Essays in Public Economics

Inauguraldissertation
zur Erlangung des akademischen Grades
eines Doktors der Wirtschaftswissenschaften
der Universität Mannheim

vorgelegt von

Karl Georg Schulz

im Frühjahrs-/Sommersemester 2022

Abteilungssprecher	Prof. Volker Nocke, PhD
Referent	Prof. Dr. Eckhard Janeba
Koreferent	Prof. Dr. Sebastian Siegloch

Tag der Verteidigung	09. Mai 2022

Acknowledgements

First and foremost, I would like to thank my supervisors, Eckhard Janeba, Sebastian Siegloch, and Nicolas Werquin, for their invaluable guidance and support throughout my time as a graduate student and many insightful discussions about each chapter in this thesis and my other research projects. All of my work greatly benefited from their feedback and experience.

My research also substantially benefited from discussions with Klaus Adam, Antoine Camous, Paweł Doligalski, Hans Peter Grüner, Dominik Sachs, Arthur Seibold, Konrad Stahl, Holger Stichnoth, Michèle Tertilt, Ernst-Ludwig von Thadden, and many others.

I also want to thank the Graduate School of Economic and Social Sciences for funding my PhD course phase and my studies at Yale University. For making my PhD life so much easier, I am thankful to Sonja Collet, Katrin Ramseier, and Sylvia Rosenkranz from the University of Mannheim, as well as Marion Lehnert, Ulrich Kehl, and Golareh Khalilpour from the graduate school.

I want to thank the amazing people I have met during my PhD studies, particularly Johannes, Timo, Alex, Johannes, Frederick, Michaela, Mark, Xi, Ruben, and Mario. Sebastian and Max, thank you for your close friendship and all your support. Lukas and Nils, thank you for being such fantastic office mates. I also thank all my friends, in particular, Alex, Jonas, Uli, Kathi, Ali, Sarah, Clemens, Rudi, and Anna, for making my non-academic life so enjoyable.

I want to thank my parents, Andrea and Karl, my sister, Elena, my brother, Florian, and my uncle, Heli, for your lifelong support. You made all of this possible. Last but not least, I want to thank Julia for all your love and support.

Contents

List of Figures

List of Tables

Preface

Both wealth and income are highly and increasingly concentrated at the top (for example, Saez and Zucman (2016) and Piketty, Saez, and Zucman (2018)). This rise in economic inequities evoked recent proposals for reforms of current tax systems towards more redistribution (see, e.g., Piketty (2014)). The incidence of such redistributive policies hinges on how these policies affect the behavior of economic agents. This classic topic in public economics is central for understanding the intended and unintended consequences of fiscal policies from a positive perspective and, ultimately, for their normative desirability. Most of the theoretical and empirical research in this area studies individuals' behavior taking pre-tax wage and return rate differentials as inherently given.

However, this assumption of exogenous skill differences is inconsistent with the cross-sectional dynamics in inequality (for instance, Gabaix et al. (2016)). Moreover, it is in contrast with the insights from the modern literature on labor and financial economics modeling the formation of wages on labor markets and return rates on financial markets. They are, to some extent, themselves economic outcomes, for example, determined by individual effort, migration responses, portfolio choices, or general equilibrium externalities. As economic outcomes, both their level and distribution are profoundly shaped by redistributive policies.

In this thesis, I make progress towards understanding redistributive policies under endogenous pre-tax incomes. In Chapter 1, I study capital taxation when the heterogeneity in return rates endogenously forms. Chapter 2 derives the optimal labor income taxation with endogenous pre-tax wages and migration responses. Finally, Chapter 3 studies the effects of business taxes, another instrument of redistribution, on the endogenous profit margins of internationally mobile firms. In the following, I describe these papers and their main results in more detail.

In Chapter 1, I investigate how the different sources of return rate inequality (see Bach, Calvet, and Sodini (2020) and Fagereng et al. (2020)) shape the equity-efficiency trade-off of redistributive tax policy. More specifically, I study capital income taxation in an economy in which households' return rates on savings correlate with ability and wealth. My main findings are as follows: Whereas the correlation of return rates with ability types, typically referred to as “type dependence,” calls for higher capital taxes, perhaps surprisingly, the correlation with wealth, labeled as “scale dependence,” lowers the optimal taxation of capital. This makes the policy implications of rising return inequality non-trivial. The intuition is that, aside from amplifying capital inequality, the endogeneity of return rates induced by scale dependence makes capital more responsive to taxes raising the efficiency costs of taxation. Therefore, it matters substantially whether return inequality reflects differences in investment talent, as in existing capital taxation models, or results from households’ effort to increase their rate of return.

In Chapter 2, which is joint work with Eckhard Janeba, I augment the classical debate on the optimal taxation of labor income (e.g., Mirrlees (1971)) and international labor migration by general equilibrium effects in production. This is an important avenue since there is ample empirical evidence of such general equilibrium effects (for instance, Card (2009)), which have been neglected in the tax and migration literature. When regarded separately, both migration and general equilibrium responses are known to limit redistribution – in the former case, due to a “threat of migration” of high-skilled workers (see Lehmann, Simula, and Trannoy (2014)) and, in the latter one, because of “trickle-down” effects in production (see Stiglitz (1982) and Sachs, Tsyvinski, and Werquin (2020)). By considering the interaction of migration and general equilibrium responses, I derive two novel results: Firstly, in the presence of international migration, the trickle-down effect is self-limiting, and secondly, in general equilibrium, the threat of migration is less severe. Therefore, I conclude that the policy prescriptions derived from a simple partial equilibrium framework without migration are more realistic than expected previously.

Chapter 3, which is joint work with Eckhard Janeba, deals with the effects of unilateral economic disintegration, such as Brexit, on the formation of national and international policies. I contribute to the tax competition literature by introducing international firm relocation and non-cooperative business tax policies into a classical multi-country, multi-sector general equilibrium trade model in a highly tractable way. Business tax policies represent any national policy that alters the spatial distribution of economic activity. Since firms can internationally relocate, their pre-tax profit margins are endogenous to business taxation. I predict that the UK will become a tax haven after Brexit and that business taxes in the remaining EU converge in the long run. Third countries, e.g., the US, appear more attractive as a business location after Brexit and, hence, can tax more. At an international level, I show that, after Brexit, EU member countries raise their degree of economic integration among each other and vis-à-vis third countries. Similarly, the UK intensifies existing trade relations with other countries. Both findings suggest a backlash to deglobalization.

In summary, this thesis demonstrates that, under endogenous pre-tax incomes, redistributive policies have unintended consequences. As a result, tax incidence analyses need to be augmented by wage, return rate, and profit responses that, in turn, alter households' and firms' incentives on an individual level and tax revenues and welfare on an aggregate level. These considerations lead to new perspectives on the normative question of how governments should optimally design tax systems. Thus, the presence of endogenous pre-tax incomes widens the classical debate on redistributive policies and creates entirely novel policy recommendations. Altogether, this thesis calls for a better understanding of the drivers of economic inequities and their empirical magnitude.

Chapter 1

Redistribution of Return Inequality

1.1 Introduction

Over the last decades, numerous countries have seen a rapid rise in wealth inequality. In the U.S., for example, the wealth share of the top 0.1% has tripled over the past forty years (Saez and Zucman (2016)). Persistent heterogeneity in the idiosyncratic returns to wealth has been successful in explaining the observed thick tail in the wealth distribution. Such "type dependence" can, for instance, plausibly arise from differences in entrepreneurial ability or investment talent. In addition to type dependence, a recent wave of empirical papers documents the prevalence of "scale dependence," referring to a positive correlation between wealth and its return.[1] Scale dependence may have various sources. Most prominently, Piketty (2014) argues that wealthier households obtain higher rates of return than poorer ones both across and within asset classes because they can take more risks and hire skilled financial advisers.

A well-known result in public finance is that exogenous inequality in return rates justifies the taxation of capital (see, e.g., Atkinson and Stiglitz (1976) and Saez (2002)). However, little is known about the policy implications of the sources of return inequality. How should capital taxation account for the presence of scale and type dependence? How should redistribution respond to a rise in inequality driven by return rates? Which sources of return inequality should a government address, and which not? Can the government alter the distribution of pre-tax return rates? To answer these questions, I study capital taxation when the inequality in return rates comes from type and scale dependence. As a leading example, I microfound scale dependence

[1]For recent empirical evidence, see Bach, Calvet, and Sodini (2020) and Fagereng, Guiso, Malacrino, and Pistaferri (2020). Moreover, it has been shown that one needs to add scale dependence to standard random growth models to account for the cross-sectional dynamics in inequality (Gabaix et al. (2016)).

on a financial market with portfolio choice and information acquisition following the argument by Piketty (2014), as originated by Arrow (1987).

According to conventional wisdom, one might expect that when the rich experience a rise in their return rates relative to the poor, this additional source of inequality provides a rationale for higher capital taxes (see, for example, Piketty (2014)). However, this chapter shows that, besides the inequality level, the source of return inequality is crucial for capital taxation: More type dependence calls for higher capital taxes, whereas scale dependence reduces the optimal capital tax.

The intuition is as follows. The optimal capital tax with return inequality features a classic trade-off between equity and efficiency. Firstly, it is inversely related to the elasticity of capital with respect to capital taxes, capturing the efficiency costs of raising taxes. Secondly, the optimal tax rises in the observed capital income inequality, which accounts for equity concerns. This trade-off is present irrespective of the source of return inequality (type and scale dependence).

However, these two measures that determine the optimal capital tax structurally depend on the source of return inequality. Type dependence amplifies the observed capital inequality, which calls for a higher capital tax. Scale dependence also increases the observed amount of capital inequality. At the same time, unlike type dependence, scale dependence raises the elasticity of capital due to a novel efficiency channel.

Under scale dependence, there is a two-way interplay between taxes and pre-tax returns: On the one hand, return rates and their distribution across households shape the wealth distribution, which serves as a critical primitive for designing a tax system. This standard channel is present in taxation models with exogenous return inequality (type dependence only). On the other hand, if the capital elasticity is non-zero, taxes affect the incentives to save. However, in the presence of scale dependence, higher savings boost pre-tax return rates, yielding a convex relationship between capital income and savings. Thus, scale dependence makes pre-tax return rates endogenous to the tax system (novel channel). I demonstrate that this convexity under scale dependence generates an inequality multiplier effect that augments the standard income and substitution effects from tax reforms.[2]

To provide an example, suppose that the government decreases the capital tax of an individual. Assuming that the substitution dominates the income effect, she saves more. However, when the amount of savings and its pre-tax return endogenously correlate, the latter also rises. In the leading example, the individual acquires more information, e.g., via financial advisory or financial education, and adjusts the financial portfolio. Now, she earns more on every dollar she

[2] Scheuer and Werning (2017) demonstrate a similar result for superstar compensation schemes and labor income taxation.

invests in future consumption. In other words, saving money pays off to a greater extent. In response, the individual saves more, which, in turn, increases her pre-tax return and so on. The own-return elasticity measures this inequality multiplier effect as it describes the responsiveness of pre-tax returns.

I show that the increase in the elasticity either offsets or dominates the rise in inequality. Perhaps surprisingly, a rise in scale dependence is, thus, either neutral or calls for a lower capital tax. In contrast, more type dependence leads to a higher optimal capital tax since it only raises the observed capital income inequality while leaving the capital elasticity unaltered, as described. Therefore, type and scale dependence have opposite policy implications. It does not only matter for tax policy whether and to which extent return rates are heterogeneous across the population, but the underlying source of return inequality is crucial for understanding how a government should respond to rising inequality. This conclusion is at odds with Piketty (2014) (Chapter 12), who uses scale dependence as an argument for more redistribution (via a progressive wealth tax).

The endogeneity of pre-tax return rates implies a Le Chatelier principle for capital (see Samuelson (1947)). Under endogenous pre-tax return rates, capital responds more elastically for flexible than for fixed return rates. As a result, an econometrician underestimates capital elasticities if she does not account for the adjustment in pre-tax return rates, for instance, by using data from a short time window in which return rates do not adjust. Even estimates from long-run data may be biased. For example, a bias materializes if one estimates elasticities from the behavior of households in the wealth distribution that do not (or only to a limited extent) feature endogenous returns. This issue is, for example, characteristic of households from the bottom of the wealth distribution who mostly hold cash and are unable to participate in the stock market due to financial constraints.

Therefore, I estimate the amount of scale dependence directly. I first provide reduced-form macro evidence by tracking the relationship between the realized return rate of the rich and their wealth in the Survey of Consumer Finances (SCF). Then, I estimate the own-return elasticity from panel data on the returns of U.S. private foundations. This idea is similar to Piketty (2014), who descriptively documents the amount of scale dependence using return data from U.S. universities. Although universities and foundations are institutional investors who potentially behave differently on the financial market, they may serve as a reasonable proxy for wealthy investors. For comparison, I also retrieve an estimate from the study by Fagereng et al. (2020) from Norway.

This procedure yields a range of estimates of the lifetime own-return elasticity between 0.1 and 0.9. A medium value of 0.5, for instance, means that doubling the amount of savings raises a household's rate of return accumulated over a lifetime by 50%. The decline in the optimal capital tax that this magnitude of scale dependence triggers is sizable. For an own-return elasticity of 0.5, the presence of scale dependence lowers the revenue-maximizing capital gains tax by more than 25%.

Aside from illustrating this inequality multiplier effect on the capital elasticity, I derive a novel, parsimonious representation of the measure of inequality that enters the formula for the optimal capital tax. The representation relates well-known inequality measures, such as the Gini coefficient of the capital income distribution, to the distribution's empirically observable shape parameter frequently considered in the inequality literature. I demonstrate how to adjust the inequality measure for scale and type dependence and calculate optimal capital taxation conditional on primitives. I also study the policy implications of the relative amount of scale and type dependence. A boost in inequality can be completely neutral for optimal taxation. For instance, holding the capital income tax fixed at 50%, doubling the own-return elasticity cancels out a surge in type dependence of 17%.

These observations hold under the partial equilibrium assumption of a small open economy and in general equilibrium. However, besides altering individual choices, in general equilibrium, tax reforms also affect aggregate variables that feed back into individual return rates. In the financial market example, the equilibrium stock price aggregates information and risk-taking that depend on aggregate wealth. Thus, an individual's pre-tax return on investment is not only a function of her own savings but also of others'. Then, aside from an inequality multiplier effect, tax reforms also induce inter-household effects. The reasoning is as follows. A tax reform changes an individual's savings and returns (due to the altered financial knowledge). As her savings adjust, in general equilibrium, the returns of others and, hence, their savings change as well. In response, this feeds back into the return of the first individual. I measure these general equilibrium effects in terms of novel cross-return elasticities. In the financial market microfoundation, I identify general equilibrium price effects that call for higher taxes in general than in partial equilibrium. Intuitively, these price effects resemble trickle-up externalities, as in a situation of rent-seeking where the rich take away return rates from the poor. The government taxes these extra rents away (see Rothschild and Scheuer (2016)).

In the foundation data, I find statistically significant but economically small cross-return elasticities. This finding suggests no or negligible general equilibrium forces. The point estimates of cross-return elasticities support some features of the financial market. For instance, there are negative effects from the top of the wealth distribution, indicating the presence of small general equilibrium effects.

Related literature. This chapter relates to four strands of the literature. Firstly, I add to the sizable literature on capital taxation. As shown by Saez (2002), return inequality provides an essential justification for why capital taxes should not be zero, unlike in Atkinson and Stiglitz (1976), Chamley (1986), and Judd (1985). So far, the focus in the literature has been on return inequality that arises from type dependence. For instance, Shourideh (2012), Saez and Stantcheva (2018), and Guvenen, Kambourov, Kuruscu, Ocampo-Diaz, and Chen (2019) allow return rates to exogenously differ across agents and study the equity and efficiency implications of capital taxation. Gerritsen, Jacobs, Rusu, and Spiritus (2020) analyze capital taxation under type and scale dependence. They show that both sources of return inequality give rise to optimal positive capital taxation and investigate the underlying mechanisms. While my model nests their established (non-)zero-capital-taxation results, I investigate how different sources of return rate inequality shape the equity-efficiency trade-off and demonstrate the opposing effects of scale and type dependence on capital taxation. Gaillard and Wangner (2021) obtain similar conclusions about the effects of type and scale dependence on wealth taxation in a quantitative model, however, restricting the policy instruments to a linear wealth tax at the top.

Moreover, I introduce scale and type dependence into two well-known taxation frameworks: the dynastic framework of linear wealth taxation by Piketty and Saez (2013) and the canonical Mirrlees (1971) model of nonlinear capital income taxation, as in Farhi and Werning (2010). Using the perturbation techniques introduced in Piketty (1997), Saez (2001), and, more recently, Golosov, Tsyvinski, and Werquin (2014), I characterize the optimal linear and nonlinear capital taxation. Besides, I allow for uncertainty (e.g., Aiyagari (1994)) and full intergenerational dynamics by restricting attention to simple tax instruments. Similarly, I separate the nonlinear taxation of labor and capital income. These restrictions allow me to derive a clear-cut characterization of the respective tax systems. However, the main conclusions regarding the presence of scale and type dependence should carry over to a fully optimal mechanism as considered in the new dynamic public finance literature (for instance, Golosov, Kocherlakota, and Tsyvinski (2003)). Moreover, I abstract from the debate on the gains from selecting different tax policy instruments, e.g., wealth vs. capital income taxation (see Guvenen et al. (2019)) or excess return taxation (for instance, Boadway and Spiritus (2021)).

Secondly, the chapter links to the literature on redistributive taxation in general equilibrium. Rothschild and Scheuer (2013), Ales, Kurnaz, and Sleet (2015), and Sachs, Tsyvinski, and Werquin (2020) extend the original framework by Stiglitz (1982). Deploying the techniques in Sachs, Tsyvinski, and Werquin (2020), I am, to the best of my knowledge, the first one to provide a thorough analysis of the nonlinear capital tax incidence and optimal capital taxation in general equilibrium. Thereby, I extend the well-known concepts of own- and cross-wage elasticities that matter for labor income taxation to pre-tax return rates in the context of capital taxation.

Thirdly, in microfounding return inequality on a financial market, I add to the literature on financial knowledge in partial (e.g., Arrow (1987) and Lusardi, Michaud, and Mitchell (2017)) and general equilibrium (e.g., Grossman and Stiglitz (1980), Verrecchia (1982), Peress (2004), Kacperczyk, Nosal, and Stevens (2019)). To the best of my knowledge, this is the first paper formalizing a link between redistribution and informational efficiency in Grossman and Stiglitz (1980) financial markets. The idea that capital taxes affect the accumulation of financial knowledge is, however, similar to the literature on taxation and human capital (for recent examples, see Krueger and Ludwig (2013), Findeisen and Sachs (2016), and Stantcheva (2017)). Also, the implications of scale dependence for capital taxation derived in this chapter are similar to those of superstar compensation schemes for labor income taxation (see Scheuer and Werning (2017)). However, the empirical evidence suggests that scale dependence is widely disseminated throughout the wealth distribution, whereas superstar effects primarily manifest at the top of the income distribution.

Fourthly, in my empirical analysis of the SCF and a large panel of U.S. foundations, I document the prevalence of scale dependence and, more generally, return inequality as in Yitzhaki (1987). More recently, Bach, Calvet, and Sodini (2020) and Fagereng et al. (2020) document scale dependence with Scandinavian data. Finally, by providing estimates of own- and cross-return elasticities, I also add to the empirical literature on estimating capital elasticities. I survey this literature in Section 1.3.1.

Outline of the chapter. The chapter is structured as follows. First, I establish the main findings in a simple conceptual framework (Section 1.2). I also describe the microfoundation and well-known extensions to the conceptual framework. In Section 1.3, I describe the empirical implications of my results and estimate own- and cross-return elasticities with data from the SCF and U.S. private foundations. In Section 1.4, I provide quantitative illustrations of the effects of type and scale dependence on the equity-efficiency trade-off and the optimal capital taxation. Section 1.5 concludes. I relegate all proofs, model extensions, and the microfoundation to the Appendix.

1.2 The Model

1.2.1 A Conceptual Framework

This section describes a simple two-period life-cycle framework to think about capital taxation under the presence of type and scale dependence. Suppose there is a unit measure of households $i \in [0,1]$ that differ in their labor earnings ability w_i. Thus, under standard monotonicity conditions, one can interpret i as a household's position in the income distribution. Aside from working l_i hours (in period 1), household i saves a_i (for period 2) to maximize lifetime utility $u_i(c_1, c_2, l)$. Households use their first-period (after-tax) labor income for consumption and savings and consume their final (after-tax) wealth in the second period. Their pre-tax return rates on savings, $r(a_i, i) \equiv r_i(a_i)$, may differ due to *type dependence* and *scale dependence*.[3] Type dependence refers to an exogenous heterogeneity in return rates ($\frac{\partial r_i(a_i)}{\partial i} > 0$): That is, some types can generate higher return rates than others, for example, because of an inherent investment talent or entrepreneurial skill. Scale dependence refers to a positive relationship between wealth and its return ($\frac{\partial r_i(a_i)}{\partial a_i} > 0$).[4] Observe that the presence of scale dependence does not rule out type dependence and vice versa. Just as in reality, both type and scale dependence may co-occur in this setting. In contrast, I refer to *type dependence only* as a setting where all the return inequality is exogenously given ($\frac{\partial r_i(a_i)}{\partial i} > 0$ and $\frac{\partial r_i(a_i)}{\partial a_i} = 0$). Let there be a linear tax rate τ_K on capital gains $a_{R,i} \equiv a_i r_i(a_i)$ and a lump-sum transfer T.[5] Suppose that utility is quasilinear in the consumption of final wealth. Utility maximization yields each household's Marshallian savings supply function $a_i = \overline{a}_i(\tau_K, r_i(a_i); i)$ and an indirect (present-value) utility $U_i(\tau_K, T)$.[6] Define the elasticity of savings with respect to the capital tax rate as $\zeta_i^{a,(1-\tau_K)} \equiv \frac{dlog(a_i)}{dlog(1-\tau_K)}$ and the capital gains elasticity as $\zeta_i^{a_R,(1-\tau_K)} \equiv \frac{dlog(a_{R,i})}{dlog(1-\tau_K)}$. Without scale dependence, households' return rates are fixed. Then, the two elasticities coincide $\tilde{\zeta}_i^{a,(1-\tau_K)} = \tilde{\zeta}_i^{a_R,(1-\tau_K)}$, where $\tilde{\zeta}_i$ indicates that the respective

[3]In this section, I focus on persistent return inequality and disregard the role of luck. In the Appendix, I deal with uncertain return rates.

[4]Later, I microfound the notion of scale dependence on a financial market with portfolio choice and financial knowledge acquisition. Wealthy households acquire more financial information and, thus, obtain higher rates of return on their financial investments than poorer households. This microfoundation generates qualitatively the same endogenous return inequality as other potential channels would do, e.g., stock market participation costs, housing, liquidity constraints, and insurance against consumption risk. Moreover, it fits well into the empirical setting I consider later. The positive and normative implications for capital taxation remain the same irrespective of the underlying mechanism that generates scale dependence.

[5]Following the capital taxation literature, I interchangeably use the terms capital income and capital gains. Similarly, I do not differentiate between realized and unrealized returns, which is in practice an important distinction. In the Appendix, I extend the exposition to the nonlinear capital income taxation and wealth taxes.

[6]This representation can also be interpreted as the static equivalent of the steady-state utility in a fully dynamic setting (see Saez and Stantcheva (2018)).

elasticity is evaluated at a fixed return rate. Under scale dependence, this is not the case. Let $\tilde{\zeta}_i^{a,r} \equiv \frac{dlog(a_i)}{dlog(r_i)}$ measure the responsiveness of savings to the rate of return. The novelty of this chapter is to explore the differential equity and efficiency effects of type and scale dependence. The *own-return elasticity* $\varepsilon_i^{r,a} \equiv \frac{dlog[r_i(a_i)]}{dlog(a_i)}$ describes the extent of scale dependence. For simplicity, let $\tilde{\zeta}_i^{a,(1-\tau_K)}$, $\tilde{\zeta}_i^{a,r}$, and $\varepsilon_i^{r,a}$ be constant in this section.[7]

A utilitarian social planner maximizes aggregate welfare at a given budget by optimally choosing the capital tax:

$$\max_{\{\tau_K,T\}} \int_i \Gamma_i U_i(\tau_K,T)\, di \ \ subject\ \ to \ \ \int_i \tau_K a_{R,i} di \geq T + \overline{E}, \tag{1.1}$$

where $\{\Gamma_i\}_{i\in[0,1]}$ are the household's (weakly decreasing) Pareto weights and $\int_i \Gamma_i di = 1$. In the following, I use this basic framework to study capital taxation under type and scale dependence, holding all the other primitives of the economy fixed (such as the savings elasticities at a given rate of return). I establish three novel findings that I summarize in Proposition 1.

Proposition 1. *Consider the optimal capital gains tax under return rate heterogeneity.*

(a) *When expressed in terms of sufficient statistics, the formula for the optimal capital tax is the same irrespective of the sources of return inequality.*

(b) *Under scale dependence, an inequality multiplier effect increases the elasticity of capital income (relative to type dependence only). This effect acts as a force for lower taxes.*

(c) *The optimal capital tax with scale dependence is either the same or lower than without scale dependence (type dependence only). By contrast, the presence of type dependence raises the optimal capital tax.*

Proof. Appendix 1.A. □

Part (a). The government's problem yields a Ramsey formula for the optimal capital tax (e.g., Diamond (1975))

$$\frac{\tau_K}{1-\tau_K} = \frac{1}{\overline{\zeta}^{a_R,(1-\tau_K)}} \mathbb{E}\left[\frac{(1-\Gamma_i)\, a_{R,i}}{\mathbb{E}(a_{R,i})}\right], \tag{1.2}$$

where $\overline{\zeta}^{a_R,(1-\tau_K)} \equiv \mathbb{E}\left[\frac{a_{R,i}}{\mathbb{E}(a_{R,i})}\zeta_i^{a_R,(1-\tau_K)}\right]$ and $\mathbb{E}\left[\frac{(1-\Gamma_i)a_{R,i}}{\mathbb{E}(a_{R,i})}\right]$ are the average elasticity of capital income and the observed capital income inequality, respectively. Irrespective of how returns form, the optimal capital income tax is decreasing in the average elasticity of capital income which quantifies the efficiency costs of raising taxes. At the same time, accounting for the society's equity concerns, the optimal tax rises in a measure of the observed capital income inequality, e.g.,

[7] The assumption that $\varepsilon_i^{r,a}$ is constant over the population finds support in my empirical analysis of Section 1.3.

the Gini coefficient of the capital income distribution (see below). Knowing these two sufficient statistics is enough to characterize the optimal capital tax, which gives part (a) of Proposition 1. However, they depend on both type and scale dependence. Therefore, I now move to a structural approach and analyze how the sources of return inequality affect the two sufficient statistics.

Part (b). I begin with the elasticity of capital. Without scale dependence (conditional on pre-tax return rates), capital income is linear in savings $a'_{R,i}(a_i)|_{\{r_i\}_{i\in[0,1]}} = r_i$ and $a''_{R,i}(a_i)|_{\{r_i\}_{i\in[0,1]}} = 0$. With scale dependence, the rate of return is endogenous. This makes capital gains convex in savings $a'_{R,i}(a_i) = r_i(a_i)$ and $a''_{R,i}(a_i) = r'_i(a_i) > 0$, where I assume that households take their equilibrium pre-tax return rates as given. To gain some intuition, consider an individual i. In a setting with type dependence only, the individual is endowed with an investment skill, allowing her to realize a return r_i. Her capital gains proportionally rise with her amount of investment. Off equilibrium, to obtain the same capital income as another individual $i' > i$, she needs to increase her savings substantially. Under scale dependence, the individual has the same return rate r_i in equilibrium as without scale dependence. However, she can reach the capital income of individual i' more easily. Still, she needs to save more. At the same time, higher savings allow her to raise the rate of return to a higher level (in the financial market by acquiring financial knowledge). This convexity boosts the savings and capital income elasticities, as I describe in the following.

Without scale dependence (with type dependence only), the average elasticity of capital income is equal to the savings elasticity for a given return $\overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}} = \tilde{\zeta}_i^{a,(1-\tau_K)}$. With scale dependence, the savings elasticity needs to account for an endogenous return adjustment. Therefore, the savings elasticity and, accordingly, the average capital income elasticity are revised upwards

$$\overline{\zeta}^{a_R,(1-\tau_K)} = \Phi_i \overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}} \tag{1.3}$$

with $\Phi_i \equiv \frac{1+\varepsilon_i^{r,a}}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}} = (1+\varepsilon_i^{r,a})\sum_{n=0}^{\infty}\left(\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}\right)^n > 1$ measuring an inequality multiplier effect. The size of the adjustment is proportional to the inequality multiplier effect Φ_i. The interpretation is straightforward: A tax cut increases a household's savings (when the substitution effect dominates the income effect). Under scale dependence, however, as savings increase, the pre-tax rate of return rises as well. The higher rate of return increases the incentives to saves. In response, rates of return adjust, and so on. Φ_i captures this infinite loop of reactions that arises with scale dependence. As a result, savings and capital gains react more elastic to tax reforms. Since the optimal capital tax is inversely related to the mean capital gains elasticity, its upward adjustment provides a force for lower capital taxes. Proposition 1 (b) follows. The result resembles the effect of superstar compensation schemes in the context labor income taxation (see Scheuer and

Werning (2017)). Thus, scale dependence is in its implications for redistribution similar to a superstar phenomenon.[8]

Part (c). How does scale dependence affect optimal capital taxes? On the one hand, as described, scale dependence raises the observed capital gains elasticity, which reduces optimal taxes. On the other hand, the presence of scale dependence has the potential to amplify capital income inequality significantly because an initial level of wealth dispersion generates more return inequality which in turn raises wealth inequality. Recall that the optimal capital income tax is increasing in the observed capital income inequality. Through this channel, one would expect higher taxes aimed at reducing inequality. Therefore, I consider the following two comparative statics exercises.

Firstly, I compare the optimal capital income tax with scale dependence, τ_K, to the tax, denoted as $\tilde{\tau}_K$, one would obtain in a baseline economy with the same but exogenous distribution of returns (type dependence only). This exercise is, in principle, non-trivial, as the measure of inequality that determines the optimal tax may be endogenous to the underlying tax code $I(\tau_K) \equiv \mathbb{E}\left[\frac{(1-\Gamma_i)a_{R,i}}{\mathbb{E}(a_{R,i})}\right]$. With constant elasticities, however, $I'(\tau_K) = 0$. Therefore, compared to an economy with type dependence that is observationally equivalent in terms of inequality ($I(\tau_K) = I(\tilde{\tau}_K)$), taxes are lower in the economy with scale dependence because the capital income elasticities are higher[9]

$$\frac{\tau_K}{1-\tau_K} = \frac{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}{1+\varepsilon_i^{r,a}}\frac{\tilde{\tau}_K}{1-\tilde{\tau}_K}. \tag{1.4}$$

To demonstrate the quantitative importance of endogenous pre-tax returns for optimal taxes, I calculate the optimal revenue-maximizing capital tax with and without scale dependence in Table 1.1. Set the elasticity of savings with respect to the rate of return equal to 0.5. Table 1.1 shows optimal capital taxes for different values of $\tilde{\zeta}_i^{a,(1-\tau_K)}$ and $\varepsilon_i^{r,a}$. As usual, the larger the savings elasticity, the lower the optimal capital tax. A novel aspect of this chapter is to have a non-zero own-return elasticity. As a benchmark, I consider $\varepsilon_i^{r,a} = 0$ in the first row (no scale dependence). The other rows differ by the magnitude of scale dependence. An own-return elasticity of 0.5, for instance, means that doubling the savings raises the rate of return accumulated over a lifetime by fifty percent. This amount of scale dependence reduces the revenue-maximizing tax rate by more than 25% (17 percentage points). In the empirical section, I identify a range of estimates

[8] One can also interpret this finding as a Le Chatelier principle for capital (see Samuelson (1947)): In the long run, when pre-tax return rates adjust, capital responds more elastic than in the short run for fixed return rates.

[9] In the dynastic economy of Section 1.C (Mirrleesian economy of Section 1.G), I show that a similar logic applies to a linear wealth tax (nonlinear capital income tax).

Table 1.1: Optimal Rawlsian Capital Tax Rate

Own-Return Elasticity	Compensated Elasticity $\tilde{\zeta}_i^{a,(1-\tau_K)} = 0.25$	$\tilde{\zeta}_i^{a,(1-\tau_K)} = 0.5$	$\tilde{\zeta}_i^{a,(1-\tau_K)} = 1$
Baseline Model (No Scale Dependence): Exogenous Inequality in r_i			
$\varepsilon_i^{r,a} = 0$	80	67	50
Microfounded Model (Scale Dependence): Endogenous Inequality in r_i			
$\varepsilon_i^{r,a} = 0.1$	78	63	46
$\varepsilon_i^{r,a} = 0.25$	74	58	41
$\varepsilon_i^{r,a} = 0.5$	67	50	33
$\varepsilon_i^{r,a} = 1$	50	33	20

of the lifetime own-return elasticity between 0.1 and 0.9. For $\tilde{\zeta}_i^{a,(1-\tau_K)} = 0.5$, this results in a reduction of the optimal capital gains tax between 6 and 40%.

Alternatively, one can interpret these back-of-the-envelope calculations as the difference between the optimal capital tax and the tax set by a politician who wrongly assumes that the inequality he observes does not come from scale dependence (but from type dependence only). Altogether, even for a relatively small amount of scale dependence, the implications for the optimal tax rate are sizable.

Secondly, instead of considering two in terms of inequality observationally equivalent economies, I compare an economy with scale dependence to one without scale dependence, holding all primitives other than scale dependence fixed. Aside from the savings elasticities at given return rates, these primitives include the wage distribution and the exogenous part of the return rate distribution that measures type dependence. One can approximate the measure of inequality as

$$I(\tau_K) \approx \frac{1}{1-\bar{i}}\zeta_i^{a_R,(1-i)}\mathbb{COV}(\Gamma_i, i),$$

where $\bar{i}$ denotes the household who earns the average capital income and

$$\zeta_i^{a_R,(1-i)} \equiv \frac{dlog\,(a_i r_i(a_i))}{dlog\,(1-i)} = \zeta_i^{a,(1-i)} + \zeta_i^{r,(1-i)} = \Phi_i \tilde{\zeta}_i^{a,(1-i)} + \tilde{\zeta}_i^{r,(1-i)}$$

defines the elasticity of capital income with respect to household rank in the income distribution. As I argue in Section 1.4, for Pareto distributed capital incomes (see, for instance, Gabaix (2009)), the latter elasticity equals the inverse of the shape parameter. Thus, this characterization provides a novel connection between the literature on optimal capital taxation and the measurement of inequality.[10]

In the context of nonlinear capital taxation (Section 1.G), I obtain an exact version of this approximation by expressing the hazard ratio of the capital income distribution in terms of type and scale dependence. In Section 1.B, I quantitatively analyze the performance of the approximation. The approach readily extends to the presence of income effects. Moreover, it allows me to remain completely agnostic about the underlying processes of return formation. Also, notice the similarity to the linearization of, e.g., steady-state equilibria in macroeconomic models.

The elasticity $\zeta_i^{a_R,(1-i)}$ consists of two terms. The first term $\Phi_i \tilde{\zeta}_i^{a,(1-i)}$ captures the described effect that, under scale dependence, any initial level of wealth inequality translates into a more pronounced degree of capital income inequality relative to a setting without scale dependence. The second term $\tilde{\zeta}_i^{r,(1-i)} \equiv \frac{\partial log(r_i)}{\partial log(1-i)}$ is the reduced form relationship between households' rank and their return rates (conditional on an amount of wealth). Therefore, it directly measures the amount of type dependence in the economy.

Thus, in the absence of type dependence ($\tilde{\zeta}_i^{r,(1-i)} = 0$), the introduction of scale dependence is completely neutral. The rise in inequality cancels the increase in the average capital income elasticity. With type dependence ($\tilde{\zeta}_i^{r,(1-i)} < 0$), scale dependence reduces optimal capital taxes. Altogether, despite its potential to boost wealth inequality, scale dependence either reduces the optimal capital tax or is entirely neutral (Proposition 1 (c)).[11] One can read this result as a possible justification for why capital taxes (e.g., in the U.S.) have not gone up, although capital income inequality has mounted (see Section 1.3.2). If this rise in inequality came from scale dependence, one should not tax more. However, if it was driven by type dependence, capital taxation should be higher since type dependence raises capital income inequality while leaving the capital elasticity unaffected. In Section 1.4, I quantitatively illustrate this insight further.

Zero-capital-taxation result. Altogether, the optimal capital tax can be expressed in terms of primitives

$$\frac{\tau_K}{1-\tau_K} \approx \frac{1}{1-\bar{i}} \frac{\tilde{\zeta}_i^{a,(1-i)} + \tilde{\zeta}_i^{r,(1-i)} / \Phi_i}{\tilde{\zeta}_i^{a,(1-\tau_K)}} \mathbb{COV}\left(\Gamma_i, i\right). \tag{1.5}$$

This novel and parsimonious representation provides four conditions under which the zero-capital-taxation result holds/does not hold. Firstly, the optimal capital tax is zero when the capital elasticity diverges $\tilde{\zeta}_i^{a,(1-\tau_K)} \to \infty$. Secondly, one obtains a zero-capital-taxation result when each

[10] See Simula and Trannoy (2020) for a related attempt in the context of optimal nonlinear labor taxation.

[11] In Section 1.G, I derive a similar neutrality result for an optimally set nonlinear capital gains tax in a life-cycle economy with nonlinear labor income taxes.

household's relative rank in the income distribution i is unrelated to the social marginal welfare weight Γ_i such that $\mathbb{COV}(\Gamma_i, i) = 0$.

Thirdly, there is an optimal zero capital tax in the absence of any initial inequality, $\tilde{\zeta}_i^{r,(1-i)} = 0$ and $\tilde{\zeta}_i^{a,(1-i)} = 0$. $\tilde{\zeta}_i^{a,(1-i)}$ describes the amount of reduced-form wealth inequality measured by the relationship between a household's wealth and the rank in the income distribution. This inequality measure depends on the presence of type dependence and the availability of a non-linear labor income tax. Without a labor income tax, $\tilde{\zeta}_i^{a,(1-i)} = \tilde{\zeta}_i^{a,lw} \tilde{\zeta}_i^{lw,(1-i)} + \tilde{\zeta}_i^{a,r} \tilde{\zeta}_i^{r,(1-i)}$, such that, absent of wage inequality ($w_i = w$, $\forall i$) and type dependence ($r_i = r$, $\forall i$), the optimal capital tax is zero. In this situation, scale dependence does not play a role since there is no underlying wealth inequality translating into endogenous return inequality. With a nonlinear labor income tax, $\tilde{\zeta}_i^{a,(1-i)} = \left(1 - T_l'(l_i w_i)\right) \tilde{\zeta}_i^{a,lw-T_l(lw)} \tilde{\zeta}_i^{lw,(1-i)} + \tilde{\zeta}_i^{a,r} \tilde{\zeta}_i^{r,(1-i)}$. Then, absent of any type dependence ($\tilde{\zeta}_i^{r,(1-i)} = 0$), the household heterogeneity is, even in the presence of scale dependence, one-dimensional (in terms of wage inequality w_i), and the capital tax is redundant as an instrument (see Atkinson and Stiglitz (1976)).[12]

Whenever there is a minimal amount of type dependence, the government wishes to levy a non-zero capital tax even if a nonlinear labor income tax is available.[13] In this case, the formula shows that the optimal capital tax rate depends on the relative magnitude of type and scale dependence $\tilde{\zeta}_i^{r,(1-i)}/\Phi_i$. This leads to the fourth condition. When the own-return elasticity converges to the inverse savings elasticity $\varepsilon_i^{r,a} \to \frac{1}{\tilde{\zeta}_i^{a,r}}$, the inequality multiplier effect diverges $\Phi_i \to \infty$. This observation is an entirely novel zero-capital-taxation result. In this case, both the capital elasticity and the observed capital income inequality go to infinity, but the elasticity diverges faster.

Corollary 1. *Under scale dependence, a rise in capital taxation compresses the distribution of pre-tax returns. However, this compression effect comes along with the cost of lowering mean pre-tax returns.*

Proof. Appendix 1.A.4. □

Interestingly, under scale dependence, the distribution of pre-tax returns is endogenous to the tax code. To see this, consider the variance of returns $\mathbb{V}(r_i)$ and a rise in the capital gains tax

[12] However, the timing of the labor income tax plays a role. If labor and capital incomes are taxed in the same period, there is no role for capital gains taxation (e.g., Gahvari and Micheletto (2016)). When the taxes are levied in different periods and the government cannot freely borrow and save, Gerritsen et al. (2020) find a positive capital tax.

[13] That is, the zero-capital-taxation result (e.g., Atkinson and Stiglitz (1976), Judd (1985), and Chamley (1986)) breaks down. The intuition is that the presence of return inequality makes household heterogeneity two-dimensional. The government, then, uses the capital gains tax as an additional screening device (e.g., Saez (2002)).

$d\tau_K > 0$. Then, under scale dependence ($\varepsilon_i^{r,a} > 0$), the variance of pre-tax returns declines

$$d\mathbb{V}(r_i) = -2\mathbb{V}(r_i)\,\varepsilon_i^{r,a}\zeta_i^{a,(1-\tau_K)}\frac{d\tau_K}{1-\tau_K} < 0.$$

In other words, the elasticity of the pre-tax return variance with respect to the retention rate is positive $\zeta^{\mathbb{V}(r),(1-\tau_K)} \equiv \frac{dlog[\mathbb{V}(r_i)]}{dlog(1-\tau_K)} > 0$. A rise in marginal taxes, therefore, reduces the pre-tax return inequality. However, this compression effect of pre-tax returns is associated with the cost of diminishing mean pre-tax returns

$$d\mathbb{E}(r_i) = -\mathbb{E}(r_i)\,\varepsilon_i^{r,a}\zeta_i^{a,(1-\tau_K)}\frac{d\tau_K}{1-\tau_K} < 0.$$

Thus, scale dependence gives rise to a new model-inherent trade-off for tax policy. On the one hand, a government that raises capital taxes can realize novel equity gains by reducing the pre-tax return inequality. But, on the other hand, there are novel efficiency costs from lowering the level of pre-tax returns.

For fully type-dependent rates of return, only the distribution of after-tax returns but not the pre-tax return distribution responds to the tax system. In the presence of scale dependence, capital taxes also affect the distribution of pre-tax returns. As a result, distributional responses of pre-tax returns provide a potential source for empirically identifying the magnitude of scale dependence. If all the return inequality came from type dependence, there should be no reaction of mean pre-tax returns and their variance to tax reforms. However, whenever there is some scale dependence, one can observe such a response. As mentioned above, the strength of the reaction is, in this simple framework, proportional to the amount of scale dependence, measured by $\varepsilon_i^{r,a}$. In Section 1.3.2, I use mean responses of the top 1% wealth group to identify scale dependence.

Proposition 2. *In the financial market microfoundation, general equilibrium price effects provide a force for a higher capital gains tax in general than in partial equilibrium.*

Proof. Appendix 1.A.5. □

In Appendix 1.E, I microfound the notion of scale dependence (for a short description, see Section 1.2.2). On a financial market, households optimally choose their portfolio and the amount of information they wish to acquire. Wealthier households invest more and, thus, have a higher incentive to gain financial knowledge than poorer investors. As a result of their better knowledge, the former obtain higher rates of return than the latter households. Portfolio returns become scale-dependent. In general equilibrium, an investor's rate of return is not only positively associated with her portfolio size but also depends on others' investment decisions $r_i\left(a_i, \{a_i\}_{i'\in[0,1]}\right)$.

The *cross-return elasticity* $\gamma_{i,i'}^{r,a} \equiv \frac{\partial log[r_i(\cdot)]}{\partial log(a_{i'})}$ measures the responsiveness of a household i's return to the amount of investment by another household i' (similar to the cross-wage elasticity in Sachs, Tsyvinski, and Werquin (2020)).[14]

I show that for linear costs of information acquisition and when everyone acquires knowledge, a change in the savings by a household i' leads to the same percentage change in the return rate of any other household i $\gamma_{i,i'}^{r,a} = \frac{1}{r_i}\delta_{i'}^{r,a}$. Moreover, $\delta_{i'}^{r,a}$ is decreasing $a_{i'}$. The semi-elasticity is positive for small values of $a_{i'}$ and negative for large ones. These general equilibrium effects are similar to trickle-up forces, where a cut in the capital income tax of the rich shifts economic rents from the bottom to the top. The intuition is as follows. When the substitution effect dominates the income effect, a tax cut on the rich's capital income increases their portfolio size and financial knowledge. Accordingly, their returns rise ($\varepsilon_i^{r,a} > 0$). This channel is also present in partial equilibrium. However, in general equilibrium, aggregate information also grows as the rich become more informed, and the value of private information declines. As a result, the reward for the relatively small amount of information the poor purchase goes down, leading to lower return rates for them ($\delta_{i'}^{r,a} < 0$). Thus, the tax cut on the rich increases their return rates but reduces those of the poor.

The formula for the optimal linear capital tax in general equilibrium is given by

$$\frac{\tau_K}{1-\tau_K} = \frac{1}{\overline{\zeta}^{a_R,(1-\tau_K)}}\mathbb{E}\left[\frac{\left(1-\Gamma_i\left(1+\gamma_i^{r,(1-\tau_K)}\right)\right)a_{R,i}}{\mathbb{E}\left(a_{R,i}\right)}\right], \tag{1.6}$$

where $\gamma_i^{r,(1-\tau_K)} \equiv \int_{i'} \gamma_{i,i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di'$ summarizes the general equilibrium effects. Suppose that the cross-return elasticities average out such that $\int_{i'}\gamma_{i',i'}^{r,a}di' = 0$.[15] Then, one can show that the average capital gains elasticity, $\overline{\zeta}^{a_R,(1-\tau_K)}$, declines relative to the partial equilibrium. Moreover, $\gamma_i^{r,(1-\tau_K)} = \frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di' < 0$. Both the general equilibrium effects and the adjustment of the capital income elasticity call for a higher capital tax.

For small general equilibrium forces ($\delta_{i'}^{r,a} \approx 0$ and $\tau_K^{GE} \approx \tau_K^{PE}$), one can use a first-order Taylor approximation to compare the optimal capital income tax in general equilibrium to the tax rate set by a politician who wrongly assumes that only partial equilibrium forces are present. Accordingly, this politician sets a tax, τ_K^{PE}, that generates a capital income distribution for which

[14] Instead of studying a financial market, one could also consider a standard production function that, for instance, exhibits decreasing returns to scale. Then, a household's return function depends on aggregate capital and, thus, on others' capital supply. In any case, the cross-return elasticity describes this dependence.

[15] In the empirical analysis (Section 1.3), I find some support for this assumption.

the tax is optimal (as proposed in Rothschild and Scheuer (2013, 2016)). The optimal general equilibrium tax rate is larger than the one in this self-confirming policy equilibrium $\tau_K^{GE} > \tau_K^{PE}$.

Alternatively, one may approximate the measure of capital income inequality in the formula for the optimal capital tax conditional on primitives as in part (c) (see Appendix 1.A.5). For the given specification, the capital income inequality is higher in general than in partial equilibrium, whereas the capital income elasticity is, again, revised downwards. Consequently, in both comparative statics exercises, general equilibrium forces call for more redistribution in general equilibrium. This result is intuitive because, as with trickle-up effects, cutting the capital tax would shift resources from poor to affluent households and lower welfare.

One may think about this as a situation of rent-seeking, where the rich take away capital income from the poor. It has been shown that it is optimal for the government to tax these rents away (see Piketty, Saez, and Stantcheva (2014) and Rothschild and Scheuer (2016)). Of course, this does not mean that the capital tax should be higher or lower than in the setting with type dependence only (without scale dependence). This comparative statics exercise only compares capital taxes in partial and general equilibrium. To evaluate whether the presence of endogenous return rates in general equilibrium should lead to more or less redistribution relative to a situation where return rates are exogenous, one needs a precise notion of the relative strength of partial and general equilibrium forces. In the empirical section, I attempt to disentangle these.

One can also interpret this result in connection with the integration of financial markets. As markets become internationally more connected, general equilibrium effects vanish ($\gamma_{i,i'}^{r,a} \rightarrow 0$). Foreign investors gain better access to a country's financial market. Vice versa, domestic investors can participate in foreign markets more easily when integration proceeds. As a result, domestic investors' impact on the return rates on the financial market is inversely related to the degree of integration. In this economy, the optimal capital gains tax and, thus, the level of redistribution declines with the international integration of financial markets. Therefore, the decline in the U.S. capital income taxation in the past decades is consistent with a reduction in general equilibrium effects due to financial market integration.

1.2.2 Microfoundation, Extensions, and Discussion

In the following, I discuss the model's main assumptions, their generality, potential extensions, and the policy implications of the framework.

Microfoundation. To begin, I describe the financial market of Appendix 1.E as *one* potential microfoundation of scale dependence. I consider a repeated Grossman and Stiglitz (1980) financial market, where households optimally choose their portfolio consisting of a risk-free bond and a risky stock and acquire information about the stochastic fundamentals that drive the stock's payoff. In the rational expectations equilibrium, the stock price clears the market for individuals' portfolios, and the implied informativeness of the price is consistent with individuals' information acquisition.[16] I incorporate taxes into this market and demonstrate the functional form of own- and cross-return elasticities in a linear example. Moreover, I include career effects and explicitly add type dependence. Interestingly, the presence of type dependence affects the distribution of own- and cross-return elasticities in the financial market.

Even though scale dependence arises, in this leading example, from information acquisition on a financial market, the exact source of scale dependence is, in principle, unimportant for tax policy. In partial equilibrium, only the magnitude of the own-return elasticity throughout the wealth distribution, $\varepsilon_i^{r,a}$, matters. Nevertheless, to identify general equilibrium effects, if present at all, the sources of scale dependence are relevant to the extent that they may enter differently into the cross-return elasticities, $\gamma_{i,i'}^{r,a}$.

