Competition:

A Fiscal Game
with Governments Acting as Middlemen

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Abstract:
In a common market with costless mobility of all factors, regional governments can attract mobile firms by granting subsidies which must be financed out of wage taxes on mobile labour. Since firms locate where subsidies are highest and workers settle where taxes are lowest, government are forced "in the splits" (double Bertrand-type tax competition). We assume that without government intervention there is unemployment in the economy. Then regional governments behave like middlemen in the (distorted) labour market and the fiscal game takes the form of competition among strategic intermediaries. Results from the theory of intermediation are applied to this framework, enabling us to explain why government size may increase rather than decline under the the pressures of ongoing economic integration, how industrial clustering may emerge from tax competition, or how unemployment can be turned into job vacancies.

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Nontechnical Summary

Tax competition may be an offspring of the combat against unemployment: Governments typically want to help their economies prosper, and with unemployment galore many believe that actively attracting investment by providing subsidies or preferential tax treatment is a good means to this end. When they luckily lured an investor to their region, politicians can proudly point to the jobs they successfully "created" and which, in their interpretation, would have been lost to other regions had they not fought so bravely in a tax competition game or a bidding war. However, largesse to mobile investors requires a deep hand in somebody’s pockets. In the absence of intergovernmental funds sharing, all infrastructure expenditures, subsidies, tax breaks, etc. have to be financed out of the budget of the government that grants them – hence out of revenues from other tax bases.

The tax bases supposed to foot the bill for attractive packages to mobile investors can themselves be mobile or immobile – which makes up for crucial differences in the strategic incentives of governments in a fiscal game. E.g., assume that governments have to finance their expenditures by a tax on labour income. If labour is immobile, then it makes a rather inelastic tax base that governments can easily resort to. This is the case most often discussed in the tax competition literature. If, however, the labour tax base is itself mobile between regions and workers will choose their residence as to avoid high taxes, governments face a trickier task: To attract investors they have to provide subsidies at a sufficiently generous level. The costs have to be covered from taxes on mobile workers who will only want to settle in a region where taxation is sufficiently low. This forces governments in the splits: They are required to finance generous expenditures out of low taxes.

To get a job, workers must settle where firms are located, i.e. where subsidies are high. High subsidies necessitate, however, high taxes to be levied on workers. Workers thus are financing their own jobs – by the intermediation of governments. Governments act as middlemen in the labour market: They "buy jobs" when rewarding firms which locate in their region with a subsidy, and they "re-sell the jobs" when charging income taxes to workers which serve to cover the governments’ budgetary needs. Since only employed workers pay taxes and the number of employed workers in a region is limited by the number of jobs created there, rationing may possibly occur in the labour market. This prevents that doing the splits may end too painful for governments.
We assume that governments are Leviathans, aiming at maximizing their budget surplusses. Furthermore, tax competition is of a double Bertrand, winner-take-all type: Firms locate where subsidies are most generous, giving governments the incentive to slightly overbid the highest of the other governments’ subsidies. Workers choose their residence among those "industrialized" regions where taxation is lowest, giving governments the incentive to slightly undercut the other governments’ tax rates.

Formally similar problems occur in the field of (financial) intermediation, e.g. when banks compete both for loans (by demanding low interest rates from borrowers) and deposits (by offering high interest rates to lenders). Given these structural similarities, we can – with minor modifications – transfer approaches and results from this strand of economic research to the field of interregional competition.

We start from a situation with unemployment, generated by too high nominal wages. Subsidies to firms boast labour demand, and wage taxes reduce workers’ incentives to supply labour. A Nash equilibrium of a fiscal game in such a setting has the following properties:

- Unemployment will never occur. Oddly enough, there even may be job vacancies.

- All Leviathan governments run a balanced budget (zero budget surplusses).

- The budget volume is huge: wage tax revenues climb to the top region of their Laffer curve (if not to its peak). Unlike in standard fiscal games, government size does not shrink due to tax competition, but rather explodes.

- Tax competition may be regarded as a giant redistribution scheme from workers to firms.

- Although all regions are identical, there may be an extreme concentration of all firms (and workers) in one region.
1 Introduction

Tax competition may be the off-spring of governments’ fight against high unemployment. Governments typically want to help their economies prosper, and with unemployment galore many believe that actively attracting investment by providing subsidies or preferential tax treatment is a good means to this end. If in an integrated economy they luckily lured an investor to their jurisdiction, politicians can proudly point to the jobs they successfully "created" and which, in their interpretation, would have been lost to other regions had they not fought so bravely against their competitors from other jurisdictions. The strive to attract businesses not too seldom takes the form of tax competition games and veritable bidding wars. Fiscal games of this type are played among national governments as well as, on the subnational level, between regional or local ones.

In any case, offering fat bribes is not without cost, and largesse to mobile investors usually requires a deep hand in some taxpayers’ pockets. Ignoring the intricacies of tax incidence, any tax cut for companies and any tax-financed subsidization of firms effectively involves a redistribution from some other taxpayers to capital owners, for whom the subsidies or special tax treatments represent pure gains. In the absence of intergovernmental funds sharing, all infrastructure expenditures, subsidies, tax breaks, tax rebates etc. have to be financed out of the budget of the government that grants them – and hence out of its own revenues from other tax bases.

The tax bases supposed to foot the bill for attractive tax and subsidy packages for mobile investors can themselves be mobile or immobile – which makes up for a crucial difference in the strategic incentives for governments when entering into a fiscal game. Assume, e.g., that there is only one such tax base available, say labour income:

- If, as it is commonly assumed in tax competition models, workers are immobile between jurisdictions, their incomes offer a rather inelastic tax base which governments can resort to when competing for investment. As a result, one finds both a high tax burden on immobile tax bases and a high subsidization of (or a low tax burden on) mobile tax issues (see, among many others, Bucovetsky/Wilson 1991). The shift of the tax burden from internationally mobile capital to more immobile labour and consumption which can be observed in all industrialized countries over the last 15 years is sometimes regarded as anecdotal evidence of this result.

- If, however, workers are also mobile between jurisdictions and choose their residence such as to avoid high tax burdens, governments face a trickier task: To attract investors they

have to provide sufficiently generous subsidies or preferential tax treatment the costs of which have to be covered from taxes on mobile households. Taxpayers, however, will only want to live in a jurisdiction when (i) there are enough jobs (say, when subsidies are high enough to make firms settle there), and when (ii) tax rates are sufficiently low. Obviously, this forces governments "in the splits": They are required to finance generous expenditures out of low taxes.

In this paper we present such a model of tax competition among governments "in the splits": Governments lure mobile investors by high subsidies and mobile households by job opportunities and low income taxes. We use "subsidy" as a generic term for any measure that is to the benefit of investors and that is – relative to an unmodelled initial state – costly for the government budget; this notion includes preferential tax treatment. Similarly, "income tax" means any labour-supply related levy on households that governments can use to raise revenues.

Under closer scrutiny, the problem of governments "in the splits" is not as paradox as we have just put it. When generous subsidies to would-be investors are financed out of residence based taxes on labour income, then, in a sense, workers may be seen as financing their own jobs – by the unwarranted intermediation of governments. If both firms and workers are mobile, then governments act like middlemen: their tax policies determine the matching of labour demands and supplies. Governments "buy jobs" when rewarding firms which locate in their jurisdiction with a subsidy, and they "re-sell the jobs" when charging an income tax to workers. Of course, only employed workers are subject to the income tax and the number of employed workers in a jurisdiction is limited by the number of jobs created there. It is this potential rationing in the labour market which prevents that doing the splits may end too painful for governments. However, the strategic considerations involved in choosing a tax-subsidy scheme are not trivial. Assume that governments aim at maximizing their budget surplusses (tax revenues minus subsidy payouts) and that tax competition is of the extreme Bertrand, winner-take-all type: Firms locate where subsidies are most generous, giving governments the incentive to slightly overbid the highest of the other governments’ subsidies. Workers choose their residence among those "industrialized" regions which impose the lightest tax burden, giving governments the incentive to slightly undercut the lowest of the other governments’ tax rates.

Formally similar problems as the one just described occur in the field of intermediation, e.g., when banks compete both for loans (by demanding low interest rates from borrowers) and deposits (by offering high interest rates to lenders). Recently, this type of competition among middlemen has received some attention in the theory of (financial) intermediation (see, e.g.,
A major question of this research is whether a large number of intermediaries can replace the rather obscure figure of the Walrasian auctioneer common in competitive equilibrium theory. Although this is far from our theme, we can – with minor modifications – transfer approaches and results from this strand of economic research to the field of fiscal competition.

Tax competition is often viewed as a taming device for Leviathan governments (e.g. Sinn 1992 or, sceptically, Edwards/Keen 1996). The mobility of tax bases is hoped to reduce the governments’ capability of exploiting them. Downward pressures on budget sizes and tax rates are perceived to be especially strong when tax competition is of the winner-take-all, Bertrand type. This reflects the fear of a “race to the bottom”, often expressed in political debates on tax competition (see e.g. OECD 1998). In our framework, where tax competition is of a double Bertrand type, results are very much in an opposite direction: Equilibria are such that the maximum of the Laffer curve for the income tax is reached. I.e., instead of putting fiscal authorities on a dietary regime, tax competition here feeds governments to obesity. However, all tax revenues go to finance generous subsidies – leaving governments with zero budget surpluses in the end. Tax competition thus installs a giant, government-administered redistribution scheme from workers to capital owners.

What does this mean for the labour market? In this paper, we start from a situation with unemployment, generated by too high a fixed nominal wage rate. Subsidies to firms decrease their labour cost (thus boosting labour demand), and income taxes subsidize leisure demand (reducing household incentives to supply labour). Hence, the redistribution scheme initiated by tax competition narrows the gap between labour supply and demand. As we will see, unemployment will completely disappear in an equilibrium. In so far, the middlemen-governments do a good job (necessary caveats will be added soon).

The paper is organized as follows: Section 2 presents the formal framework for a double-Bertrand tax competition game. Section 3 analyses the Nash equilibrium of this game under the assumption that firms are slightly myopic. The unique Nash equilibrium may involve industrial clustering in one region, and its income tax rate is the maximizer of the income tax Laffer curve. Section 4 presents two variations of the model, assuming more farsighted firms. This triggers serious existence problems for equilibria. If they exist, equilibria still are located in the peak region of the Laffer curve for the income tax. Section 5 relates the results presented here to findings of the literature and concludes.

\[2\]Hellwig (1991) and Allen/Santomero (1997) provide surveys of the theory of (financial) intermediation.
2 The model

We consider a common market formed by a given and finite number of \( n > 1 \) (\( n \in \mathbb{N} \)) jurisdictions, called regions. Within the common market, there is free mobility of all goods and factors without any impediment. To focus purely on tax and subsidy competition we assume that all regions are (ex ante) identical in any respect. Private agents (i.e., households and firms) do not have any initial attachment to a certain region; their utility and profit functions are not location specific.

There is a continuum of profit maximizing single-output firms. All firms have the same neoclassical production function \( f(e, k) \), where \( e \) and \( k \) denote labour and capital input, respectively. Both inputs are essential: \( f(0, k) = f(e, 0) = f(0, 0) = 0 \) for all \( e, k > 0 \). \( f \) satisfies the INADA limit conditions. Furthermore we assume that

\[
    f_e > 0, \quad f_{ee} < 0, \quad f_k > 0, \quad f_{kk} < 0, \quad f_{ek} > 0 \quad \text{and} \quad f_{ee} f_{kk} - (f_{ek})^2 \geq 0.
\]

Capital and labour have positive, but decreasing marginal returns and are complementary inputs. \( f \) is concave.

Firms have exogenous capital endowments (assets) \( \kappa \). There are no other sources for outside finance and hence a firm’s capital input in production is equal to \( \kappa \). We will identify \( \kappa \) with the size of the investment project which a firm with this endowment plans to undertake. We assume that \( k \) is continuously distributed on an interval \( K := [\underline{k}, \overline{k}] \) with \( 0 < \underline{k} < \overline{k} < \infty \). By \( H(k) \) we denote the distribution function of \( k \). We assume that \( H \) is differentiable and that \( H'(\kappa) > 0 \) for all \( \kappa \in K \).

As capital input is already fixed, firms only have to decide on their location and their labour input. Locational choices depend on regional wage subsidies granted by regional governments. These wage subsidies can be interpreted as premia given to firms per created job. If firm \( \kappa \) obtains a job subsidy \( s \geq 0 \), its profits amount to

\[
    \Pi = f(e, \kappa) - (w - s) \cdot e,
\]

where \( w > 0 \) is the wage rate. \( w \) is exogenous and equal for all regions (see below).

Labour is endogenously supplied by households. There is a continuum of price-taking households with linear preferences on the consumption-leisure space:

\[
    u(c, 1 - \ell, \theta) = c + \theta \cdot (1 - \ell)
\]

where \( c \) and \( \ell \in [0, 1] \) denote consumption and labour supply, respectively. \( \theta \) is the constant marginal rate of substitution between leisure and consumption and will be identified as the
household's type. It is distributed on an interval $\Theta = [\underline{\theta}, \bar{\theta}]$ where $0 < \underline{\theta} < \bar{\theta}$. $G(\theta)$ is the differentiable and atomless distribution function of $\theta$. Households decide on their region of residence and on their labour supply. From a household's viewpoint, regions differ in the job opportunities they offer (see below) and the income taxes they levy. If a household earns the gross wage $w$ and is subject to a (regional) income tax $t \in [0, w)$, her net wage per hour of work is $c = w - t$. Abusing standard taxonomy, we will often call $t$ a tax rate. Clearly, the optimal labour choice is of the bang-bang type with

$$\ell(t, \theta) = \begin{cases} 1 & \text{iff } \theta \leq w - t \\ 0 & \text{else.} \end{cases}$$

We assume that households can only work in their region of residence and, consequently, any firm can only hire workers who actually live at its location (no commuting).

Each of the identical regions $i = 1, \ldots, n$ is ruled by a government which aims at maximizing its (expected) budget surplus. Each regional government $i$ uses as policy variables the regional income tax $t_i$ and the regional job subsidy $s_i$. Governments behave non-cooperatively. A strategy of government $i$ is a pair $\sigma_i = (s_i, t_i)$. We write vectors in bold type (such as $x = (x_1, \ldots, x_n)$). $\mathbf{0}$ and $\mathbf{1}$ denote the $n$-dimensional vectors of zeros and ones, respectively.

We argued at the outset that subsidizing firms is especially attractive for governments when there is unemployment. Moreover, we argued that the measure widely held appropriate by politicians for the success of their policies is the number of new jobs created. Against this background it seems odd not to give employment any explicit weight in the governments’ objective functions, but rather to resort to the Leviathan hypothesis of budget surplus maximization. Recall, however, that we may interpret governments as intermediaries in the labour market. Such as ordinary salesmen seek to sell all their stocks and thus to balance purchases and sales, so do middleman-governments have a genuine interest to bring all their residents to work. An unemployed does not pay any wage taxes and thus is of no value for Leviathan governments. One might regard this as a cynical attitude towards the unemployment problem, but it demonstrates that the Leviathan assumption is not as unrelated to the employment issue as one might prima facie suspect.

The timing of the game will be of crucial importance for the results. For the beginning, we adopt the following sequence of moves:

1. Regional governments move first by simultaneously deciding on their strategies $\sigma_i = (s_i, t_i)$. Their choices $\sigma$ are observed by the private agents.

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There is no uncertainty in our model. However, at this point we must take provisions for random equilibrium selection issues. Hence, the term "expected".
2. Next, firms decide on their locations and announce the numbers of jobs they plan to create. These announcements are binding commitments.

3. Households choose their regions of residence and make labour-consumption choices.

4. All decisions become effective: Taxes are collected, subsidies are paid out, and final profit and utility levels will be realized.