Discussion and extensions. The chapter's message is not that taxes should be lower with return inequality than without. Instead, I analyze the differential policy implications of type and scale dependence. Also, lower taxes in the presence of scale dependence do not mean that the government should let wealth and return inequality grow indefinitely. Firstly, there might be an upper bound on households' long-run return rates, naturally limiting the amount of scale dependence and, thus, the upward adjustment in capital income elasticities. Secondly, the optimal capital tax rises with observed capital income inequality to combat rising inequality.

As already mentioned, most of the simplifying assumptions in this section are inessential for the main results. In Section 1.C, I introduce type and scale dependence into the dynamic bequest taxation framework of Piketty and Saez (2013), in which a government taxes the intergenerational transmission of wealth. The environment features income effects, the presence of labor income taxation, uncertain returns, and dynastic considerations. I demonstrate that Proposition 1, Corollary 1, and Proposition 2 carry over to this setting. However, the formulas for optimal wealth taxation now include aggregate wealth and labor income elasticities as well as distributional parameters of labor income and of received and left-over wealth. Moreover, the sufficient statistics are adjusted by another version of the own-return elasticity $\varepsilon_{i,t}^{1+r,a} \equiv \frac{\partial log[1+r_{i,t}(a_{i,t})]}{\partial log(a_{i,t})}$. Similarly, a modification of the cross-return elasticity $\gamma_{i,i',t}^{1+r,a} \equiv \frac{\partial log\left[1+r_{i,t}\left(a_{i,t},\left\{a_{i',t}\right\}_{i'\in[0,1]}\right)\right]}{\partial log\left(a_{i',t}\right)}$ enters the formula for the optimal wealth tax in general equilibrium.

In Section 1.G, I consider the nonlinear taxation of capital income in a canonical two-period life-cycle framework (Farhi and Werning (2010)). Using standard perturbation techniques, I

[16] For a more detailed exposition, I refer to Section 1.E.

derive the nonlinear incidence of capital taxes in partial and general equilibrium and the optimal tax system. Also, in this environment, Proposition 1 continues to hold. Moreover, by expressing the hazard ratio of the capital income distribution in terms of type and scale dependence, I derive, in the context of optimal nonlinear taxation, an exact version of the approximative formula for the optimal linear tax in part (c) of Proposition 1.

As noted by contributors to the literature (e.g., Guvenen et al. (2019)), a capital income and a wealth tax do not coincide with return heterogeneity. In the Guvenen et al. (2019) framework, type dependence generates return inequality between potentially liquidity-constrained entrepreneurs. A wealth tax can raise efficiency relative to a uniform capital gains tax as the former effectively levies a lower (higher) tax on capital incomes of individuals with a higher (lower) entrepreneurial talent and an exogenously higher (lower) rate of return. In their framework, return rates are independent of the amount of savings for unconstrained entrepreneurs and, given their calibration of the production function, even decreasing in the amount of savings for constrained ones. Therefore, the positive correlation between return rates and wealth in Guvenen et al. (2019) solely arises from type dependence. My model nests this type of return inequality. With scale dependence in the setting of Guvenen et al. (2019), there would be additional efficiency gains from wealth taxes because the lower tax on high-return individuals increases their pre-tax returns and further expands the tax base. This chapter aims to study the effects of scale and type dependence on redistribution where efficiency is one (but not the only) important dimension.

Furthermore, I deal with other policies such as financial education consistent with the leading financial market example (see Section 1.G). Although such policies may be better suited to address return inequality directly, empirical evidence suggests, nonetheless, a residual amount of return inequality governments cannot shut down. The reason is that these policies are also costly, giving rise to a trade-off between equity (reduction in return inequality) and efficiency (education costs). Similarly, a government would also face information acquisition costs if it provided a sovereign wealth fund open to everyone and large enough to absorb all private investment rents. Aside from information costs, such a fund may give rise to other inefficiencies, for example, agency frictions and diversification limits.

Even if these costs declined substantially, it seems unlikely that scale dependence would vanish. For example, in the financial market, the unpredictability of stock market returns may prevent the dissolution of scale dependence. Therefore, in this chapter, I take as given existing inefficiencies that create a residual amount of inequality and analyze tax policy for this given residual inequality.

Finally, the welfare weights may be endogenous to the amount of scale dependence. In the spirit of Saez and Stantcheva (2016), one may generalize the notion of social marginal welfare weights. For example, equity considerations may lead to even lower taxes when a given amount of return inequality comes from scale dependence instead of type dependence only. In the latter case, rich individuals obtain higher rates of return than the poor, for instance, because of an inherent talent they received from their parents. Under scale dependence, individuals may inherit a sizable fortune that allows them, for example, to hire skilled financial advisers the poor cannot afford. More generally, they are gifted by their parents with the absence of frictions the poor have to face. However, the rich still need to incur costs to obtain higher rates of return than the poor, for example, by taking effort. Thus, to some degree, these higher returns reflect a fair compensation for costs the rich undertake. In that sense, scale dependence may reduce inequality concerns in a society, thus, further lowering the optimal capital tax from an equity perspective.

At the same time, political economy considerations may counteract this force. For example, suppose the political power in a society is endogenous to an individual's wealth. In that case, the amplification of wealth inequality by scale dependence causes may create a rich elite that either directly influences tax policy by running for a political office or indirectly by lobbying. Thus, as Saez and Zucman (2019) proposed, it may be desirable in the interest of sustaining democracy to set wealth taxes higher than the revenue-maximizing rate to prevent an "oligarchic drift." From this perspective, return inequality may provide a rationale for higher capital taxes irrespective of its source.

1.3 Empirical Analysis

As described, scale dependence affects optimal tax policy by altering the observed capital inequality and standard elasticity measures. Therefore, in this section, I analyze the empirical implications of scale dependence. First, I describe the conceptual issues posed by scale dependence for estimating capital gains and savings elasticities and revisit estimates from the literature (Section 1.3.1). Next, Section 1.3.2 provides reduced-form macro evidence of scale dependence using the Survey of Consumer Finances (SCF). Finally, I directly estimate own- and cross-return elasticities using panel data on U.S. foundations (Section 1.3.3).

1.3.1 Empirical Implications of Scale Dependence

Conceptual description. First, I describe the implications of scale dependence for estimating capital income and savings elasticities. The standard procedure to estimate the elasticity of capital ($\tilde{\zeta}_i^{a,(1-\tau_K)}$) or capital income ($\tilde{\zeta}_i^{a_R,(1-\tau_K)}$) is to consider individual- or region-specific time-variation in tax rates and study the effects on capital or capital income. Along these lines, consider a tax reform, $d\tau_K$, and abstract from income effects. Then, under scale dependence, the percentage change in the capital holdings of household (or region) i is given by $\frac{da_i}{a_i} = -\tilde{\zeta}_i^{a,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K} + \zeta_i^{a,r} \frac{dr_i}{r_i}$. Similarly, the change in the household's capital income reads as $\frac{da_{R,i}}{a_{R,i}} = -\tilde{\zeta}_i^{a_R,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K} + (1+\zeta_i^{a,r}) \frac{dr_i}{r_i}$.

This formulation immediately reveals the econometric implication of scale dependence for the estimation of *long-run* capital elasticities. In the presence of scale dependence, estimates from data that implicitly hold the return rate fixed ($dr_i = 0$) suffer from an omitted variable bias when trying to identify the long-run elasticities. Then, the estimation misses the adjustment of returns ($dr_i > 0$), and the error term has a non-zero expectation, conditional on the covariates, violating a critical identifying assumption in empirical studies. If $\zeta_i^{a,r} > 0$, the point estimates are biased downward. In other words, wealth and capital income appear to be less responsive than they are in reality.

In the following, I describe three scenarios where this may be the case. Firstly, estimates may be biased when the empiricist does not correctly observe fluctuations in return rates. For data from a short time window, this is likely the case. In the short run, a household's return rate forms, for instance, conditional on her financial knowledge or advisers and a given financial portfolio. In the longer term, she may react to tax reforms, e.g., by hiring other financial advisers, altering the portfolio allocation, and adjusting the pre-tax return rate.

Secondly, using data from tax records, the empiricist misses unrealized capital gains. These are not only but particularly relevant for households from upper parts in the wealth distribution, who, for instance, buy stocks or private equity and hold them for a long time. Therefore, the empiricist does not observe substantial parts of the adjustment in their capital income in response to a tax reform even if the data cover an extended period. This problem also applies to housing, intangible properties, and other assets whose market value only reveals when sold.

Another issue is extrapolating estimates from one group to another in the wealth distribution, even if they are unbiased for the former group. This non-comparability is because portfolios and their flexibility differ significantly across the population. Households from low parts of the wealth distribution mostly hold cash and do not participate in the stock market. Median families have mostly housing. For wealthy households, financial and business assets are pervasive. Therefore, one cannot infer estimates of capital income elasticities from the poor to the rich and vice versa. To overcome these issues, one may directly estimate own- and cross-return elasticities throughout the wealth distribution (see next sections).

Relation to the empirical literature. Now, I summarize two strands of the empirical literature in the light of the described issues. The first strand regards the estimation of the capital income elasticity with respect to the capital gains tax. In the second strand of the literature, contributors estimate the elasticity of capital to wealth taxes. Unfortunately, the number of studies that explicitly address scale dependence is limited. This does, of course, not mean that the other estimates are wrong, but their scope of application depends on the nature of policies under consideration.

Contributors to the empirical literature on the capital income elasticity, starting from Feldstein, Slemrod, and Yitzhaki (1980), employ microdata and time-series mainly from the U.S. In this literature, the focus lies on the estimation of *realization elasticities* (for recent contributions, see Bakija and Gentry (2014), Dowd, McClelland, and Muthitacharoen (2015), and Agersnap and Zidar (2021)). The authors distinguish between transitory and permanent responses. Permanent responses seem to be more relevant for long-run tax policy. However, the estimates also need to account for scale dependence in return rates to apply to long-run capital taxation. Existing studies may not capture them because they are from a short-time window and only include capital gains realizations.

Unlike the sizable research on the elasticity of taxable income, only a few studies have, so far, attempted to estimate the elasticity of capital with respect to wealth taxes. Zoutman (2015) studies the impact of a capital tax reform on wealth accumulation in the Netherlands, noting that the portfolio composition changes over time and responds to the tax reform. However, the data only include cash returns (e.g., dividends and interest), thereby lacking a measure of actual returns. Brülhart, Gruber, Krapf, and Schmidheiny (2016) analyze Swiss time-series and microdata. Since capital gains on movable assets are untaxed in Switzerland, they cannot directly observe individual return rates. Interestingly, their capital elasticity estimates appear stable over the (upper parts of the) wealth distribution, suggesting uniform own-return elasticities aside from homogeneous intertemporal substitution and tax evasion responses.

Seim (2017) provides evidence of bunching at exemption thresholds in Sweden. Whereas being suited for identifying avoidance and evasion responses, such estimates need to be interpreted locally for the respective wealth group and may not represent real responses in the long run (see Kleven (2016)). In Denmark, Jakobsen, Jakobsen, Kleven, and Zucman (2020) estimate the wealth elasticity in a difference-in-difference setup. The estimates do not represent the entire population since the Danish wealth tax only applies to wealthy households in the observation period. To sum up, this literature pays closer attention to unrealized capital gains, which is natural, given its objective to estimate the wealth elasticity. However, the estimates are not readily generalizable to long-run wealth elasticities across the wealth distribution without knowing the amount of scale dependence.

1.3.2 Macro Evidence

In the following, I propose two approaches to directly estimate the amount of scale dependence that one can use to accommodate the capital elasticity and inequality measures appearing in optimal tax formulas. As I demonstrate in Sections 1.2 and 1.4, adjusting these measures for the estimated magnitude of scale dependence is important.

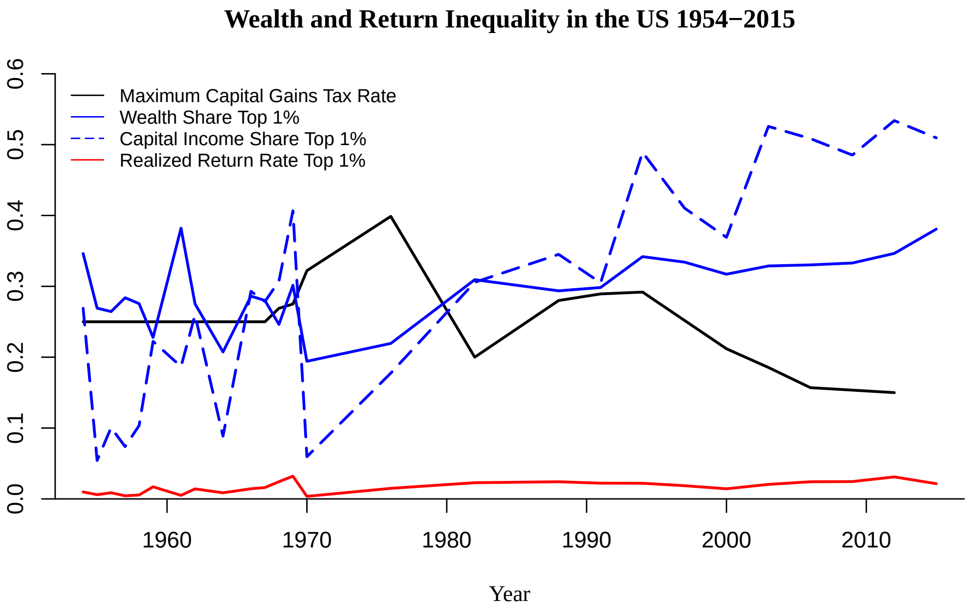
Survey of Consumer Finances. In the spirit of Corollary 1, I estimate scale dependence using the time series of a population group's average return rate and wealth. I extract the household-level asset data from the SCF for 1949-2016 provided by Kuhn, Schularick, and Steins (2020). The representative repeated cross-section contains detailed information on household wealth, portfolio composition, demographic characteristics, and capital income in the U.S. I define net wealth as the market value of all financial and non-financial assets net of the value of total debt. Since income from pension funds and life insurance is exempt from capital taxation, I exclude these assets from the wealth concept.

I divide a household's capital income by the invested capital to calculate each household's *realized return rate* in a given year. Note that capital income is reported retrospectively in the sample. Therefore, to avoid reverse causality, I approximate the invested capital in a period by subtracting the past year's capital income from the current year's net wealth. Moreover, I compute a time series of capital income and wealth shares of the top 1% in the wealth distribution. I also add data on capital taxation from the U.S. Department of the Treasury.

Figure 1.1 displays the evolution of wealth and return inequality in the U.S. over the past decades. The rise in the top 1% capital income share (blue dashed line) is more pronounced than the increase in the respective wealth share (solid blue line). Thus, pre-tax return inequality has grown, consistent with the rising average realized return rate for the top 1% (red line). Simultaneously, the rich experienced a reduction in the capital gains tax rate (black line). Altogether, this suggests that the decline in capital taxation in the U.S. intensified pre-tax return inequality, which is in line with Corollary 1 in Section 1.2.

Estimation of own-return elasticity. The proposition also suggests using responses of aggregate variables, e.g., mean return rates, to identify scale dependence. In this spirit, I regress the average realized return rate of the top 1% on their log mean wealth, giving a highly significant estimate

Figure 1.1: Evolution of Inequality and Capital Taxation in the U.S.



of 0.01.[17] For an average realized return rate of 1.5%, this estimate translates into a lifetime own-return elasticity of $\widehat{\varepsilon_i^{r,a}} \approx 0.78$ (see below for a more detailed description).

1.3.3 Micro Evidence

As argued in Section 1.3.1, using realized return rates to identify scale dependence may be problematic because a substantial share of capital gains is unrealized. Therefore, in the following, I provide evidence from microdata that captures realized and unrealized returns.

Foundation data. I use the publicly available panel data on U.S. foundations that annually report their wealth and income to the IRS in the 990-PF form. The stratified random sample covers approximately 10% of the foundation population. This procedure is similar to Piketty (2014), who uses pooled returns data of U.S. universities. The micro-files on foundations cover the years 1986 to 2016. They include market-valued wealth levels, portfolio compositions, and capital income. All observations are on an individual level.

The foundation data set has three main advantages. Firstly, it allows me to follow the relation between return rates and wealth on an individual (foundation) level over a long period. Sec-

[17] With the maximum capital gains tax rate as an instrument, an IV regression yields a similar but at the 5%-level insignificant point estimate.

ondly, although foundations are institutional investors who potentially behave differently on the financial and non-financial markets, they may serve as a reasonable proxy for wealthy investors. Their portfolios' size is similar, and their assets are also partly shifted to legal entities instead of private bank accounts. Thirdly, as mentioned, the data set contains both realized and unrealized capital gains, and foundations explicitly report donations and withdrawals.

The main disadvantage of the data set, e.g., compared to the SCF, is its limited generalizability to household behavior. The average foundation has a substantially larger endowment than the average household, and, even conditional on the same wealth level, investment behavior may differ. However, one may argue that foundations provide a reasonable proxy for the rich with a similar portfolio size who partly shifts their assets to these entities. Nonetheless, one should be cautious when interpreting the findings in the context of households.

As in Fagereng et al. (2020) and Bach, Calvet, and Sodini (2020), one can directly calculate the investment return of foundation i during a period h, $r_{i,h}$, as the market-value capital income (both realized and unrealized) divided by the average invested capital in that period. Denote foundation i's assets at market value at the beginning of year h as $a_{i,h}$. All the observations are in 2016 dollars. By construction of the empirical specifications below, I only use foundation-year observations with positive beginning-of-year assets. Moreover, to avoid outliers, I exclude foundation-year observations with return rates above 25% and below -25%.[18] As in Saez and Zucman (2016), I classify foundations by their market-value wealth at the beginning of each year into wealth groups $g = 1, ..., 7$ (index set I_g). In Table 1.2, I display descriptive statistics for these different wealth groups.

The first three (four) wealth groups capture the bottom 50% (90%) of foundations. The last two groups cover the top 1% and the top 0.1%, respectively. Foundations achieve a median return rate of 4.9%, with a median portfolio size of \$6 978 721. There is a substantial degree of heterogeneity. Foundations differ in their endowment size (wealth inequality) and their investment returns (return inequality). Whereas small foundations (below \$100k) attain an annual return of 2.7%, the top 1% of foundations gain 5.8% on their investments. A 1% increase in the endowment size is associated with a reduced-form rise in the annual return rate of 0.2%. Notice that the average foundation is substantially wealthier than the typical household. At the same time, their return rates are comparable. Accordingly, the amount of scale dependence is likely to be underestimated in the data, and the resulting estimates can be considered conservative.

[18] If one leaves out foundation-year observations with return rates below the 2.5th and above the 97.5th percentile, the results will be similar. The uncut sample features a kurtosis (above 100 000) far beyond any threshold proposed in the literature for evaluating outliers and fat tails (e.g., see Kline (2015)). After cutting the sample in the proposed manner, the kurtosis drops to 3.7, thus resembling a normal distribution's tail behavior.

Table 1.2: Summary Statistics

Wealth Group I_g	g	Relative Group Size	Wealth Level Mean	Return Rate Mean	Return Rate Std Dev
Below \$100k	1	7.9%	40 574	2.7%	(0.072)
\$100k to \$1m	2	20.6%	416 464	4.9%	(0.076)
\$1m to \$10m	3	24.6%	3 708 144	5.1%	(0.084)
\$10m to \$100m	4	39.9%	31 065 438	5.0%	(0.088)
\$100m to \$500m	5	5.7%	197 396 730	5.4%	(0.091)
\$500m to \$5bn	6	1.1%	1 207 748 745	5.8%	(0.090)
Above \$5bn	7	0.1%	10 238 369 724	5.7%	(0.097)

Note: 254 570 Observations.

Estimation of own-return elasticities. To disentangle the role of type and scale dependence for this return inequality, I utilize the data's panel structure in the following. Like Fagereng et al. (2020), I regress return rates on *beginning-of-year* net wealth

$$log\left(1+r_{i,h}\right)=\varepsilon\cdot log\left(a_{i,h}\right)+f_i+f_h+u_{i,h}, \tag{1.7}$$

where f_i and f_h are individual and time fixed effects. ε measures scale dependence, whereas f_i captures the amount of type dependence. Therefore, individual-specific time variation in wealth identifies scale dependence that arises from any direct or indirect source (e.g., portfolio choice, financial information, stock market participation costs, or liquidity). For instance, donations or withdrawals trigger such time variation in portfolio size.

There may be nonlinearities in scale dependence. In the example of the financial market in Section 1.E, there are decreasing returns to scale. There, the own-return elasticity decreases with wealth. To capture these nonlinearities, I estimate an alternative specification

$$log\left(1+r_{i,h}\right)=\varepsilon\cdot log\left(a_{i,h}\right)+\sum_{g'=2}^{7}\varepsilon_{g'}\cdot log\left(a_{i,h}\right)\cdot D_{g_{i,h},g'}+f_i+f_h+u_{i,h}, \tag{1.8}$$

where $D_{g_{i,h},g'}$ is a dummy variable, indicating a foundation i's affiliation to group g' in period h.

In Table 1.3, I report the estimated coefficients of specifications (1.7) and (1.8). They reveal a highly significant amount of scale dependence. Doubling a foundation's endowment raises its annual return rate by 0.23 percentage points (annual own-return semi-elasticity). There is no evidence for increasing or decreasing returns to scale. Interestingly, for high foundations sizes, the point estimates of (1.8) show (slightly non-significant) decreasing returns to scale that would

Table 1.3: Own-Effects Regressions

	Constant Returns to Scale			Incr./Decr. Returns to Scale
	(1.7)	(1.9)	(1.10)	(1.8)
ε	0.0023***	0.0022***	0.0001***	0.0027***
	(0.0004)	(0.0004)	(0.0000)	(0.0005)
ε_2				0.0002
				(0.0001)
ε_3				0.0002
				(0.0002)
ε_4				0.0001
				(0.0002)
ε_5				−0.0002
				(0.0002)
ε_6				−0.0003
				(0.0002)
ε_7				−0.0008
				(0.0006)
Individual FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
Observations	254 570	254 570	254 570	254 570

Note: Standard Errors (in Parentheses) Clustered by Foundation; $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

be in line with the financial market example in Section 1.E. Thus, whereas the specification cannot confirm the parametrization in the financial market, it does neither reject it.

In specifications (1.9) and (1.10), I replace log net wealth in (1.7) by foundation's group affiliation and percentile in the wealth distribution, both based on foundations' beginning-of-year net wealth:

$$log\left(1+r_{i,h}\right) = \varepsilon \cdot g_{i,h} + f_i + f_h + u_{i,h}, \tag{1.9}$$

and

$$log\left(1+r_{i,h}\right) = \varepsilon \cdot p_{i,h} + f_i + f_h + u_{i,h}, \tag{1.10}$$

where $g_{i,h}$ and $p_{i,h}$ measure foundation i's wealth group affiliation and percentile in period h. Again, there is significant scale dependence.

Including lagged foundation wealth into (1.7) does not change the results qualitatively. Instrumenting foundation wealth in (1.7) with three-year lagged donations yields the same results. Estimating (1.7) separately for boom and bust years (1990, 2001, 2008, and 2009) shows that scale dependence is driven by boom years. Large foundations realize large capital gains (losses) during boom (bust) years because they take more risks than small ones (higher return variance).

Now, I translate the scale dependence estimated with (1.7) into a value for the lifetime own-return elasticity $\varepsilon_i^{r,a}$. Multiply the estimate of (1.7) by $\frac{1+r_{m,h}}{r_{m,h}}$, where $r_{m,h} = 4.9\%$ is the median return rate, to get an estimate of the period-h own-return elasticity of a representative foundation ($\widehat{\varepsilon_{i,h}^{r,a}} \approx 0.05$). To compute the lifetime own-return elasticity, consider the compound return rate $R_m = \left(1+r_{m,h}\right)^H - 1$. Accordingly, one obtains an expression for the lifetime own-return elasticity $\widehat{\varepsilon_i^{r,a}} = \frac{H \cdot \left(1+r_{m,h}\right)^{H-1}}{R_{m,h}} \frac{dlog\left(1+r_{i,h}\right)}{dlog\left(a_{i,h}\right)}$. For $r_{m,h} = 4.9\%$ and $H = 30$, this yields an estimate of $\widehat{\varepsilon_i^{r,a}} \approx 0.1$. Even this conservatively estimated effect leads to a notable adjustment of the optimal capital tax.

Comparison to Fagereng et al. (2020). By comparing the estimate of (1.10) to the one in Fagereng et al. (2020), one can immediately see that the predictions from the foundations' data set may severely understate the amount of scale dependence among households. Based on the wealth distribution in Norway and the estimated scale dependence in Fagereng et al. (2020), I calculate an estimate of the lifetime own-return elasticity of $\widehat{\varepsilon_i^{r,a}} \approx 0.9$ in their data set,[19] which is

[19] Using the wealth distribution reported in Table 1A of Fagereng et al. (2020), I regress the household percentile on log wealth ($\widehat{\frac{dp_{i,h}}{dlog\left(a_{i,h}\right)}} = 0.1443$). Then, note that $\widehat{\frac{dr_{i,h}}{dlog\left(a_{i,h}\right)}} = \widehat{\frac{dr_{i,h}}{dp_{i,h}}} \widehat{\frac{dp_{i,h}}{dlog\left(a_{i,h}\right)}}$, where $\widehat{\frac{dr_{i,h}}{dp_{i,h}}} = 0.1383$ (see Table 9 in Fagereng et al. (2020)), to obtain an estimate for the period-h own-return semi-elasticity. Finally, for $r_{m,h} = 3.2\%$ (reported in Table 3 of Fagereng et al. (2020)) and $H = 30$, I obtain a period-h own-return elasticity of $\widehat{\varepsilon_{i,h}^{r,a}} \approx 0.6$ and a lifetime own-return elasticity of $\widehat{\varepsilon_i^{r,a}} \approx 0.9$. For $r_{m,h} = 5.6\%$, as in the SCF data, $\widehat{\varepsilon_{i,h}^{r,a}} \approx 0.4$ and $\widehat{\varepsilon_i^{r,a}} \approx 0.7$.

higher than the estimate obtained in the microdata but close to the reduced-form macro estimate (Section 1.3.2).

Estimation of cross-return elasticities. Recall that, in general equilibrium, a household's return rate, $r_i\left(a_i,\{a_{i'}\}_{i'\in[0,1]}\right)$, depends not only on one's own savings but also on those of others. A change in the savings by household i' also affects household i's return $\frac{d\left(1+r_{i,h}\right)}{1+r_{i,h}} = \varepsilon_{i,h}^{1+r,a}\cdot\frac{da_{i,h}}{a_{i,h}} + \int_{i'}\gamma_{i,i',h}^{1+r,a}\cdot\frac{da_{i',h}}{a_{i',h}}di'$. To bring this formulation closer to the data, consider the discrete counterpart $\frac{d\left(1+r_{i,h}\right)}{1+r_{i,h}} = \varepsilon_{i,h}^{1+r,a}\cdot\frac{da_{i,h}}{a_{i,h}} + \sum_{i'}\gamma_{i,i',h}^{1+r,a}\cdot\frac{da_{i',h}}{a_{i',h}}$.

In the following, I estimate the magnitude of general equilibrium effects (for each wealth group). To be able to identify cross effects, I impose more structure on these effects. I assume that they are constant over time and multiplicatively separable $\gamma_{i,i',h}^{1+r,a} = \frac{1}{1+r_{i,h}}\delta_{i',h}^{r,a}$, as in the financial market example (Section 1.E). Moreover, let general equilibrium effects be similar within a wealth group $\delta_{i',h}^{r,a}\approx\delta_{g',h}^{r,a}$ for all $i'\in I_{g'}$ and let $\delta_{g',h}^{r,a}$ be small ($\delta_{g',h}^{r,a}\approx 0$). In the estimation, I verify the latter assumption. Define the mean return in wealth group g as $\mathbb{E}_g\left(r_{i,h}\right)$. Then one can write the effect on returns as

$$\frac{d\left(1+r_{i,h}\right)}{1+r_{i,h}} = \varepsilon_{i,h}^{1+r,a}\cdot\frac{da_{i,h}}{a_{i,h}} + \sum_{g'=1}^{7}\delta_{g',h}^{r,a}\cdot\sum_{i'\in I_{g'}}\frac{da_{i',h}}{a_{i',h}}\cdot\frac{1}{1+\mathbb{E}_g\left(r_{i,h}\right)} + u_{i,h}$$

with a bias term $u_{i,h}\equiv\sum_{g'=1}^{7}\frac{\sum_{i'\in I_{g'}}\left[\left(1+\mathbb{E}_g\left(r_{i,h}\right)\right)\left(\delta_{i',h}^{r,a}-\delta_{g',h}^{r,a}\right)+\left(\mathbb{E}_g\left(r_{i,h}\right)-r_{i,h}\right)\delta_{g',h}^{r,a}\right]}{\left(1+r_{i,h}\right)\left(1+\mathbb{E}_g\left(r_{i,h}\right)\right)}\frac{da_{i',h}}{a_{i',h}}$.

For small cross effects ($\delta_{g',h}^{r,a}$) and return rates ($r_{i,h}-\mathbb{E}_g\left(r_{i,h}\right)$) and similar cross-effects in each wealth group ($\delta_{i',h}^{r,a}\approx\delta_{g',h}^{r,a}$), the bias term becomes negligible $u_{i,h}\approx 0$. Therefore, I specify the econometric model by augmenting (1.7) with cross effects

$$log\left(1+r_{i,h}\right) = \varepsilon\cdot log\left(a_{i,h}\right) + \sum_{g'=1}^{7}\delta_{g'}\cdot\overline{log\left(a_{g',h}\right)}\cdot g_{i,h} + f_i + f_h + u_{i,h} \tag{1.11}$$

and, controlling for group-specific effects,

$$log\left(1+r_{i,h}\right) = \varepsilon\cdot log\left(a_{i,h}\right) + \beta\cdot g_{i,h} + \sum_{g'=1}^{7}\delta_{g'}\cdot\overline{log\left(a_{g',h}\right)}\cdot g_{i,h} + f_i + f_h + u_{i,h} \tag{1.12}$$

where, again, $g_{i,h}$ indicates foundation i's group affiliation and $\overline{log\left(a_{g',h}\right)}\equiv\sum_{i'\in I_{g'}}log\left(a_{i',h}\right)$ measures the wealth level of group g in period h.

There are two sources for identifying $\delta_{g'}$: movements in the groups' wealth levels and foundations' mobility between wealth groups. Changes in the foundations' group affiliation arise

Table 1.4: Inter-Group Mobility

	$g_{i,h}=1$	$g_{i,h}=2$	$g_{i,h}=3$	$g_{i,h}=4$	$g_{i,h}=5$	$g_{i,h}=6$	$g_{i,h}=7$
$g_{i,h-1}=1$	15370	599	70	19	1	1	0
$g_{i,h-1}=2$	1073	43217	1239	30	1	1	0
$g_{i,h-1}=3$	29	1106	52820	1553	13	2	0
$g_{i,h-1}=4$	6	10	1035	89911	1151	16	0
$g_{i,h-1}=5$	0	0	1	718	12519	218	1
$g_{i,h-1}=6$	0	0	0	1	134	2536	14
$g_{i,h-1}=7$	0	0	0	0	0	6	216

Note: 225637 Observations.

from donations, withdrawals, and investment returns in the past. In Table 1.4, I display the amount of inter-group mobility.

As the diagonal of this mobility matrix reveals, there is a substantial group persistence (96% of observations). The majority of foundation mobility is between adjacent wealth groups. There is slightly more upward than downward mobility. Overall, 9048 foundation-year group movements identify the inter-group cross effects.

In Table 1.5, I show the coefficients estimated from (1.11) and (1.12). The estimated scale dependence (own-return elasticity) remains relatively stable. Moreover, there are statistically significant cross effects. The estimates reveal no clear relationship between $\delta_{g'}$ and g'. In the financial market example of Section 1.E, this relation would be negative. However, the sizes of the significant coefficients are, from an economic point of view, negligible. The estimates justify using small general equilibrium forces ($\delta^{r,a}_{i,h} \approx 0$) in the comparative statics (Sections 1.2 and 1.C) and validate the identifying assumption that $u_{i,h} \approx 0$ (for $\delta^{r,a}_{i',h} \approx \delta^{r,a}_{g',h}$).

If at all, the estimates of δ_7 are economically relevant. As group 7 represents the top 0.1% of foundations, this indicates the presence of negative effects from the top, as in the general equilibrium financial market. Moreover, using the estimated coefficients from specification (1.12), a simple Wald test does not reject the hypothesis that $\int_{i'} \gamma^{r,a}_{i',i'} di' = 0$ at the 5% level, which is in line with the assumption in the theoretical section.

To account for potential group-specific nonlinearities in cross effects, $\delta_{g',g''}$, I re-estimate equations (1.11) and (1.12). Again, the estimated cross effects are economically small. Moreover, the estimates do not reveal noteworthy nonlinearities. Therefore, I abstain from reporting them separately.

Altogether, I find a statistically significant and economically meaningful amount of scale dependence. The preferred estimate leads to an own-return elasticity of 0.1. Using the statis-

Table 1.5: Cross-Effects Regressions

	Constant Returns to Scale		Constant Returns to Scale	
	(1.11)		(1.12)	
ε	0.0008^{*}		0.0025^{***}	
	(0.0004)		(0.0004)	
δ_1	-0.0005^{*}	$\times 10^{-3}$	0.0007^{***}	$\times 10^{-3}$
	(0.0003)	$\times 10^{-3}$	(0.0002)	$\times 10^{-3}$
δ_2	0.0006	$\times 10^{-4}$	-0.0031^{***}	$\times 10^{-4}$
	(0.0010)	$\times 10^{-4}$	(0.0009)	$\times 10^{-4}$
δ_3	0.0030^{***}	$\times 10^{-4}$	0.0049^{***}	$\times 10^{-4}$
	(0.0007)	$\times 10^{-4}$	(0.0007)	$\times 10^{-4}$
δ_4	-0.0023^{***}	$\times 10^{-4}$	-0.0038^{***}	$\times 10^{-4}$
	(0.0007)	$\times 10^{-4}$	(0.0008)	$\times 10^{-4}$
δ_5	0.0076	$\times 10^{-4}$	0.0306^{***}	$\times 10^{-4}$
	(0.0049)	$\times 10^{-4}$	(0.0055)	$\times 10^{-4}$
δ_6	0.0036^{***}	$\times 10^{-3}$	0.0024^{**}	$\times 10^{-3}$
	(0.0011)	$\times 10^{-3}$	(0.0011)	$\times 10^{-3}$
δ_7	-0.0022^{***}	$\times 10^{-2}$	-0.0037^{***}	$\times 10^{-2}$
	(0.0007)	$\times 10^{-2}$	(0.0007)	$\times 10^{-2}$
Individual FE	Y		Y	
Time FE	Y		Y	
Observations	254 570		254 570	

Note: Standard Errors (in Parentheses) Clustered by Foundation; $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

tics reported in Fagereng et al. (2020), I retrieve an estimate of 0.9 in their data set close to the reduced-form estimate from the SCF in Section 1.3.2 (0.8). In all cases, the resulting adjustment of capital elasticities and inequality measures and the implications for tax policy are quantitatively important. The cross-effects estimates are statistically significant but economically unimportant, suggesting either no or only small general equilibrium effects. Some of the cross-effects estimates seem to be in line with the specified general equilibrium financial market model. More research is needed to assess how far the estimates from foundations apply to household data and whether general equilibrium effects are present.

1.4 Quantitative Illustration

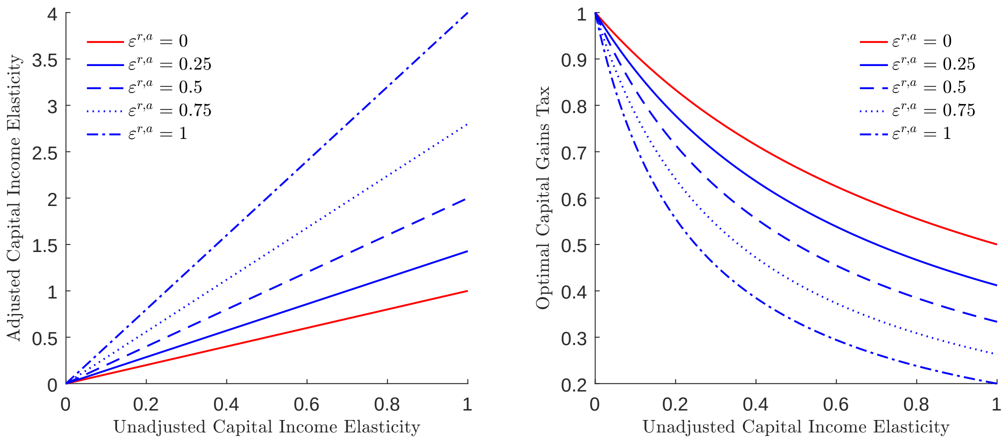
Now, I provide a quantitative exploration of the theoretical results in Section 1.2 for an empirically plausible range of type and scale dependence (see Section 1.3). First, I illustrate the inequality multiplier effect showing the efficiency effects of scale dependence. Then, I show how to adjust capital income inequality measures for scale and type dependence. Finally, I conduct comparative statics exercises of optimal capital gains taxes with respect to type and scale dependence and demonstrate how their relative magnitude matters for optimal taxation.

1.4.1 Inequality Multiplier Effect

As I show in Proposition 1 (b), scale dependence gives rise to an inequality multiplier effect that amplifies the capital income elasticity and reduces the optimal capital gains tax. To demonstrate the quantitative importance of this multiplier effect, I consider, as a first exercise, a set of economies that differ in the magnitude of scale dependence measured by the own-return elasticity. From now on, I set the reduced-form savings elasticity with respect to the rate of return to $\tilde{\zeta}_i^{a,r} = 0.5$.

The Left Panel of Figure 1.2 displays the relationship between unadjusted and adjusted capital income elasticities. The former can be interpreted as the reduced-form elasticity $\overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}}$ that describes the short-run responses of capital income, holding return rates fixed. As explained, the adjusted capital elasticity $\overline{\zeta}^{a_R,(1-\tau_K)}$ accounts for the endogeneity of return rates that makes capital more responsive. As one can see from the figure, the upward adjustment for a given unadjusted elasticity is notable even for smaller values of the own-return elasticity. The higher the underlying unadjusted elasticity, the higher this difference. Moreover, for a given adjusted elasticity measure, the unadjusted elasticity substantially declines in the value of scale dependence. Altogether, scale dependence increases the efficiency costs of raising capital taxes.

Figure 1.2: Elasticity Adjustment



Left Panel: Unadjusted vs Adjusted Capital Income Elasticites; Right Panel: Optimal Rawlsian Capital Gains Taxes

Whereas the measure of capital income inequality, such as the Gini coefficient, is (relatively) easy to observe, the degree to which portfolio returns are scale-dependent and, thus, the size of the correct adjustment of the capital gains elasticity is not. Therefore, I demonstrate the effects of observing the elasticity incorrectly, given that scale dependence shapes the capital income distribution. The Right Panel calculates optimal revenue-maximizing capital gains taxes ($\Gamma_i = 0$) for different values of $\overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}}$. For this objective function, the measure of capital income inequality that carries equity concerns plays no role in the optimal capital gains taxation. The red line is the benchmark where are all return inequality is exogenous (type dependence only). One can observe that scale dependence substantially reduces the optimal capital gains tax (blue lines). The higher the unadjusted elasticity and the greater the amount of scale dependence, the larger the adjustment in the optimal capital gains tax. For instance, an own-return elasticity of 0.25 leads to an adjustment in the capital elasticity of more than 42% . For an unadjusted capital income elasticity of 0.25, the resulting scale-dependence induced reduction in the optimal capital gains tax is around 8%.

1.4.2 Capital Income Inequality

In the previous exercise, I studied the effects of scale dependence on efficiency. However, type and scale dependence also affect a society's equity concerns. In the formula for the optimal capital gains tax, equity concerns are summarized by the inequality measure

$$\mathbb{E}\left[\frac{(1-\Gamma_i)\,a_{R,i}}{\mathbb{E}\,(a_{R,i})}\right] \approx \frac{1}{1-\bar{i}}\zeta_i^{a_R,(1-i)}\mathbb{COV}\,(\Gamma_i, i)\,.$$

This exercise aims to demonstrate how to adjust the measure of capital income inequality by the presence of scale and type dependence.

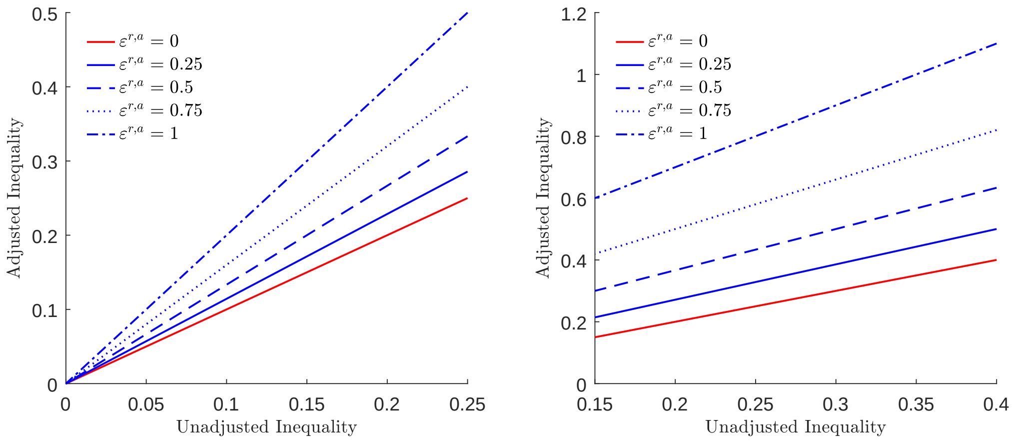
To illustrate this, I set the Pareto weights to $\Gamma_i = 2(1-i)$. This rank-dependent function is known as the Gini social welfare function introduced by Sen (1974). In line with the literature on wealth and income inequality, suppose that capital incomes are Pareto distributed (e.g., Gabaix (2009)). Then, the elasticity of capital income with respect to the household rank is constant $\zeta_i^{a_R,(1-i)} = -1/\lambda_{a_R}$, where λ_{a_R} is the shape parameter of the (steady-state) capital income distribution. Moreover, the inequality measure $I(\tau_K)$ is equal to the Gini coefficient. Using the parametrization of welfare weights, $\mathbb{COV}\,(\Gamma_i, i) = -1/6$. Moreover, I set $\bar{i} = 0.8$. This value is consistent with a shape parameter of $\lambda_{a_R} = 1.6$ (see Saez and Stantcheva (2018)). In Appendix 1.B, I analyze the performance of the approximation for different levels of inequality (λ_{a_R}) and alternative social welfare functions (Γ_i). Moreover, I omit the endogeneity of $\bar{i}$ in this section. In the robustness analysis of Appendix 1.B, I take the endogeneity explicitly into account.

Using this parametrization, I now demonstrate how the inequality measure (Gini coefficient) differs in the short run (unadjusted inequality) and the long run (adjusted inequality). Notice that one can decompose capital income inequality into wealth and return inequality, since $\zeta_i^{a_R,(1-i)} = \zeta_i^{a,(1-i)} + \zeta_i^{r,(1-i)}$. Return inequality $\zeta_i^{r,(1-i)} = \varepsilon_i^{r,a}\zeta_i^{a,(1-i)} + \tilde{\zeta}_i^{r,(1-i)}$ depends on the degree to which it is driven by scale dependence, $\varepsilon_i^{r,a}\zeta_i^{a,(1-i)}$, and a measure of type dependence, $\tilde{\zeta}_i^{r,(1-i)}$, that describes the exogenous differences in return rates. The former depends on the amount of wealth inequality $\zeta_i^{a,(1-i)} = \phi_i\tilde{\zeta}_i^{a,(1-i)} = \frac{\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} + \tilde{\zeta}_i^{a,r}\tilde{\zeta}_i^{r,(1-i)}}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}$ that is also a function of type and scale dependence ($\tilde{\zeta}_i^{r,(1-i)}$ and $\varepsilon_i^{r,a}$).

On the left-hand side of Figure 1.3, I display a situation where all the capital income and wealth inequality is entirely driven by return inequality ($\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = 0$). Without any underlying inequality ($\tilde{\zeta}_i^{r,(1-i)} = 0$), both the adjusted and the unadjusted inequality measures and, thus, the optimal capital tax rates are equal to zero for any amount of scale dependence. A positive amount of type dependence gives rise to capital income inequality. With scale dependence ($\varepsilon^{r,a} > 0$), the adjusted inequality is larger than the unadjusted one. This adjustment is moderate in the absence of any additional source of inequality ($\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = 0$).

However, the difference between the two inequality measures is more sizable when there is another source of wealth inequality, as shown in the Right Panel, where I set $\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} =$

Figure 1.3: Inequality Adjustment



Left Panel: Unadjusted vs Adjusted Inequality ($\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = 0$); Right Panel: Unadjusted vs Adjusted Inequality ($\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = -0.18$)

-0.18.[20] Therefore, a high long-run Gini coefficient of capital income (adjusted inequality) is not necessarily driven by growing type dependence (unadjusted inequality). Under scale dependence, the rich become richer because they are rich. Their accumulation of wealth makes capital income more unequal in the long run than in the short run. This effect is more substantial when there is more underlying inequality.

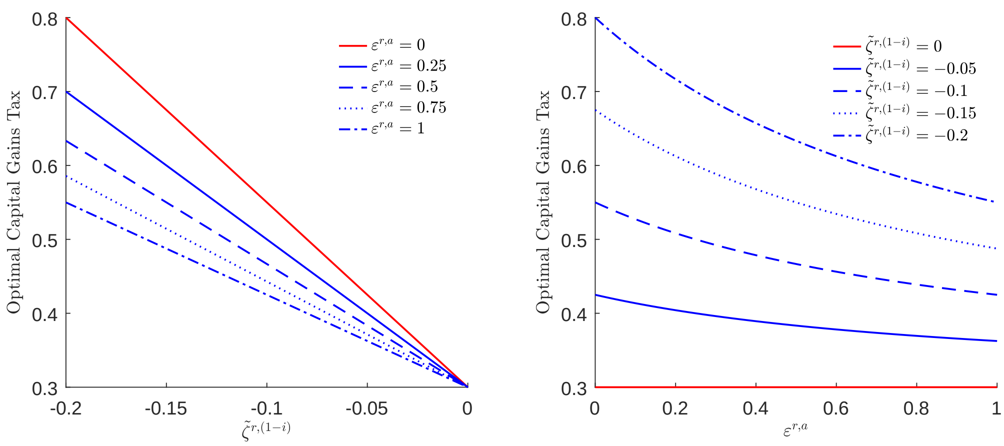
To provide an example, without an underlying inequality ($\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = 0$), an own-return elasticity of 0.25 adjusts the Gini coefficient from 0.2 to 0.23. For $\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} = -0.18$, the adjustment is more pronounced (from 0.2 to 0.27).

1.4.3 Optimal Capital Gains Taxation

Altogether, scale dependence raises the elasticity of capital income and amplifies capital income inequality. Although type dependence also contributes to capital income inequality, it does not

[20] I obtain this value in two steps. First, I calculate the shape parameter of the return rate distribution $\zeta_i^{r,(1-i)}$. In line with Saez and Stantcheva (2018), let $\zeta_i^{a_R,(1-i)} = -1/1.6$. Moreover, I set $\zeta_i^{a,(1-i)} = -1/\lambda_a = -1/3$, which is consistent with the fact that the top 10% wealth share is twice the top 1% wealth share and four times the top 0.1% wealth share (see, for instance, Piketty (2014)). Accordingly, $\zeta_i^{r,(1-i)} = \zeta_i^{a_R,(1-i)} - \zeta_i^{a,(1-i)} \approx -0.3$. Secondly, I assume that, in the baseline scenario, half of the return inequality is driven by type dependence, $\tilde{\zeta}_i^{r,(1-i)} = \frac{1}{2}\zeta_i^{r,(1-i)}$, and the other half is due to scale dependence, $\varepsilon_i^{r,a}\zeta_i^{a,(1-i)} = \frac{1}{2}\zeta_i^{r,(1-i)}$. This assumption implies an own-return elasticity of $\varepsilon_i^{r,a} \approx 0.45$ that is just in the middle of the range identified in Section 1.3. Moreover, it gives the value of $\tilde{\zeta}_i^{a,R_1}\tilde{\zeta}_i^{R_1,(1-i)} \approx -0.18$.

Figure 1.4: Optimal Capital Income Taxation



Left Panel: Optimal Capital Income Taxes vs Type Dependence; Right Panel: Optimal Capital Income Taxes vs Scale Dependence

alter the capital income elasticity. Therefore, the implications for tax policy of rising return inequality are non-trivial.

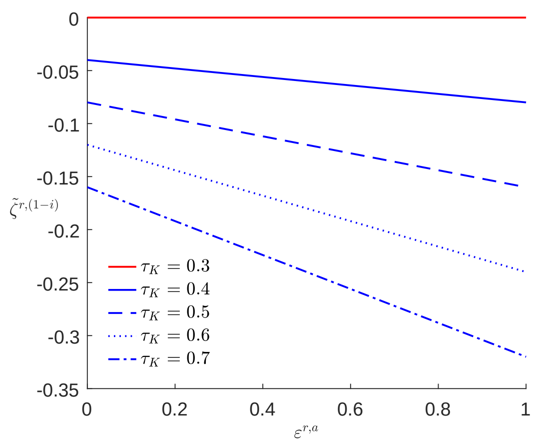
In this numerical exercise, I explore the role of type and scale dependence for the optimal taxation of capital gains, holding all other primitives fixed. This illustrates Proposition 1 (c). Again, let $\tilde{\zeta}_i^{a,r} = \tilde{\zeta}_i^{a,(1-\tau_K)} = 0.5$, $\tilde{\zeta}_i^{a,R_1} \tilde{\zeta}_i^{R_1,(1-i)} = -0.18$, $\bar{i} = 0.8$, and $\Gamma_i = 2(1-i)$.

In the Left Panel of Figure 1.4, I present the optimal capital gains tax as a function of the reduced-form return inequality that measures type dependence, $\tilde{\zeta}_i^{r,(1-i)}$. As argued theoretically, more type dependence (smaller $\tilde{\zeta}_i^{r,(1-i)}$) translates into a greater capital income inequality. Thus, a rise in inequality that is induced by type dependence calls for higher capital taxation. However, this adjustment in the optimal tax is weaker the greater the magnitude of scale dependence.