Governments are Stackelberg leaders with respect to the private agents, and play a Nash game among themselves. All private agents are small price-takers. The game structure and all moves are common knowledge. Since we consider randomized tax and subsidy schemes as unrealistic, we only discuss pure strategies.

3 A Fiscal Game with Myopic Firms

We do not require that supply of and demand for labour balance on the regional (or the systemwide) labour markets. Moreover, we explicitly allow for rationing, i.e., for involuntary unemployment or job vacancies. This requires some assumptions on how private agents account for possible disequilibria in the labour markets:

We invariably assume that households (who move last in all our games) take into account that they can only be employed in a region where job offers are in sufficient supply.

Firms, who are second to move after governments, can in principle anticipate the residential choices and labour supply decisions of households. They could thus find out whether all job opportunities they offer have a chance to be filled with workers. In this section, however, we analyse the game under the assumption that firms do not take into account that they may be rationed; they behave as if it were certain that they can always hire as many workers as they ideally want to. To partly justify this assumption (to be dropped in Section 4) note that without government intervention there will be unemployment in the economy (see Assumption $U$ below). Hence, firms may see the case of too small a labour force simply as irrelevant.

3.1 Firms’ Decisions

Firms and households make their choices in knowledge of the governments’ strategies $\sigma = (s, t)$. Since they do not care what households do, firms are only interested in the subsidy part $s$ of $\sigma$.

Firms decide on their location and on their labour demand.

It is obvious from the profit function (1) that firms select their location among those regions which offer the highest subsidy. Given a subsidy vector $s$, denote by $\hat{s} := \max_k \{s_k\}$ and by $M(s) := \{i \mid s_i = \hat{s}\}$ the highest subsidy and the set of regions which offer this, respectively. We
will call $M(s)$ the set of \textit{industrialized} regions because only regions in $M$ attract businesses. Suppose firm $\kappa$ decides to locate in region $i$, obtaining a job subsidy of $s_i$. Then its labour demand $e(w - s_i, \kappa)$ is uniquely determined by the profit maximizing condition

$$fe(e, \kappa) = w - s_i.$$ 

Calculate that

$$\frac{\partial e(w - s_i, \kappa)}{\partial s_i} = \frac{1}{fe} > 0 \quad \text{and} \quad \frac{\partial e(w - s_i, \kappa)}{\partial k} = -\frac{f_{ek}}{fe} > 0. \quad (3)$$

A firm’s labour demand is the higher the larger is the job subsidy and the larger is its capital endowment. Due to the simplicity of locational choices, total (systemwide) labour demand $L_d$ only depends on the maximal element $\hat{s}$ of a subsidy vector $s$:

$$L_d(\hat{s}) := \int_K e(w - \hat{s}, \kappa) \, dH(\kappa). \quad (4)$$

The distribution of firms across regions is irrelevant. Clearly, from (3)

$$\frac{dL_d}{d\hat{s}} > 0.$$

W.r.t. the regional distribution of firms we assume that if several regions offer the same maximum subsidy $\hat{s}$, they each face an equal chance of attracting firms. Hence, with a vector $s$ of subsidies the expected amount of jobs created in region $i$ is given by

$$L_d^i(s) = \begin{cases} \frac{1}{\sharp M(s)} \cdot L_d(\hat{s}) & \text{if } i \in M(s) \\ 0 & \text{else,} \end{cases} \quad (5)$$

where $\sharp$ denotes the cardinality of a set. From a regional government’s perspective, competition for business investments is of the Bertrand, winner-take-all type: Firms can only be attracted if the own subsidy is as least as high as any of the other regional subsidies.

### 3.2 Household Choices

When households choose their residence and labour supply, governments have already decided on $\sigma$ and firms have already found their locations and announced their job openings. Given $\sigma$, every household knows that he can only get a job in an industrialized region. Among these regions, households prefer low-tax ones. If all households (or a representative sample of them) faced the same tax rate $t \in [0, w]$, \textit{total} labour supply (possibly adjusted by the sample size) would by (2) amount to

$$L_s(t) = \int_{\Theta} \ell(t, \theta) \, dG(\theta) = G(w - t). \quad (6)$$

Check that the function $L_s(t)$ cuts both axes in a tax rate/labour supply diagram:

$$L_s(0) = G(w) > 0 = G(\emptyset) = L_s(w - \emptyset).$$
3.3 Regional Employment

We assume that if several regions in $M(s)$ offer the same lowest income tax rate, households arbitrarily choose one of these regions to apply for a job there. It may well happen that labour demand is not sufficient to cover labour supply at the smallest tax rate. Then the rejected households turn to the industrialized region with the second (third, fourth, ...) lowest tax rate where they apply with a labour supply correspondingly adjusted according to (6). We assume that households always move in representative samples, i.e., each government faces the same distribution $G(\theta)$ of households. We can now calculate the number $L^i_s(\sigma)$ of workers applying for a job in region $i$, given governments’ strategies $\sigma$:

- If $i \notin M(s)$, then labour supply is zero in this region, regardless of its tax rate.

- If region $i$ is industrialized and sets a tax rate $t_i$, the number of households applying for a job in $i$ is given by

$$L^i_s(\sigma) = \max \left\{ 0, \frac{1}{\lambda} \{ k \in M(s) \mid t_k = t_i \} \left[ G(w - t_i) - \lambda \{ k \in M(s) \mid t_k < t_i \} \frac{L^i_s(s)}{\lambda M(s)} \right] \right\}.$$  \hfill (7)

Equation (7) deserves some explanation: Any industrialized region offers $L^i_s(s)/\lambda M(s)$ jobs (see (5)). Assume for a moment that all industrialized regions chose different tax rates and that region $i$ is the unique lowest-tax region among them: $t_i < t_k$ for all $k \in M(s)$ and $k \neq i$. By (6), region $i$ faces a labour supply of $G(w - t_i)$ which it can fully serve when labour is in sufficient demand there, i.e., $G(w - t_i) \leq L^i_s(s)/\lambda M(s)$. If, however, not all households willing to work can get a job in $i$, i.e., if $G(w - t_i) > L^i_s(s)/\lambda M(s)$, then the rejected households will move to that industrialized region, say $j$, which has the next higher tax rate (i.e., where $t_i < t_j < t_k$ for all $k \neq i, j$). At a tax rate $t_j$ total labour supply would be $G(w - t_j)$. An amount of $L^j_s(s)/\lambda M(s)$ workers has already been employed in region $i$, such that region $j$ only faces an actual labour supply of $G(w - t_j) - L^j_s(s)/\lambda M(s)$ if this is greater than zero and of zero else. If there are still workers who are involuntarily unemployed, these households move to the region with the next lowest tax rate where labour supply is determined in an analogous manner.

For the case that several regions set the same tax rate we assume that there will be an equal split of labour supply amongst these regions. This is expressed by the fraction in front of the square brackets in equation (7).

In a natural way, employment $E^i(\sigma)$ in region $i$ is defined as the minimum of regional labour supply (7) and labour demand (5):

$$E^i(\sigma) = \min \{ L^i_s(s), L^i_s(\sigma) \}.$$  \hfill (8)

To have an interesting problem we make the following
Assumption $\mathcal{U}$: $L_d(0) < L_s(0)$.

Assumption $\mathcal{U}$ says that without government intervention (i.e., with $\sigma = (0,0)$), total labour supply exceeds total labour demand: There is systemwide unemployment. Since with identical strategies in all regions firms and households distribute equally across the economy, we have unemployment in every region when all governments are passive:

$$E^i(0,0) = L^i_d(0) = L_d(0)/n < L_s(0)/n = L^i_s(0,0)$$

for all $i$. Assumption $\mathcal{U}$ can equivalently be written as: $\int_0^w e(w, \kappa) dH(\kappa) < G(w)$, stating that the exogenous and constant (gross) wage rate $w$ is too high to clear the labour market. A story fabricated to motivate Assumption $\mathcal{U}$ may go as follows: The systemwide wage rate $w$ has been fixed outside the model by trade unions and employer lobbies in wage negotiations. These agents have agreed upon a wage rate above the market clearing level (which is not too unrealistic). As a consequence, there is unemployment in a "laissez-faire" regime. Regional governments observe this and try to promote employment conditions in their jurisdictions by attracting businesses. Since both $L_s$ and $L_d$ are continuous, $L_s(t)$ cuts both axes, and $L_d(s)$ is strictly increasing in $s$, we get the following obvious, but still helpful lemma:

**Lemma 1** Under Assumption $\mathcal{U}$, there exists a unique $\alpha \in [0, w - \delta]$ such that:

$$L_s(\alpha) = L_d(\alpha).$$  \hspace{1cm} (9)

The number $\alpha$ introduced in Lemma 1 has the following property: If firms are subsidized at rate $\alpha$ per job and simultaneously workers are taxed at the same rate $\alpha$, then total labour supply and total labour demand will balance. Hence, $w - \alpha$ is the competitive (Walrasian) price of labour.

### 3.4 Governments’ Payoffs

Governments are assumed to maximize their budget surplusses, i.e. the differences between tax revenues and subsidies. Given a strategy vector $\sigma$, the budget surplus $B^i$ of government $i$ amounts to

$$B^i(\sigma) := t_i \cdot E^i(\sigma) - s_i \cdot L^i_d(s),$$  \hspace{1cm} (10)

where $E^i$ and $L^i_d$ are given by (8) and (7). Note in (10) that governments pay subsidies for every workplace created in their jurisdictions (i.e., for $L^i_d$), not only for those actually employed ($E^i$).

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4Centralized wage bargaining is not uncommon even in federal economies (cf. e.g. Germany). Usually, however, wage negotiations are assumed to go about net wages which already incorporate tax effects. In our story, the tax system is determined after gross wages.
Since with identical policies of all regions firms and households distribute equally across regions, all budgets balance if all governments choose $t_i = s_i = \alpha$:

$$B^i(\alpha \cdot (1, 1)) = \alpha \cdot E^i(\alpha \cdot (1, 1)) - \alpha \cdot L^i_d(\alpha \cdot 1) = \alpha \cdot \frac{L^i_d(\alpha)}{n} - \alpha \cdot \frac{L^i_d(\alpha)}{n} = 0.$$

### 3.5 The Nash Equilibrium

Our double Bertrand tax competition game has been crafted such as to exhibit the same structure as a competition between profit maximizing intermediaries on goods or financial markets. Recently, YANELLE (1996) examined the features of double Bertrand competition between intermediaries. Her main result (Proposition 2.2) applies immediately to our scenario:

**Result 1** The tax competition game has a symmetric and (up to a permutation of regions) unique Nash equilibrium $\sigma^* = (s^* \cdot 1, t^* \cdot 1)$. In this equilibrium all governments choose a tax rate

$$t^* = \arg \max_{t \geq \alpha} t \cdot L^i_d(t).$$

Furthermore, all governments choose the subsidy $s^*$ such that

$$t^* \cdot L^i_d(t^*) = s^* \cdot L^i_d(s^*).$$

If $t^* \neq \alpha$, then in an equilibrium all firms locate in the same region.

**Proof:** We essentially follow YANELLE (1996, pp. 15f).

1. $\sigma^*$ is an equilibrium:

   Suppose, all governments $j \neq i$ have chosen $t_j = t^*$ and $s_j = s^*$. Government $i$’s considerations concerning a best reply are the following: With $s_i < s^*$ no firm would settle in region $i$. With $s_i > s^*$ government $i$ would attract all firms, but its subsidy payments would be greater than the maximum amount of income taxes it could ever raise:

   $$s_i \cdot L^i_d(s_i) > s^* \cdot L^i_d(s^*) = t^* \cdot L^i_d(t^*) = \max_{t \geq \alpha} t \cdot L^i_d(t)$$

   if $s_i > s^*$ (since $L^i_d(s)$ strictly increases in $s$). Hence, $B^i < 0$. Therefore, $s_i = s^*$ is the best choice. Given $s^*$, however, it is optimal to set $t_i = t^*$, for government $i$ would face $B^i < 0$ with any other tax rate.

2. $\sigma^*$ is the unique equilibrium:

   Suppose there were another equilibrium $\bar{\sigma} = (\bar{s} \cdot 1, \bar{t} \cdot 1) \neq \sigma^*$ (due to the symmetry of the model, only symmetric equilibria have to be considered).
(i) If \( \bar{s} > s^* \), then governments’ expenditures would exceed the maximum earnings possible: \( B_i < 0 \) for all \( i \). This cannot be an equilibrium.

(ii) If \( \bar{s} < s^* \) and \( \bar{t} = t^* \), then any government \( i \) could increase its budget surplus by choosing \( s_i = \bar{s} + \epsilon < s^* \). It would attract all firms, consequently all workers, and earn a strictly higher surplus than with \( \bar{\sigma} \):

\[
B_i \left( (\bar{s} + \epsilon, t^*), (\bar{s}, t^*) \cdot (1)^{n-1} \right) = t^* \cdot L_i(t^*) - (\bar{s} + \epsilon) \cdot L_d(\bar{s} + \epsilon)
\]

\[
> \frac{1}{n} \left[ t^* \cdot L_i(t^*) - \bar{s} \cdot L_d(\bar{s}) \right]
\]

for \( \epsilon > 0 \), but small enough.

(iii) If \( \bar{s} < s^* \) and \( \bar{t} < t^* \), then any government \( i \) could increase its budget surplus by increasing both \( t_i \) and \( s_i \). E.g., let \( \sigma_i = (\bar{s} + \epsilon, t^*) \). Then

\[
B_i \left( \sigma_i, (\bar{s}, t^*) \cdot (1)^{n-1} \right) = t^* \cdot L_i(t^*) - (\bar{s} + \epsilon) \cdot L_d(\bar{s} + \epsilon)
\]

\[
> \frac{1}{n} \left[ t^* \cdot L_i(t^*) - \bar{s} \cdot L_d(\bar{s}) \right]
\]

for \( \epsilon > 0 \) small enough.

(iv) If \( \bar{s} < s^* \) and \( \bar{t} > t^* \), then any government \( i \) could increase its budget surplus by increasing \( s_i \) and lowering \( t_i \). E.g., let \( \sigma_i = (\bar{s} + \epsilon_1, \bar{t} - \epsilon_2) \). Then

\[
B_i \left( \sigma_i, (\bar{s}, \bar{t}) \cdot (1)^{n-1} \right) = (\bar{t} - \epsilon_2) \cdot L_i(\bar{t} - \epsilon_2) - (\bar{s} + \epsilon_1) \cdot L_d(\bar{s} + \epsilon_1)
\]

\[
> \frac{1}{n} \left[ \bar{t} \cdot L_i(\bar{t}) - \bar{s} \cdot L_d(\bar{s}) \right]
\]

for \( \epsilon_1, \epsilon_2 > 0 \) small enough.

3. If \( t^* \neq \alpha \), then in the equilibrium all firms locate in the same region:

Suppose that governments have chosen \( \sigma^* \) and that there are firms in more than one region. Recall that \( B^i(\sigma^*) = 0 \) for all \( i \). Now, each of the industrialized regions has an incentive to choose \( \sigma_i = (s^*, t^* - \epsilon) \). With this deviation we get \( B_i > 0 \) because the full job capacity created in region \( i \) can be employed (recall that we have full employment or overcapacities with \( \sigma^* \)).

\[Q.E.D.\]

Result 1 has several immediate implications:

**Corollary:** In the Nash equilibrium of the tax competition game:

1. all regional budgets balance;

2. \( t^* = \alpha \) if and only if \( \alpha \) is the revenue maximizing rate of the income tax;
3. if \( t^* \neq \alpha \), total and regional labour demands exceeds total and, resp., labour supplies: 
\[ L_d(s^*) > L_s(t^*) \text{ and } L_d^i(s^* \cdot 1) > L_s^i(\sigma^*) \];

4. if \( t^* = \alpha \), the systemwide labour market and all regional labour markets clear: 
\[ L_d(s^*) = L_s(t^*) \text{ and } L_d^i(s^* \cdot 1) = L_s^i(\sigma^*) \text{ for all } i = 1, \ldots, n; \]

5. neither \( t^* \) nor \( s^* \) are ever smaller than \( \alpha \).