For example, consider an increase in type dependence from -0.1 to -0.2. Without scale dependence, this leads to a rise in the optimal capital gains tax by 45%. For an own-return elasticity of 0.25, the respective increase is 40%.

The Right Panel of the figure shows the optimal capital tax for different values of scale dependence, measured by the own-return elasticity. A rise in inequality triggered by more scale dependence tends to reduce the optimal capital gains tax rate. The adjustment due to scale dependence depends on the amount of type dependence. Without type dependence, there is no adjustment (full neutrality of scale dependence, red line). Thus, the larger the underlying type dependence, the stronger is the reduction in optimal capital income taxes due to a scale

Figure 1.5: Type vs Scale Dependence



dependence-induced rise in return inequality. For example, given a type dependence of $\tilde{\zeta}_i^{r,(1-i)} = -0.1$, a rise in $\varepsilon^{r,a}$ from 0.2 to 0.4 lowers the optimal capital gains tax by around 6%. This reduction is more pronounced (more than 8%) when type dependence is, for instance, $\tilde{\zeta}_i^{r,(1-i)} = -0.2$.

To further explore the relative importance of scale and type dependence, I finally compute the isoquants of the optimal tax function in Figure 1.5. Consider a rise in capital income inequality that is driven by both type and scale dependence. The figure reveals that a rise in inequality does not necessarily alter optimal capital taxation. There are combinations of type and scale dependence for which rising inequality is entirely neutral for tax policy. For example, at an optimal capital tax of 50%, a rise in scale dependence from $\varepsilon^{r,a} = 0.2$ to $\varepsilon^{r,a} = 0.4$ cancels out an increase in type dependence by 17%.

In this section, I quantitatively explored the consequences of scale and type dependence for common sufficient statics, such as the elasticity and inequality of capital income, as well as the optimal capital taxation. Altogether, these measures may substantially differ in the short and long run due to the endogeneity of pre-tax return rates. Moreover, type and scale dependence have opposing effects on optimal taxation. While a higher scale dependence tends to reduce taxes because the adjustment in the efficiency costs dominates the inequality rise, type dependence raises optimal taxes as the efficiency channel is absent. Overall, one cannot infer from an increase in capital income inequality that capital taxes should rise. Depending on the source of this inequality rise, it can have qualitatively opposing policy implications. This difference can be quantitatively considerable.

1.5 Conclusion

This chapter introduces scale and type dependence into the optimal taxation of capital. I show that it does not only matter if and how much return inequality there is, but the source of inequality is essential for tax policy. Both type and scale dependence raise capital inequality. Scale dependence, however, makes capital more elastic to tax reforms, expanding the efficiency costs of capital taxation. I show how to adjust standard sufficient statistics that determine the capital elasticity for scale dependence. These need to account for an inequality multiplier effect between wealth and its pre-tax return.

When scale dependence raises inequality, optimal capital taxes decline because the inequality multiplier effect offsets the increase in the observed level of inequality. Conversely, capital tax rates should increase if the same rise in inequality were driven by type dependence. As a consequence, a government should address changes in capital inequality very differently depending on their source. Therefore, exploring the magnitude of scale dependence relative to type dependence is an important avenue for future research.

Appendix

1.A Proofs for Section 1.2

1.A.1 Part (a) of Proposition 1

With and without scale dependence, the government solves $\max_{\tau_K} \int_i \Gamma_i U(\tau_K; i)\, di$ subject to $\int_i \tau_K a_{R,i} di \geq \overline{E}$. Assume that the optimization problem is concave. Taking the derivative of the Lagrangian function $\mathscr{L} = \int_i \Gamma_i U(\tau_K; i)\, di + \lambda \left[\int_i \tau_K a_{R,i} di - \overline{E}\right]$ with respect to τ_K, the first-order condition reads as

$$\int_i (\Gamma_i/\lambda) \frac{dU(\tau_K; i)}{d\tau_K} di + \int_i a_{R,i} di = \frac{\tau_K}{1-\tau_K} \int_i a_{R,i} \zeta_i^{a_R,(1-\tau_K)} di. \tag{1.13}$$

With a utility function that is quasilinear in the consumption of final wealth, the first-order effect on household utility is given by $\frac{dU(\tau_K;i)}{d\tau_K} = -a_{R,i}$ and the shadow value of public funds is equal to $\lambda = \int_i \Gamma_i di = 1$. Simplify (1.13) to obtain the Ramsey formula for the optimal capital gains tax.

1.A.2 Part (b) of Proposition 1

Without scale dependence, the average elasticity of capital income simplifies to

$$\overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}} = \int_i \frac{a_{R,i}}{\mathbb{E}(a_{R,i})} \tilde{\zeta}_i^{a_R,(1-\tau_K)} di = \tilde{\zeta}_i^{a_R,(1-\tau_K)} = \tilde{\zeta}_i^{a,(1-\tau_K)} \tag{1.14}$$

for constant elasticities. Define $\phi_i \equiv \frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}$ and $\Phi_i \equiv \left(1+\varepsilon_i^{r,a}\right)\phi_i$. With scale dependence, the household elasticity of savings

$$\begin{aligned}
\zeta_i^{a,(1-\tau_K)} &\equiv \frac{dlog(a_i)}{dlog(1-\tau_K)} = \frac{dlog(a_i)}{dlog(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}} + \frac{dlog(a_i)}{dlog(r_i)} \frac{dlog[r_i(a_i)]}{dlog(a_i)} \frac{dlog(a_i)}{dlog(1-\tau_K)} \\
&= \tilde{\zeta}_i^{a,(1-\tau_K)} + \tilde{\zeta}_i^{a,r} \varepsilon_i^{r,a} \zeta_i^{a,(1-\tau_K)} = \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}
\end{aligned}$$

and the capital income elasticity

$$\zeta_i^{a_R,(1-\tau_K)} \equiv \frac{dlog\left[a_i r_i\left(a_i\right)\right]}{dlog\left(1-\tau_K\right)} = \frac{dlog\left(a_i\right)}{dlog\left(1-\tau_K\right)} + \frac{dlog\left[r_i\left(a_i\right)\right]}{dlog\left(a_i\right)} \frac{dlog\left(a_i\right)}{dlog\left(1-\tau_K\right)}$$
$$= \left(1+\varepsilon_i^{r,a}\right) \zeta_i^{a,(1-\tau_K)} = \left(1+\varepsilon_i^{r,a}\right) \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}$$

both account for the endogenous return rate. Then, the average capital income elasticity with scale dependence

$$\overline{\zeta}^{a_R,(1-\tau_K)} = \int_i \frac{a_{R,i}}{\mathbb{E}\left(a_{R,i}\right)} \left(1+\varepsilon_i^{r,a}\right) \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)} di = \frac{1+\varepsilon_i^{r,a}}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}} \tilde{\zeta}_i^{a,(1-\tau_K)} \tag{1.15}$$

is larger than the one without $\overline{\zeta}^{a_R,(1-\tau_K)} > \overline{\zeta}^{a_R,(1-\tau_K)}|_{\{r_i\}_{i\in[0,1]}}$ for $\varepsilon_i^{r,a} > 0$.

1.A.3 Part (c) of Proposition 1

The response of the inequality measure $I\left(\tau_K\right)$ can be written as

$$I'\left(\tau_K\right) = -\frac{1}{1-\tau_K} \frac{\int_i a_{R,i} di \cdot \int_i \left(1-\Gamma_i\right) a_{R,i} \zeta_i^{a_R,(1-\tau_K)} di - \int_i \zeta_i^{a_R,(1-\tau_K)} a_{R,i} di \cdot \int_i \left(1-\Gamma_i\right) a_{R,i} di}{\left(\int_i a_{R,i} di\right)^2}.$$

For constant elasticities $\tilde{\zeta}_i^{a,(1-\tau_K)}$, $\tilde{\zeta}_i^{a,r}$, and $\varepsilon_i^{r,a}$, the capital income elasticity, $\zeta_i^{a_R,(1-\tau_K)}$, is also uniform across the population. Accordingly, the denominator of $I'\left(\tau_K\right)$ is equal to zero.

Define $\bar{i}$ such that $a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right) = \mathbb{E}\left(a_i r_i\left(a_i\right)\right)$. Then, approximate each household's capital income around the one of household $\bar{i}$, $a_i r_i = a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right) - \frac{d a_i r_i}{d(1-i)} \frac{1-\bar{i}}{a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right)} \frac{i-\bar{i}}{1-\bar{i}} a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right) + o\left(i-\bar{i}\right)$, such that $\frac{a_i r_i - a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right)}{a_{\bar{i}} r_{\bar{i}}\left(a_{\bar{i}}\right)} = \frac{a_{R,i} - \mathbb{E}\left(a_{R,i}\right)}{\mathbb{E}\left(a_{R,i}\right)} = -\zeta_i^{a_R,(1-i)} \frac{i-\bar{i}}{1-\bar{i}} + o\left(i-\bar{i}\right)$, where $\zeta_i^{a_R,i} \equiv \frac{dlog\left(a_{R,i}\right)}{dlog(1-i)}$. Therefore, for constant elasticities,

$$I\left(\tau_K\right) = \mathbb{E}\left[\left(1-\Gamma_i\right) \frac{a_{R,i}}{\mathbb{E}\left(a_{R,i}\right)}\right] = \mathbb{E}\left[\left(1-\Gamma_i\right) \frac{a_{R,i} - \mathbb{E}\left(a_{R,i}\right)}{\mathbb{E}\left(a_{R,i}\right)}\right]$$
$$\approx -\frac{1}{1-\bar{i}} \zeta_i^{a_R,(1-i)} \mathbb{E}\left[\left(1-\Gamma_i\right)\left(i-\bar{i}\right)\right] = \frac{1}{1-\bar{i}} \zeta_i^{a_R,(1-i)} \mathbb{COV}\left(\Gamma_i, i\right).$$

Moreover, the wealth elasticity with respect to the household rank can be written as

$$\zeta_i^{a,(1-i)} = \frac{dlog\left(a_i\right)}{dlog\left(R_{1,i}\right)} \frac{dlog\left(R_{1,i}\right)}{dlog\left(1-i\right)}|_{\{r_i\}_{i\in[0,1]}} + \frac{dlog\left(a_i\right)}{dlog\left(r_i\right)} \frac{dlog\left(r_i\left(a_i\right)\right)}{dlog\left(1-i\right)}|_{\{r_i\}_{i\in[0,1]}}$$
$$+ \frac{dlog\left(a_i\right)}{dlog\left(r_i\right)} \frac{dlog\left(r_i\left(a_i\right)\right)}{dlog\left(a_i\right)} \frac{dlog\left(a_i\right)}{dlog\left(1-i\right)} = \phi_i \left(\tilde{\zeta}_i^{a,R_1} \tilde{\zeta}_i^{R_1,(1-i)} + \tilde{\zeta}_i^{a,r} \tilde{\zeta}_i^{r,(1-i)}\right) = \phi_i \tilde{\zeta}_i^{a,(1-i)},$$

where $R_{1,i}$ is a household's first-period (after-tax) labor income. Hence, the respective capital income elasticity is given by

$$\zeta_i^{a_R,(1-i)} = \frac{dlog\left(a_i r_i\left(a_i\right)\right)}{dlog\left(1-i\right)} = \left(1+\varepsilon_i^{r,a}\right)\zeta_i^{a,(1-i)} + \frac{\partial log\left(r_i\left(a_i\right)\right)}{\partial log\left(1-i\right)}|_{\{r_i\}_{i\in[0,1]}} = \Phi_i \tilde{\zeta}_i^{a,(1-i)} + \tilde{\zeta}_i^{r,(1-i)}$$

and the optimal tax rate reads as

$$\frac{\tau_K}{1-\tau_K} \approx \frac{1}{1-\bar{i}} \frac{\Phi_i \tilde{\zeta}_i^{a,(1-i)} + \tilde{\zeta}_i^{r,(1-i)}}{\Phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}} \mathbb{COV}\left(\Gamma_i, i\right).$$

Note that, for given elasticities and Pareto weights, this formula is not entirely in closed form since the household that earns the mean capital income $\bar{i}$ is an endogenous variable. I omit this endogeneity for simplicity.

1.A.4 Corollary 1

The change in mean returns, $\mathbb{E}\left(r_i\right) = \int_i r_i\left(a_i\right) di$, from a tax reform $d\tau_K$ can be expressed as

$$\begin{aligned} d\mathbb{E}\left(r_i\right) &= -\int_i r_i\left(a_i\right) \frac{dlog\left[r_i\left(a_i\right)\right]}{dlog\left(a_i\right)} \frac{dlog\left(a_i\right)}{dlog\left(1-\tau_K\right)} di \cdot \frac{d\tau_K}{1-\tau_K} \\ &= -\mathbb{E}\left(r_i\right) \varepsilon_i^{r,a} \zeta_i^{a,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K}. \end{aligned}$$

Similarly, differentiate the variance of returns, $\mathbb{V}\left(r_i\right) = \mathbb{E}\left(r_i^2\right) - \mathbb{E}\left(r_i\right)^2$,

$$\begin{aligned} d\mathbb{V}\left(r_i\right) &= -2\mathbb{E}\left(r_i^2\right) \varepsilon_i^{r,a} \zeta_i^{a,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K} + 2\mathbb{E}\left(r_i\right)^2 \varepsilon_i^{r,a} \zeta_i^{a,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K} \\ &= -2\mathbb{V}\left(r_i\right) \varepsilon_i^{r,a} \zeta_i^{a,(1-\tau_K)} \frac{d\tau_K}{1-\tau_K}. \end{aligned}$$

Whenever $\zeta_i^{r,a} > 0$, $d\mathbb{E}\left(r_i\right) < 0$ and $d\mathbb{V}\left(r_i\right) < 0$.

1.A.5 Proposition 2

Optimal taxation in general equilibrium. As in 1.A.1, one calculates the social planner's first-order condition

$$\int_i \left(\Gamma_i/\lambda\right) \frac{dU\left(\tau_K;i\right)}{d\tau_K} di + \int_i \left(\Gamma_i/\lambda\right) \frac{dU\left(\tau_K;i\right)}{dr_i} \int_{i'} \frac{dr_i}{da_{i'}} \frac{da_{i'}}{d\tau_K} di' di + \int_i a_{R,i} di = \frac{\tau_K}{1-\tau_K} \int_i a_{R,i} \zeta_i^{a_R,(1-\tau_K)} di, \tag{1.16}$$

where the second term on the left-hand side of (1.16) collects cross-effects in each households' return rates. Note that by the quasilinearity of the utility function $\frac{dU(\tau_K;i)}{dr_i} = (1-\tau_K)\,a_i$. Using the definition of cross-return elasticities, the first-order inter-household effects simplify to

$$\int_i (\Gamma_i/\lambda)\,\frac{dU(\tau_K;i)}{dr_i}\int_{i'}\frac{dr_i}{da_{i'}}\frac{da_{i'}}{d\tau_K}di'di = -\int_i \Gamma_i a_{R,i}\int_{i'}\gamma_{i,i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di'di,$$

leading to the optimal capital gains tax in general equilibrium.

Elasticities in general equilibrium. Observe that, aside from collecting general equilibrium effects, one needs to adjust the elasticities. With multiplicatively separable cross-return elasticities $\gamma_{i,i'}^{r,a} = \frac{1}{r_i}\delta_{i'}^{r,a}$, the savings elasticity is

$$\begin{aligned}\zeta_i^{a,(1-\tau_K)} &= \tilde{\zeta}_i^{a,(1-\tau_K)} + \tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}\zeta_i^{a,(1-\tau_K)} + \int_{i'}\frac{dlog\,(a_i)}{dlog\,(r_i)}\frac{dlog\,[r_i\,(\cdot)]}{dlog\,(a_{i'})}\frac{dlog\,(a_{i'})}{dlog\,(1-\tau_K)}di' \\ &= \phi_i\tilde{\zeta}_i^{a,(1-\tau_K)} + \phi_i\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di'.\end{aligned}$$

Multiply the left-hand side by δ_i and integrate out to get

$$\begin{aligned}\int_{i'}\delta_{i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di' &= \int_{i'}\delta_{i'}^{r,a}di'\cdot\phi_i\tilde{\zeta}_i^{a,(1-\tau_K)} + \phi_i\tilde{\zeta}_i^{a,r}\int_{i'}\delta_{i'}^{r,a}\frac{1}{r_{i'}}di'\cdot\int_{i'}\delta_{i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di' \\ &= \int_{i'}\delta_{i'}^{r,a}di'\cdot\phi_i\tilde{\zeta}_i^{a,(1-\tau_K)},\end{aligned}$$

where the second equality follows by the simplifying assumption that cross-effects average out $\int_{i'}\gamma_{i',i'}^{r,a}di' = 0$. Moreover, if $\delta_{i'}^{r,a}$ decreases in i' (whereas return rates increase in i'),

$$\underbrace{\mathbb{COV}\left(\frac{1}{r_{i'}},\delta_{i'}^{r,a}\right)}_{>0} = \int_{i'}\gamma_{i',i'}^{r,a}di' - \mathbb{E}\left(\frac{1}{r_{i'}}\right)\mathbb{E}\left(\delta_{i'}^{r,a}\right) = \underbrace{-\mathbb{E}\left(\frac{1}{r_{i'}}\right)}_{<0}\mathbb{E}\left(\delta_{i'}^{r,a}\right).$$

Then, $\mathbb{E}\left(\delta_{i'}^{r,a}\right) = \int_{i'}\delta_{i'}^{r,a}di'$ must be negative and the elasticity is smaller in general than in partial equilibrium

$$\zeta_i^{a,(1-\tau_K)} = \left(1+\tilde{\zeta}_i^{a,r}\phi_i\frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}di'\right)\phi_i\tilde{\zeta}_i^{a,(1-\tau_K)} < \phi_i\tilde{\zeta}_i^{a,(1-\tau_K)}. \tag{1.17}$$

Notice that the savings elasticity is increasing in i.

The elasticity of capital income can be written as

$$\begin{aligned}\zeta_i^{a_R,(1-\tau_K)} &= \left(1+\varepsilon_i^{r,a}\right)\zeta_i^{a,(1-\tau_K)} + \int_{i'}\frac{dlog\,(a_ir_i)}{dlog\,(r_i)}\frac{dlog\,[r_i\,(\cdot)]}{dlog\,(a_{i'})}\frac{dlog\,(a_{i'})}{dlog\,(1-\tau_K)}di' \\ &= \left(1+\varepsilon_i^{r,a}\right)\zeta_i^{a,(1-\tau_K)} + \left(1+\tilde{\zeta}_i^{a,r}\right)\frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\zeta_{i'}^{a,(1-\tau_K)}di'.\end{aligned} \tag{1.18}$$

Assuming positive savings elasticities, the second term on the right-hand side is, again, negative since

$$\int_{i'} \delta_{i'}^{r,a} \zeta_{i'}^{a,(1-\tau_K)} di' = \underbrace{\mathbb{COV}\left(\delta_{i'}^{r,a}, \zeta_{i'}^{a,(1-\tau_K)}\right)}_{<0} + \underbrace{\mathbb{E}\left(\delta_{i'}^{r,a}\right)}_{<0} \underbrace{\mathbb{E}\left(\zeta_{i'}^{a,(1-\tau_K)}\right)}_{>0} < 0.$$

Thus, in general equilibrium, one needs to downward adjust the capital income elasticity

$$\zeta_i^{a_R,(1-\tau_K)} < \left(1+\varepsilon_i^{r,a}\right) \zeta_i^{a,(1-\tau_K)} < \left(1+\varepsilon_i^{r,a}\right) \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}.$$

Furthermore, the general equilibrium welfare effects $\gamma_i^{r,(1-\tau_K)} = \frac{1}{r_i} \int_{i'} \delta_{i'}^{r,a} \zeta_{i'}^{a,(1-\tau_K)} di'$ are negative because $\int_{i'} \delta_{i'}^{r,a} \zeta_{i'}^{a,(1-\tau_K)} di' < 0$.

Comparative statics 1. This comparative statics compares the optimal general equilibrium capital tax to one in a self-confirming policy equilibrium. Firstly, express the capital gains elasticity in Equation (1.18) as

$$\zeta_i^{a_R,(1-\tau_K)} = \underbrace{\left(1+\varepsilon_i^{r,a}\right) \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}}_{c_1} + \underbrace{\left(1+\varepsilon_i^{r,a} + \tilde{\zeta}_i^{a,r} \phi_i\right) \phi_i \tilde{\zeta}_i^{a,(1-\tau_K)} \int_{i'} \delta_{i'}^{r,a} di' \cdot \frac{1}{r_i}}_{c_2}, \tag{1.19}$$

where $c_1 > 0$ and $c_2 < 0$ are constants. Use this expression to write the measure of inequality that serves as a sufficient statistic for the optimal capital income tax as

$$I'\left(\tau_K^{GE}\right) = \frac{-c_2}{1-\tau_K^{GE}} \frac{\mathbb{E}\left(a_i\right) \mathbb{E}\left(\Gamma_i a_{R,i}\right) - \mathbb{E}\left(a_{R,i}\right) \mathbb{E}\left(\Gamma_i a_i\right)}{\left[\mathbb{E}\left(a_{R,i}\right)\right]^2}.$$

Notice that $\mathbb{COV}\left(\Gamma_i, a_{R,i}\right) < 0$, $\mathbb{COV}\left(\Gamma_i, a_i\right) < 0$, and, by the fact that capital income is convex in savings, $\mathbb{COV}\left(\Gamma_i, a_{R,i}\right) < \mathbb{COV}\left(\Gamma_i, a_i\right)$. Therefore, $I'\left(\tau_K^{GE}\right)$ is negative since

$$\begin{aligned} \mathbb{E}\left(a_i\right) \mathbb{E}\left(\Gamma_i a_{R,i}\right) - \mathbb{E}\left(a_{R,i}\right) \mathbb{E}\left(\Gamma_i a_i\right) &= \mathbb{E}\left(a_i\right) \mathbb{COV}\left(\Gamma_i, a_{R,i}\right) - \mathbb{E}\left(a_{R,i}\right) \mathbb{COV}\left(\Gamma_i, a_i\right) \\ &= \underbrace{\mathbb{E}\left(a_{R,i}\right)}_{>0} \underbrace{\left[\mathbb{COV}\left(\Gamma_i, a_{R,i}\right) - \mathbb{COV}\left(\Gamma_i, a_i\right)\right]}_{<0} \\ &\quad + \underbrace{\mathbb{E}\left(\left(1-r_i\right) a_i\right)}_{>0} \underbrace{\mathbb{COV}\left(\Gamma_i, a_{R,i}\right)}_{<0} \end{aligned}$$

for $r_i \in [0,1]$.

In the following, I approximate individual and aggregate variables in general equilibrium (and evaluated at the general equilibrium tax) around the values one would obtain when having the partial equilibrium tax rate. In other words, to show that $\tau_K^{GE} > \tau_K^{PE}$, for small general equilibrium forces ($\delta_{i'}^{r,a} \approx 0$ and $\tau_K^{GE} \approx \tau_K^{PE}$), I apply a Taylor expansion to the optimal capital income tax

$$\frac{\tau_K^{GE}}{1-\tau_K^{GE}} = \frac{\mathbb{E}\left[\left(1-\Gamma_i\left(1+\gamma_i^{r,(1-\tau_W)}\right)\right)a_{R,i}\left(\tau_K^{GE}\right)\right]}{\mathbb{E}\left[\left(c_1+\frac{1}{r_i^{GE}}c_2\right)a_{R,i}\left(\tau_K^{GE}\right)\right]}.$$

A household's capital income in general equilibrium is approximately

$$\begin{aligned}
a_{R,i}\left(\tau_K^{GE}\right) &= a_{R,i}\left(\tau_K^{PE}\right) - \left(\tau_K^{GE}-\tau_K^{PE}\right)\frac{da_{R,i}}{d\left(1-\tau_K\right)} + o\left(\tau_K^{GE}-\tau_K^{PE}\right) \\
&= a_{R,i}\left(\tau_K^{PE}\right) - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{GE}}\zeta_i^{a_R,(1-\tau_K)}a_{R,i}\left(\tau_K^{PE}\right) + o\left(\tau_K^{GE}-\tau_K^{PE}\right) \\
&= a_{R,i}\left(\tau_K^{PE}\right) - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{PE}}c_1 a_{R,i}\left(\tau_K^{PE}\right) + o\left(\tau_K^{GE}-\tau_K^{PE}\right),
\end{aligned}$$

keeping in mind that the elasticities are evaluated in general equilibrium. Similarly, approximate aggregate variables

$$\begin{aligned}
\mathbb{E}\left[\zeta_i^{a_R,(1-\tau_K)}a_{R,i}\left(\tau_K^{GE}\right)\right] &= c_1\mathbb{E}\left[a_{R,i}\left(\tau_K^{PE}\right)\right] + c_2\mathbb{E}\left[a_i\left(\tau_K^{PE}\right)\right] \\
&\quad - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{PE}}c_1^2\mathbb{E}\left[a_{R,i}\left(\tau_K^{PE}\right)\right] + o\left(\tau_K^{GE}-\tau_K^{PE}\right)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}\left[\left(1-\Gamma_i\left(1+\gamma_i^{r,(1-\tau_W)}\right)\right)a_{R,i}\left(\tau_K^{GE}\right)\right] &= \mathbb{E}\left[\left(1-\Gamma_i\left(1+\gamma_i^{r,(1-\tau_W)}\right)\right)a_{R,i}\left(\tau_K^{PE}\right)\right] \\
&\quad - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{PE}}c_1\mathbb{E}\left[\left(1-\Gamma_i\right)a_{R,i}\left(\tau_K^{PE}\right)\right] + o\left(\tau_K^{GE}-\tau_K^{PE}\right).
\end{aligned}$$

Use the fact that, in the self-confirming policy equilibrium, $\frac{\tau_K^{PE}}{1-\tau_K^{PE}} = \frac{\mathbb{E}\left[(1-\Gamma_i)a_{R,i}\left(\tau_K^{PE}\right)\right]}{c_1\mathbb{E}\left[a_{R,i}\left(\tau_K^{PE}\right)\right]}$ to express the general equilibrium tax in terms of the one in partial equilibrium

$$\frac{\tau_K^{GE}}{1-\tau_K^{GE}} = \frac{\tau_K^{PE}}{1-\tau_K^{PE}}\cdot\Delta + o\left(\tau_K^{GE}-\tau_K^{PE}\right), \tag{1.20}$$

where

$$\Delta \equiv \frac{1-\frac{\mathbb{E}\left[\Gamma_i\gamma_i^{r,(1-\tau_W)}a_{R,i}\left(\tau_K^{PE}\right)\right]}{\mathbb{E}\left[(1-\Gamma_i)a_{R,i}\left(\tau_K^{PE}\right)\right]} - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{PE}}c_1}{1+\frac{c_2\mathbb{E}\left[a_i\left(\tau_K^{PE}\right)\right]}{c_1\mathbb{E}\left[a_{R,i}\left(\tau_K^{PE}\right)\right]} - \frac{\tau_K^{GE}-\tau_K^{PE}}{1-\tau_K^{PE}}c_1}.$$

Noting that $\Delta > 1$ since $\gamma_i^{r,(1-\tau_W)} < 0$ and $c_2 < 0$, as defined in Equation (1.18), concludes the proof.

Comparative statics 2. In this exercise, I compare the optimal capital gains tax in general equilibrium to the one in partial equilibrium holding all other primitives of the economy fixed. As in part (c), define $\bar{i}$ as the household who earns the average income $a_{R,\bar{i}} = \mathbb{E}(a_{R,i})$ and approximate each household's capital income around the income of $\bar{i}$ such that $\frac{a_{R,i}}{\mathbb{E}(a_{R,i})} = 1 + \zeta_i^{a_R,i} \frac{i-\bar{i}}{\bar{i}} + o\left(i - \bar{i}\right)$. Then, notice that, in general equilibrium,

$$\begin{aligned}
\zeta_i^{a,i} &= \tilde{\zeta}_i^{a,i} + \tilde{\zeta}_i^{a,r} \varepsilon_i^{r,a} \zeta_i^{a,i} + \tilde{\zeta}_i^{a,r} \int_{i'} \frac{dlog(r_i)}{dlog(a_{i'})} \frac{dlog(a_{i'})}{dlog(r_{i'})} \frac{dlog(r_{i'})}{dlog(a_i)} \zeta_i^{a,i} di \\
&= \tilde{\zeta}_i^{a,i} + \tilde{\zeta}_i^{a,r} \varepsilon_i^{r,a} \zeta_i^{a,i} + \tilde{\zeta}_i^{a,r} \int_{i'} \frac{1}{r_i} \delta_{i'}^{r,a} \tilde{\zeta}_{i'}^{a,r} \frac{1}{r_{i'}} \delta_i^{r,a} \zeta_i^{a,i} di
\end{aligned}$$

Assuming that $\zeta_i^{a,i}$ is constant and using the fact that $\int_{i'} \delta_{i'}^{r,a} \frac{1}{r_{i'}} di = 0$, $\zeta_i^{a,i} = \tilde{\zeta}_i^{a,i} + \tilde{\zeta}_i^{a,r} \varepsilon_i^{r,a} \zeta_i^{a,i} = \phi_i \tilde{\zeta}_i^{a,i}$ which confirms the conjecture. Similarly,

$$\begin{aligned}
\zeta_i^{a_R,i} &= \left(1 + \varepsilon_i^{r,a}\right) \zeta_i^{a,i} + \tilde{\zeta}_i^{r,i} + \int_{i'} \frac{dlog(r_i)}{dlog(a_{i'})} \frac{dlog(a_{i'})}{dlog(r_{i'})} \frac{dlog(r_{i'})}{dlog(a_i)} \zeta_i^{a,i} di \\
&= \left(1 + \varepsilon_i^{r,a}\right) \zeta_i^{a,i} + \tilde{\zeta}_i^{r,i} = \Phi_i \tilde{\zeta}_i^{a,i} + \tilde{\zeta}_i^{r,i}.
\end{aligned}$$

Therefore, the relationship between household percentiles and their wealth and capital income remains unchanged because general equilibrium effects cancel out.

Then, one can approximate the capital income inequality in general equilibrium as

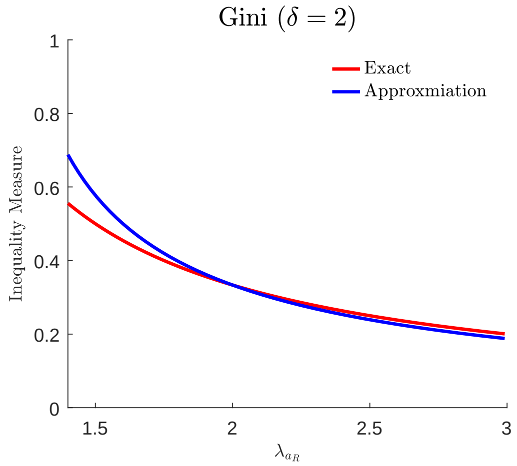
$$\begin{aligned}
\mathbb{E}\left[\frac{\left(1 - \Gamma_i \left(1 + \gamma_i^{r,(1-\tau_K)}\right)\right) a_{R,i}}{\mathbb{E}(a_{R,i})}\right] &\approx \mathbb{E}\left[\left(1 - \Gamma_i \left(1 + \gamma_i^{r,(1-\tau_K)}\right)\right) \left(1 + \zeta_i^{a_R,i} \frac{i - \bar{i}}{\bar{i}}\right)\right] \\
&= \zeta_i^{a_R,i} \mathbb{E}\left[(1 - \Gamma_i) \frac{i}{\bar{i}}\right] - \mathbb{E}\left[\Gamma_i \gamma_i^{r,(1-\tau_K)} \left(1 + \zeta_i^{a_R,i} \frac{i - \bar{i}}{\bar{i}}\right)\right],
\end{aligned}$$

which is larger than $\zeta_i^{a_R,i} \mathbb{E}\left[(1 - \Gamma_i) \frac{i}{\bar{i}}\right]$ since $\gamma_i^{r,(1-\tau_K)} < 0$. Therefore, capital income inequality is larger in general than in partial equilibrium. Using the above-derived relationships and observing that $\mathbb{E}\left(\delta_i^{r,a}\right) < 0$, the average capital income elasticity

$$\begin{aligned}
\overline{\zeta}^{a_R,(1-\tau_K)} &= \int_i \frac{a_{R,i}}{\mathbb{E}(a_{R,i})} \zeta_i^{a_R,(1-\tau_K)} di = \int_i \frac{a_{R,i}}{\mathbb{E}(a_{R,i})} \left[\left(1 + \varepsilon_i^{r,a}\right) \zeta_i^{a,(1-\tau_K)} + \left(1 + \tilde{\zeta}_i^{a,r}\right) \frac{1}{r_i} \int_{i'} \delta_{i'}^{r,a} \zeta_{i'}^{a,(1-\tau_K)} di'\right] di \\
&= \Phi_i \tilde{\zeta}_i^{a,(1-\tau_K)} \left(1 + \frac{1 + \tilde{\zeta}_i^{a,r}(1 + \Phi_i)}{1 + \varepsilon_i^{r,a}} \frac{\mathbb{E}(a_i)}{\mathbb{E}(a_{R,i})} \mathbb{E}\left(\delta_i^{r,a}\right)\right) < \Phi_i \tilde{\zeta}_i^{a,(1-\tau_K)}
\end{aligned}$$

is revised downwards in general equilibrium. Altogether, $\tau_K^{GE} > \tau_K^{PE}$ holding all primitives other than the cross-return elasticities fixed.

Figure 1.B.1: Approximation Error (Gini Social Welfare Function)



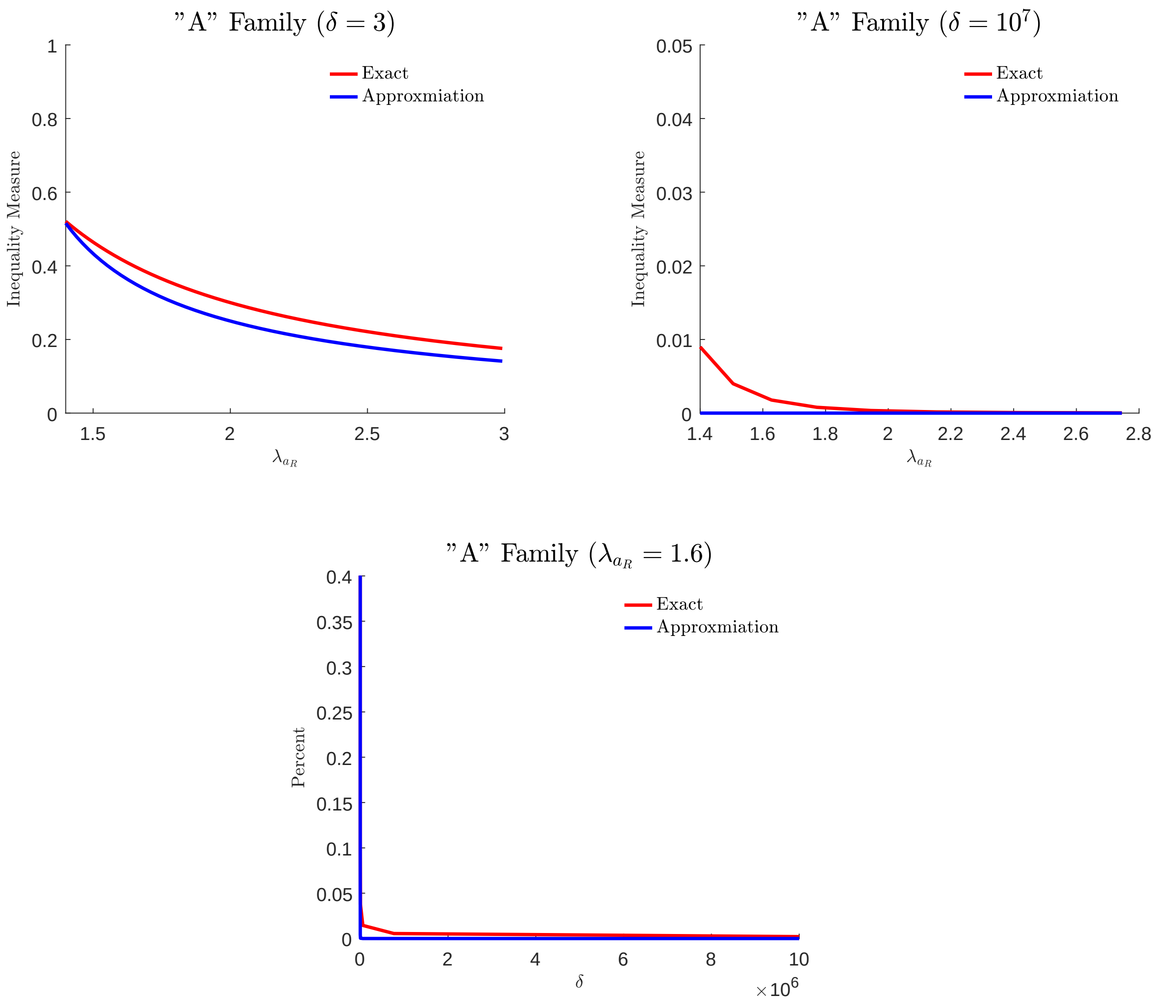
1.B Approximation Error

In Sections 1.2 and 1.4, I employ a first-order approximation of the measure of capital income inequality, allowing to remain agnostic about the distributions of wealth and return rates. The meaningfulness of this approach hinges on the performance of the approximation. Therefore, I now make specific distributional assumptions and compare the resulting exact measures of capital income inequality to their approximation. In the following approximations, I also account for the endogeneity of the household that earns the average capital income.

Following the literature on wealth and income inequality (see, for instance, Gabaix (2009)), suppose that capital income is Pareto distributed $\mathbb{P}(a_{R,i} \geq a_R) = a_{min}^{\lambda_{a_R}} a_R^{-\lambda_{a_R}}$. Observe that, under this assumption, $\zeta_i^{a_R,(1-i)} = -\frac{1}{\lambda_{a_R}}$. In the quantitative illustration of Section 1.4, I assume the Gini social welfare function, due to Sen (1974): $\Gamma_i = 2(1-i)$. This section extends the exposition to a more general class of rank-dependent social welfare functions $\Gamma_i = \frac{\delta}{\delta-1}\left(1 - i^{\delta-1}\right)$ where $\delta \geq 2$. This class is the so-called Lorenz or "A" family introduced by Aaberge (2000) that nests the Gini social welfare function ($\delta = 2$). Using these assumptions, one can derive the exact measure of capital income inequality. For $\delta = 2$, the inequality measure is exactly equal to the Gini coefficient $\mathbb{E}\left[(1-\Gamma_i)\frac{a_{R,i}}{\mathbb{E}(a_{R,i})}\right] = \frac{1}{2\lambda_{a_R}-1}$.

Figure 1.B.1 compares this exact expression to the proposed approximation for a range of values for λ_{a_R}. Overall the approximation performs well. Especially for a lower amount of inequality (larger λ_{a_R}), the approximation and the exact version of the Gini coefficient coincide. For a high level of inequality (small λ_{a_R}), the approximation will overstate the Gini measure and,

Figure 1.B.2: Approximation Error ("A" Family)



thus, the optimal tax rate. The approximation error is, however, limited (≈ 0.045) for the value used in Section 1.4 ($\lambda_{a_R} = 1.6$).

One can also study the approximation performance for other welfare functions. A higher value of δ means more weight on higher percentiles. In the limit ($\delta \to \infty$), the "A" family tends to the purely utilitarian case ($\Gamma_i = 1$). Figure 1.B.2 displays the approximation error for different values of δ. The upper two panels show the relationship between the inequality measures and the shape parameter for $\delta = 3$ and $\delta = 10^7$, respectively. Now, the approximation understates the measure of capital income inequality. However, the bias appears unsystematic over the range of shape parameters $\lambda_{a_R}$and is declining in δ. I further illustrate this observation in the lower panel by depicting the approximation error for different values of δ, holding the shape parameter fixed.

1.C A Dynamic Economy

In this section, I incorporate type and scale dependence into the dynamic bequest taxation model of Piketty and Saez (2013) that can be interpreted as a theory of capital taxation. I show that the main results from the previous section carry over. I discuss the main differences arising from a fully dynamic setting relative to the conceptual framework of Section 1.2. Moreover, I derive the optimal tax in general equilibrium. Finally, I deal with the role of uncertainty, which is present in the financial market of Section 1.E.

1.C.1 Environment

First, I describe the economic environment closely following Piketty and Saez (2013). Consider a discrete set of periods $t \in \{0, 1, ...\}$. In each period, there lives a generation of measure one.

Preferences and technology. Each household i, t from dynasty $i \in [0, 1]$ in generation t differs in a labor skill $w_{i,t}$, which may correlate across generations. Let the distribution of skills be stationary and ergodic. Individual i, t supplies labor $l_{i,t}$ to earn a pre-tax labor income $y_{L,i,t} \equiv w_{i,t} l_{i,t}$ which is taxed linearly at rate $\tau_{L,t}$. Let E_t be an exogenous transfer. At the beginning of a period, each household receives a capital endowment (inheritance) $a_{i,t} \geq 0$ from the previous generation that carries a yield of $r_{i,t}$ and is taxed at rate, $\tau_{W,t}$.[21] Suppose the initial distribution of $a_{i,0}$ is exogenously given.

Households can take effort $x_{i,t+1}$ at a cost $v(x_{i,t+1})$ to increase the rate of return $r'_{i,t}(x_{i,t}) > 0$ (e.g., financial advisory or financial knowledge acquisition). Let the usual monotonicity conditions hold. That is, effort choices, as well as savings, and, hence, labor and capital income are increasing the index i.[22] Intuitively, the higher an individual's hourly wage, the more she will work, and the more resources she can transfer to the retirement period. Moreover, an individual's incentives to take efforts to increase her capital gains rise with her position in the pre-tax wage distribution. Accordingly, there is scale dependence. That is, larger portfolios earn higher rates of return than smaller ones $r_{i,t} \equiv r_{i,t}(a_{i,t})$ where $r'_{i,t}(a_{i,t}) > 0$ and $r''_{i,t}(a_{i,t}) < 0$. When the costs are deductible from the tax base, define $r_{i,t} \equiv r_{i,t}(x_{i,t}) - v(x_{i,t})/a_{i,t}$. Return rates may also differ

[21] With return heterogeneity, it has been noted that a tax on wealth is not equivalent to a tax on capital income, $\tau_{K,t+1}$. They yield different implications for efficiency (Guvenen et al. (2019)). That is, only when $r_{i,t+1} = r_{t+1}$ for all i, $a_{R,i,t+1}(1 - \tau_{W,t+1}) = a_{i,t+1}[1 + (1 - \tau_{K,t+1}) r_{t+1}]$ if and only if $\tau_{K,t+1} = \tau_{W,t+1} \frac{1+r_{t+1}}{r_{t+1}}$. In this chapter, I disregard the important debate, which of the two policy instruments is more suitable in a given situation, and focus instead on the implications of endogenously formed return inequality for redistribution. Formally, with heterogeneous returns, a rise in the wealth tax by $d\tau_{W,t+1}$ also shifts the implied personal capital gains tax for any individual i upwards: $d\tau_{W,t+1} = d\tau_{K,i,t+1} \frac{r_{i,t+1}}{1+r_{i,t+1}} > 0$.

[22] In Section 1.G, I address monotonicity more formally.

exogenously due to type dependence: $\frac{\partial r_{i,t}}{\partial i} \geq 0$. In Section 1.E, I microfound this setup: There, returns form on a financial market in general equilibrium, making returns a function of one's own and everyone else's choices, $r_{i,t}\left(a_{i,t}, \left\{a_{j,t}\right\}_{j\in[0,1]}\right)$. For the moment, I shut down general equilibrium effects.

Household problem. Households optimally supply labor and use their after-tax, disposable income for consumption, $c_{i,t}$, and transfers into the next period (bequests), $a_{i,t+1}$, to maximize their utility $U_{i,t}\left(c_{i,t}, \underline{a}_{i,t+1}, l_{i,t}\right)$, where $a_{R,i,t+1} \equiv a_{i,t+1}\left(1+r_{i,t+1}\right)$ and $\underline{a}_{i,t+1} \equiv a_{R,i,t+1}\left(1-\tau_{W,t+1}\right)$ are the pre- and after-tax final wealth. Altogether, households solve

$$\max_{c_{i,t}, l_{i,t}, a_{i,t+1}, x_{i,t+1}} U_{i,t}\left(c_{i,t}, a_{R,i,t+1}\left(1-\tau_{W,t+1}\right), l_{i,t}, x_{i,t+1}\right) \tag{1.21}$$

subject to their budget constraint $c_{i,t} + a_{i,t+1} = a_{R,i,t}\left(1-\tau_{W,t}\right) + w_{i,t} l_{i,t}\left(1-\tau_{L,t}\right) + E_t$. As returns result from effort choices ($x_{i,t+1}$), households take their rate of return $r_{i,t+1}$ as given, when choosing $a_{i,t+1}$. The first-order condition for the optimal level of $a_{i,t+1}$ is given by $\frac{\partial U_{i,t}(\cdot)}{\partial c_{i,t}} = \frac{\partial U_{i,t}(\cdot)}{\partial \underline{a}_{i,t+1}}\left(1-\tau_{W,t+1}\right)\left(1+r_{i,t+1}\right)$.

Denote $a_t \equiv \int_i a_{i,t} di$, $a_{R,t} \equiv \int_i a_{R,i,t} di$, $c_t \equiv \int_i c_{i,t} di$, and $y_{L,t} \equiv \int_i y_{i,t} di$ as the aggregate variables in period t. Suppose that the economy converges to a unique equilibrium with ergodic steady-state distributions of earnings and wealth that are independent from the initial endowments $a_{i,0}$.

1.C.2 Optimal Taxation in Partial Equilibrium

In the following, consider the optimal long-run tax policy in the steady-state equilibrium, (τ_W, τ_L, E). Again, denote $\Gamma_{i,t} \geq 0$ as the Pareto weights. The government maximizes the sum of weighted utilities

$$\begin{aligned} \max_{\tau_W, \tau_L} \int_i & \Gamma_{i,t} U_{i,t}\left(a_{i,t}\left(1+r_{i,t}\right)\left(1-\tau_W\right) + w_{i,t} l_{i,t}\left(1-\tau_L\right) + E - a_{i,t+1},\right. \\ & \left. a_{i,t+1}\left(1+r_{i,t+1}\right)\left(1-\tau_W\right), l_{i,t}\right) di \end{aligned} \tag{1.22}$$

subject to the balanced period budget $\tau_W a_{R,t} + \tau_L y_{L,t} = E$ and scale dependence $r_{i,t} \equiv r_{i,t}\left(a_{i,t}\right)$. Observe that, for a given amount of E, τ_W and τ_L are directly linked to each other. For a budget neutral reform of the tax system, a change in τ_W triggers an according adjustment in τ_L and vice versa.

Elasticities. As before, denote the savings elasticity as $\zeta_{i,t}^{a,r} \equiv \frac{\partial log(a_{i,t})}{\partial log(r_{i,t})}$, the *own-return elasticity* as $\varepsilon_{i,t}^{r,a} \equiv \frac{\partial log[r_{i,t}(a_{i,t})]}{\partial log(a_{i,t})}$ and $\phi_{i,t} \equiv \frac{1}{1-\zeta_{i,t}^{a,r}\varepsilon_{i,t}^{r,a}} > 0$ as the measure of the *inequality multiplier effect*. It is useful to define another version of the own-return elasticity as $\varepsilon_{i,t}^{1+r,a} \equiv \frac{\partial log[1+r_{i,t}(a_{i,t})]}{\partial log(a_{i,t})}$.

With exogenous rates of return (type dependence), the elasticity of savings and initial wealth of household i reads as

$$\tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \equiv \frac{dlog\,(a_{i,t})}{dlog\,(1-\tau_W)}|_{E,r_{i,t}} = \frac{dlog\,[a_{i,t}\,(1+r_{i,t})]}{dlog\,(1-\tau_W)}|_{E,r_{i,t}} > 0.$$

With endogenously formed returns (scale dependence), the elasticity of initial wealth before and after interest are given by

$$\zeta_{i,t}^{a,(1-\tau_W)} \equiv \frac{dlog\,(a_{i,t})}{dlog\,(1-\tau_W)}|_E = \phi_{i,t}\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}$$

and

$$\zeta_{i,t}^{a_R,(1-\tau_W)} \equiv \frac{dlog\,[a_{i,t}\,(1+r_{i,t}\,(a_{i,t}))]}{dlog\,(1-\tau_W)}|_E = \left(1+\varepsilon_{i,t}^{1+r,a}\right)\zeta_{i,t}^{a,(1-\tau_W)},$$

respectively. Observe that, due to the endogeneity of returns, $\zeta_{i,t}^{a_R,(1-\tau_W)} > \zeta_{i,t}^{a,(1-\tau_W)} > \tilde{\zeta}_{i,t}^{a,(1-\tau_W)}$. Moreover, define the long-run elasticity of aggregate wealth and labor income with respect to their retention rate as

$$\zeta^{a_R,(1-\tau_W)} \equiv \frac{dlog\,(a_{R,t})}{dlog\,(1-\tau_W)}|_E$$

and

$$\zeta^{y_L,(1-\tau_L)} \equiv \frac{dlog\,(y_{L,t})}{dlog\,(1-\tau_L)}|_E.$$

As in Hendren (2016), these policy elasticities e_W and e_L include own- and cross-price effects as they feature behavioral responses to a budget-neutral reform of both τ_W and τ_L. Observe that one can decompose $\zeta^{a_R,(1-\tau_W)} = \zeta_R^{a_R,(1-\tau_W)} + \zeta_H^{a_R,(1-\tau_W)} + \zeta_E^{a_R,(1-\tau_W)}$, where

$$\zeta_R^{a_R,(1-\tau_W)} \equiv \frac{1}{a_{R,t}}\int_i (1+R)\,a_{i,t}\,\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}di$$

is the elasticity of savings at the mean rate of return $R \equiv \int_i r_{i,t}\,(a_{i,t})\,di$,

$$\zeta_H^{a_R,(1-\tau_W)} \equiv \frac{1}{a_{R,t}}\int_i [r_{i,t}\,(a_{i,t})-R]\,a_{i,t}\,\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}di$$

captures the reaction of savings with return heterogeneity, and

$$\zeta_E^{a_R,(1-\tau_W)} \equiv \frac{1}{a_{R,t}} \int_i \left[\zeta_{i,t}^{a_R,(1-\tau_W)} - \tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \right] \left[1 + r_{i,t}\left(a_{i,t}\right)\right] a_{i,t} di$$

characterizes the effects from the endogeneity in returns. This decomposition nests the setting of Piketty and Saez (2013) in which $\zeta_H^{a_R,(1-\tau_W)} = 0$ and $\zeta_E^{a_R,(1-\tau_W)} = 0$. Observe that $\zeta_E^{a_R,(1-\tau_W)} > 0$ for $r'_{i,t}\left(a_{i,t}\right) > 0$. Hence, for a given distribution of wealth and returns the elasticity of wealth, $\zeta^{a_R,(1-\tau_W)}$, is larger under scale dependence (when returns form endogenously) than under type dependence (part (b) of Proposition 1). Also note that, by the construction of scale dependence, Corollary 1 applies: $\zeta_t^{\mathbb{V}(r),(1-\tau_W)} = 2\varepsilon_{i,t}^{r,a} \zeta_{i,t}^{a,(1-\tau_W)} > 0$ and $\zeta_t^{\mathbb{E}(r),(1-\tau_W)} = \varepsilon_{i,t}^{r,a} \zeta_{i,t}^{a,(1-\tau_W)} > 0$ for constant elasticities.