For a better understanding, it may be helpful to write Result 1 in a different form. Let
\[ \hat{t} := \arg \max_{t \geq 0} (t \cdot L_s(t)) \]
be the unconstraint maximizer of the Laffer curve for the income tax. Note that \( \hat{t} \) may differ from \( t^* \) as defined in (11). There are two possible equilibrium configurations:

- If \( \hat{t} \leq \alpha \), then \( t_i^* = s_i^* = \alpha \) for all \( i \).
- If \( \hat{t} > \alpha \), then \( t_i^* = \hat{t} \) and \( s_i^* = \hat{s} \) for all \( i \), where \( \hat{s} \) solves \( \hat{t} \cdot L_s(\hat{t}) = s \cdot L_d(s) \).

These two configurations are depicted in Figure 1 and 2, respectively, where we chose labour demand and supply functions to be linear for simplicity.\(^5\)

---

**Figures 1 and 2 go here**

All regional governments choose their income tax rate \( t \) such as to maximize attainable tax revenues from labour. This maximum need not coincide with the unconstraint maximum of the income tax Laffer curve, which is reached at the tax rate \( \hat{t} \). Using the definition of \( \alpha \), we can rewrite (11) to obtain
\[ t^* = \arg \max_x \{ x \cdot \min\{L_s(x), L_d(x)\} \}, \tag{13} \]
which exhibits that governments tax employable (as contrasted to supplied) labour to the highest degree possible.

The original version of Result 1 in YANELLE (1996) is used to demonstrate that the figure of the Walrasian auctioneer who operates the market processing in competitive equilibrium theory cannot always be replaced by intermediaries who engage in a Bertrand-game (also see YANELLE 1997). This interpretation carries over to the application of YANELLE’s result here (see below). Moreover, Result 1 offers a bunch of new interpretations in the fiscal competition context:

\(^5\)Strictly speaking, the curves \( t \cdot L_s \) and \( s \cdot L_d \) in the figures are labelled incorrectly. The graphs actually depict the inverse mappings of these functions. Being precise in that respect would, however, require more notation without noticeably adding to clarity.
Result 1 says that in the Nash equilibrium of a fiscal game between middleman-governments the feasible budget size is maximized: Leviathans grow fat to their maximum height. It seems to be a stylized fact that the size of real-world governments (measured, e.g., by government spending as a percentage of GDP) is ever growing. Once celebrated in Wagner’s Law as evidence for social progress, this upward trend has since long become a matter of serious concern in most industrialized countries. Globalization, deeper economic integration, and tax competition are commonly thought to be powerful devices to tame fiscal hunger. However, this intuition has so far hardly any support from reality: Although in the 1980s and 1990s the breeze of globalization has become a strong wind, government sizes are still going up. Our model may offer an explanation for this conundrum. With free factor flows, governments may be pushed into the role of intermediary salesmen. Result 1 then says that fiscal competition makes governments grow as fat as possible.

According to (12), equilibrium job subsidies are adapted such as to balance governments’ budgets. While the budget size is maximal, all revenues are spent, leaving governments with nothing to use for their own purposes. The tax competition equilibrium thus involves the most massive transfers from workers to firms. Workers are “exploited” to the greatest possible extent, and, as all of this money goes to firms, subsidies reach their highest affordable level, too. ”Exploitation” should not be taken literally, since, by assumption $U$, net incomes $w$ of the initial situation are too high relative to the competitive wage level. Tax competition corrects for this deterioration from the equilibrium. We have already pointed out that a major cause for high unemployment in the real world is too high labour cost. In a sense, tax competition brings about a government-financed cut in labour cost which in an equilibrium amount to $w - s^\ast$. In order to enable governments to finance the cuts, income taxes have to be increased from zero to $t^\ast$. This is reminding of a tax incidence hypothesis for the labour market model, suggested e.g. by Tyrväinen (1995), that the non-wage cost of labour (the tax wedge) is half borne by wage-earners (reducing labour supply) and half by firms (lowering labour demand).

Recall that $\alpha$ can be interpreted as the competitive level of taxes or subsidies (since the labour market clears at a net wage of $w - \alpha$). According to item 5 in the Corollary, tax rates and subsidies are never lower than $\alpha$. Often they are higher (namely, if $\alpha$ hence lies in the increasing part of the Laffer curve for the income tax). I.e., compared to the competitive level, workers are taxed too heavily whereas firms are subsidized too generously. This sounds identical to an important observation of the ”standard” tax competition literature (e.g., Bucovetsky/Wilson 1990). However, the analogy is misleading. In the standard

\[ \text{Recall that } \alpha \text{ can be interpreted as the competitive level of taxes or subsidies (since the labour market clears at a net wage of } w - \alpha). \]
models capital is mobile and labour immobile. Furthermore, the benchmarks are different: Here it is a labour market equilibrium, there it is the benefit principle of taxation.

- The industrialized region(s) may face overcapacities in the equilibrium: If $\alpha$ is in the increasing part of the income tax Laffer curve, then there will be job vacancies in the equilibrium ($L_d > L_s$). Otherwise, full employment will emerge. Hence, middlemen-governments often "buy" too many jobs. Starting from a situation with unemployment (cf. Assumption 1), fiscal competition among Leviathans may lead to an oversupply of job opportunities.

This result is certainly odd seen against reality which has unemployment galore. In our model, it is due to the myopic behaviour of firms which, spurred by vast subsidies, "create" jobs without checking whether these can be employed. One might also blame governments for being responsible for the oddity of overcapacities. They subsidize workplaces, not job matches. Put differently, employed workers do not only (partially) finance their own jobs, but also some vacancies. We will address to this issue in Section 4.

- In the Nash equilibrium, there may be an extreme industrial concentration: Firms and consequently workers all settle in one region. Surprisingly, this result emerges without imposing any spatial structure on the model. Especially, there are no agglomeration advantages or disadvantages which may explain the result. The locational structure is pure chance: the region where the industry clusters is chosen randomly (with equal probability for all regions). In its spirit (but not in its origins), the clustering result here is reminiscent of a similar result in Arthur (1990) who explains locational patterns in economies as the eventual outcomes emerging from some historical processes. In his Theorem 1, Arthur (1990) shows that an extreme spatial concentration may occur in an economy under free mobility of firms with identical behavioural patterns, but without any advantages or disadvantages of agglomeration. Clustering simply reflects homogeneity of the firms' needs for workers. The similarity in the two models lies in the observation that (dis-)economies of agglomeration or a specific spatial structure in an economy are not essential in order to explain dense industrial concentrations.

4 Two Alternative Scenarios

4.1 Firms Care about Rationing

Apart from the clustering of firms in a single region, an oddity of the Nash equilibrium in Result 1 is that firms may be inducted to offer more jobs than can actually be employed. The basic reason for this peculiarity is the myopic behaviour of firms which can most easily be seen from the profit function (1). There no distinction between announced and actual employment is
made. Firms thus behave as if they were to pay wages for and produce with workers they in fact do not (and cannot) hire. In this section we drop this assumption and turn to a more rational firm behaviour. We modify stages 2 and 4 of the game as follows:

1. ...

2'. Firms make a locational choice for one region \( i \) and an announcement to create a certain number of jobs there.

3. ...

4'. All decisions become effective: Actual employment decisions are made, taxes are collected, subsidies are paid out, and final profit and utility levels will be realized.

Firms can only announce jobs in the region where they settle. Announcement are binding commitments in so far as firms must not install fewer workplaces than announced except for the case that labour supply in the region is too small to find enough staff. As before, subsidies are paid per announced job, whether vacant or not (Governments still are naïve).