Distributional parameters. Denote $g_{i,t} \equiv \Gamma_{i,t} \frac{\partial U_{i,t}(\cdot)}{\partial c_{i,t}} / \int_{i'} \Gamma_{i',t} \frac{\partial U_{i',t}(\cdot)}{\partial c_{i',t}} di'$ as the social marginal welfare weight of an individual i,t in monetary units. Define the ratios

$$\overline{a}^{initial} \equiv \int_i g_{i,t} \frac{\left[1 + r_{i,t}\left(a_{i,t}\right)\right] a_{i,t}}{a_{R,t}} di$$

and

$$\overline{a}^{final} \equiv \int_i g_{i,t} \frac{a_{i,t+1}}{a_{R,t}} di$$

as the distributional parameter of initial and final wealth before interest (received and left bequests). Similarly, define the distributional parameter of labor income $\overline{y}_L \equiv \int_i g_{i,t} \frac{y_{L,i,t}}{y_{L,t}} di$. For a given unweighted population mean, a small distributional parameter indicates a strong taste for redistribution. Alternatively, fix the redistributive goal of the society. Then, a high concentration of the respective variable leads to a low value of the distributional parameter.

Steady state. To derive the optimal tax formula, one needs to find the combination of tax rates that leads to no first-order welfare gain for any budget-neutral tax reform. First, I describe the set of budget-neutral tax reforms $(d\tau_W, d\tau_L, dE)$ with $dE = 0$. Accordingly, it follows from the government budget constraint that $d\tau_W$ and $d\tau_L$ relate to each other in the following fashion

$$a_{R,t} d\tau_W \left(1 - \zeta^{a_R,(1-\tau_W)} \frac{\tau_W}{1-\tau_W}\right) = -y_{L,t} d\tau_L \left(1 - \zeta^{y_L,(1-\tau_L)} \frac{\tau_L}{1-\tau_L}\right). \tag{1.23}$$

Using the envelope theorem and imposing that the first-order change in welfare equals zero $dSWF = 0$, yields an optimality condition for the capital tax

$$\int_i g_{i,t} \left[-\left(1 + \zeta_{i,t}^{a_R,(1-\tau_W)}\right) a_{R,i,t} d\tau_W + \frac{y_{L,i,t}}{y_{L,t}} \frac{1 - \zeta^{a_R,(1-\tau_W)} \frac{\tau_W}{1-\tau_W}}{1 - \zeta^{y_L,(1-\tau_L)} \frac{\tau_L}{1-\tau_L}} a_{R,t} d\tau_W - \frac{a_{i,t+1}}{1-\tau_W} d\tau_W \right] di = 0. \tag{1.24}$$

There are three effects of a rise in the capital tax. The first one describes the negative effect on initial wealth, whereas the third term the one on final wealth. The second term is the positive effect of the reduction in the labor income tax resulting from budget neutrality. Use the definitions of aggregates and distributional parameters to rewrite Equation (1.24)

$$-\overline{a}^{initial}\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}\right)+\frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}}\overline{y}_L-\frac{1}{1-\tau_W}\overline{a}^{final}=0 \tag{1.25}$$

where $\hat{\zeta}^{a_R,(1-\tau_W)}=\int_i\zeta_{i,t}^{a_R,(1-\tau_W)}g_{i,t}\frac{a_{R,i,t}}{a_{R,t}}di/\int_i g_{i,t}\frac{a_{R,i,t}}{a_{R,t}}di$ is the welfare-weighted average initial wealth elasticity. From these arguments, Proposition 3 directly follows.

Proposition 3 (Optimal capital tax in steady state)**.** *The optimal capital tax in the long-run steady-state equilibrium is*

$$\tau_W=\frac{1-\frac{\overline{a}^{initial}}{\overline{y}_L}\left(1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}+\frac{\overline{a}^{final}}{\overline{a}^{initial}}\right)}{1+\zeta^{a_R,(1-\tau_W)}-\frac{\overline{a}^{initial}}{\overline{y}_L}\left(1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}\right)} \tag{1.26}$$

for a given labor income tax τ_L.

Proof. Appendix 1.D.1. □

This proposition replicates the tax formula by Piketty and Saez (2013). Hence, I obtain a version of the neutrality result (Proposition 1 (a)) in the previous section: The sufficient statistics that describe the optimal capital tax are the same with and without scale dependence. As already mentioned, these sufficient statistics are, however, endogenous to the process of return formation and to the capital tax.

Comparative statics. To establish part (c) of Proposition 1 in this economy, I introduce (a small amount of) scale dependence into an economy without scale dependence that is otherwise observationally equivalent. Thus, I focus on the first comparative statics exercise in Proposition 1 (c), holding observables fixed. In this setting, this means fixing the labor supply elasticity ($\zeta^{y_L,(1-\tau_L)}$), the distribution of labor income ($\overline{y}_L$), labor taxes (τ_L), and the social marginal welfare weights ($g_{i,t}$). Let the individual wealth elasticities be uncorrelated with the marginal welfare weights such that $\hat{\zeta}^{a_R,(1-\tau_W)}=\zeta^{a_R,(1-\tau_W)}$. Moreover, I take the above-described elasticities of returns ($\varepsilon_i^{r,a}$ and $\varepsilon_i^{1+r,a}$) and savings ($\tilde{\zeta}_i^{a,r}$ and $\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}$) as given and omit distributional effects on the aggregate elasticity ($\zeta^{a_R,(1-\tau_W)}$) that may, for instance, arise when there is a correlation between elasticities and wealth. Of course, these simplifications neglect the endogeneity of these measures to capital taxes and the allocations that will change when introducing scale dependence.

However, they allow for a tractable analysis of taxes with and without scale dependence (τ_W and $\tilde{\tau}_W$, respectively).

As described, under scale dependence, the wealth elasticity has to be upward revised (part (b) of Proposition 1), providing a force for lower wealth taxes. Formally, $\zeta^{a_R,(1-\tau_W)} > \zeta^{a_R,(1-\tau_W)}|_{\{r_i\}_{i\in[0,1]}}$ since $\zeta_E^{a_R,(1-\tau_W)} > 0$. The economic intuition for this result is the same as in Section 1.2. Capital gains are convex under scale dependence. This convexity makes household wealth more elastic to tax reforms. Since the optimal tax rate is inversely related to this elasticity, this channel calls for lower capital taxes. For example, when wealth is infinitely concentrated ($\frac{\overline{a}^{initial}}{\overline{y}_L} \to 0$ and $\frac{\overline{a}^{final}}{\overline{y}_L} \to 0$), the capital tax rate reduces to $\tau_W = \frac{1}{1+\zeta^{a_R,(1-\tau_W)}}$. All the distributional effects on the optimal capital tax vanish. Relative to an economy with type dependence that is otherwise observationally equivalent in its wealth and returns distribution, the presence of scale dependence raises the wealth elasticity ($\zeta^{a_R,(1-\tau_W)} > \zeta^{a_R,(1-\tau_W)}|_{\{r_i\}_{i\in[0,1]}}$). As a result, $\tau_W < \tilde{\tau}_W$.

Nonetheless, scale dependence may raise wealth inequality relative to type dependence. A lower tax under scale dependence may decrease $\overline{a}^{final}$. This channel calls for higher taxes. In other words, the expression in Proposition 3 is not in closed form. For small policy changes ($\tau_W \approx \tilde{\tau}_W$) from introducing a small amount of scale dependence ($\varepsilon_{i,t}^{r,a} \approx 0$),[23] one can use a first-order Taylor expansion to approximate aggregate wealth

$$a_{R,t}(\tau_W) = a_{R,t}(\tilde{\tau}_W)\left[1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\zeta^{a_R,(1-\tau_W)}\right] + o(\tau_W - \tilde{\tau}_W), \tag{1.27}$$

bearing in mind that the elasticity $\zeta^{a_R,(1-\tau_W)}$ needs to account for scale dependence. Therefore a rise in the wealth tax diminishes the aggregate wealth level in the economy. Formally, $a_{R,t}(\tau_W) > a_{R,t}(\tilde{\tau}_W)$ for $\tilde{\tau}_W > \tau_W$.

Simultaneously, the wealth inequality in the society ultimately declines in response to a rise in the capital tax

$$\overline{a}^{final}(\tau_W) = \overline{a}^{final}(\tilde{\tau}_W)\frac{1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\zeta^{a,(1-\tau_W)}}{1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\zeta^{a_R,(1-\tau_W)}} + o(\tau_W - \tilde{\tau}_W). \tag{1.28}$$

If $\tilde{\tau}_W > \tau_W$, $\overline{a}^{final}(\tau_W) < \overline{a}^{final}(\tilde{\tau}_W)$ since the elasticity of aggregate wealth is larger than the aggregate savings elasticity $\zeta^{a_R,(1-\tau_W)} > \zeta^{a,(1-\tau_W)}$. Therefore, rise in the capital tax lowers the concentration of final wealth (higher $\overline{a}^{final}$). However, when one only introduces a small amount

[23] In Section 1.G, I deal with a similar comparative statics exercise without imposing any assumption on the size of policy changes.

of scale dependence, this effects disappears

$$\overline{a}^{final}(\tau_W) = \overline{a}^{final}(\tilde{\tau}_W) + o(\tau_W - \tilde{\tau}_W).$$

Interestingly, the initial (weighted) inequality is also unaffected by the tax scheme

$$\overline{a}^{initial}(\tau_W) = \overline{a}^{initial}(\tilde{\tau}_W) + o(\tau_W - \tilde{\tau}_W). \tag{1.29}$$

The reason is that, in this specification, the decline in aggregate wealth just offsets the rise in unweighted initial inequality when individual wealth elasticities do not correlate with marginal welfare weights ($\hat{\zeta}^{a_R,(1-\tau_W)} = \zeta^{a_R,(1-\tau_W)}$). Consequently, Proposition 1 (c) approximately holds in this economy: The wealth tax in an economy with a small amount scale dependence is lower than the one in an (in terms of $\overline{a}^{initial}$ and $\overline{a}^{final}$) observationally equivalent economy without scale dependence as in the former the elasticity of capital is higher.[24]

Dynamic efficiency. Suppose that the government chooses $(\tau_{W,t}, \tau_{L,t})$ to maximize

$$\begin{aligned} SWF = \sum_{t=0}^{\infty} \beta^t \int_i \Gamma_{i,t} U_{i,t} & (a_{i,t}(1+r_{i,t})(1-\tau_{W,t}) + w_{i,t} l_{i,t}(1-\tau_{L,t}) + E_t - a_{i,t+1}, \\ & a_{i,t+1}(1+r_{i,t+1})(1-\tau_{W,t+1}), l_{i,t})\, di \end{aligned} \tag{1.30}$$

subject to the set of period budget constraints $\tau_{W,t} a_{R,t} + \tau_{L,t} y_{L,t} = E_t$ and scale dependence $r_{i,t} \equiv r_{i,t}(a_{i,t})$, where $\beta \in [0,1]$ denotes the generational discount rate.

To solve for the optimal policy, consider a uniform, budget-neutral reform of the tax code at a distant future point in time, T, when all variables have converged. That is $(d\tau_{W,t}, d\tau_{L,t}) = (d\tau_W, d\tau_L)$ for all $t \geq T$. Imposing that the reform has no first-order effect on social welfare, $dSWF = 0$, one obtains a dynamic version of the optimality condition from above

$$-\overline{a}^{initial}\left(1 + (1-\beta)\sum_{t=T}^{\infty} \beta^{t-T} \zeta_t^{a_R,(1-\tau_W)}\right) + \overline{y}_L (1-\beta) \sum_{t=T}^{\infty} \beta^{t-T} \frac{1 - \zeta_t^{a_R,(1-\tau_W)} \frac{\tau_W}{1-\tau_W}}{1 - \zeta_t^{y_L,(1-\tau_W)} \frac{\tau_L}{1-\tau_L}} - \frac{1}{1-\tau_W} \frac{1}{\beta} \overline{a}^{final} = 0. \tag{1.31}$$

Hence, in the optimal dynamic tax formula, the steady-state elasticity is now replaced with discounted elasticities. All the intuitions from the steady-state economy carry over.

1.C.3 Optimal Taxation in General Equilibrium

Reconsider the steady-state economy from before. Now, assume that returns are formed in general equilibrium. That is, $r_{i,t}\left(a_{i,t}, \{a_{i',t}\}_{i' \in [0,1]}\right)$. As in Section 1.2, define the *cross-return elas-*

[24] By similar techniques, one may analyze the impact of a small change in the amount of scale dependence.

ticity as $\gamma^{r,a}_{i,i',t} \equiv \frac{\partial log\left(r_{i,t}\right)}{\partial log\left(a_{i',t}\right)}$.[25] Let the cross-elasticity be multiplicatively separable $\gamma^{r,a}_{i,i',t} = \frac{1}{r_{i,t}}\delta^{r,a}_{i',t}$ (similar to the CES example of Sachs, Tsyvinski, and Werquin (2020)). That is, a change in the savings by a household i' leads to the same change the returns of any other household i in the percentage points. In the financial market setting of Section 1.E, this assumption holds when the costs of information acquisition are linear and all households acquire financial information. It is useful to also define another version of the cross-return elasticity $\gamma^{1+r,a}_{i,i',t} \equiv \frac{\partial log\left(1+r_{i,t}\right)}{\partial log\left(a_{i',t}\right)} = \frac{r_{i,t}}{1+r_{i,t}}\gamma^{r,a}_{i,i',t}$.

First, note that the elasticity of wealth before and after interest are augmented by general equilibrium effects

$$\zeta^{a,(1-\tau_W)}_{i,t} = \phi_{i,t}\tilde{\zeta}^{a,(1-\tau_W)}_{i,t} + \phi_{i,t}\zeta^{a,r}_{i,t}\frac{\int_{i'}\gamma^{r,a}_{i,i',t}\phi_{i',t}\tilde{\zeta}^{a,(1-\tau_W)}_{i',t}di'}{1-\int_{i'}\gamma^{r,a}_{i',i',t}\phi_{i',t}\zeta^{a,r}_{i',t}di'}$$

and

$$\zeta^{a_R,(1-\tau_W)}_{i,t} = \left(1+\zeta^{1+r,a}_{i,t}\right)\zeta^{a,(1-\tau_W)}_{i,t} + \left(1+\zeta^{a,1+r}_{i,t}\right)\int_{i'}\gamma^{1+r,a}_{i,i',t}\zeta^{a,(1-\tau_W)}_{i',t}di',$$

respectively. The aggregate and distributional variables are defined as before. The sign and the distribution of cross-return (semi-)elasticities, $\delta^{r,a}_{i',t}$, determine how the wealth elasticities are adjusted. In the model of Section 1.E with linear information costs, $\delta^{r,a}_{i',t}$ is positive for small values of $a_{i',t}$ and negative for large ones. This resembles a situation of trickle-up, in which cutting the top tax shifts economic rents from the bottom to the top.

To illustrate the implications for the wealth elasticities, assume constant elasticities $\tilde{\zeta}^{a,(1-\tau_W)}_{i,t} = \tilde{\zeta}^{a,(1-\tau_W)}_{i',t}$, $\zeta^{a,r}_{i,t} = \zeta^{a,r}_{i',t}$, and $\varepsilon^{r,a}_{i,t} = \varepsilon^{r,a}_{i',t}$ and suppose that cross-return elasticities average out such that $\int_{i'}\gamma^{r,a}_{i',i',t}di' = 0$. Then,

$$\zeta^{a,(1-\tau_W)}_{i,t} = \phi_{i,t}\tilde{\zeta}^{a,(1-\tau_W)}_{i,t}\left(1+\zeta^{a,r}_{i,t}\phi_{i,t}\frac{1}{r_{i,t}}\int_{i'}\delta^{r,a}_{i',t}di'\right) < \phi_{i,t}\tilde{\zeta}^{a,(1-\tau_W)}_{i,t}$$

and

$$\zeta^{a_R,(1-\tau_W)}_{i,t} = \left(1+\varepsilon^{1+r,a}_{i,t}\right)\zeta^{a,(1-\tau_W)}_{i,t} + \frac{1+\zeta^{a,1+r}_{i,t}}{1+r_{i,t}}\int_{i'}\delta^{r,a}_{i',t}\zeta^{a,(1-\tau_W)}_{i',t}di' < \left(1+\varepsilon^{1+r,a}_{i,t}\right)\phi_{i,t}\tilde{\zeta}^{a,(1-\tau_W)}_{i,t}.$$

[25] Less heuristically, one may define the cross-return elasticity as the Gateaux derivative of the return functional $r_i\left(a_i,\{a_j\}_{j\in[0,1]}\right)$. That is, perturb $\{a_j\}_{j\in[0,1]}$ by the Dirac measure at i', $\delta(i')$,

$$\gamma^{r,a}_{i,i'} \equiv \lim_{\mu\to 0}\frac{d}{d\mu}r_i\left(a_i,\{a_j\}_{j\in[0,1]}+\mu\delta(i')\right).$$

The formulation of the return functional $r_i(\cdot)$ is such that there are no discontinuous jumps of $\gamma^{r,a}_{i,i'}$ at $i'=i$. Any non-infinitesimal effect of a_i on the return functional is collected in the first argument of $r_i(\cdot)$.

Therefore, in this general equilibrium specification, wealth reacts less elastically to tax reforms relative to the partial equilibrium setting.

Taking stock of all general equilibrium effects, the optimal tax rate is defined by the optimality condition

$$\int_i g_{i,t}\left[-\left(1+\zeta_{i,t}^{a_R,(1-\tau_W)}\right)a_{R,i,t}+\frac{y_{L,i,t}}{y_{L,t}}\frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}}a_{R,t}\right.$$
$$\left.-\frac{a_{i,t+1}}{1-\tau_W}\left(1+\int_{i'}\gamma_{i,i',t+1}^{1+r,a}\zeta_{i',t+1}^{a,(1-\tau_W)}di'\right)\right]di=0.$$

which can be written as

$$-\overline{a}^{initial}\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}\right)+\frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}}\overline{y}_L-\frac{1}{1-\tau_W}\overline{a}^{final}\left(1+\hat{\gamma}^{1+r,(1-\tau_W)}\right)=0 \tag{1.32}$$

using the notation from above and defining

$$\hat{\gamma}^{1+r,(1-\tau_W)}\equiv\int_i\left(1+\zeta_{i,t}^{a,1+r}\right)\left(\int_{i'}\gamma_{i,i',t+1}^{1+r,a}\zeta_{i',t+1}^{a,(1-\tau_W)}di'\right)g_{i,t}\frac{a_{i,t+1}}{a_{R,t}}di\Big/\int_i g_{i,t}\frac{a_{i,t+1}}{a_{R,t}}di.$$

Note that $\hat{\gamma}^{1+r,(1-\tau_W)}<0$. Thus, the general equilibrium spillovers do not only indirectly enter the cost-benefit analysis through the downward-adjusted aggregate elasticities $\zeta^{a_R,(1-\tau_W)}$ and $\hat{\zeta}^{a_R,(1-\tau_W)}$ (Proposition 2), but also directly through $\hat{\gamma}^{1+r,(1-\tau_W)}$. The latter term accounts for a first-order spillover effect on final wealth that reduces the aggregate costs of taxing wealth. This effect adds to the reduction in aggregate elasticities. To sum up, I state Proposition 4.

Proposition 4 (Optimal capital tax in general equilibrium)**.** *The optimal capital tax in the long-run steady-state general equilibrium is*

$$\tau_W^{GE}=\frac{1-\frac{\overline{a}^{initial}}{\overline{y}_L}\left(1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}+\frac{\overline{a}^{final}}{\overline{a}^{initial}}\left(1+\hat{\gamma}^{1+r,(1-\tau_W)}\right)\right)}{1+\zeta^{a_R,(1-\tau_W)}-\frac{\overline{a}^{initial}}{\overline{y}_L}\left(1-e_L\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}\right)}. \tag{1.33}$$

for a given labor income tax τ_L.

Proof. Appendix 1.D.2. □

Comparative statics. To establish the comparative statics of optimal capital taxation, as in Proposition 2, I follow the reasoning in Section 1.C.2. I introduce (a small amount of) general equilibrium effects into the partial equilibrium economy with scale dependence that is otherwise

observationally equivalent. I fix the labor supply elasticity ($\zeta^{y_L,(1-\tau_L)}$), the distribution of labor income ($\bar{y}_L$), labor taxes (τ_L), and the social marginal welfare weights ($g_{i,t}$). Suppose that the individual wealth elasticities do not correlate with the marginal welfare weights such that $\hat{\zeta}^{a_R,(1-\tau_W)} = \zeta^{a_R,(1-\tau_W)}$, and hold the above-described elasticities of returns ($\varepsilon_i^{r,a}$ and $\varepsilon_i^{1+r,a}$) and savings ($\tilde{\zeta}_i^{a,r}$ and $\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}$) constant. Moreover, I omit any distributional effects on the aggregate wealth elasticity ($\zeta^{a_R,(1-\tau_W)}$). Let the amount of scale dependence and general equilibrium forces be small ($\zeta_{i,t}^{r,a} \approx 0$ and $\delta_{i,t}^{r,a} \approx 0$).
To compare the wealth tax in partial equilibrium, τ_W^{PE}, to the one in general equilibrium, τ_W^{GE}, I approximate the endogenous distributional variables on the right-hand side of Equation (1.33). Again, a higher capital tax (e.g., $\tau_W^{GE} > \tau_W^{PE}$) reduces aggregate wealth (e.g., $a_{R,t}\left(\tau_W^{GE}\right) < a_{R,t}\left(\tau_W^{PE}\right)$)

$$a_{R,t}\left(\tau_W^{GE}\right) = a_{R,t}\left(\tau_W^{PE}\right)\left[1 + \frac{\tau_W^{PE} - \tau_W^{GE}}{1-\tau_W^{PE}}\zeta^{a_R,(1-\tau_W)}\right] + o\left(\tau_W^{GE} - \tau_W^{PE}\right). \tag{1.34}$$

However, under the assumptions mentioned above, there are no first-order effects on initial and final wealth inequality: $\bar{a}^{initial}\left(\tau_W^{GE}\right) = \bar{a}^{initial}\left(\tau_W^{PE}\right) + o\left(\tau_W^{GE} - \tau_W^{PE}\right)$ and $\bar{a}^{final}\left(\tau_W^{GE}\right) = \bar{a}^{final}\left(\tau_W^{PE}\right) + o\left(\tau_W^{GE} - \tau_W^{PE}\right)$. Accordingly, only the adjustment in the aggregate wealth elasticity, $\zeta^{a_R,(1-\tau_W)}$, and the general equilibrium effect, $\hat{\gamma}^{1+r,(1-\tau_W)}$, affect the optimal capital tax rate. To sum up, when general equilibrium forces and scale dependence are small, the optimal capital tax is higher in general equilibrium compared to the self-confirming tax in an (in terms of $\bar{a}^{initial}$ and $\bar{a}^{final}$) observationally equivalent partial equilibrium economy.[26] This result is intuitive given the presence of trickle up.

1.C.4 Uncertainty

In this section, I consider the Barro-Becker dynastic model extension in Piketty and Saez (2013), which allows for uncertainty in the rates of return $r_{i,t}$. In this framework, individuals do not only care about their well-being, but also about the one of their children. As before, the government chooses a linear, deterministic tax system $(\tau_{L,t}, \tau_{W,t}, E_t)$. Household i in period t optimally chooses $(l_{i,t}, a_{i,t+1}, e_{i,t})$ to maximize $U_{i,t} = u_{i,t}(c,l,e) + \beta\mathbb{E}_t\left[U_{i,t+1}\right]$, where $\beta < 1$, subject to $c_{i,t} + a_{i,t+1} = (1-\tau_{W,t})a_{R,i,t} + (1-\tau_{L,t})y_{L,i,t} + E_t$. For any $a_{i,t+1} \geq 0$, the Euler equation reads as $\frac{\partial u_{i,t}(\cdot)}{\partial c_{i,t}}a_{i,t+1} = \beta\left(1-\tau_{W,t+1}\right)\mathbb{E}_t\left[a_{R,i,t+1}\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\right]$. In the beginning of period $t+1$, stochastic returns have realized so that one can summarize the set of Euler equations as $\bar{a}_{t+1}^{final} =$

[26] Using similar approximations, one may evaluate the impact of a small change in the amount of general equilibrium forces.

$\beta(1-\tau_{W,t+1})\overline{a}_{t+1}^{initial}$ with the definitions from the deterministic version of the model $\overline{a}_{t+1}^{initial} \equiv \int_i g_{i,0} \frac{a_{R,i,t+1}}{a_{R,t+1}} di$ and $\overline{a}_{t+1}^{final} \equiv \int_i g_{i,0} \frac{a_{i,t+1}}{a_{R,t+1}} di$ and Pareto weights $\left\{\Gamma_{0,i}\right\}_{i\in[0,1]}$.

Suppose that the economy features an ergodic equilibrium with long-run variables independent from initial values. Let tax policies as well as individual choices converge. In the following, I consider the utilitarian ($\Gamma_{0,i}=1$) optimal long-run policy in the ergodic steady-state equilibrium. Suppose, without loss of generality, that this equilibrium is reached in period 0. The government chooses (τ_L, τ_W, E) to maximize the steady-state discounted expected social welfare

$$SWF_\infty \equiv \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u_{i,t}\left((1-\tau_W)a_{R,i,t} + (1-\tau_L)y_{L,i,t} + E - a_{i,t+1}, l_{i,t}\right)\right] \tag{1.35}$$

subject to $\tau_W a_{R,t} + \tau_L y_{L,t} = E$. The optimal tax system can be described by the optimality condition

$$\begin{aligned} dSWF_\infty = 0 = \mathbb{E}\left[\frac{\partial u_{i,0}(\cdot)}{\partial c_{i,0}}(1-\tau_W)da_{R,i,0}\right] - \mathbb{E}\left[\frac{\partial u_{i,0}(\cdot)}{\partial c_{i,0}}a_{R,i,0}d\tau_W\right] \\ - \sum_{t=0}^{\infty}\beta^{t+1}\mathbb{E}\left[\frac{\partial u_{i,t+1}(\cdot)}{\partial c_{i,t+1}}a_{R,i,t}d\tau_W\right] - \sum_{t=0}^{\infty}\beta^t\mathbb{E}\left[\frac{\partial u_{i,t}(\cdot)}{\partial c_{i,t}}y_{L,i,t}d\tau_L\right] \end{aligned}$$

which, using the individual's first-order conditions and budget neutrality of the tax reform and defining $\zeta_i^{a_R,(1-\tau_W)} \equiv \frac{dlog\left(a_{R,i,0}\right)}{dlog(1-\tau_W)}$, simplifies to

$$\begin{aligned} 0 = -\sum_{t=0}^{\infty}\beta^t\mathbb{E}\left[\frac{\partial u_{i,0}(\cdot)}{\partial c_{i,0}}a_{R,i,0}\left(1+\zeta_i^{a_R,(1-\tau_W)}\right)\right] \\ -\sum_{t=0}^{\infty}\beta^t\mathbb{E}\left[\frac{\partial u_{i,t}(\cdot)}{\partial c_{i,t}}\frac{a_{i,t+1}}{1-\tau_W} + \frac{\partial u_{i,t}(\cdot)}{\partial c_{i,t}}a_{R,t}\frac{\left(1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}\right)}{\left(1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}\right)}\frac{y_{L,i,t}}{y_{L,t}}\right] \end{aligned} \tag{1.36}$$

Since the economy is in the ergodic steady state, the optimal tax formula reads as

$$\tau_W = \frac{1 - \frac{(1-\beta)\overline{a}^{initial}}{\overline{y}_L}\left(1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)} + \frac{\overline{a}^{final}}{(1-\beta)\overline{a}^{initial}}\right)}{1+\zeta^{a_R,(1-\tau_W)} - \frac{(1-\beta)\overline{a}^{initial}}{\overline{y}_L}\left(1-\zeta^{y_L,(1-\tau_W)}\frac{\tau_L}{1-\tau_L}\right)\left(1+\hat{\zeta}^{a_R,(1-\tau_W)}\right)} \tag{1.37}$$

with the only difference to Proposition 3 that $\overline{a}^{initial}$ is weighted by $(1-\beta)$ to account for the fact that one discounts the costs of taxing future generations. Altogether, including uncertainty into the economy does not alter the implications of endogenous return inequality.

1.D Proofs for Section 1.C

1.D.1 Optimal Linear Wealth Taxation in Partial Equilibrium

Elasticities. In the presence of scale dependence, the elasticity of initial wealth before and after interest can be derived as

$$\zeta_{i,t}^{a,(1-\tau_W)} = \frac{dlog\left(a_{i,t}\right)}{dlog\left(1-\tau_W\right)}|_{E,r_{i,t}} + \frac{dlog\left(a_{i,t}\right)}{dlog\left(r_{i,t}\right)}\frac{dlog\left[r_{i,t}\left(a_{i,t}\right)\right]}{dlog\left(a_{i,t}\right)}\frac{dlog\left(a_{i,t}\right)}{dlog\left(1-\tau_W\right)}|_E = \phi_{i,t}\tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \tag{1.38}$$

and

$$\zeta_{i,t}^{a_R,(1-\tau_W)} = \frac{dlog\left[1+r_{i,t}\left(a_{i,t}\right)\right]}{dlog\left(1-\tau_W\right)} + \zeta_{i,t}^{a,(1-\tau_W)} = \left(1+\varepsilon_{i,t}^{1+r,a}\right)\zeta_{i,t}^{a,(1-\tau_W)}, \tag{1.39}$$

respectively.

Optimal capital tax in steady state. Budget neutrality of the tax reform implies

$$\begin{gathered} d\tau_W a_{R,t} + \tau_W da_{R,t} = -d\tau_L y_{L,t} - \tau_L dy_{L,t} \\ \Longleftrightarrow\ d\tau_W a_{R,t}\left(1-\frac{1-\tau_W}{a_{R,t}}\frac{da_{R,t}}{d\left(1-\tau_W\right)}\frac{\tau_W}{1-\tau_W}\right) = -d\tau_L y_{L,t}\left(1-\frac{1-\tau_L}{y_{L,t}}\frac{dy_{L,t}}{d\left(1-\tau_L\right)}\frac{\tau_L}{1-\tau_L}\right), \end{gathered}$$

which simplifies to Equation (1.23).

To obtain Equation (1.24), plug the households' first-order conditions $\frac{\partial U_{i,t}(\cdot)}{\partial c_{i,t}} = \frac{\partial U_{i,t}(\cdot)}{\partial \underline{a}_{i,t+1}}\frac{\left(1-\tau_{W,t+1}\right)a_{R,i,t+1}}{a_{i,t+1}}$ and Equation (1.23) into

$$\begin{aligned} dSWF &= \int_i \Gamma_{i,t}\frac{\partial U_{i,t}}{\partial c_{i,t}}\left[\left(1-\tau_W\right)da_{R,i,t} - a_{R,i,t}d\tau_W - w_{i,t}l_{i,t}d\tau_L\right]di - \int_i \Gamma_{i,t}\frac{\partial U_{i,t}}{\partial \underline{a}_{i,t+1}}a_{R,i,t+1}d\tau_W di \\ &= \int_i g_{i,t}\left[\frac{1-\tau_W}{a_{R,i,t}}\frac{da_{R,i,t}}{d\tau_W}a_{R,i,t}d\tau_W - a_{R,i,t}d\tau_W + \frac{y_{L,i,t}}{y_{L,t}}\frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}}a_{R,t}d\tau_W - \frac{a_{i,t+1}}{1-\tau_W}d\tau_W\right]di, \end{aligned}$$

and set this expression equal to zero. Equation (1.25) follows from

$$\begin{aligned} 0 &= \int_i g_{i,t}\left[-\left(1+\zeta_{i,t}^{a_R,(1-\tau_W)}\right)a_{R,i,t}d\tau_W + \frac{y_{L,i,t}}{y_{L,t}}\frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}}a_{R,t}d\tau_W - \frac{a_{i,t+1}}{1-\tau_W}d\tau_W\right]di \\ &= -\int_i g_{i,t}\frac{a_{R,i,t}}{a_{R,t}}di\left(1+\frac{\int_i g_{i,t}\zeta_{i,t}^{a_R,(1-\tau_W)}\frac{a_{R,i,t}}{a_{R,t}}di}{\int_i g_{i,t}\frac{a_{R,i,t}}{a_{R,t}}di}\right) + \frac{1-\zeta^{a_R,(1-\tau_W)}\frac{\tau_W}{1-\tau_W}}{1-\zeta^{y_L,(1-\tau_L)}\frac{\tau_L}{1-\tau_L}}\int_i g_{i,t}\frac{y_{L,i,t}}{y_{L,t}}di \\ &\quad - \frac{1}{1-\tau_W}\int_i \frac{g_{i,t}a_{i,t+1}}{a_{R,t}}di. \end{aligned}$$

Rearrange this equation to get the optimal wealth tax in Proposition 3.

Comparative statics. Now, I approximate individual and aggregate variables in in the presence of scale dependence (and evaluated at optimal tax rate) around the values that would emerge without scale dependence. Memorizing that the elasticities account for the presence of scale dependence, household wealth is approximately given by

$$\begin{aligned} a_{R,i,t}(\tau_W) &= a_{R,i,t}(\tilde{\tau}_W) + (\tau_W - \tilde{\tau}_W)\frac{da_{R,i,t}}{d\tau_W} + o(\tau_W - \tilde{\tau}_W) \\ &= a_{R,i,t}(\tilde{\tau}_W)\left[1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\zeta_{i,t}^{a_R,(1-\tau_W)}\right] + o(\tau_W - \tilde{\tau}_W). \end{aligned}$$

Integrate out to get Equation (1.27)

$$a_{R,t}(\tau_W) = a_{R,t}(\tilde{\tau}_W) + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\int_i \zeta_{i,t}^{a_R,(1-\tau_W)} a_{R,i,t}(\tilde{\tau}_W)\, di + o(\tau_W - \tilde{\tau}_W).$$

Plug Equation (1.27) and

$$\int_i g_{i,t} a_{i,t}(\tau_W)\, di = \int_i g_{i,t} a_{i,t}(\tilde{\tau}_W)\, di + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\int_i g_{i,t}\zeta_{i,t}^{a,(1-\tau_W)} a_{i,t}(\tilde{\tau}_W)\, di + o(\tau_W - \tilde{\tau}_W)$$

into

$$\begin{aligned} \overline{a}^{final}(\tau_W) &= \frac{a_{R,t}(\tilde{\tau}_W)}{a_{R,t}(\tau_W)}\left[\overline{a}^{final}(\tilde{\tau}_W) + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\int_i g_{i,t}\zeta_{i,t}^{a,(1-\tau_W)}\frac{a_{i,t}(\tilde{\tau}_W)}{a_{R,t}(\tilde{\tau}_W)} di\right] + o(\tau_W - \tilde{\tau}_W) \\ &= \overline{a}^{final}(\tilde{\tau}_W)\frac{1}{1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\zeta^{a_R,(1-\tau_W)}}\left[1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\hat{\zeta}^{a,(1-\tau_W)}\right] + o(\tau_W - \tilde{\tau}_W), \end{aligned}$$

where I use the definition of $\hat{\zeta}^{a,(1-\tau_W)} \equiv \int_i \zeta_{i,t}^{a,(1-\tau_W)} g_{i,t}\frac{a_{i,t}}{a_{R,t}} di / \int_i g_{i,t}\frac{a_{i,t}}{a_{R,t}} di$. Assuming that the savings elasticities are uncorrelated with the marginal welfare weights $\hat{\zeta}^{a,(1-\tau_W)} = \zeta^{a,(1-\tau_W)}$, Equation (1.28) follows. Proceed along the same lines, to obtain Equation (1.29)

$$\overline{a}^{initial}(\tau_W) = \overline{a}^{initial}(\tilde{\tau}_W)\frac{a_{R,t}(\tilde{\tau}_W)}{a_{R,t}(\tau_W)}\left[1 + \frac{\tilde{\tau}_W - \tau_W}{1-\tilde{\tau}_W}\hat{\zeta}^{a_R,(1-\tau_W)}\right] + o(\tau_W - \tilde{\tau}_W).$$

Then, $\overline{a}^{initial}(\tau_W) = \overline{a}^{initial}(\tilde{\tau}_W) + o(\tau_W - \tilde{\tau}_W)$, for $\hat{\zeta}^{a_R,(1-\tau_W)} = \zeta^{a_R,(1-\tau_W)}$.

Dynamic efficiency. Plug the households' first order conditions and Equation (1.23) into $dSWF = 0$ to get

$$
\begin{aligned}
0 &= \sum_{t=T}^{\infty} \beta^t \int_i \Gamma_{i,t} \frac{\partial U_{i,t}}{\partial c_{i,t}} \left[(1-\tau_W)\, da_{R,i,t} - a_{R,i,t} d\tau_W - y_{L,i,t} d\tau_L \right] di - \sum_{t=T-1}^{\infty} \beta^t \int_i \Gamma_{i,t} \frac{\partial U_{i,t}}{\partial \underline{a}_{i,t+1}} a_{R,i,t+1} d\tau_W di \\
&= -\sum_{t=T}^{\infty} \beta^t \int_i g_{i,t} \left[\frac{a_{R,i,t}}{a_{R,t}} \left(1 + \zeta_{i,t}^{a_R,(1-\tau_W)} \right) + \frac{y_{L,i,t}}{y_L} \frac{1 - \zeta_t^{a_R,(1-\tau_W)} \frac{\tau_W}{1-\tau_W}}{1 - \zeta_t^{y_L,(1-\tau_W)} \frac{\tau_L}{1-\tau_L}} \right] di \\
&\quad - \frac{1}{1-\tau_W} \sum_{t=T-1}^{\infty} \beta^t \int_i g_{i,t} \frac{a_{i,t+1}}{a_{R,t}} di
\end{aligned}
$$

and use the definitions of the distributional parameters to show Equation (1.31).

1.D.2 Optimal Linear Wealth Taxation in General Equilibrium

Elasticities. The general equilibrium savings elasticity is given by

$$
\begin{aligned}
\zeta_{i,t}^{a,(1-\tau_W)} &= \frac{dlog\,(a_{i,t})}{dlog\,(1-\tau_W)}|_{E,r_{i,t}} + \zeta_{i,t}^{a,r} \varepsilon_{i,t}^{r,a} \zeta_{i,t}^{a,(1-\tau_W)} + \zeta_{i,t}^{a,r} \int_{i'} \gamma_{i,i',t}^{r,a} \zeta_{i',t}^{a,(1-\tau_W)} di' \\
&= \phi_{i,t} \tilde{\zeta}_{i,t}^{a,(1-\tau_W)} + \frac{1}{r_{i,t}} \phi_{i,t} \zeta_{i,t}^{a,r} \int_{i'} \delta_{i'}^{r,a} \zeta_{i',t}^{a,(1-\tau_W)} di',
\end{aligned}
$$

using the multiplicatively separable cross-return elasticities $\gamma_{i,i',t}^{r,a} = \frac{1}{r_{i,t}} \delta_{i'}^{r,a}$. One can simplify the second term on the right-hand side to

$$
\begin{aligned}
\int_{i'} \delta_{i'}^{r,a} \zeta_{i',t}^{a,(1-\tau_W)} di' &= \int_{i'} \delta_{i',t}^{r,a} \phi_{i',t} \tilde{\zeta}_{i',t}^{a,(1-\tau_W)} di' + \int_{i'} \delta_{i',t}^{r,a} \phi_{i',t} \zeta_{i',t}^{a,r} \frac{1}{r_{i',t}} di' \int_{i''} \delta_{i'',t}^{r,a} \tilde{\zeta}_{i'',t}^{a,(1-\tau_W)} di'' \\
&= \frac{1}{1 - \int_{i'} \delta_{i',t}^{r,a} \phi_{i',t} \zeta_{i',t}^{a,r} \frac{1}{r_{i',t}} di' \int_{i''}} \int_{i'} \delta_{i',t}^{r,a} \phi_{i',t} \tilde{\zeta}_{i',t}^{a,(1-\tau_W)} di'.
\end{aligned}
$$

The wealth elasticity can be derived as

$$
\begin{aligned}
\zeta_{i,t}^{a_R,(1-\tau_W)} &= \left(1 + \varepsilon_{i,t}^{1+r,a} \right) \zeta_{i,t}^{a,(1-\tau_W)} + \int_{i'} \frac{dlog\,(a_{i,t}\,(1 + r_{i,t}\,(a_{i,t})))}{dlog\,(1 + r_{i,t})} \frac{dlog\,[1 + r_{i,t}\,(\cdot)]}{dlog\,(a_{i',t})} \frac{dlog\,(a_{i',t})}{dlog\,(1-\tau_W)}|_E\, di' \\
&= \left(1 + \varepsilon_{i,t}^{1+r,a} \right) \zeta_{i,t}^{a,(1-\tau_W)} + \left(1 + \zeta_{i,t}^{a,1+r} \right) \int_{i'} \gamma_{i,i',t}^{1+r,a} \zeta_{i',t}^{a,(1-\tau_W)} di'.
\end{aligned}
$$

Under the assumption that $\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}$, $\zeta_{i,t}^{a,r}$, and $\varepsilon_{i,t}^{r,a}$ are constant and cross-return elasticities average out $\int_{i'} \gamma_{i',i',t}^{r,a} di' = 0$, these expressions simplify to

$$
\zeta_{i,t}^{a,(1-\tau_W)} = \phi_{i,t} \tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \left(1 + \frac{1}{r_{i,t}} \phi_{i,t} \zeta_{i,t}^{a,r} \int_{i'} \delta_{i'}^{r,a} di' \right) < \phi_{i,t} \tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \tag{1.40}
$$

and

$$\zeta_{i,t}^{a_R,(1-\tau_W)} = \left(1+\varepsilon_{i,t}^{1+r,a}\right)\zeta_{i,t}^{a,(1-\tau_W)} + \frac{1+\zeta_{i,t}^{a,1+r}}{1+r_{i,t}}\phi_{i,t}\tilde{\zeta}_{i,t}^{a,(1-\tau_W)}\int_{i'}\delta_{i',t}^{r,a}di' < \left(1+\varepsilon_{i,t}^{1+r,a}\right)\phi_{i,t}\tilde{\zeta}_{i,t}^{a,(1-\tau_W)} \quad (1.41)$$

since $\int_{i'}\delta_{i'}^{r,a}di' < 0$ for $\int_{i'}\gamma_{i',i',t}^{r,a}di' = \underbrace{\mathbb{COV}\left(\frac{1}{r_{i',t}},\delta_{i'}^{r,a}\right)}_{>0} + \int_{i'}\delta_{i'}^{r,a}di' \cdot \underbrace{\int_{i'}\frac{1}{r_{i',t}}di'}_{>0} = 0.$

Optimal capital tax in steady state. Observe that there are inter-household welfare effects from the endogeneity of each household's return rate in other households' savings

$$\begin{aligned}&\frac{\partial U_{i,t}}{\partial \underline{a}_{i,t+1}}a_{R,i,t+1}\int_{i'}\frac{dlog\left[a_{i,t}\left(1+r_{i,t}\left(a_{i,t}\right)\right)\right]}{dlog\left(1+r_{i,t}\right)}\frac{dlog\left[1+r_{i,t}\left(\cdot\right)\right]}{dlog\left(a_{i',t}\right)}\frac{dlog\left(a_{i',t}\right)}{dlog\left(1-\tau_W\right)}|_E\, di' \\ =&\frac{a_{i,t+1}}{1-\tau_W}\frac{\partial U_{i,t}\left(\cdot\right)}{\partial c_{i,t}}\left(1+\zeta_{i,t}^{a,1+r}\right)\int_{i'}\gamma_{i,i',t}^{1+r,a}\zeta_{i',t}^{a,(1-\tau_W)}di'.\end{aligned}$$

Insert this equation and, as before, the households' first-order conditions and Equation (1.23) into

$$\begin{aligned}dSWF =& \int_i \Gamma_{i,t}\frac{\partial U_{i,t}}{\partial c_{i,t}}\left[\left(1-\tau_W\right)da_{R,i,t} - a_{R,i,t}d\tau_W - w_{i,t}l_{i,t}d\tau_L\right]di - \int_i \Gamma_{i,t}\frac{\partial U_{i,t}}{\partial \underline{a}_{i,t+1}}a_{R,i,t}d\tau_W di \\ &+ \int_i \Gamma_{i,t}\frac{\partial U_{i,t}}{\partial \underline{a}_{i,t+1}}a_{R,i,t+1}\int_{i'}\frac{dlog\left[a_{i,t}\left(1+r_{i,t}\left(a_{i,t}\right)\right)\right]}{dlog\left(1+r_{i,t}\right)}\frac{dlog\left[1+r_{i,t}\left(\cdot\right)\right]}{dlog\left(a_{i',t}\right)}\frac{dlog\left(a_{i',t}\right)}{dlog\left(1-\tau_W\right)}|_E\, di'di.\end{aligned}$$

Set this expression equal to zero and use the definitions of the distributional parameters to get Equation (1.32). Equation (1.33) follows from rearranging Equation (1.32).

Comparative statics. As in the partial equilibrium, approximate household savings and wealth in general equilibrium as

$$a_{i,t}\left(\tau_W^{GE}\right) = a_{i,t}\left(\tau_W^{PE}\right) + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}}a_{i,t}\left(\tau_W^{PE}\right)\zeta_{i,t}^{a,(1-\tau_W)} + o\left(\tau_W^{GE}-\tau_W^{PE}\right)$$

and

$$a_{R,i,t}\left(\tau_W^{GE}\right) = a_{R,i,t}^{PE}\left(\tau_W^{PE}\right) + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}}a_{R,i,t}\left(\tau_W^{PE}\right)\zeta_{i,t}^{a_R,(1-\tau_W)} + o\left(\tau_W^{GE}-\tau_W^{PE}\right),$$

where, again, the elasticities are evaluated in general equilibrium. Integrate out the second expression to get Equation (1.34).

Moreover, initial wealth can be written as

$$\begin{aligned}\overline{a}^{initial}\left(\tau^{GE}\right) &= \frac{\int_i g_{i,t} a_{R,i,t}\left(\tau^{PE}\right) di + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \int_i g_{i,t} a_{R,i,t}\left(\tau^{PE}\right) \zeta_{i,t}^{a_R,(1-\tau_W)} di}{\int_i a_{R,i,t}\left(\tau^{PE}\right) di + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \int_i a_{R,i,t}\left(\tau^{PE}\right) \zeta_{i,t}^{a_R,(1-\tau_W)} di} + o\left(\tau_W^{GE}-\tau_W^{PE}\right) \\ &= \overline{a}^{initial}\left(\tau^{PE}\right) \frac{1+\frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \hat{\zeta}^{a_R,(1-\tau_W)}}{+\frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \zeta^{a_R,(1-\tau)}} + o\left(\tau_W^{GE}-\tau_W^{PE}\right).\end{aligned}$$

Therefore, for $\hat{\zeta}^{a_R,(1-\tau_W)} = \zeta^{a_R,(1-\tau_W)}$, $\overline{a}^{initial}\left(\tau^{GE}\right) = \overline{a}^{initial}\left(\tau^{PE}\right) + o\left(\tau_W^{GE}-\tau_W^{PE}\right)$. Similarly, final wealth

$$\begin{aligned}\overline{a}^{final}\left(\tau^{GE}\right) &= \frac{\int_i g_{i,t} a_{i,t}\left(\tau^{PE}\right) di + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \int_i g_{i,t} a_{i,t}\left(\tau^{PE}\right) \zeta_{i,t}^{a,(1-\tau_W)} di}{\int_i a_{R,i,t}\left(\tau^{PE}\right) di + \frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \int_i a_{R,i,t}\left(\tau^{PE}\right) \zeta_{i,t}^{a_R,(1-\tau_W)} di} + o\left(\tau_W^{GE}-\tau_W^{PE}\right) \\ &= \overline{a}^{final}\left(\tau^{PE}\right) \frac{1+\frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \hat{\zeta}^{a,(1-\tau_W)}}{1+\frac{\tau_W^{PE}-\tau_W^{GE}}{1-\tau_W^{PE}} \zeta^{a_R,(1-\tau)}} + o\left(\tau_W^{GE}-\tau_W^{PE}\right)\end{aligned}$$

simplifies to $\overline{a}^{initial}\left(\tau^{GE}\right) = \overline{a}^{initial}\left(\tau^{PE}\right) + o\left(\tau_W^{GE}-\tau_W^{PE}\right)$ for $\varepsilon_{i,t}^{r,a} \approx 0$ and $\delta_{i,t}^{r,a} \approx 0$.

1.E The Financial Market

In this section, I develop a general equilibrium financial market model, which serves as a microfoundation for the endogenous formation of return inequality (scale dependence). I also show how to incorporate type dependence. Recall that households work in the first period and can transfer resources into the next period by saving parts of their labor income. In the following setting, the returns on savings form on a financial market with imperfect information. For a given amount of savings, households choose their optimal investment portfolio and can acquire information about the stochastic returns on the financial market. This setting gives rise to inequality in the returns to investment. As high-income individuals decide to save more than low-income individuals, they have an incentive to acquire more financial knowledge, which allows them to generate higher (risk-adjusted) returns.