We now must distinguish between the announced and the actual value of a variable. Announced variables will wear a tilde. Let \( \bar{e}(i, \sigma, \kappa) \) be the number of jobs a type-\( \kappa \) firm announces to create in region \( i \) when governments choose \( \sigma \) as their tax-and-subsidy policies. Clearly, if a firm does not locate in \( i \), its labour demand in that region is zero. Unlike labour demand \( e \) in the previous section, the variable \( \bar{e} \) does not only depend on the highest subsidy, but on the whole strategy vector \( \sigma \). Define

\[
\bar{L}_d^i(\sigma) := \int_K \bar{e}(i, \sigma, \kappa) \, dH(\kappa)
\]

as total announced labour demand in region \( i \) and denote by

\[
\bar{M}(\sigma) := \left\{ i \left| \bar{L}_d^i(\sigma) > 0 \right. \right\}
\]

the set of potentially industrialized regions, i.e., the set of regions where firms plan to install workbenches.

Next check that households are not affected by the change in the rules of the game. They can still only apply for a job in a potentially industrialized region. Rationing of the firms is irrelevant for them. Households can treat announced jobs as if they were actually created. Hence, neither their migration nor their labour supply decisions differ from those in Section 3. Eqs. (6) and (7) remain valid if we replace \( L_d(s) \) by \( \bar{L}_d^i(\sigma) \) and \( M(s) \) by \( \bar{M}(\sigma) \) in (7):

\[
L_d^i(\sigma) = \max \left\{ 0, \frac{1}{\| \{ k \in \bar{M}(\sigma) \mid t_k = t_i \} \|} \cdot \left[ G(w - t_i) - \sum_{k \in \bar{M}(\sigma) \text{ with } t_k < t_i} \bar{L}_d^k(\sigma) \right] \right\}. \quad (14)
\]
As by assumption firms never hire more workers than they announce, actual employment $E^i$ coincides with effective labour demand $L^d_i$, which never exceeds the smaller of announced job capacities and labour supply:

$$E^i(\sigma) \equiv L^d_i(\sigma) = \min\{\tilde{L}^d_i(\sigma), L^i(\sigma)\}. \tag{15}$$

Regional governments collect taxes per unit of labour employed and spend subsidies per job announcement. The budget surplus of government $i$ is given by:

$$B^i(\sigma) = t_i \cdot E^i(\sigma) - s_i \cdot \tilde{L}^d_i(\sigma). \tag{16}$$

We assume that, if rationing occurs in region $i$, then it hits all firms in that region in equal proportions. Hence, if it announces to create a number of $\bar{e}(\cdot)$ jobs in $i$, then a firm will actually employ a number of

$$e(\cdot) = \min \left\{ 1, \frac{L^i(\sigma)}{L^d_i(\sigma)} \right\} \cdot \bar{e}(\cdot) \tag{17}$$

workers. Hence, firm $\kappa$’s profits when it locates in region $i$ amount to:

$$\Pi = f(e, \kappa) - w \cdot e + s_i \cdot \bar{e}$$

$$= f(e, \kappa) - \left[ w - s_i \cdot \max \left\{ 1, \frac{L^i(\sigma)}{L^d_i(\sigma)} \right\} \right] \cdot e,$$

where we used (17). Unlike in the previous scenario, firms do not necessarily locate in the region with the highest subsidy. Rather than being rationed a firm may prefer a less generous region without rationing. Firms also have to take into account the tax rates $t$, not only the subsidy part of $\sigma$. The relevant parameter is the effective subsidy rate

$$\tilde{s}_i := s_i \cdot \max \left\{ 1, \frac{\bar{L}^i(\sigma)}{L^d_i(\sigma)} \right\}$$

which is higher than the statutory one if rationing occurs in region $i$. (Recall that subsidies are paid per job opening, not per hiring contract.) The set of potentially industrialized regions is then:

$$\tilde{M}(\sigma) = \{ i \mid \tilde{s}_i \geq s_k \text{ for all } k \}.$$

Next we need an assumption about how firms infer the effective subsidies $\tilde{s}_i$ from the observed governmental strategies $\sigma$. In doing so they must anticipate households’ decisions and the other firms’ behaviour. To keep matters simple, we assume that all firms ignore their influence on all $\tilde{s}_i$ (which in fact is marginal) and that their forecasting methods are identical. Given $\sigma$, all firms thus unanimously foresee the same vector of $\tilde{s}_i$. Somehow vaguely, these calculations shall
be based on the assumption that all agents behave rationally in the pursuit of their objectives. Especially – and this is indeed the only conjectural assumption we need –, if the strategy vector chosen by the governments does not necessitate any rationing, then firms should not expect rationing to happen. In that case, they will behave as outlined in Section 3.

We are again interested in the Nash equilibria of the subsidy-and-tax competition game among regional governments. The problem now is a bit uglier. As a first step, we show

**Lemma 2** In a Nash equilibrium \( \sigma^* \), labour supply in all potentially industrialized regions does not exceed announced job capacities, i.e.:

\[
i \in \bar{M}(\sigma^*) \implies L_i(\sigma^*) \leq \bar{L}_i(\sigma^*). (18)
\]

The proof of Lemma 2 is relegated to the Appendix. Lemma 2 has a series of important consequences:

1. **In a Nash equilibrium \( \sigma^* \), all industrialized regions levy the same tax rate:**

\[
t_i^* = t^* \text{ for all } i \in \bar{M}(\sigma^*).
\]

**Proof:** From Lemma 2, there is never oversupply of labour in any industrialized region and thus economywide. If a region levies a tax rate higher than the minimum tax rate of all other regions, than it will not attract any labour and thus will run a budget deficit. Hence, from (14), we get that in an equilibrium

\[
L_i^1(\sigma^*) = \frac{G(w - t^*)}{\bar{g}(\sigma^*)} \text{ for all } i \in \bar{M}(\sigma^*).
\]

2. **In a Nash equilibrium \( \sigma^* \), total labour demand equals total announced labour demand:**

\[
G(w - t^*) = \sum_i \bar{L}_i(\sigma^*).
\]

**Proof:** From Lemma 2, total labour demand cannot exceed announced labour supply in an equilibrium. Now suppose that it were strictly smaller. Then at least one region pays subsidies for job announcements which remain vacant. By slightly lowering its tax rate, this region could attract and employ more workers, and thus raise higher revenues at constant expenditures.\(^7\)

3. **Consequently, in a Nash equilibrium, announced labour demand and labour supply coincide in every region:**

\[
\bar{L}_i(\sigma^*) = L_i(\sigma^*) \text{ for all } i.
\]

This in turn implies:

\(^7\)Note that this argument also applies when all firms cluster in a single region (which, from Result 1, cannot be excluded).
• As there is no rationing, in a Nash equilibrium announced and effective labour demand will coincide for every firm:

\[ \tilde{L}_d^i(\sigma^*) = L_d^i(\sigma^*) \Rightarrow \bar{e}(i, \sigma^*, \kappa) = \bar{\tau}(i, \sigma^*, \kappa) \text{ for all firms } \kappa. \]

• In a Nash equilibrium, \( \tilde{s}_i^* = s_i^* \). Consequently, all potentially industrialized regions must offer the same subsidy:

\[ s_i^* = s_j^* = s^* \text{ for all } i, j \in \tilde{M}(\sigma^*). \]

• The Nash equilibrium budget surplus of every region \( i \in \tilde{M} \) amounts to

\[ B^i = (t^* - s^*) \cdot G(w - t^*). \]

4. As a consequence, in a Nash equilibrium there is no rationing in any region:

\[ \tilde{L}_d^i(\sigma^*) = L_d^i(\sigma^*) = L_d^i(\sigma^*) \text{ for all } i. \]

Recall the assumption that, if the strategy vector \( \sigma \) does not necessitate any rationing, then firms will behave such as in Section 3. We so far have shown that in a Nash equilibrium rationing cannot occur, but that the labour market will clear. Hence, the only candidate vector for a Nash equilibrium is the "Walrasian" vector \( \sigma^* = \alpha \cdot (1, 1) \). From Result 1 we know, however, that \( \alpha \cdot (1, 1) \) can only be a Nash equilibrium if \( \alpha \) is at the same time a revenue maximizing tax rate. This observation carries over to the present scenario:

**Result 2** Let \( L_d(s) \) and \( L_a(t) \) be as originally defined in (4) and (6) and be \( \alpha \) such that \( L_a(\alpha) = L_d(\alpha) \). If

\[ \alpha = \arg\max \{x \cdot \min\{L_d(x), L_d(x)\}\}, \tag{20} \]

then the tax competition game possesses a unique and symmetric Nash equilibrium which is given by

\[ \sigma^* = \alpha \cdot (1, 1). \]

Otherwise, no Nash equilibrium exists.

**Proof:** Under condition (20) the Nash equilibrium derived in Result 1 does not imply rationing, but has clearing labour markets. Hence, it is also an equilibrium of the modified game. If condition (20) does not hold, the Nash equilibrium of Result 1 involves rationing. Now suppose that \( \alpha \cdot (1, 1) \) is a Nash equilibrium. Then there exists \( \hat{t} \neq \alpha \) such that

\[ \hat{t} \cdot L_a(\hat{t}) > \alpha \cdot L_a(\alpha) = \alpha \cdot L_d(\alpha). \]
Let government $j$ choose $s_j = \alpha + \epsilon < \hat{t} = t_j$. Since with this strategy firms do not expect to be rationed in region $j$, the budget surplus of government $j$ amounts to:

$$B^j = \hat{t} \cdot G(w - \hat{t}) - (\alpha + \epsilon) \cdot L_d(\alpha + \epsilon) > 0,$$

which is a profitable deviation from the zero-surplus situation $(\alpha, \alpha)$. \textit{Q.E.D.}

Condition (20) requires the labour market clearing tax rate to be a revenue maximizing one. If condition (20) holds, then a unique Nash equilibrium exists which is symmetric, entails clearing regional and systemwide labour markets and puts governments on top of the Laffer curve for the "employment tax". This equilibrium does not differ from the one depicted in Figure 1 above (compare (20) and (13), which describes the Nash equilibrium tax rate in the setting of Section 3). If condition (20) does not hold, then no Nash equilibrium exists.

\textsc{Yanelle (1996)} analyses Bertrand competition among intermediaries who, in addition to the usual buying and selling prices can also choose capacities they maximally will buy from sellers. A Nash equilibrium then only exists if the competitive price in the market is also a revenue maximizing one. In this case, buying and selling at the \textit{Walrasian} price and creating capacities equal to the trade volume at this price is the equilibrium strategy (Propositions 3.1 and 3.2 in \textsc{Yanelle 1996}). Result 2 above is in the same spirit: By fixing tax rates and subsidy levels, regional governments induce firms to make job announcements that can be interpreted as capacities. In a Nash equilibrium the piling up of over-capacities cannot occur, i.e., no region would let firms announce a greater number of subsidized jobs than can later be occupied (cf. (19)). However, such an equilibrium may fail to exist.

For interpretations and implications of Result 2 we refer to our discussion of Result 1. Note, however, that a Nash equilibrium in the actual scenario cannot lead to industrial clustering in one region. For the cases where Result 1 predicts such an extreme pattern, Result 2 states the non-existence of an equilibrium. Hence, if a Nash equilibrium exists in the actual scenario, firms and workers distribute equally across the regional system.

4.2 Governments Move Twice

We briefly consider a second modification of the original game. In the previous section we made a distinction between announced and actual numbers of workplaces, thus rendering firms' reasoning "more rational". Here, we adopt a different sequence of moves: First, regional governments non-cooperatively fix their subsidies $s$. Second, firms make their locational choices and thus reveal how much jobs they want to open. Third, governments non-cooperatively set income tax rates $t$. Finally, households decide on their residences and employers. When it comes to
choosing tax rates, regional governments already know their budgetary needs: In the first stage they have committed to a "buying price" for jobs created in their jurisdiction, and in the second one firms have imposed the corresponding quantities. Obviously, we have a two-stage game among governments. We therefore look for subgame-perfect Nash equilibria.

Formally, the setting just described is identical to that in Stahl (1988). Two main results of Stahl's approach on winner-take-all competition among merchants can thus readily be transferred to the interregional framework presented here:

**Result 3 (Stahl (1988))** Let $L_d(s)$ and $L_s(t)$ be as originally defined in (4) and (6) and be $\alpha$ such that $L_s(\alpha) = L_d(\alpha)$. If

$$\alpha = \arg \max_x \{ x \cdot \min \{ L_s(x), L_d(x) \} \},$$

then the tax competition game possesses a unique and symmetric subgame-perfect Nash equilibrium which is given by

$$\sigma^* = \alpha \cdot (1,1).$$

Otherwise, no subgame-perfect Nash equilibrium exists.

The proof is a straightforward adaptation of the proofs for Propositions 1 and 2 in Stahl (1988).

The reader may feel to encounter a déjà-vue when coming across Result 3: Apart from the tiny "subgame-perfect" added, Result 3 coincides with Result 2. Consequently, in case it exists an equilibrium is such as in Figure 1, part a) and has the properties and implications already discussed in Section 3.

The alternative scenarios discussed in this section reveal that severe existence problems for Nash equilibria can be expected in Bertrand tax competition. This is in line with results obtained by Schulze/Koch (1994) and Koch/Schulze (1998) for one-sided Bertrand tax competition. There, existence problems are due to asymmetries or to the availability of "outside options" for taxation. In the double-Bertrand setting discussed here, neither of these assumptions is needed to trigger non-existence.

If an equilibrium exists in either of the two scenarios discussed in this section, then it involves full employment. This is reminiscent of a result obtained by Gabszewicz/Van Ypersele (1996) for a fiscal game where the strategic variable of governments is the minimum wage. In this model, if there is too high a wage rate in a region, mobile capital will flee the region and thereby trigger

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*Note that these results (especially the non-existence part) crucially hinge upon the tie-breaking rule which – in our context – says that with equal tax rates all industrialized regions share labour supply equally.*
unemployment. Therefore the minimum wage rate will (often) be set on the (low) competitive level, although the policy makers’ preferences are strongly biased towards workers’ interests. In the model presented here, where regional governments in fact also determine the wage rate, the competitive level will emerge, too. Although the mechanisms here and in Gabszewicz/Van Ypersele (1996) are quite diverse, they share a common message: Fiscal competition drives labour markets towards the equilibrium, which, for employed workers, comes at the price of a lower wage rate.

5 Discussion and Conclusion

We set up a model of a tax competition game where governments choose their tax-and-subsidy strategies as to corner both sides of a distorted labour market. It seems worthwhile to compare the findings reported here to those emerging from the more standard tax competition literature which usually assumes that there exists at least one immobile item in the economy. As Koch/Schulze (1998) point out, the core model of tax competition uses (among other ingredients) capital as a generically mobile factor, whereas all other factors cannot cross regional borders. Even in richer models with several mobile items (such as Burbidge/Myers 1994) there is one immobile tax issue (plausibly enough, land is often chosen for this role). Only few authors examine the case of both capital and labour mobility, albeit with imperfect household mobility (see, e.g., Mansoorian/Myers 1993, 1997, or Eggert 1995). In all these models, regional governments optimize an objective function related to the utility of the owners of the immobile factors. The main message says that the Nash equilibria of such games are inefficient: Free mobility of capital imposes an arbitrage condition upon the economy and thus gives off-spring to interregional externalities that are ignored by self-concerned regional governments. The approach taken here differs from the standard model in several respects:

- Governments in our model aim at maximizing their budget surpluses rather than regional welfare. This Leviathan assumption is not uncommon in the literature, but most of these models (e.g., Kanbur/Keen 1993, Schulze/Koch 1994, Koch/Schulze 1998 or Janeba/Peters 1999) incorporate only one (namely the revenue) side of the government budget and thus implicitly assume that expenditures are fixed. Moreover, governments in these models can resort to an (elastic or inelastic) ”inside option” for taxation and thus do not entirely depend on mobile tax bases to cover their (unmodeled) fiscal needs. Sinn (1992) discusses the taming effects of tax competition on Leviathans in a constitutional framework (however, the model is not fully specified). In Edwards/Keen (1996) government objectives encompass pure Leviathan behaviour as a special case. Revenues from taxes on mobile capital can be used to finance either local public goods, which benefit local citizens, or socially wasteful activities, which benefit only the policy maker. Ed-
WARDS/KEEN (1996) find that a policy coordination among regional governments may be harmful to the utility of a representative citizen which happens if and only if the elasticity of the tax base falls below Leviathan's marginal propensity to engage in wasteful activities. This indicates that for tax competition coming close to the Bertrand-type policy coordination is always beneficial, as (in an equilibrium) the tax base reacts infinitely elastic with respect to marginal tax changes.