As standard in generational models (e.g., Piketty and Saez (2013)), I subdivide the investment period into $h = 1,...,H+1$ subperiods. For instance, for $H = 30$, the working life has a duration of 30 years. In the following environment, this means that, during their working life, households repeatedly interact on the financial market. In particular, they can adjust their portfolio and their financial knowledge. Between subperiods, there is no time discounting.

1.E.1 Environment

I model the financial market in each subperiod h as in Peress (2004) version of the Grossman and Stiglitz (1980) economy. The general equilibrium model features individuals, who differ in their initial wealth, $a_{i,h}$, which depends on initial savings $a_{i,1}$ and returns realized before h, a financial market with public and private signals about stochastic returns, and endogenous inequality in investment returns. The main goal is to justify the reduced form of investment returns as a function of initial savings $r\left(a_i, \{a_j\}_{j\in[0,1]}\right)$. Whenever I drop the subperiod index h, I refer to the first subperiod ($h = 1$).

Payoff structure. In subperiod h, household $i \in [0,1]$ invests $a_{i,h}$ on a financial market. As in Grossman and Stiglitz (1980), there are two assets: one risk-free asset (bond) and one risky asset (stock). In each subperiod h, households purchase a costly private signal about the stock's payoff and observe a public signal (price). After that, they decide on how much to invest in the risky and the risk-free asset. In this class of models, there exists no closed-form solution for the rational expectations equilibrium in settings that go beyond constant absolute risk aversion (CARA) utilities. In these models, this issue is also present when one considers redistributive taxation. Therefore, I adopt the idea by Peress (2004) who scales the economy with a parameter z. For a small z, one can approximately solve the model in closed form for arbitrary preferences and nonlinear taxes.[27]

In each subperiod, there is a risk-free asset in infinitely elastic supply that delivers a return of $r_h^f z$. The risky asset has an endogenous price P_h and a random payoff Π_h that is log-normally distributed with mean $b_h z$ and variance $\sigma^2 z$, where $log(\Pi_h) \equiv \pi_h z$. The mean payoff is normally distributed $b_h \sim \mathcal{N}\left(\mathbb{E}(b), \sigma_b^2\right)$. In other words, in each subperiod, nature draws a stochastic fundamental of the economy that drives stock returns. For simplicity, I assume that the draws of b_h are uncorrelated over time.

Define $r_{i,h}^p z$ as the realized investment return of household i in subperiod h. For a small z (e.g., $z = 1/H$), $r_{i,h}^p z$ is small so that one can neglect nonlinearities as follows. The compound rate of return can be approximated by $R_i \equiv \left(1 + r_{i,1}^p z\right) \cdot \ldots \cdot \left(1 + r_{i,H}^p z\right) - 1 = \sum_{h=1}^H r_{i,h}^p z + o(z)$. Capital income reads as $R_i a_{i,1} = \sum_{h=1}^H a_{i,h} r_{i,h}^p z + o(z)$. Therefore, when $z = 1/H$, the investment return r_i denotes the average return. Consider the setting in Section 1.C, where the government taxes final wealth linearly according to τ_W.[28]

[27] This procedure is similar to the time increment dt in continuous-time models.

[28] Analyzing a nonlinear capital gains tax, $T_k(\cdot)$, with $T_k(0) = 0$, $T_k'(0) = 0$, and $T_k''(0) = 0$, leads to the same conclusions (see Appendix 1.F.4). Therefore, the financial market, described here, also microfounds the formation of

Information structure. As standard in the literature, assume that there are noise traders who have access to other investment technologies, such as human capital, or make random errors in their forecast of payoffs. The existence of noise traders prevents the full revelation of private information via the publicly-observed price and, as a result, a fully efficient financial market. Otherwise, nobody would have an incentive to purchase the private signal in the first place (Grossman-Stiglitz paradox). Accordingly, the net supply of risky assets, θ_h, is random. Assume that the net supply is normally distributed, $\theta_h \sim \mathscr{N}\left(\mathbb{E}(\theta), \sigma_\theta^2\right)$, and independent from payoffs. This technicality ensures that the equilibrium price is a noisy signal about the fundamentals of the economy.

Households can acquire financial knowledge, for example, by conducting research, obtaining financial education, or employing financial advisers. In particular, they observe a noisy private signal $s_{i,h} = b_h + \vartheta_{i,h}$ with $\vartheta_{i,h} \sim \mathscr{N}\left(0, \frac{1}{x_{i,h}}\right)$ and can purchase a signal precision of $x_{i,h} \in \mathbb{R}_+ \cup \{0\}$ at cost $v\left(x_{i,h}\right) z$, measured in monetary units, where $v(\cdot)$ is increasing, convex, twice continuously differentiable and $v(0) = 0$. That is, information acquisition becomes more and more costly. This assumption is in line with the idea that households obtain pieces of information, and each extra piece correlates with the previous ones. Nonetheless, this model gives rise to increasing returns to information acquisition. Moreover, assume that private signals are uncorrelated across households and that households cannot resell their information. As in reality, agency problems may constrain information resale or sharing.[29]

Timing. The timing of each subperiod is as follows. For a given amount of savings, households purchase financial knowledge $x_{i,h}$. Then, they observe the private and the public price signal. Households form rational expectations about the payoff of the risky asset given the observed signals and decide how much of their savings to invest in the risky asset. Formally, an investor i chooses a share of stocks, $\varsigma_{i,h}$, and a bond share, $\left(1-\varsigma_{i,h}\right)$, given her expectation $\mathbb{E}_{i,h}\left(\cdot|\mathscr{F}_{i,h}\right)$ conditional on the information set $\mathscr{F}_{i,h}$ where $\mathscr{F}_{i,h} = \left\{s_{i,h}, P_h\right\}$, if a signal has been acquired, and $\mathscr{F}_{i,h} = \{P_h\}$, else. Finally, payoffs realize.

Household problem. Given the portfolio choice $\varsigma_{i,h}$, the return of the portfolio reads as

$$r_{i,h}^p z = \varsigma_{i,h} \frac{\Pi_h - P_h}{P_h} + (1-\varsigma_{i,h}) r_h^f z \tag{1.42}$$

returns in the analysis of nonlinear taxes in Section 1.G. Similarly, one can consider a linear capital gains tax as in Section 1.2.

[29] Observe the implicit assumption that knowledge fully depreciates intertemporally. Any departure from this assumption would, just as non-convex cost functions, strengthens the main results.

per unit of savings $a_{i,h-1}$. At the end of the subperiod household i's wealth is the portfolio's gross return net of costs of information acquisition

$$a_{i,h} = a_{i,h-1}\left(1 + r^{p}_{i,h-1}z\right) - v\left(x_{i,h-1}\right)z.$$

I assume that the costs of information acquisition are monetary, realize at the end, and are deductible from the base of the capital tax.

Due to the model approximation used here, the main result that the portfolio return increases with wealth, derived in the next section, is robust to various permutations of these assumptions on the information costs. In particular, it does not matter when the monetary costs accrue. Moreover, when the costs of information acquisition are non-monetary, the key results will carry over with a minor constraint on the shape of the cost function.

Final wealth, $a_{i,H+1}$, can be recursively written as

$$a_{i,H+1} = a_{i,1}\left(1 + \sum_{h=1}^{H} r^{p}_{i,h}z\right) - \sum_{h=1}^{H} v\left(x_{i,h}\right)z + o\left(z\right).$$

I assume that utility from final, after-tax wealth, $\underline{a}_{i,t+1}$, is linearly separable and isoelastic $u\left(\underline{a}_{i,t+1}\right) = \frac{\underline{a}_{i,t+1}^{1-\rho}-1}{1-\rho}$. Then, this utility is approximately given by

$$u\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+R_i\right) - v\left(X_i\right)\right)\right] + o\left(z\right)$$

where $R_i \equiv \sum_{h=1}^{H} r^{p}_{i,h}z$ and $v\left(X_i\right) \equiv \sum_{h=1}^{H} v\left(x_{i,h}\right)z$. This justifies the preference structure in the dynamic economy of Section 1.C.

It remains to show that $r_{i,h} = r\left(a_{i,h}, \left\{a_{j,h}\right\}_{j\in[0,1]}\right)$. Firstly, note that utility from final wealth can also be written as

$$H \cdot u\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1 + r^{p}_{i,1}z\right) - v\left(x_{i,1}\right)z\right)\right] + o\left(1\right) \tag{1.43}$$

Hence, for a given distribution of initial wealth, $a_{i,1}$, the repeated financial market interaction in subperiod h is up to a constant *fully static*.[30] Therefore, in the following, I drop time indices in individual and aggregate variables for notational convenience. Accordingly, in each subperiod, households maximize their expected utility

$$\max_{x} \mathbb{E}_i\left(\max_{\varsigma} \mathbb{E}_i\left(u\left[\left(1-\tau_W\right)\left(a_i\left(1 + r^{p}_{i}z\right) - v\left(x_i\right)z\right)\right] \mid \mathscr{F}_i\right)\right) \tag{1.44}$$

[30]Incorporating dynamic aspects, e.g., the accumulation of wealth and the resulting spread in the wealth distribution, would only strengthen the main result that wealthier households obtain higher rates of return than poorer ones.

The set of optimal choices by household i on the financial market reads as $\{\varsigma_i, x_i\}$ which will be functions of initial savings. Moreover, denote the p.d.f. of savings as $g(a_i)$ and the c.d.f. as $G(a_i)$, respectively.

A side-effect of the model approximation is that one can rewrite the stochastic period utility in deterministic units

$$\begin{aligned}\mathbb{E}_i\left(u\left[(1-\tau_W)\left(a_i\left(1+r_i^p z\right)-v(x_i)z\right)\right]\right) &= u\left[(1-\tau_W)\left(a_i\left(1+\mathbb{E}\left(r_i^p z\right)\right)-v(x_i)z\right)\right] \\ &\quad + \frac{1}{2}u''\left[(1-\tau_W)a_i\right](1-\tau_W)^2\,\mathbb{V}\left(a_i r_i^p z\right)+o(z).\end{aligned}$$

To get this expression, approximate the expected utility around the mean portfolio return. Thus, second-period utility features a deterministic mean-variance trade-off in the spirit of Markowitz (1952, 1959). Households trade off endogenous ex ante risk and returns. I derive these measures in the following. Therefore, the tax analysis with deterministic returns is sufficient.

Aggregate variables. Denote the risk tolerance of a household, who invests a_i, as the inverse coefficient of absolute risk aversion $\psi(a_i) \equiv \frac{-u'(a_i)}{u''(a_i)}$. With the specified utility function, $\psi(a_i) = a_i/\rho$. In principle, $\psi'(a_i) > 0$ would be sufficient to obtain scale dependence.

Moreover, dropping the time index on the aggregate variables, define the aggregate risk-taking by $\mathscr{T} \equiv \int_i \mathscr{T}_i di \equiv \int_i \psi(a_i)\,di$, the aggregate noisiness by $\mathscr{N} \equiv \int_i \mathscr{N}_i di \equiv \int_i \frac{\psi(a_i)}{h_0(\mathscr{I})+x_i}di$, and the aggregate informativeness of the price by $\mathscr{I} \equiv \int_i \mathscr{I}_i di \equiv \frac{1}{\sigma^2}\int_i \frac{x_i\psi(a_i)}{h_0(\mathscr{I})+x_i}di$, where $h_0(\mathscr{I}) \equiv \frac{1}{\sigma_b^2}+\frac{\mathscr{I}^2}{\sigma_\theta^2}$ measures the precision of the public signal. Therefore, the variable $\mathscr{T}$ aggregates risk tolerance or risk-taking of all households. $\mathscr{N}$ summarizes the noisiness (inverse precision) of the prior, the stock price, and the private signals of households, whereas $\mathscr{I}$ measures the total signal precision relative to the total precision. Both $\mathscr{N}$ and $\mathscr{I}$ are weighted by the risk tolerance. The definition of these three variables will prove convenient when deriving the equilibrium of the economy. Now, one can define the rational expectations equilibrium of the financial market.

Rational expectations equilibrium. Define a rational expectations equilibrium as the set of choices $\{\varsigma_i, x_i\}$, the stock's price as a function of Π and θ and the informativeness $\mathscr{I}$ such that

(1) households optimally choose their portfolio and signal precision

$$\varsigma_i = \varsigma(S_i, x_i, a_i; P, \mathscr{I}) \equiv arg\max_{\varsigma}\mathbb{E}_i\left(u\left[(1-\tau_W)\left(a_i\left(1+r_i^p z\right)-v(x_i)z\right)\right]|\mathscr{F}_i\right) \tag{1.45}$$

and

$$x_i = x(a_i;\mathscr{I}) \equiv arg\max_{x}\mathbb{E}_i\left[\max_{\varsigma}\mathbb{E}_i\left(u\left[(1-\tau_W)\left(a_i\left(1+r_i^p z\right)-v(x_i)z\right)\right]|\mathscr{F}_i\right)\right], \tag{1.46}$$

(2) P clears the stock market

$$\int_i \frac{\varsigma_i a_i}{P} di = \theta, \tag{1.47}$$

and

(3) the implied informativeness of the price is consistent with observed choices of individual information precision

$$\mathscr{I} = \frac{1}{\sigma^2} \int_i \frac{x(a_i;\mathscr{I})\,\psi(a_i)}{h_0(\mathscr{I}) + x(a_i;\mathscr{I})} di. \tag{1.48}$$

1.E.2 The Equilibrium

In the following, I show that, in the approximated Grossman and Stiglitz (1980) economy, investment returns and their distribution depend on capital justifying the reduced form assumption on the capital gains functional in the sections before. I solve the model by backward induction. First, one shows that there exists a log-linear rational expectations equilibrium and derive portfolio choices and the equilibrium stock price. Then, to demonstrate that the amount of information acquisition, x_i, increases in the portfolio size, a_i, one characterizes the demand for information by the first-order condition

$$v'(x_i) = \frac{1}{2\rho} a_i \mathscr{S}'(x_i;\mathscr{I}), \tag{1.49}$$

where $\mathscr{S}(x_i;\mathscr{I})$ is the expected squared Sharpe ratio of an investor. Wealthy investors purchase more information than poorer ones. There exists a threshold value $a_i^*(\mathscr{I})$ below which nobody obtains information. There is a congestion effect. The threshold wealth $a_i^*(\mathscr{I})$ is increasing in $\mathscr{I}$. Hence, a rise in the aggregate informativeness lowers the number of investors who choose to purchase information.

Furthermore, note that information is a strategic substitute. That is, $x(a_i;\mathscr{I})$ is a decreasing function of $\mathscr{I}$. The higher the informativeness of the public signal (price), the lower is the need for acquiring private information. In other words, the information acquisition by all investors imposes an effect on an individual investor via the equilibrium price. Investors do not internalize this effect. Finally, it can be shown that there exists a unique scalar for $\mathscr{I}$ and, thus, for $\mathscr{N}$. Therefore, the log-linear equilibrium is unique.

Portfolio Returns and Sharpe Ratio

Now, I present the implications of information acquisition for portfolio returns. As we have seen, wealthier investors acquire more information, even though each extra piece of information becomes more and more costly. Does this information advantage help investors to generate higher rates of return? To answer this question, define the excess return of investor i's portfolio $r_i^{pe} z \equiv r_i^p z - r^f z$.

Lemma 1 (Returns, variance, and Sharpe ratio)**.** *The expected excess return, its variance, and the Sharpe ratio are increasing in x_i which rises in a_i:*

$$\mathbb{E}\left(r_i^{pe} z\right) = \mathbb{E}\left(r_i^p z\right) - r^f z = \frac{1}{\rho} \mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right) z + o(z), \tag{1.50}$$

$$\mathbb{V}\left(r_i^{pe} z\right) = \mathbb{V}\left(r_i^p z\right) = \frac{1}{\rho^2} \mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right) z + o(z), \tag{1.51}$$

and

$$\frac{\mathbb{E}\left(r_i^{pe} z\right)}{\sqrt{\mathbb{V}\left(r_i^{pe} z\right)}} = \sqrt{\mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right) z} + o(1). \tag{1.52}$$

Proof. See Appendix 1.F.2. ☐

Lemma 1 reveals how the portfolio returns (and its risk) relate to the individual's signal precision x_i, portfolio sizes a_i, and the relative risk aversion $1/\rho$. Both the expected excess return and its standard deviation are declining in the relative risk aversion. Moreover, these variables increase in the degree of individual information that rises in the portfolio size. Hence, wealthier investors obtain higher returns and are willing to take more risk relative to poorer households. Moreover, returns depend on aggregate information.

To sum up, an individual's demand for stocks and information, as well as her (risk-adjusted) return, depend on her amount of investment and, through the equilibrium price, on others' investments. Households become richer because they are rich. As a result, the final wealth distribution is more unequal than the initial one. This insight originates from Arrow (1987).

Moreover, an investor's return does not directly depend on her capital tax. This feature derives from the linear approximation of the economy and the CRRA utility function. Altogether, this financial market interaction justifies the reduced form assumption on the endogenous return inequality in Section 1.C and 1.G, $r_i\left(a_i, \{a_j\}_{j\in[0,1]}\right)$.

An Example

Suppose, for simplicity, that $\mathbb{E}(\theta) = 0$ and $v(x_i) = \kappa x_i$. Due to the linearity of costs, the rents from private signal extraction are constant conditional on a given amount of investment. A higher degree of public information reduces one-to-one the demand for private information. Moreover, let $a_0 > a_i^*(\mathscr{I})$. Then, the elasticity of the return (in a given subperiod) with respect to the amount of investment is positive

$$\varepsilon_i^{\mathbb{E}(r^p z),a} \equiv \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial log\left(a_i\right)} = \frac{\sqrt{\rho\kappa/\left(2\sigma^2 a_i\right)}}{\mathscr{S}\left(a_i, \left\{a_j\right\}_{j\in[0,1]}\right)/\rho + r^f} > 0.$$

Also, note that the expected return is concave in the amount of investment. Therefore, own-return elasticity decreases with a_i.

The cross-return elasticity reads as

$$\gamma_{i,i'}^{\mathbb{E}(r^p z),a} \equiv \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial log\left(a_{i'}\right)} = \sum_{\mathscr{A}\in\{\mathscr{T},\mathscr{N},\mathscr{I}\}} \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial \mathscr{A}} \frac{\partial \mathscr{A}_{i'}}{\partial log\left(a_{i'}\right)}.$$

One can show that by the linearity of the cost function it is multiplicatively separable. That is, $\gamma_{i,i'}^{\mathbb{E}(r^p z),a} = \frac{1}{\mathbb{E}\left(r_i^p z\right)}\delta_{i'}^{\mathbb{E}(r^p z),a}$.

Observe that the cross-return elasticity carries risk and information effects. Investors are rewarded for risk that they are willing to take on the stock market. The variability of the price measures this risk: $\mathbb{V}\left(log\left(P\right)\right) = \mathbb{V}\left(p_\xi \xi\right)$ where ξ is the public signal and p_ξ is the responsiveness of the price to the public signal. Two channels affect the amount of this aggregate risk and, as a result, individual returns.

Firstly, a rise in aggregate information, $\mathscr{I}$, lowers the variance of ξ and, therefore, lowers portfolio returns. Secondly, the sensitivity of the stock price to the price signal, p_ξ, is determined in general equilibrium. As the aggregate noisiness, $\mathscr{N}$, declines, the equilibrium stock price becomes more sensitive to the price signal so that p_ξ increases. Similarly, a rise in risk tolerance, $\mathscr{T}$, increases the demand for stocks intensifying the relation between the price and the public signal. Hence, a rise in $\mathscr{T}$ (a reduction in $\mathscr{N}$) increases the variability of the public signal.

Altogether, a rise in portfolio size $a_{i'}$ (and, therefore, in information $x_{i'}$) has opposing effects on the return of household i. For simplicity, let $\sigma^2 = \sigma_b^2 = \sigma_\theta^2 = 1$. Then, one can show that $\delta_{i'}^{\mathbb{E}(r^p z),a} \geq 0$ for $a_{i'} \leq \tilde{a}$ and $\delta_{i'}^{\mathbb{E}(r^p z),a} < 0$ for $a_{i'} > \tilde{a}$. Whereas an investor's marginal contribution to risk is constant, contributions to information are nonlinear in the amount of investment. For instance, the impact of wealthy investors on information is larger than the one of poorer investors (i.e., $\frac{\partial^2 \mathscr{I}_{i'}}{\partial a_{i'}^2} > 0$). They contribute marginally more to the level of aggregate information, which reduces uncertainty and, hence, the idiosyncratic reward for risk ($\mathbb{E}\left(r_i^p z\right)$).

Consequently, this setting is analogous to trickle-up. Consider a tax cut on the wealth of the rich. As a reaction, wealthy investors increase their portfolio size which allows them to generate higher rates of return because they acquire more information ($\varepsilon_i^{\mathbb{E}(r^p z),a} > 0$). At the same time, the level of aggregate information increases. As a consequence, the value of private information decreases. The reward for the small amount of private information, that poorer households acquire, declines ($\delta_{i'}^{\mathbb{E}(r^p z),a} < 0$). Therefore, the tax cut shifts capital income from the bottom to the top.

Of course, this observation holds when all households, even the poor, invest in financial knowledge (i.e., $a_0 > a_i^*(\mathscr{I})$). Suppose that $a_i = a_i^*(\mathscr{I})$ for some $i \in (0,1)$. Then, the poor, who do not invest in information, may benefit from a tax cut for the rich, as they only rely on public information. In this situation, only the middle class suffers from a loss in their rents from private information acquisition.

1.E.3 Extensions

In this section, I extend the financial market model by considering two practically relevant modifications of the financial market model. First, I consider career effects. In the second extension, I deal with type dependence. Throughout this section, suppose the assumptions from the example hold. That is, let $\mathbb{E}(\theta) = 0$, $\sigma^2 = \sigma_b^2 = \sigma_\theta^2 = 1$, and $v(x_i) = \kappa x_i$. Moreover, assume that $a_0 > a_i^*(\mathscr{I})$.

Career Effects

Wealthy households may not only obtain high financial knowledge since their portfolios are sizable but also because of the professional network they build during their career. In other words, as they earn a high income and, as a result, become wealthy, they gain access to specialist knowledge about financial markets either because they work in the finance industry or they get to know financial experts. This channel additionally boosts their portfolio returns.

To formalize this, let $v(x_i, y_i)$ where $\frac{\partial^2 v(x_i,y_i)}{\partial y_i^2} > 0$ and $\frac{\partial v(x_i,y_i)}{\partial x_i \partial y_i} < 0$. The marginal costs of purchasing information decrease with an individual's income y_i. Then, the Sharpe ratio

$$\mathscr{S}\left(a_i, l_i, \{a_j\}_{j\in[0,1]}, \{l_j\}_{j\in[0,1]}\right)$$

and, accordingly, the expected rate of return, as well as its variance, increase with an individual's labor supply.[31] As labor supply increases with i, this force amplifies the main feature of the model of endogenous return inequality. Put differently, $\varepsilon_i^{\mathbb{E}(r^p z),l} \equiv \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial log(l_i)} > 0$. In general equilibrium, $\gamma_{i,i'}^{\mathbb{E}(r^p z),l} \equiv \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial log\left(l_{i'}\right)} \neq 0$.

31 $\frac{\partial v(x_i,y_i)}{\partial x_i \partial y_i} < 0$ implies by the second fundamental theorem of calculus that $\frac{\partial v(x_i,y_i)}{\partial y_i} \neq 0$. Therefore, the labor supply elasticities are modified by an additional marginal effect on information costs.

Type Dependence

As noted in the literature on inequality (e.g., Benhabib, Bisin, and Zhu (2011)), type dependence explains the thick tail in the distribution of wealth observed in many countries. Applied to the financial market setting, this refers to a situation where the rich are also talented in investing their money.

The easiest way to incorporate type dependence is to let κ_i vary by type. That is, suppose κ_i is decreasing in the index i. Thus, there is heterogeneity not only in hourly wages, but also in the marginal costs of information acquisition. Accordingly, an investor's Sharpe ratio $\mathscr{S}_i(\cdot)$ is indexed by i. The presence of cost heterogeneity amplifies the inequality in returns. The reasoning is as follows. Wealthy, talented investors acquire more financial knowledge than without type dependence, as it is cheaper for them. Therefore, they earn higher returns. In turn, the incentives to save rise such that their portfolio increases in size. Because of scale dependence, this further boosts their returns.

Moreover, the distribution of own-return elasticities is affected. To see this, compare own-return semi-elasticities of household i and j where $i > j$: $\frac{\mathbb{E}\left(r_i^p z\right)\varepsilon_i^{\mathbb{E}(r^p z),a}}{\mathbb{E}\left(r_j^p z\right)\varepsilon_j^{\mathbb{E}(r^p z),a}} = \sqrt{\frac{a_j}{a_i}}\sqrt{\frac{\kappa_i}{\kappa_j}}$. There are two effects that compress the distribution of own-return semi-elasticities. Firstly, $\sqrt{\frac{\kappa_i}{\kappa_j}} < 1$. Secondly, type dependence leads to more return inequality which boosts wealth inequality. Thus, $\sqrt{\frac{a_j}{a_i}}$ is lower in the presence of type dependence. The effect on the distribution of own-return elasticities is even larger $\frac{\varepsilon_i^{\mathbb{E}(r^p z),a}}{\varepsilon_j^{\mathbb{E}(r^p z),a}} = \frac{\mathbb{E}\left(r_j^p z\right)}{\mathbb{E}\left(r_i^p z\right)}\sqrt{\frac{a_j}{a_i}}\sqrt{\frac{\kappa_i}{\kappa_j}}$ because return inequality, $\frac{\mathbb{E}\left(r_j^p z\right)}{\mathbb{E}\left(r_i^p z\right)}$, directly enters the expression. Therefore, the presence of type dependence compresses the distribution of own-return elasticities.

In general equilibrium, the distribution of cross-return semi-elasticities is unaffected by type dependence, whereas the effect on the distribution of cross-return elasticities depends on the effects on return inequality $\frac{\gamma_{i,i'}^{\mathbb{E}(r^p z),a}}{\gamma_{j,i'}^{\mathbb{E}(r^p z),a}} = \frac{\mathbb{E}\left(r_j^p z\right)}{\mathbb{E}\left(r_i^p z\right)}$. If type dependence triggers a rise in return inequality, the distribution of cross-return elasticities will flatten.

1.F Proofs for Section 1.E

1.F.1 Approximations

I show, by induction, that the statement $P(H) : \Pi_{h=1}^{H}\left(1 + r_{i,h}^p z\right) = 1 + \sum_{h=1}^{H} r_{i,h}^p z + o(z)$ holds for any $H \geq 1$. The base case, $P(1)$, is trivially fulfilled. For the inductive step, let $P(k)$ hold. Then,

$P(k+1)$ is also true since

$$\Pi_{h=1}^{k}\left(1+r_{i,h}^{p}z\right)\cdot\left(1+r_{i,k+1}^{p}z\right)=\left(1+\sum_{h=1}^{k}r_{i,h}^{p}z+o(z)\right)\left(1+r_{i,k+1}^{p}z\right)$$
$$=1+\sum_{h=1}^{k}r_{i,h}^{p}z+\left(1+\sum_{h=1}^{k}r_{i,h}^{p}z\right)r_{i,k+1}^{p}z+o(z)=1+\sum_{h=1}^{k+1}r_{i,h}^{p}z+o(z)\,.$$

Using this expression, period-h wealth can be written as

$$a_{i,h}=a_{i,h-1}\left(1+r_{i,h-1}^{p}z\right)-v\left(x_{i,h-1}\right)z=\left[a_{i,h-2}\left(1+r_{i,h-2}^{p}z\right)-v\left(x_{i,h-2}\right)z\right]\left(1+r_{i,h-1}^{p}z\right)-v\left(x_{i,h-1}\right)z$$
$$=a_{i,h-2}\left(1+r_{i,h-1}^{p}z+r_{i,h-2}^{p}z\right)-v\left(x_{i,h-2}\right)z-v\left(x_{i,h-1}\right)z+o(z)=\ldots$$
$$=a_{i,1}\left(1+\sum_{j=1}^{h-1}r_{i,h-j}^{p}z\right)-\sum_{j=1}^{h-1}v\left(x_{i,h-j}\right)z+o(z)=a_{i}\left(1+\sum_{s=1}^{h-1}r_{i,s}^{p}z\right)-\sum_{s=1}^{h-1}v\left(x_{i,s}\right)z+o(z)$$

for any $h=1,\ldots,H+1$. Capital income is given by

$$R_{i}a_{i,1}=\sum_{h=1}^{H}a_{i,1}r_{i,h}^{p}z+o(z)=\sum_{h=1}^{H}a_{i,h}r_{i,h}^{p}z+\sum_{h=1}^{H}\left(a_{i,1}-a_{i,h}\right)r_{i,h}^{p}z+o(z)=\sum_{h=1}^{H}a_{i,h}r_{i,h}^{p}z+o(z)\,.$$

Defining the overall information effort as $x_i\equiv\sum_{h=1}^{H}x_{i,h}z$, the information costs can be approximated by

$$v\left(X_{i}\right)\equiv v\left(\sum_{h=1}^{H}x_{i,h}z\right)\equiv v\left(x_{i,1}z,\ldots,x_{i,H}z\right)=v(0,\ldots,0)+\sum_{h=1}^{H}\frac{\partial v(0,\ldots,0)}{\partial x_{i,h}}\left(x_{i,h}z-0\right)+o(z)$$
$$=v'(0)x_{i,1}z+\ldots+v'(0)x_{i,H}z+o(z)=\sum_{h=1}^{H}\left(v\left(x_{i,h}\right)z-v(0)\right)+o(z)=\sum_{h=1}^{H}v\left(x_{i,h}\right)z+o(z)$$

Therefore, one can rewrite the utility from final wealth as

$$u\left(a_{i,H+1}\right)=u\left[\left(1-\tau_{W}\right)\left(a_{i,1}\left(1+\sum_{h=1}^{H}r_{i,h}^{p}z\right)-\sum_{h=1}^{H}v\left(x_{i,h}\right)z\right)\right]+o(z)$$
$$=u\left[\left(1-\tau_{W}\right)\left(a_{i,1}\left(1+R_{i}\right)-v\left(X_{i}\right)\right)\right]+o(z)\,,$$

which justifies the preference structure in Section 1.C. Alternatively, one can express the utility from final wealth as

$$\begin{aligned}
u\left(a_{i,H+1}\right) &= u\left(a_{i,1}+\sum_{h=1}^{H}\Delta a_h\right)+o\left(z\right)=u\left(\Delta a_1,...,\Delta a_H\right)+o\left(z\right)\\
&= u\left(0,...,0\right)+\sum_{h=1}^{H}\frac{\partial u\left(0,...,0\right)}{\partial \Delta a_h}\Delta a_h+o\left(z\right)=u\left(a_{i,1}\right)+\sum_{h=1}^{H}\left(u\left(a_{i,1}+\Delta a_h\right)-u\left(a_{i,1}\right)\right)+o\left(z\right)\\
&= \sum_{h=1}^{H}u\left(a_{i,1}\left(1+r_{i,h}^{p}z\right)-v\left(x_{i,h}\right)z\right)+o\left(1\right),
\end{aligned}$$

where I defined $\Delta a_h \equiv a_{i,1}r_{i,h}^{p}z-v\left(x_{i,h}\right)z$. By the simplifying assumption that knowledge fully depreciates intertemporally, Equation (1.43) follows.

As I will show later, any moment higher than the return variance is negligible. Accordingly, expected period-utility can be approximated around the utiltity from expected wealth as follows

$$\begin{aligned}
\mathbb{E}_i\left(u\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+r_{i,1}^{p}z\right)-v\left(x_{i,1}\right)z\right)\right]\right) &= \mathbb{E}_i\left(u\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+\mathbb{E}\left(r_{i,1}^{p}z\right)\right)-v\left(x_{i,1}\right)z\right)\right]\right)\\
&+\left(1-\tau_W\right)\mathbb{E}_i\left(u'\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+\mathbb{E}\left(r_{i,1}^{p}z\right)\right)-v\left(x_{i,1}\right)z\right)\right]\left[a_{i,1}r_{i,1}^{p}z-a_{i,1}\mathbb{E}\left(r_{i,1}^{p}z\right)\right]\right)\\
+\frac{1}{2}\left(1-\tau_W\right)^2\mathbb{E}_i&\left(u''\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+\mathbb{E}\left(r_{i,1}^{p}z\right)\right)-v\left(x_{i,1}\right)z\right)\right]\left[a_{i,1}r_{i,1}^{p}z-a_{i,1}\mathbb{E}\left(r_{i,1}^{p}z\right)\right]^2\right)+o\left(z\right)\\
=u\left[\left(1-\tau_W\right)\left(a_{i,1}\left(1+\mathbb{E}\left(r_{i,1}^{p}z\right)\right)-v\left(x_{i,1}\right)z\right)\right]&+\frac{1}{2}u''\left[\left(1-\tau_W\right)a_{i,1}\right]\left(1-\tau_W\right)^2\mathbb{V}\left(a_{i,1}r_{i,1}^{p}z\right)+o\left(z\right).
\end{aligned}$$

1.F.2 The Financial Market Equilibrium and Linear Taxation

Equilibrium price, existence, and demand for stocks. In the following, I characterize the financial market equilibrium in subperiod 1 (and, therefore, for each subperiod h). Therefore, I completely drop time indices in this section. I start with portfolio choices and derive the equilibrium stock price. Lemma 2 summarizes the results.

Lemma 2 (Existence of equilibrium, equilibrium price, and portfolio choice)**.** *Assume z is small. Then, there exists a log-linear rational expectations equilibrium. The equilibrium price is linear in $\xi \equiv b-\frac{1}{\mathscr{I}}\theta$*

$$log\left(P\right)=pz=\left(p_0+p_\xi\xi-r^f\right)z+o\left(z\right) \tag{1.53}$$

where $p_0 \equiv \frac{\mathscr{N}}{\mathscr{T}}\left[\frac{\mathbb{E}(b)}{\sigma_b^2}+\frac{\mathscr{I}\mathbb{E}(\theta)}{\sigma_\theta^2}\right]+\frac{1}{2}\sigma^2$ and $p_\xi \equiv 1-\frac{\mathscr{N}}{\mathscr{T}\sigma_b^2}$. The optimal investment in the risky asset is given by

$$\varsigma_i=\frac{1}{\rho\sigma\sqrt{z}}\lambda_i+o\left(1\right) \tag{1.54}$$

where $\lambda_i \equiv \frac{\sqrt{z}}{\sigma}\left[\frac{1}{h_0(\mathscr{I})+x_i}\left(\frac{\mathbb{E}(b)}{\sigma_b^2}+\frac{\mathscr{I}\mathbb{E}(\theta)}{\sigma_\theta^2}+\frac{\mathscr{I}^2}{\sigma_\theta^2}\xi+x_is_i\right)+\frac{1}{2}\sigma^2-p-r^f\right]$ is the investor's Sharpe ratio.

The proof of Lemma 2 involves three steps. Conjecturing the log-linear equilibrium price (Equation (1.53)), determine the conditional variance and expectation of payoffs (step 1), derive the optimal portfolio (step 2), and determine the equilibrium price using the stock market clearing confirming the price conjecture (step 3).

Step 1: By the law of total conditional variance and expectation, the conditional variance of payoff and the conditional expected payoff read as

$$\begin{aligned}\mathbb{V}_i(\pi z|\mathscr{F}_i) &= \mathbb{E}_i(\mathbb{V}_i(\pi z|b,\mathscr{F}_i)|\mathscr{F}_i) + \mathbb{V}_i(\mathbb{E}_i(\pi z|b,\mathscr{F}_i)|\mathscr{F}_i)\\ &= \mathbb{E}_i(\mathbb{V}_i(\pi z|b)|\mathscr{F}_i) + \mathbb{V}_i(bz|\mathscr{F}_i) = \sigma^2 z + o(z)\end{aligned}$$

and, using $b \equiv \xi + \frac{1}{\mathscr{I}}\theta$ in Lemma 2,

$$\begin{aligned}\mathbb{E}_i(\pi z|\mathscr{F}_i) &= \mathbb{E}_i(\mathbb{E}_i(\pi z|b,\mathscr{F}_i)|\mathscr{F}_i) = \mathbb{E}_i(bz|\mathscr{F}_i)\\ &= \frac{1}{h_0(\mathscr{I}) + x_i}\left[\frac{1}{\sigma_b^2}\mathbb{E}(b) + \frac{\mathscr{I}}{\sigma_\theta^2}\mathbb{E}(\theta) + \frac{\mathscr{I}^2}{\sigma_\theta^2}\xi + x_i s_i\right] z + o(z).\end{aligned}$$

Step 2: In the following, I approximate the household's Euler equation

$$\begin{aligned}0 &= \mathbb{E}_i\left[u'\left((1-\tau_W)\left(a_i\left(1+\varsigma_i\frac{\Pi-P}{P} + (1-\varsigma_i)r^f z\right) - v(x_i)z\right)\right)\left(\frac{\Pi-P}{P} - r^f z\right)|\mathscr{F}_i\right]\\ &= u'((1-\tau_W)a_i)\mathbb{E}_i\left[\left(\frac{\Pi-P}{P} - r^f z\right)|\mathscr{F}_i\right]\\ &\quad + (1-\tau_W)a_i\varsigma_i u''((1-\tau_W)a_i)\mathbb{E}_i\left[\left(\frac{\Pi-P}{P} - r^f z\right)^2|\mathscr{F}_i\right] + o(z) \end{aligned}\tag{1.55}$$

that determines the optimal portfolio choice. Note that

$$\begin{aligned}\mathbb{E}_i\left[\left(\frac{\Pi-P}{P} - r^f z\right)|\mathscr{F}_i\right] &= \mathbb{E}_i\left[\left(\frac{exp(\pi z) - exp(pz)}{exp(pz)} - r^f z\right)|\mathscr{F}_i\right]\\ &= \mathbb{E}_i\left[\left(\frac{1+\pi z + \frac{1}{2}(\pi z)^2 + o(z^2) - 1 - pz - \frac{1}{2}(pz)^2 - o(z^2)}{1+pz+o(z)}\right)|\mathscr{F}_i\right] - r^f z\\ &= \mathbb{E}_i(\pi z|\mathscr{F}_i) + \frac{1}{2}\mathbb{E}_i\left[\left((\pi z)^2 - (pz)^2\right)|\mathscr{F}_i\right] - pz - r^f z + o(z)\\ &= \mathbb{E}_i(bz|\mathscr{F}_i) + \frac{1}{2}\sigma^2 z - pz - r^f z + o(z)\end{aligned}\tag{1.56}$$

and

$$\mathbb{E}_i\left[\left(\frac{\Pi-P}{P}-r^f z\right)^2|\mathscr{F}_i\right]=\mathbb{E}_i\left[\left(\pi z+\frac{1}{2}(\pi z)^2-pz-\frac{1}{2}(pz)^2-r^f z\right)^2|\mathscr{F}_i\right]+o(z)$$
$$=\mathbb{E}_i\left[(\pi z)^2|\mathscr{F}_i\right]+o(z)=\sigma^2 z+o(z). \tag{1.57}$$

Plug these expressions and the conjectured equilibrium price into Equation (1.55). To get Equation (1.54), rearrange the resulting expression and observe that $\frac{-u'((1-\tau_W)a_i)}{(1-\tau_W)a_i u''((1-\tau_W)a_i)}=\frac{1}{\rho}$.

Step 3: Plug Equation (1.54) and the definitions of the aggregate variables into the stock market clearing condition (Equation (1.45)) to get

$$\theta=\frac{1}{\sigma^2}\left[\left(\frac{1}{\sigma_b^2}E(b)+\frac{\mathscr{I}}{\sigma_\theta^2}E(\theta)+\frac{\mathscr{I}^2}{\sigma_\theta^2}\left(b-\frac{1}{\mathscr{I}}\theta\right)\right)\mathscr{N}+b\sigma^2\mathscr{I}+\mathscr{T}\left(\frac{1}{2}\sigma^2-p-r^f\right)\right]+o(1).$$

Rearrange to conclude that Equation (1.53) is fulfilled.

Demand for information and equilibrium uniqueness. In Lemma 3, I characterize the demand for information and confirm the uniqueness of the equilibrium.

Lemma 3 (Demand for information, equilibrium informativeness, and uniqueness of equilibrium)**.** *Assume z is small. There exists a threshold wealth $a_i^*(\mathscr{I})\equiv 2\rho v'(0)\sigma^2 h_0(\mathscr{I})^2$, above which there is positive information acquisition, x_i, that increases in a_i according to the first-order condition*

$$v'(x_i)=\frac{a_i}{2\rho}\mathscr{S}'(x_i;\mathscr{I}), \tag{1.58}$$

where $\mathscr{S}(x_i;\mathscr{I},\mathscr{N},\mathscr{T})z=\mathbb{E}_i\left(\lambda_i^2\right)$ is expected squared Sharpe ratio of an investor. $\mathscr{S}(x_i;\mathscr{I},\mathscr{N},\mathscr{T})$ is increasing and concave in the precision of the private signal x_i. Therefore, the informativeness of the price can be written as

$$\mathscr{I}=\frac{1}{\rho\sigma^2}\int_{a_i^*(\mathscr{I})}^{a_1}\frac{a_i x_i(a_i;\mathscr{I})}{h_0(\mathscr{I})+x_i(a_i;\mathscr{I})}dG(a_i). \tag{1.59}$$

There exists a unique log-linear equilibrium.

To proof Lemma 3, observe that a households expected squared Sharpe ratio is given by

$$\mathscr{S}(x_{i,1};\mathscr{I})z\equiv\mathbb{E}_i\left(\lambda_i^2\right)=\mathbb{V}_i(\lambda_i)+\mathbb{E}_i(\lambda_i)^2$$
$$=-\frac{z}{\sigma^2}\frac{1}{h_0(\mathscr{I})+x_i}+\frac{z}{\sigma^2}\left[\frac{\sigma_\theta^2}{\mathscr{I}^2}p_\xi^2+\sigma_b^2\left(1-p_\xi\right)^2+\frac{E(\theta)^2}{\mathscr{I}^2}\left(1-h_0(\mathscr{I})\frac{\mathscr{N}}{\mathscr{T}}\right)^2\right]. \tag{1.60}$$

Similar to above, one approximates

$$\begin{aligned}\mathbb{E}_i\left[u(\cdot)\,|\mathscr{F}_i\right] &= u\left((1-\tau_W)\,a_i\right) + (1-\tau_W)\,u'\left((1-\tau_W)\,a_i\right)\\ &\cdot\left[a_i\varsigma_i\left(\mathbb{E}_i\left(\pi z|\mathscr{F}_i\right) + \frac{1}{2}\mathbb{V}_i\left(\pi z|\mathscr{F}_i\right) - pz - r^f z\right) + a_i r^f z - v\left(x_i\right)z\right]\\ &+ \frac{1}{2}\left(1-\tau_W\right)^2 u''\left((1-\tau_W)\,a_i\right)a_i^2\varsigma_i^2\mathbb{V}_i\left(\pi z|\mathscr{F}_i\right) + o\left(z\right)\end{aligned}$$

to obtain a non-stochastic expression for

$$\mathbb{E}_i\left[u(\cdot)\right] = u\left((1-\tau_W)\,a_i\right) + (1-\tau_W)\,u'\left((1-\tau_W)\,a_i\right)z\left(\frac{a_i}{2\rho}\frac{\mathbb{E}_i\left(\lambda_i^2\right)}{z} + a_i r^f - v\left(x_i\right)\right) + o\left(z\right)$$

Then, optimize over signal precision x_i. The first-order condition

$$v'\left(x_i\right) = \frac{1}{2\rho}a_i\mathscr{S}'\left(x_i;\mathscr{I}\right) = \frac{a_i}{2\rho\sigma^2}\left(\frac{1}{\sigma_b^2} + \frac{\mathscr{I}^2}{\sigma_\theta^2} + x_i\right)^{-2}$$

is sufficient by the second-order condition

$$\frac{\partial^2\mathbb{E}_i\left(u(\cdot)\right)}{\partial x_i^2} = \frac{1}{2\rho}a_i\mathscr{S}''\left(x_i;\mathscr{I}\right) - v''\left(x_i\right) < 0$$

and, hence, characterizes the unique solution to the household information acquisition problem.

By the implicit function theorem, information acquisition rises with wealth

$$\frac{dx_i}{da_i} \propto \frac{\partial^2\mathbb{E}_i\left(u(\cdot)\right)}{\partial x_i\partial a_i} = \frac{1}{2\rho}\mathscr{S}'\left(x_i;\mathscr{I}\right) > 0$$

and $a_i^*\left(\mathscr{I}\right) \equiv 2\rho v'\left(0\right)\sigma^2\mathscr{S}\left(0;\mathscr{I}\right)^{-1} = 2\rho v'\left(0\right)\sigma^2 h_0\left(\mathscr{I}\right)^2$ is the threshold wealth level above which there is information acquisition. Denote i^* as the respective threshold household. Again, use the implicit function theorem to show that

$$\frac{dx_i}{d\mathscr{I}} \propto \frac{\partial^2\mathbb{E}_i\left(u(\cdot)\right)}{\partial x_i\partial\mathscr{I}} = \frac{\partial\mathscr{S}'\left(x_i;\mathscr{I}\right)}{\partial\mathscr{I}} = -\frac{2a_i}{\rho\sigma^2\left(\frac{1}{\sigma_b^2} + \frac{\mathscr{I}^2}{\sigma_\theta^2} + x_i\right)^3}\frac{\mathscr{I}}{\sigma_\theta^2} < 0.$$

Finally, one needs to show that the equilibrium information $\mathscr{I}$ is uniquely determined for a given distribution of wealth. Define

$$\sum\left(\mathscr{I}\right) \equiv \mathscr{I} - \frac{1}{\rho\sigma^2}\int_{i^*}^{1}\frac{a_i x_i\left(a_i;\mathscr{I}\right)}{h_0\left(\mathscr{I}\right) + x_i\left(a_i;\mathscr{I}\right)}di.$$

One can demonstrate that the differential of this expression is positive

$$\frac{d\sum(\mathscr{I})}{d\mathscr{I}} = 1 - \frac{1}{\rho\sigma^2}\int_{i^*}^{1} \frac{h_0(\mathscr{I})\frac{dx_i(a_i;\mathscr{I})}{d\mathscr{I}} - x_i(a_i;\mathscr{I})\frac{dh_0(\mathscr{I})}{d\mathscr{I}}}{\left(h_0(\mathscr{I}) + x_i(a_i;\mathscr{I})\right)^2} di > 0.$$

Moreover, $\sum(0) \leq 0$, since $x_i(a_i;0) \geq 0$, and $\sum(\infty) \geq 0$, as $x_i(a_i;\infty) = 0$. By the continuity of $\sum(\mathscr{I})$, there is a unique $\mathscr{I}$ such that $\sum(\mathscr{I}) = 0$. Therefore, $\mathscr{N}$ and $\mathscr{T}$ are also uniquely defined.

Returns, variance, and Sharpe ratio. Lastly, I derive the key moments of return rates conditional on the amount of investment. Excess portfolio returns are given by $r_i^{pe}z \equiv r_i^p z - r^f z = \varsigma_i\left(\frac{\Pi-P}{P} - r^f z\right)$. Using Equations (1.56) and (1.57) and the definition in Equation (1.60), by the law of total expectation, expected returns read as

$$\begin{aligned}
\mathbb{E}\left(r_i^{pe}z\right) &= \mathbb{E}\left[\mathbb{E}\left(\varsigma_i\left(\frac{\Pi-P}{P} - r^f z\right)|\mathscr{F}_i\right)\right] \\
&= \mathbb{E}\left[\left(\frac{1}{\rho\sigma\sqrt{z}}\lambda_i + o(1)\right)\left(\mathbb{E}_i(bz|\mathscr{F}_i) + \frac{1}{2}\sigma^2 z - pz - r^f z + o(z)\right)\right] \\
&= \frac{1}{\rho}\mathbb{E}\left(\lambda_i^2\right) + o(z) = \frac{1}{\rho}\mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right)z + o(z)
\end{aligned}$$

and the return variance is given by

$$\begin{aligned}
\mathbb{V}\left(r_i^{pe}z\right) &= \mathbb{V}\left(r_i^p z\right) = \mathbb{E}\left[\left(r_i^{pe}z\right)^2\right] - \mathbb{E}\left(r_i^{pe}z\right)^2 = \mathbb{E}\left[\left(r_i^{pe}z\right)^2\right] \\
&= \mathbb{E}\left[\varsigma_i^2\mathbb{E}\left(\left(\frac{\Pi-P}{P} - r^f z\right)^2|\mathscr{F}_i\right)\right] = \mathbb{E}\left[\left(\frac{1}{\rho\sigma\sqrt{z}}\lambda_i + o(1)\right)^2\left(\sigma^2 z + o(z)\right)\right] \\
&= \frac{1}{\rho^2}\mathbb{E}\left(\lambda_i^2\right) + o(z) = \frac{1}{\rho^2}\mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right)z + o(z),
\end{aligned}$$

which shows Equations (1.50) and (1.51). Equation (1.52) follows from dividing (1.50) by the square root of (1.51). Observe that both $\mathbb{E}\left(r_i^{pe}z\right)$ and $\mathbb{V}\left(r_i^{pe}z\right)$, rise in a_i because $\mathbb{E}\left(\lambda_i^2\right)$ is an increasing function of x_i.