- Unlike in great parts of the fiscal federalism literature, there are no immobile items in our model (except for the ever-lasting governments). Let us briefly consider what would happen if either households or firms were immobile:

1. If each region is inhabited by a sufficiently large number of households, luring an investor into the region will create a marginal budget surplus of $t - s_i$ per job. For simplicity, let $t$ be a uniform tax rate in all regions. Firms still locate where regional subsidies $s_i$ are highest. Consequently, a race between governments towards higher subsidies will start. In the end subsidies are as high as to eat up the whole marginal surplus: $s_i^* = t$ for all regions.

2. Now assume that firms are irreversibly established and have installed workbenches in sufficient supply (assume an equal distribution across regions). If mobile households can be attracted to work at these workbenches by low tax rates, then governments engage in a tax cutting race and the zero tax rate is the unique Nash equilibrium of such a game. If no budget deficit is allowed, this requires zero job subsidies as well.

In any case governments end up with a zero budget surplus – such as in the present paper. All these balanced budgets are, however, reached by quite different tax-subsidy mixes. The interesting element in our observations is that governments choose tax rates as to climb on top of the Laffer curve, and spend all funds to footloose firms. A simple intuition runs as follows: Attracting sufficiently many firms by high subsidies is prerequisite for governments to earn sufficiently high revenues from the income tax. As a consequence, the demand side of the labour market can fully exploit the supply side. Of course, this is an implication of the order of moves in our game: firms move before households. An interesting suggestion for further research would be to reverse the order of moves. Intuition tells us that if households settle before firms choose their locations, this will lead to either a Walrasian outcome or to a zero tax-and-subsidy equilibrium – thus perpetuating unemployment. One might certainly question the assumption of perfect mobility of both firms and workers on the grounds of reality. Somehow surprisingly, empirical evidence for tax-induced migration responses seems to be more clearcut for labour than for capital. Meanwhile, however, improved statistical analysis finds that both factors do in fact migrate to tax-favoured
locations (see Inman/Rubinfeld 1996 or Hines 1996 for details and further references). Hence, the mobility assumption adopted here is certainly extreme, but not totally far-fetched.

- All regions in our model are identical. Hence, it does not come as a surprise when all equilibria are symmetric. Similarly, if regions differed in size, preferences, or resource endowments, equilibria of a tax competition games would reflect these differences (see e.g. Hausler/Wooton 1999 or Huizinga/Nielsen 1997). However, recent empirical findings seem to support a convergence hypothesis for tax rates in the EU – albeit on a high overall level of taxation (see e.g. Chennells/Griffith 1997). Tanzi/Zee (1998) report that in 1997 the tax rates on individual incomes (which correspond to the \( t_i \) of our model) in the EU averaged at rather high 47.5\%, but exhibited a rather small coefficient of variation of 18.7\%, which is only a third of that among the 50 states of the U.S. They interpret their observation as some sort of ”decentralized harmonization”, working through economic integration and factor mobility in the EU. In principle, the present model, which predicts high and equal income tax rates in a tax competition game may theoretically underpin such an interpretation. However, since data on the non-wage cost of labour – which would correspond to (the negative of) the \( s_i \) in our model – are not available, and furthermore our model lacks a lot of reality’s features, we refrain from drawing too many parallels to European reality.

Finally recall the main idea of this paper: Governments in an integrated economic framework operate as middlemen. They use their contestable tax monopoly for correcting for possible distortions which hinder markets from fully revealing their allocative prowess. Who thinks this an overly optimistic view, will see selfish, merchant governments as ”selling the state away” in bidding wars and tax competition. Anyway, huge, Leviathan-like governments are not necessarily doomed to extinction in the ongoing process of globalization and integration; governments of that type may even grow to obesity.
Appendix: Proof of Lemma 2

Suppose the contrary of the assertion. I.e., there exists $j$ with strategy $(s_j^*, t_j^*)$ such that $\tilde{L}_d^j(\sigma^*) > 0$ and $\tilde{L}_d^j(\sigma^*) > \tilde{L}_d^j(\sigma^*) = E^j(\sigma^*)$.

First verify that then in all regions $k \in \tilde{M}(\sigma^*)$ with $t_k^* \leq t_j^*$ we must have

$$L_k^*(\sigma^*) > \tilde{L}_d^k(\sigma^*),$$

(22)
too. Now consider a marginal increase of region $j$'s tax rate from $t_j^*$ to $t_j^* + \epsilon$ and analyse what happens to $L_d^j$ as given by (14). Two cases have to be distinguished:

- There is no $k \neq j$ such that $t_k^* = t_j^*$. Then:

$$L_d^j(\sigma^*) = G(w - t_j^*) - \sum_{h \in \tilde{M}(\sigma^*) \text{ with } t_h^* < t_j^*} \tilde{L}_d^h(\sigma^*).$$

By a marginal increase in $t_j^*$ the sum term in this expression does not change (especially check that no firm will change its behaviour). Only will the value of $G$ change, but, due to continuity, marginally. I.e., $\epsilon > 0$ can be chosen such that $L_d^j$ still exceeds $\tilde{L}_d^j$.

- There exists $k \in \tilde{M}(\sigma^*)$ other than $j$ such that $t_k^* = t_j^*$. Let $q$ be the number of such regions. Then:

$$L_d^j(\sigma^*) = L_k^*(\sigma^*) = \frac{1}{q} \cdot \left\{ G(w - t_j^*) - A \right\}.$$

where we defined

$$A := \sum_{h \in \tilde{M}(\sigma^*) \text{ with } t_h^* < t_j^*} \tilde{L}_d^h(\sigma^*).$$

By an increase in $t_j^*$ this becomes:

$$L_d^j(\cdot) = G(w - t_j^* - \epsilon) - A - \sum_{k \neq j, k \in \tilde{M}(\sigma^*) \text{ with } t_k^* = t_j^*} \tilde{L}_d^k(\sigma^*)$$

$$> G(w - t_j^* - \epsilon) - A - (q - 1) \cdot L_k^*(\sigma^*)$$

$$= G(w - t_j^* - \epsilon) - A - \frac{q - 1}{q} \cdot \left( G(w - t_j^*) - A \right)$$

$$> \frac{1}{q} \cdot \left( G(w - t_j^*) - A \right)$$

$$= L_d^j(\sigma^*) > \tilde{L}_d^j(\sigma^*).$$

The first of these inequalities follows from (22), the second holds for $\epsilon$ small enough due to the continuity of $G$, and the third one holds by assumption.

Hence, by an appropriate increase in $t_j^*$, government $j$ would not affect its tax base. Employment in region $j$ is still $E^j = \tilde{L}_d^j$. However, as the tax rate is higher, so are revenues. Hence, $\sigma^*$ cannot be an equilibrium.

$Q.E.D.$


Figure 1: $\dot{t} \leq \alpha$
Figure 2: $\dot{t} > \alpha$