1.F.3 An Example

Own-return elasticity. Let $\mathbb{E}(\theta) = 0$, $v(x_i) = \kappa x_i$, and $a_0 > a_i^*(\mathscr{I})$. Then, Equation (1.49) that pins down the demand for information, simplifies to $h_0(\mathscr{I}) + x_i = \sqrt{\frac{a_i}{2\rho\kappa\sigma^2}}$. By Equations (1.50) and (1.60)

$$\begin{aligned}\mathbb{E}\left(r_i^{pe}z\right) &= -\frac{z}{\rho\sigma^2}\frac{1}{h_0\left(\mathscr{I}\right)+x_i} + \frac{z}{\rho\sigma^2}\left[\frac{\sigma_\theta^2}{\mathscr{I}^2}p_\xi^2 + \sigma_b^2\left(1-p_\xi\right)^2\right] + o\left(z\right) \\ &= -\frac{z}{\rho\sigma}\sqrt{\frac{2\rho\kappa}{a_i}} + \frac{z}{\rho\sigma^2}\left[\frac{\sigma_\theta^2}{\mathscr{I}^2}\left(1-\frac{\mathscr{N}}{\mathscr{T}\sigma_b^2}\right)^2 + \sigma_b^2\left(\frac{\mathscr{N}}{\mathscr{T}\sigma_b^2}\right)^2\right] + o\left(z\right).\end{aligned}$$

The return function is increasing and concave in a_i: $\frac{d\mathbb{E}\left(r_i^p z\right)}{da_i} = \frac{z}{\sigma}\sqrt{\frac{\kappa}{2\rho a_i^3}} > 0$ and $\frac{d^2\mathbb{E}\left(r_i^p z\right)}{da_i^2} = -\frac{3}{2}\frac{z}{\sigma}\sqrt{\frac{\kappa}{2\rho a_i^5}} < 0$. Consequently, the own-return elasticity elasticity in a given period is

$$\varepsilon_i^{\mathbb{E}(r^p z),a} \equiv \frac{a_i}{\mathbb{E}\left(r_i^p z\right)}\frac{d\mathbb{E}\left(r_i^p z\right)}{da_i} = \frac{1}{\mathbb{E}\left(r_i^p z\right)}\frac{z}{\sigma}\sqrt{\frac{\kappa}{2\rho a_i}} = \frac{\sqrt{\kappa/\left(2\rho\sigma^2 a_i\right)}}{\mathscr{S}\left(a_i,\left\{a_j\right\}_{j\in[0,1]}\right)/\rho + r^f}. \tag{1.61}$$

Since $\frac{\partial\mathscr{S}\left(a_i,\left\{a_j\right\}_{j\in[0,1]}\right)}{\partial a_i} > 0$ and $\frac{\partial\sqrt{\kappa/\left(2\rho\sigma^2 a_i\right)}}{\partial a_i} < 0$, the own-return elasticity decreases in a_i.

Cross-return elasticity. It is more tedious to derive the cross-return elasticity. I focus on the case, where $\sigma^2 = \sigma_b^2 = \sigma_\theta^2 = 1$. In the following, I show that

$$\begin{aligned}\gamma_{i,i'}^{\mathbb{E}(r^p z),a} &\equiv \frac{a_{i'}}{\mathbb{E}\left(r_i^p z\right)}\frac{d\mathbb{E}\left(r_i^p z\right)}{da_{i'}} \\ &= \frac{a_{i'}}{\mathbb{E}\left(r_i^p z\right)}\left(\frac{\partial\mathbb{E}\left(r_i^p z\right)}{\partial\mathscr{T}}\frac{\partial\mathscr{T}_{i'}}{\partial a_{i'}} + \frac{\partial\mathbb{E}\left(r_i^p z\right)}{\partial\mathscr{N}}\frac{\partial\mathscr{N}_{i'}}{\partial a_{i'}} + \frac{\partial\mathbb{E}\left(r_i^p z\right)}{\partial\mathscr{I}}\frac{\partial\mathscr{I}_{i'}}{\partial a_{i'}}\right) \equiv \frac{1}{\mathbb{E}\left(r_i^p z\right)}\delta_{i'}^{\mathbb{E}(r^p z),a},\end{aligned}$$

where $\delta_{i'}^{\mathbb{E}(r^p z),a}$ is decreasing in $a_{i'}$ and $\delta_{i'}^{\mathbb{E}(r^p z),a} \geq 0$ for $a_{i'} \leq \tilde{a}$. Then, $\delta_{i'}^{\mathbb{E}(r^p z),a} < 0$ for $a_{i'} > \tilde{a}$ trivially follows by the continuity of the return function. Recall the definitions of the aggregate variables $\mathscr{I} \equiv \int_{i'}\mathscr{I}_{i'}di'$, $\mathscr{N} \equiv \int_{i'}\mathscr{N}_{i'}di'$, and $\mathscr{T} \equiv \int_{i'}\mathscr{T}_{i'}di$. For the given parametrization, $\frac{\partial\mathscr{N}_{i'}}{\partial a_{i'}} = \sqrt{\kappa/\left(2a_{i'}\rho\right)}$ and $\frac{\partial\mathscr{T}_{i'}}{\partial a_{i'}} = 1/\rho$,. Use $\mathscr{I} = \mathscr{T} - h_0\left(\mathscr{I}\right)\mathscr{N}$ to show that

$$\frac{\partial\mathscr{I}_{i'}}{\partial a_{i'}} = \frac{\frac{\partial\mathscr{T}_{i'}}{\partial a_{i'}} - h_0\left(\mathscr{I}\right)\frac{\partial\mathscr{N}_{i'}}{\partial a_{i'}}}{1+2\mathscr{I}\mathscr{N}} = \frac{1/\rho - \left(1+\mathscr{I}^2\right)\sqrt{\kappa/\left(2a_{i'}\rho\right)}}{1+2\mathscr{I}\mathscr{N}}.$$

Since

$$\begin{aligned}\frac{\partial\mathbb{E}\left(r_i^p z\right)}{\partial p_\xi} &= \frac{2z}{\rho\sigma^2}\left[\frac{\sigma_\theta^2}{\mathscr{I}^2}p_\xi + \sigma_b^2 p_\xi - \sigma_b^2\right] = \frac{2z}{\rho\sigma^2}\frac{\sigma_\theta^2}{\mathscr{I}^2}\left[1 - \frac{\mathscr{N}}{\mathscr{T}\sigma_b^2} + \frac{\mathscr{I}^2}{\sigma_\theta^2}\frac{\mathscr{N}}{\mathscr{T}}\right] \\ &= \frac{2z}{\rho\sigma^2}\frac{\sigma_\theta^2}{\mathscr{I}^2}\left[1 - h_0\left(\mathscr{I}\right)\frac{\mathscr{N}}{\mathscr{T}}\right] = \frac{2z}{\rho\sigma^2}\frac{\sigma_\theta^2}{\mathscr{I}^2}\frac{\mathscr{I}}{\mathscr{T}} = \frac{2z}{\rho\mathscr{I}\mathscr{T}},\end{aligned}$$

$$\frac{\partial \mathbb{E}\left(r_i^p z\right)}{\partial \mathscr{N}} = \frac{\partial \mathbb{E}\left(r_i^p z\right)}{\partial p_\xi}\frac{\partial p_\xi}{\partial \mathscr{N}} = -\frac{2z}{\rho \mathscr{I} \mathscr{T}^2}$$

and

$$\frac{\partial \mathbb{E}\left(r_i^p z\right)}{\partial \mathscr{T}} = \frac{\partial \mathbb{E}\left(r_i^p z\right)}{\partial p_\xi}\frac{\partial p_\xi}{\partial \mathscr{T}} = \frac{2z}{\rho \mathscr{I} \mathscr{T}^2}\frac{\mathscr{N}}{\mathscr{T}}.$$

Furthermore, $\frac{\partial \mathbb{E}\left(r_i^p z\right)}{\partial \mathscr{I}} = -\frac{2z}{\rho \mathscr{I}^3}\left(1-\frac{\mathscr{N}}{\mathscr{T}\sigma_b^2}\right)^2$. Collecting all terms, the cross-return semi-elasticity in a subperiod can be written as

$$\begin{aligned}
\delta_{i'}^{\mathbb{E}(r^p z),a} &= a_{i'}\frac{2z}{\rho \mathscr{I}}\left(\frac{1}{\mathscr{T}^2}\frac{\mathscr{N}}{\mathscr{T}}\cdot\frac{\partial \mathscr{T}_{i'}}{\partial a_{i'}} - \frac{1}{\mathscr{T}^2}\cdot\frac{\partial \mathscr{N}_{i'}}{\partial a_{i'}} - \frac{1}{\mathscr{I}^2}\frac{\partial \mathscr{I}_{i'}}{\partial a_{i'}}\right)\\
&= a_{i'}\frac{2z}{\rho \mathscr{I}}\left[\left(\frac{1}{\mathscr{T}^2}\frac{\mathscr{N}}{\mathscr{T}} - \frac{(1-\mathscr{N}/\mathscr{T})^2}{\mathscr{I}^2(1+2\mathscr{I}\mathscr{N})}\right)\cdot\frac{\partial \mathscr{T}_{i'}}{\partial a_{i'}} + \left(\frac{(1-\mathscr{N}/\mathscr{T})^2}{\mathscr{I}^2(1+2\mathscr{I}\mathscr{N})}\left(1+\mathscr{I}^2\right) - \frac{1}{\mathscr{T}^2}\right)\cdot\frac{\partial \mathscr{N}_{i'}}{\partial a_{i'}}\right]\\
&= \frac{2za_{i'}}{\rho \mathscr{T}^2\mathscr{I}^3(1+2\mathscr{I}\mathscr{N})}\left[\Omega_{\mathscr{T}}\cdot\frac{1}{\rho} + \Omega_{\mathscr{N}}\cdot\sqrt{\kappa/(2a_{i'}\rho)}\right], \qquad (1.62)
\end{aligned}$$

where

$$\begin{aligned}
\Omega_{\mathscr{T}} &\equiv \mathscr{I}^2(1+2\mathscr{I}\mathscr{N})\frac{\mathscr{N}}{\mathscr{T}} - \mathscr{T}^2(1-\mathscr{N}/\mathscr{T})^2\\
&= -\mathscr{I}^2\left[(1+2\mathscr{I}\mathscr{N})\left(\frac{\mathscr{I}}{\mathscr{T}} + \mathscr{I}^2\frac{\mathscr{N}}{\mathscr{T}}\right) + \mathscr{I}^2\mathscr{N}^2\right] < 0
\end{aligned}$$

and

$$\begin{aligned}
\Omega_{\mathscr{N}} &\equiv \mathscr{T}^2(1-\mathscr{N}/\mathscr{T})^2\left(1+\mathscr{I}^2\right) - \mathscr{I}^2(1+2\mathscr{I}\mathscr{N})\\
&= \mathscr{I}^4\left[(1+\mathscr{I}\mathscr{N})^2 + \mathscr{N}^2\right] > 0.
\end{aligned}$$

This semi-elasticity declines with $a_{i'}$

$$\frac{\partial \delta_{i'}^{\mathbb{E}(r^p z),a}}{\partial a_{i'}} = \frac{2z}{\rho \mathscr{T}^2\mathscr{I}^3(1+2\mathscr{I}\mathscr{N})}\left[\Omega_{\mathscr{T}}\cdot\frac{1}{\rho} + \frac{1}{2}\Omega_{\mathscr{N}}\cdot\sqrt{\kappa/(2a_{i'}\rho)}\right] < 0.$$

To show that $\left[\Omega_{\mathscr{T}}\cdot\frac{1}{\rho} + \frac{1}{2}\Omega_{\mathscr{N}}\cdot\sqrt{\kappa/(2a_{i'}\rho)}\right] < 0$, first, rearrange

$$4\sqrt{a_{i'}/(2\kappa\rho)}\left[(1+2\mathscr{I}\mathscr{N})\left(\frac{\mathscr{I}}{\mathscr{T}} + \mathscr{I}^2\frac{\mathscr{N}}{\mathscr{T}}\right) + \mathscr{I}^2\mathscr{N}^2\right] > \mathscr{I}^2\left[(1+\mathscr{I}\mathscr{N})^2 + \mathscr{N}^2\right].$$

Since, $a_{i'} > a_i^*(\mathscr{I})\ \forall i'$, $\sqrt{\frac{a_{i'}}{2\rho\kappa}} > \left(1+\mathscr{I}^2\right)\ \forall i'$. Therefore,

$$\begin{aligned} 4\sqrt{\frac{a_{i'}}{2\rho\kappa}}\left[(1+2\mathscr{I}\mathscr{N})\left(\frac{\mathscr{I}}{\mathscr{T}}+\mathscr{I}^2\frac{\mathscr{N}}{\mathscr{T}}\right)+\mathscr{I}^2\mathscr{N}^2\right] &> 4\left(1+\mathscr{I}^2\right)\left[(1+2\mathscr{I}\mathscr{N})\left(\frac{\mathscr{I}}{\mathscr{T}}+\mathscr{I}^2\frac{\mathscr{N}}{\mathscr{T}}\right)+\mathscr{I}^2\mathscr{N}^2\right] \\ &= 4(1+2\mathscr{I}\mathscr{N})\frac{\mathscr{I}}{\mathscr{T}}+3\mathscr{I}^2\mathscr{N}^2+3\mathscr{I}^2+8\mathscr{I}^3\mathscr{N}+3\mathscr{I}^4\mathscr{N}^2 \\ &\quad -2\mathscr{I}^3\mathscr{N}^2+\mathscr{I}^2\left[(1+\mathscr{I}\mathscr{N})^2+\mathscr{N}^2\right] > \mathscr{I}^2\left[(1+\mathscr{I}\mathscr{N})^2+\mathscr{N}^2\right] \end{aligned}$$

where the last inequality follows from the fact that $3\mathscr{I}^4\mathscr{N}^2 > 2\mathscr{I}^3\mathscr{N}^2$ for $\mathscr{I} \geq 1$ and $3\mathscr{I}^2\mathscr{N}^2 > 2\mathscr{I}^3\mathscr{N}^2$ for $\mathscr{I} < 1$. Finally, define $\tilde{a} : \Omega_{\mathscr{N}} \cdot \sqrt{\kappa/(2\tilde{a}\rho)} = -\Omega_{\mathscr{T}} \cdot \frac{1}{\rho}$. By the continuity of the cross-return semi-elasticity, $\delta_{i'}^{\mathbb{E}(r^p z),a} \geq 0$ for all $a_{i'} \leq \tilde{a}$ and $\delta_{i'}^{\mathbb{E}(r^p z),a} < 0$ for all $a_{i'} > \tilde{a}$.

1.F.4 The Financial Market Equilibrium and Nonlinear Taxation

In this section, I shortly demonstrate that the financial market also microfounds scale dependence when there is a nonlinear capital gains tax, $T_k(\cdot)$, instead of a linear wealth tax. Assume that $T_k(0) = T_k''(0) = 0$. For a nonlinear capital gains tax, it does not matter whether or not information costs are deductible from the tax base.

The reasoning is the same as before (Appendix 1.F.2). Again, the repeated financial market interaction is static such that households optimize

$$\underset{x}{max}\, \mathbb{E}_i\left(\underset{\varsigma}{max}\, \mathbb{E}_i\left(u\left[a_i\left(1+r_i^p z\right)-T_k\left(a_i r_i^p z\right)-v(x_i)z\right]|\mathscr{F}_i\right)\right)$$

in each period. There exists a log-linear rational expectations equilibrium equilibrium in which the price and the optimal investment in the risky asset can be derived

$$log(P) = pz = \left(p_0+p_\xi\xi-r^f\right)z+o(z)$$

and

$$\varsigma_i = \frac{1}{\rho\sigma\sqrt{z}}\frac{1}{1-T_k'(0)}\lambda_i+o(1),$$

using the same approximations as before. Similarly, the demand for information and the equilibrium information read as

$$v'(x_i) = \frac{a_i}{2\rho}\left(1+T_k'(0)\right)\mathscr{S}'(x_i;\mathscr{I})$$

and

$$\mathscr{I} = \frac{1}{\rho\sigma^2}\frac{1}{1-T_k'(0)}\int_{a_i^*(\mathscr{I})}^{a_1}\frac{a_i x_i(a_i;\mathscr{I})}{h_0(\mathscr{I})+x_i(a_i;\mathscr{I})}dG(a_i),$$

where $a_i^* (\mathscr{I}) \equiv 2v'(0)\rho\sigma^2 h_0(\mathscr{I})^2 / \left(1+T_k'(0)\right)$ denotes the threshold wealth. The equilibrium is, again, unique.

Taking stock of equilibrium choices, expected returns and the variance of returns are given by

$$\mathbb{E}\left(r_i^{pe}z\right) = \mathbb{E}\left(r_i^{p}z\right) - r^f z = \frac{1}{\rho}\frac{1}{1-T_k'(0)}\mathscr{S}\left(a_i, \{a_j\}_{j\in[0,1]}\right)$$

and

$$\mathbb{V}\left(r_i^{pe}z\right) = \mathbb{V}\left(r_i^{p}z\right) = \frac{1}{\rho}\frac{1}{1-T_k'(0)}\mathbb{E}\left(r_i^{pe}z\right),$$

respectively. For $T_k'(0) = 0$, all expressions coincide with those in Appendix 1.F.2.

1.G A Life-Cycle Economy

In this section, I develop a standard two-period life-cycle framework, as introduced by Farhi and Werning (2010), for studying nonlinear capital taxation with scale and type dependence. Using this framework, I study the nonlinear tax incidence and optimal taxation in partial and in general equilibrium. Moreover, I deal with the presence of other policies. Firstly, I consider a subsidy on the costs of information acquisition (financial advisory). Secondly, I study a financial education program.

1.G.1 Environment

In the following, I describe the economic environment. The objective is to provide an accessible setting that reveals the main insights about the nonlinear taxation of capital gains. As in Mirrlees (1971), the economy is populated by a continuum of households $i \in [0,1]$. The first source of heterogeneity is the productivity of labor. Agent i's earnings ability $w_i \in \mathbb{R}_+$ is distributed according to a c.d.f. F and a p.d.f. f. Without loss of generality, one can order household indices such that wages increase in i. Then, one may interpret i as the household's position in the pre-tax wage distribution.

Time is discrete, and there are two periods $t = 0,1$. In the first period, households supply labor, consume and save. In the second period, they consume their savings. Therefore, the first period may be interpreted as an individual's working life with duration H, whereas, in the second period, she is retired. Individuals may take efforts to increase their returns on investment. The resulting return function increases in the amount of savings. In Section 1.E, I show how this relationship emerges in a financial market setting with optimal portfolio choice and information acquisition. This setting gives rise to inequality in the returns to investment. In the financial market, high-income individuals decide to save more and acquire more information than low-income individuals. This information advantage allows them to generate higher (risk-adjusted) returns than households from lower parts of the income and wealth distribution.

Preferences and technology. Households have Greenwood, Hercowitz, and Huffman (1988) preferences

$$u(l_i, a_i, e_i; w_i) \equiv u_0(c_{i,0} - v_0(l_i)) + \beta u_1(c_{i,1} - v_1(x_i)), \tag{1.63}$$

where $\beta \in (0,1]$ denotes the households' discount factor, $u_t(\cdot)$ is a concave and increasing period utility, and $v_t(\cdot)$ denotes the convex and increasing disutility from effort. A household of type i can transfer resources across periods by saving assets a_i. In the first (working) period, households supply labor $l_i > 0$ and earn after-tax income $y_i - T_l(y_i)$, where $T_l(\cdot)$ denotes the government's nonlinear tax on labor income $y_i \equiv w_i l_i$. To increase the returns on the investment of assets a_i, a household can take effort $x_i > 0$. Assume that the costs of this effort accrue in the second (retirement) period. Capital gains are, for the moment, given by the reduced form relation $\tilde{r}_i\left(e_i, \{e_j\}_{j\in[0,1]}\right) \equiv r_i > 0$ where $r(\cdot)$ is increasing and concave in its first argument.

A straightforward interpretation is that households acquire financial knowledge by employing financial advisers to raise the rate of return on their investment. In partial equilibrium, an individual's investment return only depends on her own effort choice, whereas, in general equilibrium, an individual's investment returns may depend on choices by everyone else. In Section 1.E, I microfound this reasoning. Capital gains, $a_{R,i} \equiv r_i a_i$, are taxed nonlinearly according to $T_k(\cdot)$. In Section 1.C, I assume that households can deduct effort costs $v_1(\cdot)$ from the tax base. For completeness, in this section, I consider the situation in which these costs are not deductible. One can show that Lemma 1 in Section 1.E ($\frac{\partial r_i}{\partial a_i} > 0$) holds in this economy (for $T_k(0) = 0$, $T_k'(0) = 0$, and $T_k''(0) = 0$) irrespective of this deductibility. Therefore, return rates exhibit scale dependence. Moreover, let $\frac{\partial r_i}{\partial i} \geq 0$ so that there may also be type dependence. Let all functions be twice continuously differentiable in their arguments.

Monotonicity. Define the local rate of tax progressivity as $p_t(y) \equiv -\frac{\partial log[1-T_t'(y)]}{\partial log(y)} = \frac{yT_t''(y)}{1-T_t'(y)}$ for $t \in \{l,k\}$. Observe that the usual monotonicity conditions will hold if labor and capital taxes are not too progressive ($p_l(y_l) < 1$ and $p_k(a_{R,i}) < 1$). That is, effort choices, as well as savings, and, hence, labor and capital income are increasing in the index i. Intuitively, the higher an individual's hourly wage, the more she will work, and the more resources she can transfer to the retirement period. Moreover, an individual's incentives to take efforts to increase her capital gains rise with her position in the pre-tax wage distribution. Due to the one-to-one mapping between wages and incomes, one may write returns as a function of savings, $\tilde{r}_i\left(e_i, \{e_j\}_{j\in[0,1]}\right) = r_i\left(a_i, \{a_j\}_{j\in[0,1]}\right)$. I will make use of this formulation later on.

Household problem. In the working period, households consume their after-tax labor income net of savings

$$c_{i,0} + a_i \leq w_i l_i - T_l\left(w_i l_i\right). \tag{1.64}$$

In the retirement period, their consumption is given by their final after-tax wealth

$$c_{i,1} \leq a_i\left(1 + r_i\right) - T_k\left(r_i a_i\right). \tag{1.65}$$

Let $\mathscr{U}_i\left(T_l, T_k\right)$ denote household i's indirect utility from optimally choosing savings, a_i, and effort levels, $\{l_i, x_i\}$, to maximize Equation (1.63) subject to Equations (1.64) and (1.65). As standard, suppose the household problem is convex. With a slight abuse of notation, let l_i and a_i denote household i's Marshallian (uncompensated) labor supply and savings functionals. The first-order conditions of the household maximization problem define these functionals implicitly.

Government problem. For simplicity, suppose that households and the government face the same discount factor. Then, the government's budget constraint reads as

$$\mathscr{R}\left(T_l, T_k\right) \equiv \int_0^1 T_l\left(w_i l_i\right) di + \beta \int_0^1 T_k\left(r_i a_i\right) di \geq \bar{E}. \tag{1.66}$$

The government has a utilitarian objective function. Consequently, it chooses a tax system $\{T_l, T_k\}$ to maximize

$$\mathscr{G}\left(T_l, T_k\right) \equiv \int_0^1 \Gamma_i \mathscr{U}_i\left(T_l, T_k\right) di \tag{1.67}$$

subject to Equation (1.66), where Γ_i denotes household i's Pareto weight with $\int_0^1 \Gamma_i di = 1$. Denote λ as the marginal value of public funds and $g_{i,t} \equiv \left(1/\lambda\right) \Gamma_i u_t'\left(c_{i,t} - v_t\left(\cdot\right)\right)$ as the marginal social welfare weight.

1.G.2 Incidence of Nonlinear Tax Reforms

In this section, I study the impact of a small reform of an arbitrary (potentially suboptimal) tax scheme, e.g., the U.S. tax code, on labor supply and savings by households, as well as on government revenues and social welfare. Technically, I derive the impact of perturbing an arbitrary tax schedule T_t, where $t \in \{l, k\}$, (e.g., the capital gains tax) on the optimal choices by an agent i and aggregate variables in partial and general equilibrium. In other words, I reform the initial tax schedule by $\hat{T}_t$ and analyze the effects on optimal choices. As a by-product, I obtain the optimal tax scheme when the aggregate marginal benefits are equal to the marginal costs.

Gateaux derivatives. To formalize this idea, define the Gateaux derivative of the functional $\mathscr{F} : \mathscr{C}(\mathbb{R}_+, \mathbb{R}) \to \mathbb{R}$ at T_t in the direction $\hat{T}_t$ by

$$\hat{\mathscr{F}}\left(T_t, \hat{T}_t\right) \equiv \lim_{\mu \to 0} \frac{d}{d\mu} \mathscr{F}\left(T_t + \mu \hat{T}_t\right).$$

Accordingly, perturb the system of first-order conditions by $\mu \hat{T}_t$ and denote $\hat{l}_i\left(T_t, \hat{T}_t\right)$ and $\hat{a}_i\left(T_t, \hat{T}_t\right)$ the Gateaux derivative of labor supply and savings in the direction $\hat{T}_t$. Similarly, perturb Equations (1.66) and (1.67) to obtain the incidence on tax revenues, $\hat{\mathscr{R}}\left(T_t, \hat{T}_t\right)$, and social welfare, $\hat{\mathscr{G}}\left(T_t, \hat{T}_t\right)$.

Elasticities. Denote $I_{0,i} \equiv y_i T_l'(y_i) - T_l(y_i)$ and $I_{1,i} \equiv a_{R,i} T_k'(a_{R,i}) - T_k(a_{R,i})$ as the virtual income of individual i in period 0 and period 1, respectively. Define $\zeta_i^{a,(1-T_t')}$ (η_i^{a,I_t}) as the compensated elasticity of household i's savings with respect to the retention rate of the tax in period t (the income effect parameter of savings with respect to income in period t) along the nonlinear budget line. The elasticities of labor supply are defined accordingly. Again, let $\tilde{\zeta}$ and $\tilde{\eta}$ indicate the elasticities at a fixed rate of return that, without scale dependence, coincide with the observed elasticities. Given the GHH preferences, labor supply is independent of the capital gains tax scheme ($\zeta_i^{l,\left(1-T_k'\right)} = \eta_i^{l,I_1} = 0$). Moreover, let $\tilde{\zeta}_i^{a,r}$ be the elasticity of savings with respect to the rate of return.[32]

The novelty of this chapter to let an individual's rate of return vary with her savings and, in general equilibrium, with the savings of others. As before, define the *own-return elasticity* as $\varepsilon_i^{r,a} \equiv \frac{\partial log[r_i(\cdot)]}{\partial log(a_i)}$. It measures the impact of one's wealth on her rate of return, thus, accounting for scale dependence originating, for example, from the variable acquisition of financial knowledge as in Section 1.E. For all $i' \in [0,1]$ the *cross-return elasticity* $\gamma_{i,i'}^{r,a} \equiv \frac{\partial log[r_i(\cdot)]}{\partial log\left(a_{i'}\right)}$ captures any kind of complementarity between households' wealth and its return. In the example of Section 1.E, it contains inter-household spillovers from financial knowledge and risk-taking. The cross-return elasticity quantifies in reduced form the impact of the portfolio size of household i' in the returns of household i. In partial equilibrium, it is equal to zero for all i, i'.

Incidence on savings. In the following, I characterize the nonlinear incidence of capital gains tax reforms on savings for a given labor tax. One may write, as an intermediate step, the percentage

[32] Similar to $\tilde{\zeta}_i^{a,\left(1-T_k'\right)}$ and $\tilde{\eta}_i^{a,I_k}$, the definition of $\tilde{\zeta}_i^{a,r}$ involves a correction factor that accounts for behavioral effects along the nonlinear budget line $\frac{1}{1+p_k\left(a_{R,i}\right)\tilde{\zeta}_i^{a,(1-\tau_k)}}$, where $\tilde{\zeta}_i^{a,(1-\tau_k)}$ is the compensated savings elasticity along the linear budget line.

change of savings in reaction to a capital gains tax reform as

$$\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)}{a_i} = -\underbrace{\tilde{\zeta}_i^{a,\left(1-T_k'\right)}}_{>0}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \underbrace{\tilde{\eta}_i^{a,I_2}}_{\leq 0}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)} + \underbrace{\tilde{\zeta}_i^{a,r}}_{>0}\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)}{r_i}. \tag{1.68}$$

The first two terms describe the standard positive income and negative substitution effect. As the last term reveals, an inequality multiplier effect from the adjustment in the rate of return, now, augments the reaction of savings.

In the following, I show how to use estimates on the elasticity of returns. The partial equilibrium return adjustment in response to the tax reform is proportional to the reaction of the portfolio size

$$\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)^{PE}}{r_i} = \varepsilon_i^{r,a}\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{PE}}{a_i}. \tag{1.69}$$

In general equilibrium, one needs to account for all kinds of spillovers

$$\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)^{GE}}{r_i} = \varepsilon_i^{r,a}\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{GE}}{a_i} + \int_{i'}\gamma_{i,i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'. \tag{1.70}$$

Thus, in both cases, one needs to upward adjust income and substitution effects of savings by an inequality multiplier effect $\phi_i \equiv \frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}} > 1$. As the adjustment in savings depends on the shape of the tax reform, the government can directly redistribute the return inequality.

In general equilibrium, there are also cross-return effects. Therefore, combining Equations (1.68) and (1.70), the incidence on savings is given by a Fredholm integral equation of the second kind that can be solved using a standard resolvent formalism.

Lemma 4 (Incidence on savings)**.** *Consider a small reform of an arbitrary capital gains tax scheme in the direction $\hat{T}_k$. Define $\phi_i \equiv \frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}$. In partial equilibrium, the first-order change in the optimal savings is given by*

$$\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{PE}}{a_i} = -\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \phi_i\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)}. \tag{1.71}$$

Let $\int_{i'}\int_i\left|\phi_i\tilde{\zeta}_i^{a,r}\gamma_{i,i'}^{r,a}\right|^2 didi' < 1$. Then, the general equilibrium adjustment is given by

$$\begin{aligned}\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{GE}}{a_i} &= -\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \phi_i\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)} \\ &\quad - \phi_i\tilde{\zeta}_i^{a,r}\int_{i'}\phi_{i'}R_{i,i'}\left[\tilde{\zeta}_{i'}^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i'}\right)}{1-T_k'\left(a_{R,i'}\right)} + \tilde{\eta}_{i'}^{a,I_2}\frac{\hat{T}_k\left(a_{R,i'}\right)}{a_{R,i'}\left(1-T_k'\left(a_{R,i'}\right)\right)}\right]di',\end{aligned} \tag{1.72}$$

where for every $i,i' \in [0,1]$ the resolvent is given by $R_{i,i'} \equiv \sum_{n=1}^{\infty} \mathscr{K}_{i,i'}^{(n)}$ with $\mathscr{K}_{i,i'}^{(1)} = \gamma_{i,i'}^{r,a}$ and, for $n \geq 2$, $\mathscr{K}_{i,i'}^{(n)} = \int_{i''} \mathscr{K}_{i,i''}^{(n-1)} \phi_{i''} \zeta_{i''}^{a,r} \gamma_{i'',i'}^{r,a} di''$.

Proof. Appendix 1.H.2. □

Lemma 4 describes the reaction of savings to a small change in the capital gains tax in terms of sufficient statistics (Chetty (2009)). All these sufficient statistics are, in principle, observable by the econometrician and serve as primitives of the model. Nonetheless, these objects are endogenous variables evaluated at a given tax scheme and equilibrium concept.

Incidence on savings in partial equilibrium. As usual, a change in an individual's capital gains tax induces an income effect and a substitution effect on savings. A rise in the marginal capital gains tax reduces the incentive to transfer resources across periods (substitution effect). At the same time, the household feels poorer in the second period and, therefore, saves more (income effect).

Relative to the case of exogenous capital gains ($\phi_i = 1$), these two effects need to be adjusted upwards by an inequality multiplier effect $\phi_i > 1$ accounting for the endogeneity of returns, which is the main difference of this chapter from the existing literature. Tax reforms generate novel inequality multiplier effects. Consider the partial equilibrium economy without income effects and suppose, for instance, that individual i faces a reduction in the marginal capital gains tax. Due to the substitution effect, she will save more. However, the scale dependence leads to an adjustment in her investment returns.

In the financial market example, considered in Section 1.E, the altered amount of savings triggers the following chain of reactions. Because the individual saves more, she invests a higher absolute amount on the stock market. The larger portfolio raises the incentives to acquire costly information about the fundamentals of the economy that drive the stocks' payoffs. As the individual makes more informed decisions on the financial market, her returns rise. Since her returns on investment become larger relative to before, the payoffs from investment and, therefore, savings increase. The higher amount of savings feeds back into the optimal knowledge acquisition and, in turn, boosts returns. This loop continues infinitely.

The term ϕ_i captures this infinite sequence of adjustments. To see this, rewrite $\phi_i = \frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}} = \sum_{n=0}^{\infty} \left(\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}\right)^n$. Therefore, one can interpret the endogeneity of portfolio returns as an amplification force. It multiplies the standard income and substitution effect. As a result, I establish a version of Proposition 1 (b): savings, just as capital income, react more elastic to reforms of the capital gains tax

$$\phi_i \tilde{\zeta}_i^{a,\left(1-T_k'\right)} > \tilde{\zeta}_i^{a,\left(1-T_k'\right)} \quad \text{and} \quad \left|\phi_i \tilde{\eta}_i^{a,I_2}\right| \geq \left|\tilde{\eta}_i^{a,I_2}\right|.$$

Incidence on savings in general equilibrium. In general equilibrium, a household's rate of return is a function of everyone's decisions. Therefore, in addition to the described inequality multiplier effects, cross-return effects come into play, which I characterize in closed form in Lemma 4. The additional term aggregates the partial-equilibrium reactions by households across the skill distribution. They are weighted by the resolvent of the integral equation and account for an infinite sequence of return adjustments due to the general equilibrium spillovers. In the financial market example, they come from the endogeneity of stock prices, which aggregate individuals' information acquisition and risk-taking. For instance, a decrease in the tax rate of the rich makes them acquire relatively more financial knowledge and alter their portfolio composition. As a result, the equilibrium price adjusts, which also affects households from the bottom of the wealth distribution, given that they participate in the stock market. However, their altered behavior feeds, in turn, back into the equilibrium price so that the rich modify their choices again.

The resolvent formalism captures this infinite sequence of reactions. The resolvent is the sum of iterated kernels. The first kernel, $\mathscr{K}_{i,i'}^{(1)}$, describes the impact of savings by household i' on the returns of i. The second kernel $\mathscr{K}_{i,i'}^{(2)} = \int_{i''} \gamma_{i,i''}^{r,a} \phi_{i''} \zeta_{i''}^{a,r} \gamma_{i'',i'}^{r,a} di'$ accounts for the effect of savings by i' on the returns and, therefore, savings of households i'' which, in turn, affect the decision making of household i. For $n = 3$ the formula describes the impact of household i' on households i'' who affect i'''. The latter, then, influence the returns generated by household i. Observe that this reasoning is in its spirit similar to Sachs, Tsyvinski, and Werquin (2020) who study general equilibrium reactions of wages and labor supply in response to a reform of the labor income tax schedule.

Whether or not the presence of general equilibrium adjustments amplifies savings responses depends on the sign and magnitude of $\gamma_{i,i'}^{r,a}$ along the wealth distribution. Suppose, for instance, that $\gamma_{i,i'}^{r,a} > 0$ for all $i' \in [0,1]$. That is, there is a positive complementarity between a household's return on investment and others' investment. Then, general equilibrium forces further amplify income and substitution effects.

Conversely, suppose households live in a small open economy. Then, they have access to an international financial market, where they interact with other, larger investors or institutions whose decisions are affected by other margins and policies. Thus, the marginal impact of the former households on prices becomes small such that $\gamma_{i,i'}^{r,a} \to 0$.

Incidence on return inequality. One may decompose the incidence on returns in closed form

$$\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)^{GE}}{r_i} = -\phi_i\varepsilon_i^{r,a}\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \phi_i\varepsilon_i^{r,a}\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)} + \phi_i CE_i, \tag{1.73}$$

where $CE_i \equiv -\int_{i'}\phi_{i'}R_{i,i'}\left[\tilde{\zeta}_{i'}^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i'}\right)}{1-T_k'\left(a_{R,i'}\right)} + \tilde{\eta}_{i'}^{a,I_2}\frac{\hat{T}_k\left(a_{R,i'}\right)}{a_{R,i'}\left(1-T_k'\left(a_{R,i'}\right)\right)}\right]di'$ summarizes the cross effects. Therefore, the return inequality directly depends on the underlying tax code and how the policy-maker wishes to reform it.

One may measure the inequality in returns by their variance, $\mathbb{V}\left(r_i\right)$. The effect of a tax reform on return inequality is, then, given by $\hat{\mathbb{V}}\left(r_i\right) = \hat{\mathbb{E}}\left(r_i^2\right) - \hat{\mathbb{E}}\left(r_i\right)^2$. For simplicity, suppose that there are no income and general equilibrium effects and that the other elasticities are constant along the wealth distribution. Let the tax rate be linear. Then, one can write the impact of a tax reform on the return inequality as

$$d\mathbb{V}\left(r_i\right) = -2\mathbb{V}\left(r_i\right)\left(\phi-1\right)\frac{\tilde{\zeta}^{a,\left(1-T_k'\right)}}{\zeta^{a,r}}\frac{d\tau_k}{1-\tau_k}.$$

Put differently, the elasticity of return inequality with respect to the capital gains tax

$$\zeta^{\mathbb{V}(r),(1-\tau_k)} \equiv -\frac{\partial log\left[\mathbb{V}\left(r_i\right)\right]}{\partial log\left(1-\tau_k\right)} = -\frac{2\varepsilon^{r,a}\zeta^{a,\left(1-T_k'\right)}}{1-\varepsilon^{r,a}\zeta^{a,r}}$$

is negative for $\varepsilon^{r,a} > 0$. Therefore, a rise in the linear tax rate (more redistribution) compresses the distribution of returns and, hence, reduces the inequality in returns. One can also show that mitigating the return inequality goes along with the cost of lowering mean returns. This is, again, Corollary 1.

Incidence on utilities. Having derived the incidence on savings and returns, we can now study the effects on utilities. In partial equilibrium, this simply reads as

$$\hat{\mathscr{U}}_i\left(T_k,\hat{T}_k\right)^{PE} = -\lambda g_{i,1}\left(\beta/\Gamma_i\right)\hat{T}_k\left(a_{R,i}\right) \tag{1.74}$$

which is a straightforward application of the envelope theorem. In general equilibrium, one needs to keep track of the spillovers, or cross-effects, from others' decisions. That is,

$$\hat{\mathscr{U}}_i\left(T_k,\hat{T}_k\right)^{GE} = -\lambda g_{i,1}\left(\beta/\Gamma_i\right)\hat{T}_k\left(a_{R,i}\right) + \lambda g_{i,1}\left(\beta/\Gamma_i\right)a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)\left(1+\tilde{\zeta}_i^{a,r}\right)CE_i. \tag{1.75}$$

For any equilibrium concept, a rise in the tax liability mechanically reduces the utility of a household. By the envelope theorem, there is no first-order effect due to a change in savings and effort choices. In general equilibrium, due to the endogeneity of portfolio returns, one needs to add the impact of others' decisions on individual investment returns. Not surprisingly, an increase in the rate of return raises the utility of a household. Whether or not returns rise, depends, as described, on the distribution of cross-return elasticities.

Incidence on government revenues and social welfare. Now, one can bring together all parts of the incidence analysis to study the change in social welfare and government revenues in response to the reform of the capital gains tax.

Lemma 5 (Incidence on revenues and welfare)**.** *Let* $\int_{i'}\int_i \left|\phi_i \zeta_i^{a,r} \gamma_{i,i'}^{r,a}\right|^2 didi' < 1$. *Denote* $EQ \in \{PE, GE\}$ *as the equilibrium concept. Then, the first-order change in social welfare in response to a small reform in* T_k *reads as*

$$\hat{\mathscr{G}}\left(T_k, \hat{T}_k\right)^{EQ} = \int_i \Gamma_i \hat{\mathscr{U}}_i \left(T_k, \hat{T}_k\right)^{EQ} di. \tag{1.76}$$

The first-order change in government revenues is given by

$$\hat{\mathscr{R}}\left(T_k, \hat{T}_k\right)^{EQ} = \beta \int_i \hat{T}_k\left(a_{R,i}\right) di + \beta \int_i T_k'\left(a_{R,i}\right) a_{R,i} \left[\frac{\hat{r}_i\left(T_k, \hat{T}_k\right)^{EQ}}{r_i} + \frac{\hat{a}_i\left(T_k, \hat{T}_k\right)^{EQ}}{a_i}\right] di. \tag{1.77}$$

Proof. Appendix 1.H.2. □

I start with the impact on revenues. Observe that there are three types of effects: mechanical, behavioral, and return effects. The mechanical and behavioral effects are standard. The first one measures the direct impact of reforming the tax scheme on revenue collection, holding the tax base fixed. The second one regards the change in household behavior in response to a tax reform. In general equilibrium, this adjustment of behavior carries the spillover effects mentioned above. The return on investment adjusts due to changes in an individual's investment size and, in general equilibrium, others' amount of investment.

The effects on welfare are similar. By the envelope theorem, there are no first-order behavioral effects. However, households suffer from a rise in the overall tax liability (mechanical effect). Furthermore, the general equilibrium adjustment of returns imposes uninternalized general equilibrium effects on individuals. In other words, since an individual's rate of return depends on everyone's choices, one needs to add this additional impact on individual utilities from the behavior of others. In the aggregate, these effects add to the standard mechanical effect on social welfare.

1.G.3 Optimal Nonlinear Taxation

In this section, I describe the optimal nonlinear capital gains tax for a given labor tax. This procedure is similar to Section 1.C, where I explicitly address the interdependence of labor and capital taxes.

Having studied the nonlinear incidence of arbitrary capital gains tax reforms on government revenues and social welfare, I obtain, as a special case, the optimal capital gains tax by equating the sum of first-order effects equal to zero (see, for example, Saez (2001)). At the optimal tax scheme, there are no first-order effects from reforming the tax scheme in the direction of $\hat{T}_k$:

$$\frac{1}{\lambda}\hat{\mathscr{G}}\left(T_k,\hat{T}_k\right)^{EQ}+\hat{\mathscr{R}}\left(T_k,\hat{T}_k\right)^{EQ}=0. \tag{1.78}$$

In other words, one cannot find a revenue-neutral tax reform that raises social welfare. Denote $h\left(a_{R,i}\right)$ as the pdf and $H\left(a_{R,i}\right)$ as the cdf of capital income $a_{R,i}$.

Optimal taxation in partial equilibrium. As a benchmark, I consider the optimal nonlinear tax scheme in partial equilibrium. That is, let $\gamma_{i,i'}^{r,a}=0$ for all $i,i'\in[0,1]$. Proposition 5 characterizes the optimal nonlinear capital gains tax.

Proposition 5 (Optimal nonlinear capital gains tax in partial equilibrium)**.** *The optimal nonlinear capital gains tax on capital gains in partial equilibrium is almost everywhere given by*

$$\begin{aligned}\frac{T_k'\left(a_{R,i}\right)^{PE}}{1-T_k'\left(a_{R,i}\right)^{PE}}=&\frac{1}{\zeta_i^{a_R,\left(1-T_k'\right)}}\frac{1-H\left(a_{R,i}\right)}{a_{R,i}h\left(a_{R,i}\right)}\\&\times\int_{a_{R,i}}^{\infty}\left(1-g_{i'',1}\right)exp\left[-\int_{a_{R,i}}^{a_{R,i'''}}\frac{\eta_{i'}^{a_R,T_k}}{\zeta_{i'}^{a_R,\left(1-T_k'\right)}}\frac{da_{R,i'}}{a_{R,i'}}\right]\frac{dH\left(a_{R,i''}\right)}{1-H\left(a_{R,i}\right)},\end{aligned} \tag{1.79}$$

where $\zeta_i^{a_R,\left(1-T_k'\right)}\equiv\Phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}$, $\eta_i^{a_R,T_k}\equiv\Phi_i\tilde{\eta}_i^{a,T_k}$, $\Phi_i\equiv\left(1+\varepsilon_i^{r,a}\right)\phi_i$, *and* $\phi_i\equiv\frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}$.

Proof. Appendix 1.H.3. □

The optimal marginal tax rate on capital gains in partial equilibrium is a version of the Diamond (1998) ABC-formula with income effects (as in Saez (2001)). It expresses the optimal tax wedge on capital gains in terms of behavioral and income effects, the hazard ration of the capital gains distribution, and the marginal social welfare weights above $a_{R,i}$.

Therefore, I obtain Proposition 1 (a). Whether or not rates of return are endogenous, the optimal capital gains tax is described by the observed income and behavioral effects and the observed capital income distribution. Nonetheless, the formation of rates of return directly affects these sufficient statistics.

Observe that Φ_i upwards adjusts the elasticities. Holding the elasticities $\tilde{\zeta}_i^{a,\left(1-T_k'\right)}$ and $\tilde{\eta}_i^{a,T_k}$ fixed, under scale dependence ($\varepsilon_i^{r,a} > 0$ and $\Phi_i > 1$), the compensated elasticity of capital gains is larger

$$\zeta_i^{a_R,\left(1-T_k'\right)} > \tilde{\zeta}_i^{a_R,\left(1-T_k'\right)} = \tilde{\zeta}_i^{a,\left(1-T_k'\right)}$$

than under type dependence only. The income effect does not alter the optimal capital tax since

$$\zeta_i^{a_R,\left(1-T_k'\right)}/\eta_i^{a_R,T_k} = \tilde{\zeta}_i^{a,\left(1-T_k'\right)}/\tilde{\eta}_i^{a,T_k}.$$

Accordingly, the adjustment of elasticities provides a force for a lower capital gains tax under scale dependence. Simultaneously, scale dependence may increase the observed capital income inequality measured by the hazard ratio $\frac{1-H\left(a_{R,i}\right)}{a_{R,i}h\left(a_{R,i}\right)}$, which calls for higher taxes. To establish part (c) of Proposition 1, I express Equation (1.79) in terms of primitives:

$$\begin{aligned}\frac{T_k'\left(a_{R,i}\right)^{PE}}{1-T_k'\left(a_{R,i}\right)^{PE}} =& \frac{\tilde{\zeta}_i^{a,I_0}\tilde{\zeta}_i^{I_0,i}+\tilde{\zeta}_i^{a,r}\tilde{\zeta}_i^{r,i}+\tilde{\zeta}_i^{r,i}/\Phi_i}{\tilde{\zeta}_i^{a,\left(1-T_k'\right)}}\frac{1-i}{i} \\ &\times\int_i^1\left(1-g_{i'',1}\right)exp\left[-\int_i^{i''}\frac{\tilde{\eta}_{i'}^{a,T_k}}{\tilde{\zeta}_{i'}^{a,\left(1-T_k'\right)}}\frac{di'}{i'}\right]\frac{di''}{1-i}.\end{aligned} \tag{1.80}$$

Therefore, when investment rates are endogenously determined (scale dependence), the capital gains tax is the same as the one without scale dependence holding all the other primitives of the economy fixed and in the absence of type dependence ($\tilde{\zeta}_i^{r,i} = 0$). The upward adjustment in the savings elasticity just offsets the rise in observed inequality. When there is type dependence ($\tilde{\zeta}_i^{r,i} > 0$), conditional on all other primitives, the capital gains tax is lower with than without scale dependence. The resulting adjustment depends on the relative strength of type and scale dependence $\tilde{\zeta}_i^{r,i}/\Phi_i$.

To further illustrate the implications for redistribution, suppose the capital gains tax is approximately linear "at the top", e.g., for the top 1% in the wealth distribution. Assume that the elasticities converge to the values $\zeta^{a_R,\left(1-T_k'\right)} = \Phi\tilde{\zeta}^{a,\left(1-T_k'\right)}$, there is no type dependence $\tilde{\zeta}_i^{r,i} = 0$ at the top and that there are no income effects $\tilde{\eta} = 0$. Suppose that, without scale dependence, capital gains in this top bracket are Pareto distributed with parameter $\tilde{a}_k > 1$. Under scale dependence, the Pareto parameter is given by $a_k = \tilde{a}_k/\Phi$. Then, the linear top tax rate reads as

$$\tau_k^{top} = \frac{1-\bar{g}_k}{1-\bar{g}_k+a_k\zeta^{a_R,\left(1-T_k'\right)}}$$

where $\bar{g}_k$ is the limiting value of the social welfare weight. Therefore, I also obtain the neutrality result at the top: This rise in capital income inequality that scale dependence triggers and the adjustment in the elasticity cancel out ($a_k \zeta^{a_R,\left(1-T_k'\right)} = \tilde{\zeta}^{a,\left(1-T_k'\right)} \tilde{a}_k$). This neutrality result provides a potential justification for why capital gains taxes (e.g., in the U.S.) have not increased despite the drastic rise in top capital income inequality.

Optimal taxation in general equilibrium. In the following, I characterize the optimal revenue-maximizing nonlinear taxation of capital gains in general equilibrium. For simplicity, abstract from income effects. Moreover, let cross-return elasticities be multiplicatively separable, as in the example of the financial market (Section 1.E). That is, $\gamma_{i,i'}^{r,a} = \frac{1}{r_i} \delta_{i'}^{r,a}$, where $\delta_{i'}^{r,a}$ decreases in i' and $\delta_{i'}^{r,a} > 0$ ($\delta_{i'}^{r,a} < 0$) for small $a_{i'}$ (large $a_{i'}$). Then, Proposition 6 describes the optimal capital tax.

Proposition 6 (Optimal nonlinear capital gains tax in general equilibrium)**.** *The optimal revenue-maximizing nonlinear capital gains tax on capital gains in general equilibrium is almost everywhere given by*

$$\frac{T_k'\left(a_{R,i}\right)^{GE}}{1-T_k'\left(a_{R,i}\right)^{GE}} = \frac{1}{\zeta_i^{a_R,\left(1-T_k'\right)}} \frac{1-H\left(a_{R,i}\right)}{a_{R,i} h\left(a_{R,i}\right)} - \frac{\delta_i^{r,a}}{r_i\left(1+\varepsilon_i^{r,a}\right)\left(1+\Psi\right)}$$
$$\times \int_{\mathbb{R}_+} \frac{1+\left(1+\zeta_{i'}^{r,a}\right)\zeta_{i'}^{a,r}}{\zeta_{i'}^{a_R,\left(1-T_k'\right)}} \frac{1-H\left(a_{R,i'}\right)}{a_{R,i'} h\left(a_{R,i'}\right)} \frac{a_{i'}\left[1-T_k'\left(a_{R,i'}\right)^{GE}\right]}{a_i\left[1-T_k'\left(a_{R,i}\right)^{GE}\right]} dH\left(a_{R,i'}\right), \tag{1.81}$$

where $\zeta_i^{a_R,\left(1-T_k'\right)} \equiv \Phi_i \tilde{\zeta}_i^{a,\left(1-T_k'\right)}$, $\Phi_i \equiv \left(1+\varepsilon_i^{r,a}\right)\phi_i$, $\phi_i \equiv \frac{1}{1-\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}}$, *and* $\Psi \equiv \int_{\mathbb{R}_+} \frac{1}{1+\varepsilon_{i'}^{r,a}} \frac{1}{r_{i'}} \delta_{i'}^{r,a} dH\left(a_{R,i'}\right)$.

Proof. Appendix 1.H.3. □

The optimal tax in general equilibrium adds an additional term on the right-hand side to the partial equilibrium tax (Equation (1.79)). Observe that the second factor of this extra term is positive (for $0 < T_k'\left(a_{R,i'}\right) < 1$ for all $a_{R,i'}$). Therefore, the sign of $\frac{-\delta_i^{r,a}}{r_i\left(1+\varepsilon_i^{r,a}\right)\left(1+\Psi\right)}$ determines how to adjust the tax rate in general equilibrium. As in Section 1.C, suppose that cross-return elasticities cancel out $\int_i \gamma_{i,i}^{r,a} di = 0$ and let $\varepsilon_i^{r,a}$ be constant such that $\Psi = 0$. Then, the sign of the adjustment depends on the one of $-\delta_i^{r,a}$.

As a benchmark, consider a politician who sets a tax scheme, $\overline{T}_k'\left(a_{R,i}\right)$, wrongly assuming that there are no general equilibrium effects for a given initial tax code.[33] Then, one can write

[33] This notion includes the self-confirming policy equilibrium, proposed by Rothschild and Scheuer (2013, 2016), where a planner implements a tax scheme that generates a capital income distribution for which this tax schedule is optimal, $\overline{T}_k'\left(a_{R,i}\right)^{SCPE}$.

the general equilibrium tax rate as

$$\frac{T_k'(a_{R,i})^{GE}}{1-T_k'(a_{R,i})^{GE}} = \frac{\overline{T}_k'(a_{R,i})}{1-\overline{T}_k'(a_{R,i})} - \frac{\delta_i^{r,a}}{r_i(1+\varepsilon_i^{r,a})(1+\Psi)}$$
$$\times \int_{\mathbb{R}_+} \left[1+\left(1+\varepsilon_{i'}^{r,a}\right)\zeta_{i'}^{a,r}\right] \frac{\overline{T}_k'(a_{R,i})}{1-\overline{T}_k'(a_{R,i})} \frac{a_{i'}\left[1-T_k'(a_{R,i'})^{GE}\right]}{a_i\left[1-T_k'(a_{R,i})^{GE}\right]} dH\left(a_{R,i'}\right)$$

Therefore, cross-effects provide a force for higher capital taxes at the top ($\delta_i^{r,a} < 0$ for large a_i) and lower taxes at the bottom making the tax code ceteris paribus more progressive than in the self-confirming policy equilibrium (Proposition 2).

1.G.4 Other Policies

In the following, I study the interaction with other policies. Consider the partial equilibrium. I distinguish two different policies. In the first case, the government reduces κ for everyone, and, in the second one, it provides a minimum level of financial information. In both cases, the government optimally chooses $\mathscr{P}$ to maximize $\mathscr{G}(T_l, T_k)$ subject to $\mathscr{R}(T_l, T_k) \geq \bar{E} + \beta C(\mathscr{P})$ where $\beta C(\mathscr{P})$ is an increasing and convex cost function. The optimal $\mathscr{P}$ is implicitly defined by

$$\frac{d}{d\mathscr{P}}\left[\frac{1}{\lambda}\mathscr{G}(T_l, T_k) + \mathscr{R}(T_l, T_k)\right] = \beta C'(\mathscr{P}). \tag{1.82}$$

Using the approximations described in Section 1.E, the first-order condition simplifies to

$$\underbrace{\frac{d}{d\mathscr{P}}\int_i (\Gamma_i/\lambda)\left[u_0(\cdot) + \beta H u_1\left[\mathbb{E}(\cdot)\right] + \frac{1}{2}\beta H u_1''(a_i)\mathbb{V}\left(a_i r_i^p z\right)\right] di + o(z)}_{\equiv \mathscr{W}\mathscr{E}_{\mathscr{P}}}$$
$$+ \underbrace{\frac{d}{d\mathscr{P}}\int_i \beta H T_k\left[a_i \mathbb{E}\left(r_i^p z\right)\right] di + o(z)}_{\equiv \mathscr{R}\mathscr{E}_{\mathscr{P}}} = \beta C'(\mathscr{P}). \tag{1.83}$$

The optimal policy, therefore, trades off first-order revenue and welfare effects. In the following, I describe the first-order condition for each policy in more detail.

Cost subsidy. In the first case, the government lowers the marginal costs of all investors ($\mathscr{P} = \Delta\kappa < 0$). This policy could take the form of a subsidy on financial advisory costs. By the envelope theorem, the first-order welfare impact reduces to the positive effect of cost savings

$$\mathscr{W}\mathscr{E}_{\kappa} = \frac{1}{\kappa}\beta H \int_i g_{i,1}\left(\mathbb{E}(\cdot)\right) x_i z di, \tag{1.84}$$

whereas the revenue differential includes behavioral effects

$$\begin{aligned}\mathscr{R}\mathscr{E}_{\kappa} = & \frac{1}{\kappa}\beta H \int_i \frac{T_k'\left[\mathbb{E}\left(a_i r_i^p z\right)\right]}{1-T_k'\left[\mathbb{E}\left(a_i r_i^p z\right)\right]}\left(1+\varepsilon_i^{r,a}\right)\eta_i^{a,I_2} v\left(x_i\right) z di \\ & -\frac{1}{\kappa}\beta H \int_i T_k'\left[\mathbb{E}\left(a_i r_i^p z\right)\right] a_i \mathbb{E}\left(r_i^p z\right)\left(1+\tilde{\zeta}_i^{a,r}\right)\zeta_i^{\mathbb{E}(r^p z),\kappa} di \end{aligned} \tag{1.85}$$

with an income effect $\eta_i^{a,I_2} \leq 0$ and the elasticity of the return rate with respect to marginal information costs $\zeta_i^{\mathbb{E}(r^p z),\kappa} \equiv \frac{\partial log\left[\mathbb{E}\left(r_i^p z\right)\right]}{\partial log(\kappa)} < 0$.

On the one hand, the reduction in κ induces a positive impact on capital incomes. Since the acquisition of information becomes relatively cheaper, households acquire more financial knowledge, which allows them to generate higher rates of return. As returns rise, households also save more.

On the other hand, the first term characterizes a negative income effect. Households feel wealthier due to the decline in information costs. As a result, they save less such that capital incomes diminish.

Financial education. In the second case, the government provides a minimum level of financial knowledge as a public good ($\mathscr{P} = \underline{x}$). This policy refers to a situation where the government offers a compulsory finance course to all high school students for free. Formally, the government ensures that $x_i \geq \underline{x}$ for all $i \in [0,1]$. Then, the costs of information acquisition read as $v(x_i) = \kappa z \cdot max\{0, x_i - \underline{x}\}$. Observe that there is a threshold household, $\underline{i}$, with wealth level, $a_{\underline{i}}$, below which households only rely on the education program. They do not acquire any private information beyond $\underline{x}$ and obtain the same rate of return $\mathbb{E}\left(r_{\underline{i}}^p z\right)$. Households above $\underline{i}$ are not affected in their decision making.

The first-order welfare change features two effects

$$\mathscr{W}\mathscr{E}_{\underline{x}} = \frac{1}{\underline{x}}\beta H \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} \int_0^{\underline{i}} \mathbb{E}\left[g_{i,1}(\cdot)\right] \frac{dlog\left[\mathbb{E}\left(u_1'(\cdot)\right)\right]}{dlog\left[\mathbb{E}\left(r_{\underline{i}}^p z\right)\right]} di + \frac{1}{\underline{x}}\beta H v(\underline{x}) z \int_{\underline{i}}^1 g_{i,1}\left[\mathbb{E}(\cdot)\right] di + o(z) \tag{1.86}$$

with $\zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} \equiv \frac{\partial log\left[\mathbb{E}\left(r_{\underline{i}}^p z\right)\right]}{\partial log(\underline{x})} > 0$. The first one describes the positive impact on utility for households below $\underline{i}$ who experience a rise in their rate of return as the government increases $\underline{x}$ ($d\underline{x} > 0$). The second effect is a mechanical cost-saving effect on households above $\underline{i}$.

The revenue effect

$$\mathscr{R}\mathscr{E}_{\underline{x}} = \frac{1}{\underline{x}}\beta H \int_0^{\underline{i}} T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right] a_i \mathbb{E}\left(r_i^p z\right) \left(1+\tilde{\zeta}_i^{a,r}\right) \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} di$$
$$+ \frac{1}{\underline{x}}\beta H \int_i \frac{T_k'\left[\mathbb{E}\left(a_i r_i^p z\right)\right]}{1-T_k'\left[\mathbb{E}\left(a_i r_i^p z\right)\right]} a_i \mathbb{E}\left(r_i^p z\right)\left(1+\varepsilon_i^{r,a}\right)\eta_i^{a,I_2} v\left(\underline{x}\right) z di + o\left(z\right) \tag{1.87}$$

characterizes income effects for all households and the effects of $\underline{x}$ on the capital incomes of households below $\underline{i}$. A rise in the minimum level of financial knowledge allows these households to obtain higher rates of return. Moreover, they save more as returns increase.

Which of the two policies the government should undertake, depends on the magnitude of the revenue and welfare effects. In particular, one needs to know about the size of the policy elasticities $\zeta_i^{\mathbb{E}(r^p z),\kappa}$ and $\zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}}$. These describe the responsiveness of individual returns with respect to a reduction in information costs and a rise in the minimum education provision by the government, respectively.

The impact of the policy also interacts with the tax code. Two identical societies that only vary in their redistributive preference may, therefore, deem very distinct policies desirable. Similarly, this is the case when they solely differ in the way how returns are formed (i.e., the importance of scale dependence relative to type dependence). Moreover, the marginal costs of policy implementation, $C'\left(\mathscr{P}\right)$, depend on the respective policy $\mathscr{P}$ and other parameters, such as the efficiency of a country's educational system.

1.H Proofs for Section 1.G

1.H.1 Preliminaries

Household choices. For the specified GHH preferences, the households' first-order conditions are given by

$$\begin{aligned}
\left[l_i\right] &: 0 = w_i\left(1-T_l'\left(w_i l_i\right)\right) - v_0'\left(l_i\right)\\
\left[a_i\right] &: 0 = \left[1+r_i\left(1-T_k'\left(r_i a_i\right)\right)\right]\beta u_1'\left(\cdot\right) - u_0'\left(\cdot\right)\\
\left[e_i\right] &: 0 = a_i\left(1-T_k'\left(r_i a_i\right)\right) r_i'\left(e_i\right) - v_1'\left(e_i\right),
\end{aligned} \tag{1.88}$$

where the optimal labor supply decisions can be decoupled from the savings and information effort choices. Let the second-order conditions hold. That is,

$$\frac{\partial^2 u\left(l_i, a_i, e_i; w_i\right)}{\partial l_i^2} = -w_i^2 T_l''\left(w_i l_i\right) - v_0''\left(l_i\right) < 0$$

and the Hessian $H = \begin{pmatrix} \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i^2} & \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} \\ \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} & \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial e_i^2} \end{pmatrix}$ is negative definite.

Monotonicity. Now, I describe the relationship between optimal choices and pre-tax wages. A household's labor supply increases with the wage rate

$$\begin{aligned}\frac{dl_i}{dw_i} &= -\frac{\partial^2 u(l_i,a_i,e_i;w_i)/(\partial l_i \partial w_i)}{\partial^2 u(l_i,a_i,e_i;w_i)/\partial l_i^2} = \frac{1-T_l'(w_il_i) - w_il_iT_l''(w_il_i)}{w_i^2 T_l''(w_il_i) + v_0''(l_i)} \\ &= \frac{l_i}{w_i} \frac{1-p_l(y_i)}{\frac{v_0''(l_i)}{v_0'(l_i)} l_i + p_l(y_i)} > 0,\end{aligned}$$

where I use the definition of the local rate of tax progressivity $p_t(y) \equiv \frac{yT_t''(y)}{1-T_t'(y)}$ for $t \in \{l,k\}$ and the assumption that $p_l(y_i) < 1$. Since $\frac{dy_i}{dw_i} = w_i \frac{dl_i}{dw_i} + l_i$, labor earnings also rise with the wage rate. Savings and effort choices depend on w_i according to

$$\begin{aligned}\begin{pmatrix} da_i/dw_i \\ de_i/dw_i \end{pmatrix} &= -H^{-1} \begin{pmatrix} -u_0''(\cdot)\, l_i \left(1-T_l'(w_il_i)\right) \\ 0 \end{pmatrix} \\ &= \frac{1}{det(H)} \begin{pmatrix} \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial e_i^2} & -\frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} \\ -\frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} & \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i^2} \end{pmatrix} \begin{pmatrix} u_0''(\cdot)\, l_i \left(1-T_l'(w_il_i)\right) \\ 0 \end{pmatrix} \\ &= \frac{u_0''(\cdot)\, l_i \left(1-T_l'(w_il_i)\right)}{det(H)} \begin{pmatrix} \frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial e_i^2} \\ -\frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} \end{pmatrix}.\end{aligned}$$

Observe that by the second-order conditions $det(H) > 0$ and $\frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial e_i^2} < 0$. Moreover, for $p_k(r_ia_i) < 1$,

$$\frac{\partial^2 u(l_i,a_i,e_i;w_i)}{\partial a_i \partial e_i} = \beta\left(1-T_k'(a_{R,i})\right)\left(1-p_k(a_{R,i})\right) r_i'(e_i) > 0.$$

Altogether, due to the concavity of $u_0(\cdot)$, $\frac{da_i}{dw_i} > 0$ and $\frac{de_i}{dw_i} > 0$. Consequently, capital income rises in the pre-tax wage $\frac{da_{R,i}}{dw_i} = a_i r'(e_i) \frac{de_i}{dw_i} + r_i \frac{da_i}{dw_i} > 0$ and in the position in the income distribution $\frac{da_{R,i}}{di} = \frac{da_{R,i}}{dw_i}\frac{dw_i}{di} + \frac{da_{R,i}}{dr_i}\frac{dr_i}{di} > 0$.

1.H.2 Incidence of Nonlinear Tax Reforms

Incidence on savings in partial equilibrium. To derive the incidence on savings in partial equilibrium, plug Equation (1.69) into (1.68)

$$\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{PE}}{a_i} = -\tilde{\zeta}_i^{a,\left(1-T_k'\right)} \frac{\hat{T}_k'(a_{R,i})}{1-T_k'(a_{R,i})} - \tilde{\eta}_i^{a,I_2} \frac{\hat{T}_k(a_{R,i})}{a_{R,i}\left(1-T_k'(a_{R,i})\right)} + \tilde{\zeta}_i^{a,r} \varepsilon_i^{r,a} \frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{PE}}{a_i}$$

Rearrange this expression to obtain Equation (1.71) in Lemma 4.

Incidence on savings in general equilibrium. To derive Equation (1.72) in Lemma 4, insert Equation (1.70) into (1.68) and rearrange

$$\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)^{GE}}{a_i}=-\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}-\phi_i\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)}+\phi_i\tilde{\zeta}_i^{a,r}\int_{i'}\gamma_{i,i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'$$

This expression is a Fredholm integral equation of the second kind. Suppose that $\int_{i'}\int_i\left|\phi_i\tilde{\zeta}_i^{a,r}\gamma_{i,i'}^{r,a}\right|^2 didi'<1$. Then, by Theorem 2.3.1 in Zemyan (2012), the unique solution to this expression is given by Equation (1.72).

Incidence on return inequality. In partial equilibrium, the effect on returns can be written as

$$\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)^{PE}}{r_i}=-\phi_i\varepsilon_i^{r,a}\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}-\phi_i\varepsilon_i^{r,a}\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)},$$

where I use Equations (1.69) and (1.71). Using the fact that

$$\int_{i'}\gamma_{i,i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'=-\int_{i'}\phi_{i'}R_{i,i'}\left[\tilde{\zeta}_{i'}^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i'}\right)}{1-T_k'\left(a_{R,i'}\right)}+\tilde{\eta}_{i'}^{a,I_2}\frac{\hat{T}_k\left(a_{R,i'}\right)}{a_{R,i'}\left(1-T_k'\left(a_{R,i'}\right)\right)}\right]di'\equiv CE_i,$$

the general equilibrium incidence on returns reads as

$$\frac{\hat{r}_i\left(T_k,\hat{T}_k\right)^{GE}}{r_i}=-\phi_i\varepsilon_i^{r,a}\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}-\phi_i\varepsilon_i^{r,a}\tilde{\eta}_i^{a,I_2}\frac{\hat{T}_k\left(a_{R,i}\right)}{a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)}+\phi_i CE_i$$

Incidence on utilities. The partial equilibrium incidence on household utilities is standard. In general equilibrium, one needs to account for cross-return effects that come from the dependence of each household's return rate on the savings of all other households

$$\begin{aligned}\hat{\mathscr{U}}_i\left(T_k,\hat{T}_k\right)^{GE}=&-\lambda g_{i,1}\left(\beta/\Gamma_i\right)\hat{T}_k\left(a_{R,i}\right)+\lambda g_{i,1}\left(\beta/\Gamma_i\right)a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)\int_{i'}\gamma_{i,i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'\\&+\lambda g_{i,1}\left(\beta/\Gamma_i\right)a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)\tilde{\zeta}_i^{a,r}\int_{i'}\gamma_{i,i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'\\=&-\lambda g_{i,1}\left(\beta/\Gamma_i\right)\hat{T}_k\left(a_{R,i}\right)+\lambda g_{i,1}\left(\beta/\Gamma_i\right)a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)\left(1+\tilde{\zeta}_i^{a,r}\right)CE_i\end{aligned}$$

Incidence on revenues and welfare. Equation (1.76) is standard. Perturb Equation (1.66)

$$\hat{\mathscr{R}}\left(T_k,\hat{T}_k\right)^{EQ}=\beta\int_i\hat{T}_k\left(a_{R,i}\right)di+\beta\int_i T_k'\left(a_{R,i}\right)\left[a_i\hat{r}_i\left(T_k,\hat{T}_k\right)^{EQ}+r_i\hat{a}_i\left(T_k,\hat{T}_k\right)^{EQ}\right]di$$

and rearrange to get (1.77).

1.H.3 Optimal Nonlinear Taxation

Optimal taxation in partial equilibrium. Setting the sum of first-order welfare and revenue effects equal to zero, the optimal nonlinear capital gains tax is characterized by

$$\begin{aligned}&\int_{a_{R,i}}\left[1-g_{i,1}-(1+\varepsilon_i^{r,a})\,\phi_i\tilde{\eta}_i^{a,I_2}\frac{T_k'(a_{R,i})}{1-T_k'(a_{R,i})}\right]\hat{T}_k(a_{R,i})\,dH(a_{R,i})\\&=\int_{a_{R,i}}a_{R,i}(1+\varepsilon_i^{r,a})\,\phi_i\tilde{\zeta}_i^{a,(1-T_k')}\frac{T_k'(a_{R,i})}{1-T_k'(a_{R,i})}\hat{T}_k'(a_{R,i})\,dH(a_{R,i}).\end{aligned}$$

Integrate the first term by parts and apply the fundamental theorem of calculus of variations to get

$$\begin{aligned}\frac{T_k'(a_{R,i})}{1-T_k'(a_{R,i})}=&\frac{1}{(1+\varepsilon_i^{r,a})\,\phi_i\tilde{\zeta}_i^{a,(1-T_k')}}\frac{1-H(a_{R,i})}{a_{R,i}h(a_{R,i})}\\&\times\int_{a_{R,i}}^{\infty}\left[1-g_{i',1}-\left(1+\varepsilon_{i'}^{r,a}\right)\phi_{i'}\tilde{\eta}_{i'}^{a,I_2}\frac{T_k'\left(a_{R,i'}\right)}{1-T_k'\left(a_{R,i'}\right)}\right]\frac{dH\left(a_{R,i'}\right)}{1-H(a_{R,i})}.\end{aligned}$$

This expression is a first-order linear differential equation. Use standard techniques (see Saez (2001)) to obtain Equation (1.79).

To express (1.79) in terms of the pre-tax wage distribution, change the variables in the integration

$$\frac{T_k'(a_{R,i})}{1-T_k'(a_{R,i})}=\frac{1}{\zeta_i^{a_R,(1-T_k')}}\frac{1-i}{a_{R,i}h(a_{R,i})}\int_i^1\left(1-g_{i'',1}\right)exp\left[-\int_i^{i''}\frac{\tilde{\eta}_{i'}^{a,T_k}}{\tilde{\zeta}_{i'}^{a,(1-T_k')}}\frac{di'}{i'}\right]\frac{di''}{1-i}. \tag{1.89}$$

Since $i=H(a_{R,i})$,

$$i=a_{R,i}h(a_{R,i})\frac{da_{R,i}}{di}\frac{i}{a_{R,i}}. \tag{1.90}$$

The elasticity of capital income with respect to the income rank is given by

$$\frac{da_{R,i}}{di}\frac{i}{a_{R,i}}=(1+\varepsilon_i^{r,a})\frac{da_i}{di}\frac{i}{a_i}+\frac{\partial r_i(a_{R,i})}{\partial i}\frac{i}{r_i(a_{R,i})}\equiv(1+\varepsilon_i^{r,a})\,\phi_i\zeta_i^{a,i}+\tilde{\zeta}_i^{r,i} \tag{1.91}$$

where the second equality follows from the fact that

$$\zeta_i^{a,i}=\tilde{\zeta}_i^{a,i}+\tilde{\zeta}_i^{a,r}\varepsilon_i^{r,a}\zeta_i^{a,i}=\phi_i\tilde{\zeta}_i^{a,i}=\phi_i\left(\tilde{\zeta}_i^{a,y-T_l(y)}\tilde{\zeta}_i^{y-T_l(y),i}+\tilde{\zeta}_i^{a,r}\tilde{\zeta}_i^{r,i}\right).$$

Plug Equations (1.90) and (1.91) into (1.89), to get Equation (1.80).

Optimal taxation in general equilibrium. First, note that, in the absence of income effects and for $\gamma_{i,i'}^{r,a} = \frac{1}{r_i}\delta_{i'}^{r,a}$, the incidence on savings can be written as

$$\begin{aligned}\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)}{a_i} &= -\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} + \tilde{\zeta}_i^{a,r}\frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)}{a_{i'}}di' \\ &= -\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \tilde{\zeta}_i^{a,r}\frac{1}{r_i}\frac{1}{1-\int_i\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}di}\int_i\delta_i^{r,a}\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}di,\end{aligned}$$

noting that

$$\begin{aligned}\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)}{a_{i'}}di' &= -\int_i\delta_i^{r,a}\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}di + \int_i\delta_i^{r,a}\tilde{\zeta}_i^{a,r}\frac{1}{r_i}di\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)}{a_{i'}}di' \\ &= -\frac{1}{1-\int_i\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}di}\int_i\delta_i^{r,a}\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}di.\end{aligned}$$

By the latter expression, the response of capital income reads as

$$\begin{aligned}\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)}{a_i} + \frac{\hat{r}_i\left(T_k,\hat{T}_k\right)}{r_i} &= \left(1+\varepsilon_i^{r,a}\right)\frac{\hat{a}_i\left(T_k,\hat{T}_k\right)}{a_i} + \frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)}{a_{i'}}di' = -\left(1+\varepsilon_i^{r,a}\right)\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)} \\ &\quad\times\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} - \left[1+\left(1+\varepsilon_i^{r,a}\right)\tilde{\zeta}_i^{a,r}\right]\frac{1}{r_i}\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)}{a_{i'}}di'\end{aligned}$$

and the incidence on utility is given by

$$\hat{\mathscr{U}}_i\left(T_k,\hat{T}_k\right) = -\lambda g_{i,1}\left(\beta/\Gamma_i\right)\hat{T}_k\left(a_{R,i}\right) + \lambda g_{i,1}\left(\beta/\Gamma_i\right)a_{R,i}\left(1-T_k'\left(a_{R,i}\right)\right)\frac{1+\tilde{\zeta}_i^{a,r}}{r_i}\int_{i'}\delta_{i'}^{r,a}\frac{\hat{a}_{i'}\left(T_k,\hat{T}_k\right)^{GE}}{a_{i'}}di'.$$

Again, impose that there is no first-order effect on the social planner's objective function, $\frac{1}{\lambda}\hat{\mathscr{U}}_i\left(T_k,\hat{T}_k\right)^{GE} + \hat{\mathscr{R}}\left(T_k,\hat{T}_k\right)^{GE} = 0$, to characterize the optimal capital gains tax

$$\begin{aligned}\int_i\left(1-g_{i,1}\right)\hat{T}_k\left(a_{R,i}\right)di &= \int_i g_{i,1}a_{R,i}\left[1-T_k'\left(a_{R,i}\right)\right]\frac{1+\tilde{\zeta}_i^{a,r}}{r_i}di\frac{1}{1-\int_i\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}di}\int_i\delta_i^{r,a}\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)}di \\ &+ \int_i T_k'\left(a_{R,i}\right)a_{R,i}\Bigg[\left(1+\varepsilon_i^{r,a}\right)\phi_i\tilde{\zeta}_i^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i}\right)}{1-T_k'\left(a_{R,i}\right)} \\ &+ \left[1+\left(1+\varepsilon_i^{r,a}\right)\tilde{\zeta}_i^{a,r}\right]\frac{1}{r_i}\frac{1}{1-\int_{i'}\zeta_{i'}^{a,r}\frac{1}{r_{i'}}\delta_{i'}^{r,a}di'}\int_{i'}\delta_{i'}^{r,a}\phi_{i'}\tilde{\zeta}_{i'}^{a,\left(1-T_k'\right)}\frac{\hat{T}_k'\left(a_{R,i'}\right)}{1-T_k'\left(a_{R,i'}\right)}di'\Bigg]di\end{aligned}$$

if and only if

$$\int_i (1-g_{i,1})\,\hat{T}_k(a_{R,i})\,di = \int_i T_k'(a_{R,i})\,a_{R,i}\,(1+\varepsilon_i^{r,a})\,\phi_i \tilde{\zeta}_i^{a,(1-T_k')}\frac{\hat{T}_k'(a_{R,i})}{1-T_k'(a_{R,i})}di$$
$$+\int_i a_i\left\{g_{i,1}\left[1-T_k'(a_{R,i})\right]\left(1+\tilde{\zeta}_i^{a,r}\right)+T_k'(a_{R,i})\left[1+(1+\varepsilon_i^{r,a})\,\tilde{\zeta}_i^{a,r}\right]\right\}di$$
$$\times\frac{1}{1-\int_i \tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}di}\int_i \delta_i^{r,a}\phi_i\tilde{\zeta}_i^{a,(1-T_k')}\frac{\hat{T}_k'(a_{R,i})}{1-T_k'(a_{R,i})}di.$$

In this setting, the easiest way to derive an expression for the optimal capital gains tax is to consider the Saez (2001) perturbation: $\hat{T}_k(a_{R,i})=1_{a_{R,i}\geq a_{R,i^*}}$ and $\hat{T}_k'(a_{R,i})=\delta_{a_{R,i^*}}(a_{R,i})$, where $\delta_{a_{R,i^*}}(a_{R,i})$ is the Dirac delta function. Then, under revenue maximization ($g_{i,1}=0$), the previous expression simplifies to

$$\frac{T_k'(a_{R,i^*})}{1-T_k'(a_{R,i^*})}=\frac{1}{(1+\zeta_{i^*}^{r,a})\,\phi_{i^*}\tilde{\zeta}_{i^*}^{a,(1-T_k')}}\frac{1-H(a_{R,i^*})}{a_{R,i^*}h(a_{R,i^*})}$$
$$-\frac{\int_{a_{R,i}} a_iT_k'(a_{R,i})\left[1+(1+\varepsilon_i^{r,a})\,\tilde{\zeta}_i^{a,r}\right]dH(a_{R,i})}{1-\int_{a_{R,i}}\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})}\frac{\frac{1}{r_{i^*}}\delta_{i^*}^{r,a}}{1+\varepsilon_{i^*}^{r,a}}\frac{1}{a_{i^*}\left(1-T_k'(a_{R,i^*})\right)},$$

where I expressed all the variables in terms of observables. Rearrange and integrate out to get

$$\int_{a_{R,i}}\left[1+(1+\varepsilon_i^{r,a})\,\tilde{\zeta}_i^{a,r}\right]\frac{a_i\left(1-T_k'(a_{R,i})\right)}{(1+\varepsilon_{i^*}^{r,a})\,\phi_{i^*}\tilde{\zeta}_{i^*}^{a,(1-T_k')}}\frac{1-H(a_{R,i})}{a_{R,i}h(a_{R,i})}dH(a_{R,i})$$
$$=\frac{\int_{a_{R,i}}\frac{1+\left(1+\varepsilon_i^{r,a}\right)\tilde{\zeta}_i^{a,r}}{1+\varepsilon_i^{r,a}}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})+1-\int_{a_{R,i}}\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})}{1-\int_{a_{R,i}}\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})}\int_{a_{R,i}}a_iT_k'(a_{R,i})\left[1+(1+\zeta_i^{r,a})\,\tilde{\zeta}_i^{a,r}\right]dH(a_{R,i})$$
$$=\left[1+\int_{a_{R,i}}\frac{1}{1+\varepsilon_i^{r,a}}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})\right]\frac{\int_{a_{R,i}}a_iT_k'(a_{R,i})\left[1+(1+\varepsilon_i^{r,a})\,\tilde{\zeta}_i^{a,r}\right]dH(a_{R,i})}{1-\int_{a_{R,i}}\tilde{\zeta}_i^{a,r}\frac{1}{r_i}\delta_i^{r,a}dH(a_{R,i})}.$$

Altogether, one can write the optimal nonlinear capital gains tax as

$$\frac{T_k'(a_{R,i})}{1-T_k'(a_{R,i})}=\frac{1}{(1+\varepsilon_i^{r,a})\,\phi_i\tilde{\zeta}_i^{a,(1-T_k')}}\frac{1-H(a_{R,i})}{a_{R,i}h(a_{R,i})}-\frac{\frac{1}{r_i}\delta_i^{r,a}}{1+\varepsilon_i^{r,a}+\int_{a_{R,i}'}\frac{1+\varepsilon_i^{r,a}}{1+\varepsilon_{i'}^{r,a}}\frac{1}{r_{i'}}\delta_{i'}^{r,a}dH\left(a_{R,i'}\right)}$$
$$\times\int_{a_{R,i'}}\frac{1+\left(1+\varepsilon_{i'}^{r,a}\right)\zeta_{i'}^{a,r}}{\left(1+\varepsilon_{i'}^{r,a}\right)\phi_{i'}\tilde{\zeta}_{i'}^{a,(1-T_k')}}\frac{1-H\left(a_{R,i'}\right)}{a_{R,i'}h\left(a_{R,i'}\right)}\frac{a_{i'}\left(1-T_k'\left(a_{R,i'}\right)\right)}{a_i\left(1-T_k'(a_{R,i})\right)}dH\left(a_{R,i'}\right),$$

which concludes the proof of Equation (1.81).

To compare this capital gains tax to the one by the exogenous technology planner, note that in the latter case

$$\frac{\overline{T_k}'(a_{R,i})}{1-\overline{T_k}'(a_{R,i})} = \frac{1}{\zeta_i^{a_{R,}(1-T_k')}} \frac{1-H(a_{R,i})}{a_{R,i}h(a_{R,i})}$$

and insert this expression into (1.81).

1.H.4 Other Policies

Preliminaries. In line with the financial market in Section 1.E, second-period expected utility, which the policy $\mathscr{P}$ affects, is given by

$$\begin{aligned} H\cdot\mathbb{E}_i\left(u_1\left[a_i\left(1+r_i^p z\right)-T_k\left(a_i r_i^p z\right)-v(x_i)z\right]\right) &= H\cdot u_1\left[a_i\left(1+\mathbb{E}\left(r_i^p z\right)\right)-T_k\left(a_i\mathbb{E}\left(r_i^p z\right)\right)-v(x_i)z\right] \\ &+H\cdot\frac{1}{2}u_1''(a_i)\mathbb{V}\left(a_i r_i^p z\right)+o(z). \end{aligned}$$

Therefore, the impact of policy $\mathscr{P}$ on welfare can be written as

$$\mathscr{W}\mathscr{E}_{\mathscr{P}} \equiv \frac{d}{d\mathscr{P}}\frac{1}{\lambda}\mathscr{G}(T_l,T_k) = \frac{d}{d\mathscr{P}}\int_i (\Gamma_i/\lambda)\left[u_0(\cdot)+\beta H u_1[\mathbb{E}(\cdot)]+\frac{1}{2}\beta H u_1''(a_i)\mathbb{V}\left(a_i r_i^p z\right)\right]di+o(z).$$

A household's tax liability can be approximated by

$$\begin{aligned} T_k(R_i a_i) &= T_k(a_i r_{i,1}z+\ldots+a_i r_{i,H}z)+o(z) \equiv T_k(a_i r_{i,1}z,\ldots,a_i r_{i,H}z)+o(z) = \sum_{h=1}^{H} T_k'(0)a_i r_{i,h}z+o(z) \\ &= T_k(a_i r_{i,1}z)-T_k(0)+\ldots+T_k(a_i r_{i,H}z)-T_k(0)+o(z) = \sum_{h=1}^{H} T_k(a_i r_{i,h}z)+o(z) \end{aligned}$$

Using this expression, the first-order effect on revenues reads as

$$\begin{aligned} \mathscr{R}\mathscr{E}_{\mathscr{P}} \equiv \frac{d}{d\mathscr{P}}\mathscr{R}(T_l,T_k) &= \frac{d}{d\mathscr{P}}\int_i \beta\mathbb{E}(T_k[R_i a_i])di = \frac{d}{d\mathscr{P}}\int_i \beta\sum_{h=1}^{H} T_k\left[a_i\mathbb{E}\left(r_{i,h}^p z\right)\right]di+o(z) \\ &+\frac{d}{d\mathscr{P}}\int_i \beta\sum_{h=1}^{H} T_k'\left[a_i\mathbb{E}\left(r_{i,h}^p z\right)\right]\mathbb{E}\left[a_i r_{i,h}^p z-a_i\mathbb{E}\left(r_{i,h}^p z\right)\right]di \\ &+\frac{1}{2}\frac{d}{d\mathscr{P}}\int_i \beta\sum_{h=1}^{H} T_k''\left[a_i\mathbb{E}\left(r_{i,h}^p z\right)\right]\mathbb{E}\left[\left(a_i r_{i,h}^p z-a_i\mathbb{E}\left(r_{i,h}^p z\right)\right)^2\right]di+o(z) \\ &= \frac{d}{d\mathscr{P}}\int_i \beta\sum_{h=1}^{H} T_k\left[a_i\mathbb{E}\left(r_{i,h}^p z\right)\right]di+o(z) = \frac{d}{d\mathscr{P}}\int_i \beta H T_k\left[a_i\mathbb{E}\left(r_i^p z\right)\right]di+o(z) \end{aligned}$$

since, in partial equilibrium,

$$\begin{aligned}
\mathbb{E}\left(r_{i,h}^{p}z\right) &= \frac{1}{\rho}\mathscr{S}\left(a_{i,h}\right)z+r^{f}z+o\left(z\right)=\frac{1}{\rho}\mathscr{S}\left(a_{i}\Pi_{s=1}^{h}\left(1+r_{i,h}^{p}z\right)\right)z+r^{f}z+o\left(z\right)\\
&=\frac{1}{\rho}\mathscr{S}\left(a_{i}+a_{i}\sum_{s=1}^{h}r_{i,h}^{p}z\right)z+r^{f}z+o\left(z\right)=\frac{1}{\rho}\mathscr{S}\left(a_{i}\right)z+r^{f}z+o\left(z\right)=\mathbb{E}\left(r_{i}^{p}z\right).
\end{aligned}$$

Cost subsidy. For a cost subsidy, $\mathscr{P}=\Delta\kappa<0$, the first-order welfare effect can be written as in Equation (1.84)

$$\mathscr{W}\mathscr{E}_{\kappa}=\int_{i}\left(\Gamma_{i}/\lambda\right)\beta Hu_{1}'\left[\mathbb{E}\left(\cdot\right)\right]x_{i}zdi\equiv\frac{1}{\kappa}\beta H\int_{i}g_{i,1}\left(\mathbb{E}\left(\cdot\right)\right)v\left(x_{i}\right)zdi+o\left(z\right)$$

and, defining the elasticity of returns with respect to marginal information costs $\zeta_{i}^{\mathbb{E}(r^{p}z),\kappa}\equiv\frac{\partial log\left[\mathbb{E}\left(r_{i}^{p}z\right)\right]}{\partial log(\kappa)}<0$, the effect on government revenue is given by Equation (1.85)

$$\begin{aligned}
\mathscr{R}\mathscr{E}_{\kappa} &= -\frac{d}{d\kappa}\int_{i}\beta HT_{k}\left[a_{i}\mathbb{E}\left(r_{i}^{p}z\right)\right]di+o\left(z\right)=-\frac{1}{\kappa}\beta H\int_{i}T_{k}'\left[\mathbb{E}\left(a_{i}r_{i}^{p}z\right)\right]a_{i}\mathbb{E}\left(r_{i}^{p}z\right)\\
&\times\left[\left(1+\varepsilon_{i}^{r,a}\right)\eta_{i}^{a,I_{2}}\frac{-x_{i}z}{\mathbb{E}\left(r_{i}^{p}z\right)\left(1-T_{k}'\left(a_{i}\mathbb{E}\left(r_{i}^{p}z\right)\right)\right)}\frac{\kappa}{a_{i}}+\left(1+\tilde{\zeta}_{i}^{a,r}\right)\zeta_{i}^{\mathbb{E}(r^{p}z),\kappa}\right]di+o\left(z\right)\\
&=\frac{1}{\kappa}\beta H\int_{i}\frac{T_{k}'\left[\mathbb{E}\left(a_{i}r_{i}^{p}z\right)\right]}{1-T_{k}'\left[\mathbb{E}\left(a_{i}r_{i}^{p}z\right)\right]}\left(1+\varepsilon_{i}^{r,a}\right)\eta_{i}^{a,I_{2}}v\left(x_{i}\right)zdi\\
&-\frac{1}{\kappa}\beta H\int_{i}T_{k}'\left[\mathbb{E}\left(a_{i}r_{i}^{p}z\right)\right]a_{i}\mathbb{E}\left(r_{i}^{p}z\right)\left(1+\tilde{\zeta}_{i}^{a,r}\right)\zeta_{i}^{\mathbb{E}(r^{p}z),\kappa}di+o\left(z\right).
\end{aligned}$$

Financial education. When the government provides a minimal level of financial knowledge, $\underline{x}$, for free, such that the information cost reads as $v\left(x_{i}\right)=\kappa z\cdot max\left\{0,x_{i}-\underline{x}\right\}$, there is a threshold household, below which households do not acquire additional information and obtain the same return rate

$$x_{i}=\sqrt{\frac{a_{i}}{\sigma\rho\kappa}}-1-\mathscr{I}\leq\underline{x}\iff a_{i}\leq\sigma\rho\kappa\left(\underline{x}+1+\mathscr{I}\right)^{2}\equiv a_{\underline{i}}.$$

Define the elasticity of returns with respect to the minimal information provided by the government as $\zeta_{\underline{i}}^{\mathbb{E}(r^{p}z),\underline{x}}\equiv\frac{dlog\left[\mathbb{E}\left(r_{\underline{i}}^{p}z\right)\right]}{dlog(\underline{x})}>0$. The effect of raising $\underline{x}$ ($d\underline{x}>0$) on welfare consists of a rise in return rates of households below $\underline{i}$ and a cost reduction for households above $\underline{i}$

$$\begin{aligned}
\mathscr{W}\mathscr{E}_{\underline{x}} &= \frac{1}{\underline{x}}\beta H \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} \int_0^{\underline{i}} (\Gamma_i/\lambda) \left[u_1' \left[\mathbb{E}(\cdot)\right] a_i \left(1 - T_k' \left[\mathbb{E}\left(a_i r_{\underline{i}}^p z\right)\right]\right) + \frac{1}{2} u_1''(a_i)\, a_i^2 \right] \mathbb{E}\left(r_{\underline{i}}^p z\right) di + o(z) \\
&+ \frac{1}{\underline{x}}\beta H v(\underline{x})\, z \int_{\underline{i}}^1 (\Gamma_i/\lambda)\, u_1' \left[a_i \left(1 + \mathbb{E}\left(r_i^p z\right)\right) - T_k\left(a_i \mathbb{E}\left(r_i^p z\right)\right) - \kappa\left(x_i - \underline{x}\right) z\right] di + o(z) \\
&= \frac{1}{\underline{x}}\beta H \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} \int_0^{\underline{i}} \mathbb{E}\left[g_{i,1}(\cdot)\right] \frac{dlog\left[\mathbb{E}\left(u_1'(\cdot)\right)\right]}{dlog\left[\mathbb{E}\left(r_{\underline{i}}^p z\right)\right]} di + \frac{1}{\underline{x}}\beta H v(\underline{x})\, z \int_{\underline{i}}^1 g_{i,1}\left[\mathbb{E}(\cdot)\right] di + o(z),
\end{aligned}$$

which shows Equation (1.86). The first-order revenue effect (Equation (1.87))

$$\begin{aligned}
\mathscr{R}\mathscr{E}_{\underline{x}} &= \frac{1}{\underline{x}}\beta H \int_0^{\underline{i}} T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right] a_i \mathbb{E}\left(r_i^p z\right) \left(1 + \tilde{\zeta}_i^{a,r}\right) \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} di \\
&+ \frac{1}{\underline{x}}\beta H \int_i T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right] a_i \mathbb{E}\left(r_i^p z\right) \left(1 + \varepsilon_i^{r,a}\right) \frac{\partial a_i}{\partial I_2} \frac{\partial I_2}{\partial \underline{x}} \frac{\underline{x}}{a_i} di + o(z) \\
&= \frac{1}{\underline{x}}\beta H \int_0^{\underline{i}} T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right] a_i \mathbb{E}\left(r_i^p z\right) \left(1 + \tilde{\zeta}_i^{a,r}\right) \zeta_{\underline{i}}^{\mathbb{E}(r^p z),\underline{x}} di \\
&+ \frac{1}{\underline{x}}\beta H \int_i \frac{T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right]}{1 - T_k' \left[\mathbb{E}\left(a_i r_i^p z\right)\right]} a_i \mathbb{E}\left(r_i^p z\right) \left(1 + \varepsilon_i^{r,a}\right) \eta_i^{a,I_2} v(\underline{x})\, z di + o(z)
\end{aligned}$$

collects the effects on the capital income of households below $\underline{i}$ and income effects for all households.

Chapter 2

Nonlinear Taxation and International Mobility in General Equilibrium

Joint with Eckhard Janeba.

2.1 Introduction

International (and inter-regional) mobility of high-income individuals has been at the center of recent theoretical and empirical research due to its far-reaching implications for the taxation of mobile individuals and the progressivity of the income tax code. Mirrlees (1971) has already recognized the importance of international migration but focuses on a closed economy case in his formal analysis.[1] The literature on optimal income taxation has demonstrated the effects of migration on the level and shape of optimal marginal income tax rates. In particular, migration tends to decrease optimal marginal tax rates over the entire income distribution (see Simula and Trannoy (2010)) and, depending on the shape of migration semi-elasticities, even lead to negative tax rates at the top (Lehmann, Simula, and Trannoy (2014)). The mobility of labor reduces tax payments by top-income workers relative to a closed economy. The driving force has been labeled the "threat of migration" and is even present in a situation in which no net migration occurs in equilibrium. A standard assumption in these models is, however, that workers' wages are exogenous.

The role of wage endogeneity for tax policy has been highlighted in another strand of the optimal income taxation literature. For instance, in the closed economy model of Stiglitz (1982), the government lowers the top tax rate to encourage the labor supply of high-skilled individuals,

[1] See Kleven et al. (2020) for a survey of the empirical literature.

thereby raising wages at the bottom and reducing them at the top. Contributors to the literature refer to this indirect redistribution of pre-tax wages as "predistribution." However, this literature has so far disregarded the presence of labor mobility.

This chapter bridges the gap between these two strands of the literature by studying optimal income taxation and international mobility in general equilibrium, i.e., with endogenous wages. We shed light on the interaction of general equilibrium and migration responses in two steps. As a first step, we introduce labor mobility (as in Lehmann, Simula, and Trannoy (2014)) into the N-type version of the Stiglitz (1982) model. We show that the optimal general equilibrium marginal tax rate at the top is higher (i.e., a lower labor subsidy) in the open than in the closed economy. Moreover, we derive conditions under which the optimal marginal tax rate at the bottom declines. Thus, migration may lead to a more progressive tax code in terms of marginal tax rates.

The intuition for this finding is that the predistribution effect, which the government achieves by taxing incomes less progressively, is self-limiting under labor mobility: Any reduction in pre-tax wage inequality comes along with lower immigration (or higher emigration) of high-skilled workers, which, in turn, raises their wages, partly offsetting the benefits of predistribution.

In addition to the finding concerning the tax rates at the ends of the ability distribution, we also characterize the effect of mobility on optimal marginal tax rates in the "middle" and show that it relates to the sign of the optimal marginal tax rate in the closed economy. While the above insights on marginal tax rates complement well the existing literature, they are only to some extent informative about the shape of the income tax scheme (see Heathcote and Tsujiyama (2021b) for a discussion). The N-type model does not reveal profound results on average tax rates that would be necessary to characterize the overall effect on tax progressivity.

Therefore, we calibrate a continuous version of the model to the U.S. economy as a second step. We depart from the Mirrleesian approach of solving for the optimal arbitrarily nonlinear tax system by restricting attention to the class of nonlinear tax functions with a constant rate of progressivity (CRP). The CRP tax scheme approximates well the current U.S. code (see Heathcote, Storesletten, and Violante (2017)) and delivers similar policy prescriptions as the fully nonlinear optimum (e.g., Heathcote and Tsujiyama (2021a)). This modified approach allows us to study the interaction of migration and general equilibrium effects throughout the entire income distribution and to speak to the impact on average tax rates that are more critical for migration decisions than marginal rates.

We discover two novel effects revealing the underlying mechanisms that limit predistribution and weaken the migration threat. Firstly, there is a *wage effect on migration*: A government can broaden the tax base by predistributing less to low-skilled workers, for example by raising tax progressivity, which amplifies wage inequality and, in turn, triggers high-skilled immigration. Secondly, a rise in tax progressivity provokes high-skilled emigration, boosting wage inequality and expanding the tax base – a *migration effect on wages*.

We demonstrate that, depending on the shape of the semi-elasticity of migration, the presence of general equilibrium effects offsets between 6% and 47% of the classic migration-induced erosion in the optimal rate of tax progressivity. In our simulations, the wage effect on migration appears quantitatively more important than the migration effect on wages. We conclude that the threat of migration is, depending on the shape of the migration semi-elasticity, exaggerated, and the canonical Mirrlees (1971) model with fixed wages and no migration provides a more realistic benchmark than previously expected.

Related literature. Our work is related to several recent contributions to the literature on optimal nonlinear income taxation. Stiglitz (1982) initiated the debate on labor income taxation with endogenous wages in a two-type setting. Generalizing Stiglitz (1982) to a continuum of types, Sachs, Tsyvinski, and Werquin (2020) consider reforms of arbitrarily nonlinear tax schedules and the optimal taxation in general equilibrium. They demonstrate that increasing tax rates in an initially progressive tax system increases government revenue more with endogenous than with exogenous wages. Therefore, depending on the initial tax code, it may be beneficial to raise tax progressivity. Our quantitative framework confirms their result for reforms of the rate of tax progressivity instead of elementary tax reforms (as in Sachs, Tsyvinski, and Werquin (2020)). However, the setting of Sachs, Tsyvinski, and Werquin (2020) ignores workers' extensive margin migration responses.

Rothschild and Scheuer (2013) examine the optimal nonlinear income tax schedule in a multi-sector Roy model with endogenous wages. Trickle-down effects are central for their finding that the optimal tax system is more progressive than in an environment without occupational choice. At first glance, one might think that their two-sector setting nests our two-country economy. The key difference to their paper is that, in our model, two governments set tax policies for each country separately. In Rothschild and Scheuer (2013), only one government chooses the tax schedule for both sectors. The latter, however, is equivalent to the coordinated tax policy setup in our model, which we consider in the Appendix.

Our discrete N-type framework connects the closed economy setup with endogenous wages by Stiglitz (1982) to the two-country environment of Lehmann, Simula, and Trannoy (2014) with internationally mobile workers and heterogeneous migration costs. Contrary to the fixed-wage economies in the tax competition literature (e.g., Lehmann, Simula, and Trannoy (2014)), in our environment, workers are imperfect substitutes in producing a composite output good under constant returns to scale.

Altogether, our continuous-type framework nests those of Mirrlees (1971), Lehmann, Simula, and Trannoy (2014), and Sachs, Tsyvinski, and Werquin (2020). We derive and quantify the traditional effects occurring in the former papers. Moreover, we discover novel effects that enlighten the interaction between migration and general equilibrium responses and compare their quantitative importance to the traditional effects.

Finally, this chapter adds to the literature on optimal taxation in the spirit of Ramsey (1927). By restricting attention to the class of CRP tax functions (developed in Feldstein (1969), Persson (1983), and Benabou (2000)), we follow Heathcote, Storesletten, and Violante (2017), who document that the current U.S. tax code is close to CRP and, then, explore this tax function in a dynamic setting with earnings risk. Heathcote and Tsujiyama (2021a) and Heathcote and Tsujiyama (2021b) study the CRP tax scheme in a Mirrlees (1971) model showing that it approximates well the full optimum.

Outline. In Section 2.2, we solve a discrete N-type Stiglitz (1982) model with general equilibrium and migration responses. We briefly discuss extensions to our framework and relate our model and findings to the empirical literature. In Section 2.3, we present our quantitative framework and derive the incidence of reforms to the rate of tax progressivity. Then, we calibrate the framework to the U.S. economy and compute the optimal rate of progressivity in four different policy regimes depending on whether migration and general equilibrium effects are taken into account. Section 2.4 concludes. We relegate all proofs to the Appendix. Moreover, in the Appendix, we provide a parameterized version of our N-type economy, leading to closed-form expressions for the optimal tax system. Finally, we study the effects of tax coordination under symmetric country sizes.

2.2 Optimal Marginal Tax Rates and International Migration in General Equilibrium

2.2.1 The N-Type Model

Economic environment. We extend the canonical model of Stiglitz (1982) to a setting where two countries or regions $i = A, B$ compete for internationally mobile workers. We go beyond the two-type setting studied in Stiglitz (1982) by considering an arbitrary set of skill or productivity types $\theta \in \Theta = \{1, ..., N\}$, which is private information of individuals and not observable to the government. Aside from studying the impact of migration and general equilibrium on bottom and top tax rates, this allows us to speak to the effects on the optimal taxation of the middle class. Without loss of generality, we order types such that their equilibrium wages are increasing in types $w_{i,\theta} > w_{i,\theta-1}$. Therefore, one can interpret a household's type as its position (e.g., percentile) in the equilibrium wage distribution. Moreover, consider a general class of utility functions, $u(c,l)$, with consumption c, labor supply l, and labor income $y \equiv wl$. Let $u(c,l)$ satisfy the Spence-Mirrlees single-crossing property, $\frac{d}{dw}\frac{-u_l(c,y/w)/w}{u_c(c,y/w)} < 0$, and suppose that labor and consumption are separable (e.g., $u(c,l) = h(c) - v(l)$).

Let $n_{i,\theta}$ be the number of natives (born) in country i with skill θ. Denote $N_{i,\theta}$ as country i's equilibrium mass of θ-type workers and $l_{i,\theta}$ as an individual's labor supply. As we explain later, both the labor supply and the equilibrium population will be endogenous to the tax system. In each country i, competitive firms produce a single composite output under a constant elasticity of substitution (CES)

$$F_i\left(\{l_{i,\theta}N_{i,\theta}\}_{\theta\in\Theta}\right) = \left[\sum_{\theta\in\Theta} a_{i,\theta}\left(l_{i,\theta}N_{i,\theta}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

for some $\sigma \in [0,\infty)$ and $a_{i,\theta} \in \mathbb{R}_+$.[2] Consequently, a worker θ's marginal product pins down her wage rate in that country

$$w_{i,\theta} = a_{i,\theta}\left[l_{i,\theta}N_{i,\theta}/F_i\left(\{l_{i,\theta}N_{i,\theta}\}_{\theta\in\Theta}\right)\right]^{-\frac{1}{\sigma}} \; for\; \theta \in \Theta, \tag{2.1}$$

which she takes as given. Let labor and goods markets clear in each country. Define $\gamma_{i,\theta,\theta} \equiv \frac{\partial w_{i,\theta}}{\partial\left(N_{i,\theta}l_{i,\theta}\right)}\frac{N_{i,\theta}l_{i,\theta}}{w_{i,\theta}} < 0$ and $\gamma_{i,\theta,\theta'} \equiv \frac{\partial w_{i,\theta}}{\partial\left(N_{i,\theta'}l_{i,\theta'}\right)}\frac{N_{i,\theta'}l_{i,\theta'}}{w_{i,\theta}} > 0$ as the own- and cross-wage elasticity. Country i's government taxes labor income $y_{i,\theta} \equiv w_{i,\theta}l_{i,\theta}$ according to a nonlinear tax scheme $T_i\left(y_{i,\theta}\right)$.

Labor supply. Conditional on living in country i, a worker optimally chooses labor supply $l_{i,\theta}$ to maximize utility $u\left(c_{i,\theta},l_{i,\theta}\right)$. Consumption is given by the after-tax income $c_{i,\theta} = y_{i,\theta} - T_i\left(y_{i,\theta}\right)$. Then, the worker's first-order condition

$$u_c\left(c_{i,\theta},l_{i,\theta}\right)w_{i,\theta}\left(1 - T_i'\left(y_{i,\theta}\right)\right) = -u_l\left(c_{i,\theta},l_{i,\theta}\right) \tag{2.2}$$

pins down optimal labor supply.

Migration. As in Lehmann, Simula, and Trannoy (2014), a worker θ born in country i draws a migration cost m from a conditional density function $G_i(m|\theta) = \int_0^m g_i(x|\theta)\,dx$, accounting for the fact that migration costs may differ between workers (even conditional on skill-type).

[2] In a two-type setup, one can extend the results to any constant-return-to-scale production function.

Then, a native in country i, for instance, migrates to country j if and only if $u\left(c_{j,\theta},l_{j,\theta}\right)-m>u\left(c_{i,\theta},l_{i,\theta}\right)$. Defining $\Delta_i\equiv u\left(c_{i,\theta},l_{i,\theta}\right)-u\left(c_{j,\theta},l_{j,\theta}\right)$, one can derive a country's equilibrium mass of θ-workers as

$$N_{i,\theta}\equiv\rho_i\left(\Delta_i|\theta\right)\equiv\begin{cases}n_{i,\theta}+G_j\left(\Delta_i|\theta\right)n_{j,\theta} & for\ \Delta_i\geq 0\\ \left(1-G_i\left(-\Delta_i|\theta\right)\right)n_{i,\theta} & for\ \Delta_i\leq 0\end{cases}. \tag{2.3}$$

Accordingly, denote the semi-elasticity of migration as $\eta_{i,\theta}\equiv\frac{\partial\rho_i(\Delta_i|\theta)}{\partial\Delta_i}\frac{1}{N_{i,\theta}}>0$.

Government problem. We consider a Nash game between the governments of the two countries. Each government chooses its nonlinear income tax schedule, taking the other country's tax schedule as given and correctly anticipating the migration and labor supply effects from its tax policy. As in Simula and Trannoy (2010) and Lehmann, Simula, and Trannoy (2014), we consider a Rawlsian objective function: The government maximizes the utility of the lowest type. The approach has the advantage that - given the government's objective of redistributing from high and medium types to the lowest type - constraints from mobility become most visible. In addition, one avoids the issue of whose utility a government should maximize (residents or natives).[3] Formally, country i's government wants to redistribute to the lowest type (i.e., $T_i\left(y_{i,\theta}\right)\geq T_i\left(y_{i,\theta-1}\right)$), and, thus, solves

$$\max_{\left\{c_{i,\theta},l_{i,\theta},N_{i,\theta}\right\}_{\theta\in\Theta}} \quad u\left(c_{i,1},l_{i,1}\right) \tag{2.4}$$

$$subject\ to \quad u\left(c_{i,\theta},l_{i,\theta}\right)\geq u\left(c_{i,\theta-1},\frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)\ for\ \theta\in\{2,...,N\}, \tag{2.5}$$

$$\sum_{\theta\in\Theta}N_{i,\theta}c_{i,\theta}\leq F_i\left(\left\{N_{i,\theta}l_{i,\theta}\right\}_{\theta\in\Theta}\right), \tag{2.6}$$

as well as subject to the endogeneity of wages (Equation (2.1)) and the equilibrium population (Equation (2.3)), and taking the other country j's tax scheme as given. Equations (2.5) and (2.6) are the high-skilled worker's incentive constraint and the government budget (no public good provision, purely redistributive tax). Observe that one can omit non-local incentive constraints where $\theta'<\theta-1$ and $\theta'>\theta+1$ (see Milgrom and Shannon (1994)). We focus on solutions where only the local downward incentive constraints bind. Also notice that we implicitly assume that governments do not discriminate between natives and immigrant workers in their taxation. Observe that this planner problem is identical to the one in Ales, Kurnaz, and Sleet (2015) except

[3] Defining Pareto weights $\left\{\psi_{i,\theta}\right\}_{\theta\in\Theta}$ with $\psi_{i,\theta-1}\geq\psi_{i,\theta}$, this exposition is very similar for a utilitarian objective. A Rawlsian government is a special case where $\psi_{i,1}=1/N_{i,1}$ and $\psi_{i,\theta}=0$ for all $\theta\geq 2$.

for the endogeneity of the equilibrium population (Equation (2.6)) and the specification of Pareto weights to a Rawlsian objective function.

2.2.2 Optimal Marginal Tax Rates

Optimal top and bottom marginal tax rate. As a benchmark, consider the marginal tax rate chosen by an exogenous technology planner who ignores migration ($\eta_{i,\theta}=0,\ \forall\theta\in\Theta$). In Proposition 7, we characterize high-skilled workers' optimal (Nash equilibrium) marginal tax rate with migration and compare it to the closed economy's optimal tax rate (without migration). For the comparison, we compute in the closed economy the optimal tax rate of an exogenous technology planner, $T_i^{ex\prime}\left(y_{i,\theta}\right)$, who ignores migration but takes as given the number of the population groups $\left\{N_{i,\theta}\right\}$ that materialize in the open economy Nash equilibrium.[4] This notion includes the self-confirming policy equilibrium proposed by Rothschild and Scheuer (2013, 2016), where the exogenous technology planner sets the tax scheme such that it generates outcomes for which it is optimal.[5]

Proposition 7. *In the Nash equilibrium of our N-type economy, the optimal marginal tax at the top is higher, $T_i'\left(y_{i,N}\right)>T_i^{ex\prime}\left(y_{i,N}\right)$, than in the closed economy.*

Proof. See Appendix 2.A. □

Perhaps surprisingly, the optimal marginal tax rate at the top is *higher* with migration than without. This finding is at odds with those from the tax competition literature, where migration leads to *lower* marginal tax rates (e.g., Lehmann, Simula, and Trannoy (2014)).[6] The reason is that general equilibrium externalities are absent in the existing partial equilibrium models (fixed wages). These trickle-down forces provide a rationale for a lower marginal tax rate at the top relative to an economy with fixed wages. With labor migration, these general equilibrium forces may still call for a lower marginal tax of high-skilled workers (compared to the partial equilibrium) but less relative to an economy without migration. In that sense, trickle-down forces are partly offset by labor migration.

[4] More directly, one may focus on the symmetric Nash equilibrium. Then, we do not have to take a stand on why a government, which does not consider migration responses in its optimization (e.g., in a Mirrlees (1971) benchmark), does not respond to migration flows when observing them.

[5] In Appendix 2.C, we demonstrate in a two-type setup that a Cobb-Douglas technology, a linear consumption utility, and an isoelastic disutility of labor yield, in the symmetric Nash equilibrium, closed-form expressions for $T_i^{ex\prime}\left(y_{i,\theta}\right)$. Then, there is no endogeneity of the right-hand side variables in the underlying tax code.

[6] The possibility that migration opportunities of the rich may increase tax rates has been noted in other situations (absent of general equilibrium wage effects), such as endogenous land quality (see Glazer, Kanniainen, and Poutvaara (2008)).

Assumption 1. *Let the migration semi-elasticity be increasing or constant, that is,* $\eta_{i,\theta+1} \geq \eta_{i,\theta}$*. Moreover, suppose that the native population is symmetric* $n_{i,\theta+1} = n_{i,\theta}$*.*

For the result on the bottom tax rate, we make Assumption 1. The first part of the assumption rules out that the migration semi-elasticity decreases for some parts of the income distribution.[7] The second part is not very restrictive. For instance, one can interpret a worker's type θ as her position in the income distribution, in which case $n_{i,\theta} = \frac{1}{N}$.

Proposition 8. *Let Assumption 1 hold. Then, in the symmetric Nash equilibrium of our N-type economy, the optimal marginal tax at the bottom is lower,* $T_i'(y_{i,1}) < T_i^{ext\prime}(y_{i,1})$*, than in the closed economy.*

Proof. See Appendix 2.A. □

With endogenous wages, the intuition for the lower bottom tax rate is, besides the standard migration elasticity argument in partial equilibrium, the same as the one that calls for a lower marginal subsidy at the top. In response to a lower marginal tax rate, low-skilled workers' labor supply rises. On the one hand, this leads to a decline in low-skilled workers' wage rates. But, on the other hand, due to the complementarity of labor, the wages of high-skilled workers increase. Altogether, the lower marginal tax at the bottom amplifies pre-tax wage inequality in the respective country trying to attract high-skilled workers. Thus, the bottom tax rate does not only decline due to the "migration threat" as described in Lehmann, Simula, and Trannoy (2014) but also due to the amplification of pre-tax wage inequality.[8]

Optimal middle tax rate. The effect of migration on the tax rate of middle incomes, $\theta \in \{2,...,N-1\}$, is case-specific. Firstly, considering the binding incentive constraint of θ-workers

$$u(c_{i,\theta}, l_{i,\theta}) = u\left(c_{i,\theta-1}, \frac{l_{i,\theta-1} w_{i,\theta-1}}{w_{i,\theta}}\right),$$

observe that the government can raise θ-workers' utility by amplifying pre-tax wage inequality between $\theta - 1$ and θ (higher $\frac{w_{i,\theta}}{w_{i,\theta-1}}$). Secondly, by the binding incentive constraint at $\theta + 1$

$$u(c_{i,\theta+1}, l_{i,\theta+1}) = u\left(c_{i,\theta}, \frac{l_{i,\theta} w_{i,\theta}}{w_{i,\theta+1}}\right),$$

[7] In Section 2.3, we also consider a decreasing semi-elasticity.

[8] To see this, set wage responses equal to zero ($\sigma = \infty$). This setup with fixed wages nests the standard migration-induced decline in bottom tax rates and the unaltered no-distortion-at-the-top-result for finite productivities (see Lehmann, Simula, and Trannoy (2014)). With endogenous wages, the migration induced tax cut at the bottom is higher in absolute terms than in partial equilibrium, reflecting the incentives to boost pre-tax wage inequality.

more pre-tax wage inequality between θ and $\theta+1$ (higher $\frac{w_{i,\theta+1}}{w_{i,\theta}}$) increases the utility of workers with type $\theta+1$.

To raise θ-workers' utility, the government has an incentive to tax type θ more to reduce their labor supply and, thereby, raise $\frac{w_{i,\theta}}{w_{i,\theta-1}}$. We call this channel "local trickle-down" since a tax cut on the relatively richer θ-workers would trickle down to the poorer workers of type $\theta-1$. At the same time, the second incentive constraint calls for a higher $\frac{w_{i,\theta+1}}{w_{i,\theta}}$, which the government can achieve by taxing θ-workers less. We label this mechanism as "local trickle-up" because the tax cut on the poor (type θ) trickles up to the rich (type $\theta+1$). To simplify the exposition, we make the following assumption.

Assumption 2. *Let the interaction between the migration semi-elasticity and general equilibrium production complementarities be small:* $\frac{1}{\sigma}\eta_{i,\theta} \to 0$ *for all* $\theta \in \Theta$.

Under Assumption 2, one can easily relate the effect of migration in general equilibrium to the exogenous technology planner's marginal tax rate. We summarize this result in Proposition 9. Notice that the proof of the proposition does not rely on Assumption 1 or on any symmetry assumption.

Proposition 9. *Let Assumption 2 hold. Then, in the Nash equilibrium of our N-type economy, the optimal marginal tax of a worker with type* θ *is higher in the open economy,* $T_i\left(y_{i,\theta}\right) > T_i^{ext\prime}\left(y_{i,\theta}\right)$*, if and only if the marginal tax rate set by the exogenous technology planner is negative,* $T_i^{ext\prime}\left(y_{i,\theta}\right) < 0$*. The optimal marginal tax of a* θ*-type worker is lower in the open economy,* $T_i\left(y_{i,\theta}\right) < T_i^{ext\prime}\left(y_{i,\theta}\right)$*, if and only if the exogenous technology planner's marginal tax rate is positive,* $T_i^{ext\prime}\left(y_{i,\theta}\right) > 0$.

Proof. See Appendix 2.B. □

Thus, if the exogenous technology planner's marginal tax rate of type θ is negative, the tax rate will rise due to the presence of migration. In this situation, the exogenous technology planner's incentive to decrease θ's tax rate to raise pre-tax wage inequality between $\theta-1$ and θ is sufficiently strong. The mechanism that calls for a rise in wage inequality $\frac{w_{i,\theta}}{w_{i,\theta-1}}$ (local trickle-down) dominates the local trickle-up mechanism that calls for a rise in $\frac{w_{i,\theta+1}}{w_{i,\theta}}$. Vice versa, if the exogenous technology planner sets a positive marginal tax rate, then the presence of migration responses has the intuitive negative impact on the optimal marginal tax rate.

Proposition 9 further illustrates Propositions 7 and 8. At the top, migration reduces the optimal tax rate since the closed economy's optimal tax rate is negative. However, the bottom tax rate rises in response to migration because the exogenous technology planner's tax rate is positive.

2.2.3 Discussion and Extensions

We start our discussion of the results with a note of caution. The presence of international migration opportunities makes optimal tax codes more progressive at the ends of the type distribution in terms of marginal tax rates. Nonetheless, the effect of migration on net-tax payments and transfers could be the opposite. To understand this ambiguity, consider the case with only two types, $N = 2$. Starting from the optimal consumption allocation without migration and holding labor supply fixed, a revenue-neutral reduction in lump-sums to low-skilled workers leads to high-skill immigration and low-skill emigration. The government can use the resulting fiscal surplus for transfers to low-skilled workers leading to a welfare improvement. However, this line of reasoning is not complete to prove lower tax burdens on high-skilled workers because labor supplies also change, thereby affecting tax payments (which are the difference between consumption and gross income). Therefore, one cannot infer from the results on the responses of marginal tax rates to the reaction of average tax rates and, thus, tax progressivity. This observation motivates the quantitative framework in Section 2.3. Moreover, the quantitative framework is free of restrictions on the migration semi-elasticities and their interaction with general equilibrium forces which are necessary to obtain theoretical results on the middle and bottom tax rates.

Our setup allows us to consider the effects of tax coordination on marginal income tax rates. Tax coordination provides a way to overcome the inefficiencies from the non-cooperative setting of tax policies, although typically in the context of representative household models. In Appendix 2.D, we show that in our framework, under cross-country symmetry, governments can restore the autarky solution by coordinating their income taxation. The intuition is that governments internalize the cross-country externalities from international labor migration when coordinating their tax policies. Thus, in general equilibrium, international coordination of income taxation leads to *less* tax progressivity in terms of marginal tax rates. This finding is in contrast to the conventional view that fiscal competition between governments limits the amount of redistribution, and tax coordination may, therefore, raise the level of tax progressivity (for a survey of the literature on tax competition and coordination, see Keen and Konrad (2013)). Moreover, notice that the two-type version of the coordinated tax policy setup is equivalent to Rothschild and Scheuer (2013). In their model, a policymaker sets a tax scheme under occupational mobility. In our coordination setting, a planner chooses the tax system in both countries subject to the international mobility of labor.

2.3 Optimal Tax Progressivity and International Migration in General Equilibrium

2.3.1 Quantitative Framework

In the following, we develop a standard framework of optimal tax progressivity and consider a Mirrleesian economy with migration and general equilibrium wage responses. However, we depart from the Mirrleesian approach of solving for an optimal arbitrarily nonlinear tax scheme and focus on finding the optimum of a parameterized nonlinear tax function. This complements and extends our analysis of the previous N-type model and allows us to go beyond statements about marginal tax rates. We, thereby, contribute to the recent literature on Ramsey vs. Mirrlees taxation (Heathcote and Tsujiyama (2021a)), optimal taxation in general equilibrium (Sachs, Tsyvinski, and Werquin (2020)), and provide a connection to optimal taxation with migration in partial equilibrium (Lehmann, Simula, and Trannoy (2014)).

Setup. We retain the two-country framework. In contrast to Section 2.2, we assume that, in country $i \in \{A, B\}$, there is a continuum of workers $\theta \in \Theta = [0,1] \sim F_i(\theta)$. Without loss of generality, a worker's type θ can be interpreted as her position in the wage distribution. Each worker's utility function is quasilinear with an isoelastic disutility from labor (measured by e)

$$u(c_{i,\theta}, l_{i,\theta}) = c_{i,\theta} - \frac{l_{i,\theta}^{1+\frac{1}{e}}}{1+\frac{1}{e}}.$$

Consumption is given by the after-tax labor income $c_{i,\theta} = y_{i,\theta} - T_i\left(y_{i,\theta}\right)$, where $y_{i,\theta} \equiv w_{i,\theta} l_{i,\theta}$ denotes a worker's labor income. Instead of deriving a Diamond-Saez formula for the optimal nonlinear tax function (see Diamond (1998) and Saez (2001)), we restrict attention to a standard constant-rate-of-progressivity (CRP) tax function

$$T_i(y_{i,\theta}) = y_{i,\theta} - \frac{1-\tau_i}{1-p_i} y_{i,\theta}^{1-p_i},$$

for $\tau_i \in \mathbb{R}$ and $p_i < 1$. The parameter p_i represents the rate of progressivity and is the key focus of our analysis. The other parameter τ_i is relevant for the level of tax revenues and therefore of less interest for our purposes). It has been shown that this CRP tax scheme not only approximates well the shape of the current U.S. tax code (see Heathcote, Storesletten, and Violante (2017)) but also delivers policy prescriptions close to the Mirrleesian optimum in a closed economy (Heathcote and Tsujiyama (2021a) and Heathcote and Tsujiyama (2021b)). Given this parametrization, each worker's optimal labor supply reads as $l_{i,\theta} = (1-\tau_i)^{\frac{e}{1+p_i e}} w_{i,\theta}^{\frac{(1-p_i)e}{1+p_i e}}$. The reduced-form elasticities of optimal labor supply along the nonlinear budget with respect to the wage rate and the tax scheme's level parameter and progressivity rate, respectively, set are given by

$$\varepsilon_{i,\theta}^{l,(1-\tau)} \equiv \frac{\partial l_{i,\theta}}{\partial\left(1-\tau_i\right)} \frac{1-\tau_i}{l_{i,\theta}} = \frac{e}{1+p_i e} = \varepsilon_i^{l,(1-\tau)},$$

$$\varepsilon_{i,\theta}^{l,w} \equiv \frac{\partial l_{i,\theta}}{\partial w_{i,\theta}} \frac{w_{i,\theta}}{l_{i,\theta}} = \left(1-p_i\right) \varepsilon_i^{l,(1-\tau)} = \varepsilon_i^{l,w},$$

and

$$\varepsilon_{i,\theta}^{l,(1-p)} \equiv \frac{\partial l_{i,\theta}}{\partial\left(1-p_i\right)} \frac{1-p_i}{l_{i,\theta}} = \varepsilon_i^{l,w} \varepsilon_i^{l,(1-\tau)} \left[\left(1+\frac{1}{e}\right) log\left(w_{i,\theta}\right) + log\left(1-\tau_i\right)\right].$$

As before and in Lehmann, Simula, and Trannoy (2014), a conditional migration cost distribution $G_i\left(m|\theta\right)$ gives rise to an endogenous equilibrium population distribution $N_{i,\theta} \equiv \rho_i\left(\Delta_{i,\theta}|\theta\right)$ for each worker θ and country i where $\Delta_{i,\theta}$ denotes the utility difference from living in country i relative to country j. Again, denote $\eta_{i,\theta} \equiv \frac{\partial \rho_i\left(\Delta_{i,\theta}|\theta\right)}{\partial \Delta_{i,\theta}} \frac{1}{N_{i,\theta}}$ as the semi-elasticity of migration. In each country i, a mass-one continuum of identical firms produces a single final output good using the labor of each worker type. Moreover, let firms produce under a constant elasticity of substitution (CES)

$$F_i\left(\left\{l_{i,\theta} N_{i,\theta}\right\}_{\theta\in\Theta}\right) = \left[\int_\Theta a_{i,\theta}\left(l_{i,\theta} N_{i,\theta}\right)^{\frac{\sigma-1}{\sigma}} d\theta\right]^{\frac{\sigma}{\sigma-1}}$$

for some $\sigma \in [0,\infty)$ and $a_{i,\theta} \in \mathbb{R}_+$. Firms earn zero profits, and, as in the discrete-type setting, workers' wages are equal to the respective marginal productivity of labor

$$w_{i,\theta} = a_{i,\theta}\left[l_{i,\theta} N_{i,\theta} / F_i\left(\left\{l_{i,\theta} N_{i,\theta}\right\}_{\theta\in\Theta}\right)\right]^{-\frac{1}{\sigma}} \; for \; \theta \in \Theta. \tag{2.7}$$

For this CES technology, $\sigma = 0$, $\sigma = 1$, and $\sigma = \infty$ correspond to Leontieff, Cobb-Douglas, and exogenous-wage technologies.

As in the N-type setting above, we focus on the optimal Rawlsian tax system. In each country i, the government chooses the progressivity of the tax schedule p_i[9] to maximize the indirect utility of the lowest worker type subject to the aggregate budget constraint (with exogenous revenue requirement E):

$$\max_{p_i} \; \mathscr{U}_{i,0} \; subject \; to \; \mathscr{R}_i \equiv \int_\Theta T_i\left(y_{i,\theta}\right) N_{i,\theta} d\theta \geq E, \tag{2.8}$$

[9]For simplicity, we ignore the optimal choice of the tax level parameter τ_i. In the absence of migration responses, one can show that wages do not respond to this parameter ($\frac{dw_{i,\theta}}{d\tau_i} = 0$, $\forall\theta \in \Theta$) and, thus, a social planner chooses the same tax level parameter irrespective of general equilibrium effects. Also see Section 2.3.3 for a more detailed discussion. By omitting τ_i as a tax instrument in this analysis, all fiscal effects are reflected in the optimal choice of the tax progressivity p_i.

as well as subject to the equilibrium supply and demand of labor, the endogenous migration responses, and taking as given the other government's tax system.

Policy scenarios. We consider a situation with symmetric countries and characterize the optimal progressivity parameter in the symmetric Nash equilibrium ($N_{i,\theta} = \rho_i(0|\theta)$), in which no migration takes place in equilibrium while the threat of migration (Lehmann, Simula, and Trannoy (2014)) is affecting tax policies. By setting $\sigma = \infty$ and $\eta_{i,\theta} = 0$, $\forall \theta \in \Theta$, accordingly, we compare the Nash equilibrium optimal policies to three well-studied, counterfactual policy environments: Mirrlees (1971) (partial equilibrium/no migration), Lehmann, Simula, and Trannoy (2014) (partial equilibrium/migration), and Sachs, Tsyvinski, and Werquin (2020) (general equilibrium/no migration).

Aggregate incidence of tax reforms. Before simulating the optimal tax progressivity, we characterize the incidence of reforming the tax progressivity in each policy scenario. This allows us to form an intuition about the underlying economic forces that drive the optimal choice of tax progressivity. As a byproduct, we obtain the planner's first-order condition that characterizes the solution to the taxation problem: Again, denoting ξ_i as the marginal value of public funds, the government chooses, in each policy scenario, the progressivity of the income tax schedule such that $\frac{d\left(\mathscr{U}_{i,0}/\xi_i+\mathscr{R}_i\right)}{dp_i} = 0$.

For any small policy reform, $dp_i > 0$, one can decompose the first-order impact on the government objective function into five terms

$$\mathscr{M}\mathscr{E}_i + \mathscr{B}\mathscr{E}_i + \mathscr{G}\mathscr{E}_i + \mathscr{M}\mathscr{M}\mathscr{E}_i + \mathscr{W}\mathscr{E}\mathscr{M}_i. \tag{2.9}$$

The first term captures the direct effect of a small rise in p_i (i.e., higher tax progressivity) on aggregate tax revenues and the indirect utility of the lowest type

$$\mathscr{M}\mathscr{E}_i \equiv \int_{\theta\in\Theta} \frac{\partial T_i(y_{i,\theta})}{\partial(1-p_i)} N_{i,\theta} d\theta \,(-dp_i) - \frac{\partial T_i(y_{i,0})}{\partial(1-p_i)} \frac{1}{\xi_i}(-dp_i). \tag{2.10}$$

In the literature (e.g., Saez (2001)), this effect is referred to as the *mechanical effect*. The second and third terms collect labor supply and demand effects. Higher tax progressivity reduces the workers' incentives to supply labor (see later for an explicit characterization of $dl_{i,\theta}$ in terms of dp_i) which is commonly labeled as the *behavioral effect*

$$\mathscr{B}\mathscr{E}_i \equiv \int_{\theta\in\Theta} \frac{dl_{i,\theta}}{l_{i,\theta}} y_{i,\theta} T_i'(y_{i,\theta}) N_{i,\theta} d\theta. \tag{2.11}$$

The combination of the mechanical and the behavioral effect leads to a Diamond-Saez formula for the optimal tax progressivity in partial equilibrium without migration.

In general equilibrium, not only labor supply but also wages (that is, labor demand) respond to tax reforms which is the third term

$$\mathscr{G}\mathscr{E}_i \equiv \int_{\theta\in\Theta} \frac{dw_{i,\theta}}{w_{i,\theta}} y_{i,\theta} T_i'(y_{i,\theta}) N_{i,\theta} d\theta + \frac{dw_{i,0}}{w_{i,0}} \frac{y_{i,0}(1-T_i'(y_{i,0}))}{\xi_i}. \tag{2.12}$$

As shown by Stiglitz (1982) and Sachs, Tsyvinski, and Werquin (2020), these wage responses ($dw_{i,\theta}$) call for lower tax progressivity when the government optimally chooses an arbitrarily nonlinear tax schedule. The intuition is that a government can lower the inequality in pre-tax wages by taxing the poor more and the rich less. This tax-induced reduction in wage inequality is called predistribution. For this specification of welfare, wage changes affect tax revenues and the welfare of the lowest ability type. When describing the wage responses below, we show that the term $\mathscr{G}\mathscr{E}_i$ also depends on the presence of migration responses.

The fourth term captures the mechanical effect of changing the tax code on the equilibrium population

$$\mathscr{M}\mathscr{M}\mathscr{E}_i \equiv -\int_{\theta\in\Theta} T_i(y_{i,\theta}) \frac{\partial N_{i,\theta}}{\partial(1-p_i)} d\theta(-dp_i) = -\int_{\theta\in\Theta} T_i(y_{i,\theta}) \frac{\partial T_i(y_{i,\theta})}{\partial(1-p_i)} \eta_{i,\theta} N_{i,\theta} d\theta(-dp_i). \tag{2.13}$$

For instance, a rise in tax progressivity leads to labor emigration and, thus, lower tax revenues. This negative impact of labor mobility limiting a government's ability to levy high taxes is typically referred to as the threat of migration (e.g., Lehmann, Simula, and Trannoy (2014)).

Finally, the fifth term captures a novel *wage effect on migration* that captures the first-order effects of wage changes on the equilibrium population

$$\mathscr{W}\mathscr{E}\mathscr{M}_i \equiv \int_{\theta\in\Theta} T_i(y_{i,\theta}) \frac{\partial N_{i,\theta}}{\partial w_{i,\theta}} (dw_{i,\theta}) d\theta = \int_{\theta\in\Theta} T_i(y_{i,\theta}) \frac{dw_{i,\theta}}{w_{i,\theta}} y_{i,\theta} \left(1-T_i'(y_{i,\theta})\right) \eta_{i,\theta} N_{i,\theta} d\theta. \tag{2.14}$$

As we show below, $\mathscr{W}\mathscr{E}\mathscr{M}_i$ is positive. The intuition is as follows: By making the tax code more progressive (in terms of p_i) and, therefore, amplifying pre-tax wage inequality, a government can trigger high-skilled immigration and raise more tax revenues. This effect weakens predistribution and works against the threat of migration that calls for lower tax progressivity in response to a rise in labor mobility. The decomposition allows us to consider the various combinations of wage settings (endogenous vs. exogenous) and migration settings (with and without migration) by shutting down one or more of the five effects shown in (2.9).

Individual incidence of tax reforms. To shed more light on the nature of aggregate responses, we now characterize individual responses to a change in the policy parameter p_i. Absent of income effects, a worker's labor supply responds in general equilibrium in two respects: directly through the behavioral effect and indirectly due to the adjustment in her wage

$$\frac{dl_{i,\theta}}{l_{i,\theta}} = \varepsilon_{i,\theta}^{l,(1-p)} \frac{-dp_i}{1-p_i} + \varepsilon_{i,\theta}^{l,w} \frac{dw_{i,\theta}}{w_{i,\theta}}. \tag{2.15}$$

Similarly, we perturb the equilibrium population to show that the response of the equilibrium population consists of a direct (mechanical) effect and a wage effect

$$\frac{dN_{i,\theta}}{N_{i,\theta}} = -\eta_{i,\theta} \frac{\partial T_i(y_{i,\theta})}{\partial(1-p_i)} (1-p_i) \frac{-dp_i}{1-p_i} + \eta_{i,\theta} y_{i,\theta} \left(1 - T_i'(y_{i,\theta})\right) \frac{dw_{i,\theta}}{w_{i,\theta}}. \tag{2.16}$$

By the envelope theorem, behavioral effects play no first-order role for the equilibrium population.

To determine the impact on the labor supply and the equilibrium population, we derive the incidence on wages by perturbing the wage equation (2.7)

$$\frac{dw_{i,\theta}}{w_{i,\theta}} = -\frac{1}{\sigma}\left(\frac{dl_{i,\theta}}{l_{i,\theta}} + \frac{dN_{i,\theta}}{N_{i,\theta}}\right) + \frac{1}{\sigma} \frac{\left[\int_{\theta\in\Theta} a_{i,\theta}\left(\frac{dl_{i,\theta}}{l_{i,\theta}} + \frac{dN_{i,\theta}}{N_{i,\theta}}\right)(l_{i,\theta}N_{i,\theta})^{\frac{\sigma-1}{\sigma}} d\theta\right]}{\left[\int_{\theta\in\Theta} a_{i,\theta}(l_{i,\theta}N_{i,\theta})^{\frac{\sigma-1}{\sigma}} d\theta\right]}. \tag{2.17}$$

By solving this system of three equations, the wage incidence can be written in closed form[10]

$$\begin{aligned}\frac{dw_{i,\theta}}{w_{i,\theta}} =& -\frac{1}{\sigma} \frac{1}{1+\frac{1}{\sigma}\left(\varepsilon_{i,\theta}^{l,w} + \eta_{i,\theta} y_{i,\theta}\left(1-T'(y_{i,\theta})\right)\right)} \left\{ \frac{dl_{i,\theta}^{PE}}{l_{i,\theta}} + \frac{dN_{i,\theta}^{PE}}{N_{i,\theta}} \right. \\ & \left. - \frac{\int_\Theta \left(\frac{dl_{i,\theta}^{PE}}{l_{i,\theta}} + \frac{dN_{i,\theta}^{PE}}{N_{i,\theta}}\right) y_{i,\theta} N_{i,\theta} / \left[1 + \frac{1}{\sigma}\left(\varepsilon_{i,\theta}^{l,w} + \eta_{i,\theta} y_{i,\theta}\left(1 - T_i'(y_{i,\theta})\right)\right)\right] d\theta}{\int_\Theta y_{i,\theta} N_{i,\theta} / \left[1 + \frac{1}{\sigma}\left(\varepsilon_{i,\theta}^{l,w} + \eta_{i,\theta} y_{i,\theta}\left(1 - T_i'(y_{i,\theta})\right)\right)\right] d\theta} \right\},\end{aligned}$$

where we denote $\frac{dl_{i,\theta}^{PE}}{l_{i,\theta}} + \frac{dN_{i,\theta}^{PE}}{N_{i,\theta}} \equiv \varepsilon_{i,\theta}^{l,(1-p)} \frac{-dp_i}{1-p_i} - \eta_{i,\theta} \frac{\partial T_i(y_{i,\theta})}{\partial(1-p_i)}(1-p_i)\frac{-dp_i}{1-p_i}$ as the partial equilibrium labor supply and migration response.

Thus, the wage responses depend on the presence of migration responses. To make this more transparent, we set $\eta_{i,\theta} = 0$ for any θ in the expression for the wage incidence and define $\widehat{\frac{dw_{i,\theta}}{w_{i,\theta}}}$ as the wage change absent of migration responses. Then, the general equilibrium effect, $\mathscr{GE}_i = \mathscr{WE}_i + \mathscr{MEW}_i$, can be decomposed into a standard wage effect that captures the aggregate (welfare and revenue) effect of wage changes absent of migration (see Sachs, Tsyvinski, and

[10]Plugging Equation (2.17) into the sum of (2.15) and (2.16) gives an inhomogeneous Fredholm integral equation of the second kind that one can solve using standard techniques.

Werquin (2020))

$$\mathscr{W}\mathscr{E}_i \equiv \frac{\widehat{dw_{i,0}}}{w_{i,0}} \frac{y_{i,0}\left(1-T_i'\left(y_{i,0}\right)\right)}{\xi_i} + \int_{\theta\in\Theta} \frac{\widehat{dw_{i,\theta}}}{w_{i,\theta}} y_{i,\theta} T_i'\left(y_{i,\theta}\right) N_{i,\theta} d\theta \tag{2.18}$$

and a novel *migration effect on wages*

$$\mathscr{M}\mathscr{E}\mathscr{W}_i \equiv \left(\frac{dw_{i,0}}{w_{i,0}} - \frac{\widehat{dw_{i,0}}}{w_{i,0}}\right) \frac{y_{i,0}\left(1-T_i'\left(y_{i,0}\right)\right)}{\xi_i} + \int_{\theta\in\Theta} \left(\frac{dw_{i,\theta}}{w_{i,\theta}} - \frac{\widehat{dw_{i,\theta}}}{w_{i,\theta}}\right) y_{i,\theta} T_i'\left(y_{i,\theta}\right) N_{i,\theta} d\theta. \tag{2.19}$$

As the *wage effect on migration*, the *migration effect on wages* also works against predistribution: By raising tax progressivity, a government triggers high-skilled emigration and, thereby, raises pre-tax wage inequality. This boost in wage inequality allows the government to collect more taxes at the top such that tax revenues rise. Altogether, both the migration effect on wages and the wage effect on migration work against the conventional migration threat and weaken predistribution.

2.3.2 Calibration

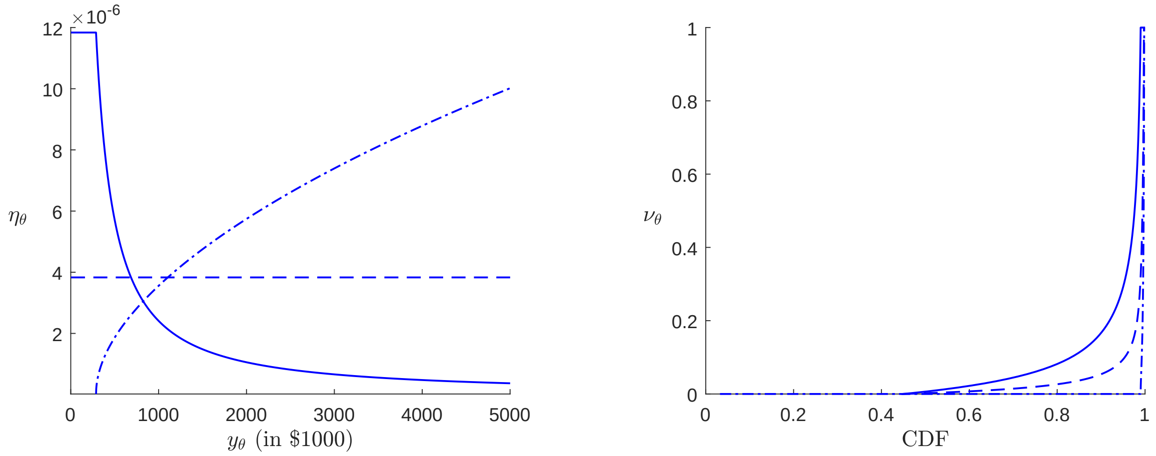
We assume that the current U.S. tax schedule can be approximated by a CRP tax schedule with parameters $p_i = 0.151$ and $\tau_i = -3$ (Sachs, Tsyvinski, and Werquin (2020), Heathcote, Storesletten, and Violante (2017)). Moreover, to match the current U.S. labor income distribution, we proceed as in Sachs, Tsyvinski, and Werquin (2020): Let earnings below \$150 000 be log-normally distributed with mean 10 and variance 0.95. Above \$150 000, we append a Pareto distribution with a tail parameter that decreases from 2.5 to 1.5 (above \$250 000). We set the Frisch elasticity of labor supply to $\varepsilon = 0.33$ (Chetty (2012)).

In the calibration, we follow Sachs, Tsyvinski, and Werquin (2020) who extend the approach of Saez (2001): We use the empirical U.S. income distribution and the individual first-order conditions to infer the wage distribution (Saez (2001)) and, for a given elasticity of substitution, to back out the underlying productivity distribution (Sachs, Tsyvinski, and Werquin (2020)), $\left\{a_{i,\theta}\right\}_{\theta\in\Theta}$. We set the elasticity of substitution to $\sigma = 1.5$ (Katz and Murphy (1992), Card and Lemieux (2001), Card (2009)).[11]

We assume an exponential migration cost distribution $G_i\left(m|\theta\right) = 1 - e^{-\delta_{i,\theta} m}$. Observe that $\eta_{i,\theta} = \frac{g_i(m|\theta)}{1-G_i(m|\theta)} = \delta_{i,\theta}$. Therefore, $\delta_{i,\theta}$ is equal to the migration semi-elasticity and, hence, a

[11] This value is considered to be at the lower bound of the likely values (Card (2009)). Setting the elasticity of substitution to a value at the upper bound, $\sigma = 2.5$ (Card (2009)), only slightly changes the quantitative results. In the literature there also exist values outside of this range, e.g., $\sigma = 0.6$ (Dustmann, Frattini, and Preston (2013)) and $\sigma = 3.1$ (Heathcote, Storesletten, and Violante (2017)).

Figure 2.3.1: Calibrated Migration (Semi-)Elasticities



model primitive. As in Lehmann, Simula, and Trannoy (2014), we consider three cases depending on whether the semi-elasticity of migration is decreasing, constant, or increasing. Thus, we depart from the assumption in the N-type model that the semi-elasticity is weakly increasing (used for Proposition 8), and allow for other possibilities.

In all three cases, we choose the semi-elasticities such that the average migration elasticity, $\nu_{i,\theta} \equiv \eta_{i,\theta} c_{i,\theta}$, of the top 1% of the income distribution is constant $\nu_{i,\theta} = \nu_{top}$, $\forall \theta \geq 0.99$ (Lehmann, Simula, and Trannoy (2014)). Following Kleven et al. (2020), we choose a medium value for the top migration elasticity: $\nu_{top} = 1.0$. In the first case of Lehmann, Simula, and Trannoy (2014), the semi-elasticity of migration is a positive constant up to the top 1% and then decreasing. In the second one, it is constant over the whole population and, in the third case, the migration semi-elasticity is zero up to the top 1% and then increasing. In Figure 2.3.1, we depict the calibrated migration elasticities and semi-elasticities. In this economy, the calibrated population-wide migration elasticity is the largest in the first case with 0.059 and smaller in the second and third case (0.026 and 0.010, respectively).

2.3.3 Numerical Simulations

We now use the calibrated model to simulate the optimal rate of tax progressivity. That is, we find the tax progressivity parameter, p_i, that solves the planner's respective first-order condition for each of the four policy scenarios (partial or general equilibrium; with or without migration). In the Mirrlees (1971) scenario, $\mathscr{ME}_i + \mathscr{BE}_i = 0$, whereas, in the setting of Lehmann, Simula,

Table 2.3.1: Optimal Tax Progressivity in Each Policy Scenario

	$p^{Mirrlees}$	$p^{Lehmann\ et\ al.}$	$p^{Sachs\ et\ al.}$	$p^{GE\ \&\ Migration}$
$\eta_{i,\theta}$ decreasing	0.268	0.214 (-20.1%)	0.268	0.217 (-19.0%)
$\eta_{i,\theta}$ constant	0.268	0.233 (-13.1%)	0.268	0.239 (-10.8%)
$\eta_{i,\theta}$ increasing	0.268	0.238 (-11.2%)	0.268	0.252 (-6.0%)
GE Effects	X	X	✓	✓
Migration	X	✓	X	✓

Note: Tax Level Parameter Set to $\tau_i = -3$; Changes in Brackets Relative to Optimal Mirrleesian Progressivity.

and Trannoy (2014), $\mathscr{ME}_i + \mathscr{BE}_i + \mathscr{MME}_i = 0$. In both partial equilibrium environments, we assume that the policymaker correctly infers the current wage distribution from the income distribution (as in Saez (2001)) but incorrectly assumes wages to be fixed. In the Sachs, Tsyvinski, and Werquin (2020) policy scenario, we choose p_i such that $\mathscr{ME}_i + \mathscr{BE}_i + \mathscr{WE}_i = 0$. Finally, in our setting with migration and general equilibrium effects, p_i solves $\mathscr{ME}_i + \mathscr{BE}_i + \mathscr{GE}_i + \mathscr{MME}_i + \mathscr{WEM}_i = 0$. Since we consider the symmetric Nash equilibrium, in which net migration flows are zero at each point in the income distribution, we do not have to adjust the population mass for the differently chosen rates of tax progressivity in the policy scenarios.

In Table 2.3.1, we display the optimal rates of tax progressivity in each of the described policy scenarios. Overall, the effects are moderate. We note that in the absence of migration responses, general equilibrium effects are negligible, leading to virtually the same optimal tax progressivity ($p^{Mirrlees} \approx p^{Sachs\ et\ al.} \approx 0.268$). This optimal rate of progressivity is in line with the values found in the literature (for example, Heathcote and Tsujiyama (2021a)).

As the decomposition below reveals, wage effects, $\mathscr{WE}_i$, are very small in this specification. There are two reasons for that: the production function and the specified preferences. Firstly, starting from the Mirrleesian optimum, the tax scheme is close to linear $T_i'\left(y_{i,\theta}\right) \approx \tau_i$ such that the general equilibrium revenue effect can be approximated by

$$\int_{\theta\in\Theta} \frac{\widehat{dw_{i,\theta}}}{w_{i,\theta}} y_{i,\theta} T_i'\left(y_{i,\theta}\right) N_{i,\theta} d\theta \approx \tau_i \int_{\theta\in\Theta} \frac{\widehat{dw_{i,\theta}}}{w_{i,\theta}} y_{i,\theta} N_{i,\theta} d\theta.$$

However, for a constant returns to scale production function $\int_{\theta\in\Theta}\widehat{\frac{dw_{i,\theta}}{w_{i,\theta}}}y_{i,\theta}N_{i,\theta}d\theta = 0$.[12] Secondly, without migration and given our assumption of an isoelastic labor disutility, the labor supply elasticity with respect to the progressivity $\varepsilon_{i,\theta}^{l,(1-p)}$ is quantitatively close to constant across skill types θ. This makes wage responses $\widehat{\frac{dw_{i,\theta}}{w_{i,\theta}}}$ small.[13]

This model feature that general equilibrium wage effects are negligible has two advantages for studying the impact of migration responses. For one, it provides us with a benchmark level of optimal progressivity that does not depend on the presence of general equilibrium effects ($p^{Mirrlees} \approx p^{Sachs\ et\ al.}$). For another, the quantitative magnitude of the interaction between general equilibrium and migration effects appears conservative, as the general equilibrium effects are initially small (absent of migration).

We now turn to the impact of migration responses. Depending on the shape of the migration semi-elasticity, in partial equilibrium, migration lowers the optimal tax progressivity by 11% to 20% (see the second column in Table 2.3.1). The migration-induced reduction is most pronounced when the semi-elasticity is decreasing because the population-wide migration elasticity is the largest in this case. This confirms the insight by Lehmann, Simula, and Trannoy (2014) that the adjustment in the tax scheme to migration responses crucially depends on the slope of the migration semi-elasticity. Similarly, the optimal tax progressivity in general equilibrium declines due to the migration responses (third and fourth columns). This observation echoes our prediction from the N-type model (Proposition 9) that migration reduces marginal tax rates (lower p) starting from a tax system with positive marginal tax rates.

However, in all three specifications of the migration-semi elasticity, the migration-induced reduction in the optimal tax progressivity is smaller in general equilibrium. Thus, irrespective of the shape of the migration semi-elasticity (increasing, constant, or decreasing), the presence of general equilibrium effects makes the migration responses less important for tax policy. In this economy, general equilibrium effects offset between 6% and 47% (e.g., from -11.2% to -6.0%) of the migration-induced reduction in the optimal tax progressivity.[14] The offsetting effect is largest for an increasing semi-elasticity. Interestingly, the interaction of migration and

[12] This observation echoes Corollary 3 in Sachs, Tsyvinski, and Werquin (2020).

[13] To further illustrate this, consider an alternative specification with income effects where consumption utility is logarithmic. Then, workers' labor supplies and their elasticities are constant, $l_{i,\theta} = (1-p_i)^{\frac{\varepsilon}{1+\varepsilon}}$ and $\varepsilon_{i,\theta}^{l,(1-p)} = \frac{\varepsilon}{1+\varepsilon}$, and wages do not depend on tax progressivity (and wage changes are exactly zero). By a similar argument, the level of tax progressivity, τ_i, plays no role in the current specification with a linear consumption utility. Absent migration, the wage response to a change in τ_i, $\widehat{\frac{dw_{i,\theta}}{w_{i,\theta}}}$, is equal to zero since $\varepsilon_{i,\theta}^{l,(1-\tau)} = \frac{e}{1+p_i e}$ is constant across workers.

[14] For a purely revenue-maximizing government, the offset ranges between 10% and 50%.

Table 2.3.2: Decomposition

	$\mathscr{ME}_i$	$\mathscr{BE}_i$	$\mathscr{WE}_i$	$\mathscr{MME}_i$	$\mathscr{MEW}_i$	$\mathscr{WEM}_i$
η_θ decreasing	49.5%	−23.9%	0.2%	−23.7%	1.1%	1.6%
η_θ constant	48.1%	−31.4%	0.2%	−16.4%	1.1%	2.8%
η_θ increasing	45.4%	−34.7%	0.2%	−13.3%	1.0%	5.4%

Note: Decomposition of Total Effect into Mechanical Effect ($\mathscr{ME}_i$), Behavioral Effect ($\mathscr{BE}_i$), Mechanical Migration Effect ($\mathscr{MME}_i$), General Equilibrium Wage Effect ($\mathscr{WE}_i$), Migration Effect on Wages ($\mathscr{MEW}_i$), and Wage Effect on Migration ($\mathscr{WEM}_i$) (all effects evaluated at the optimum and relative to total absolute effects).

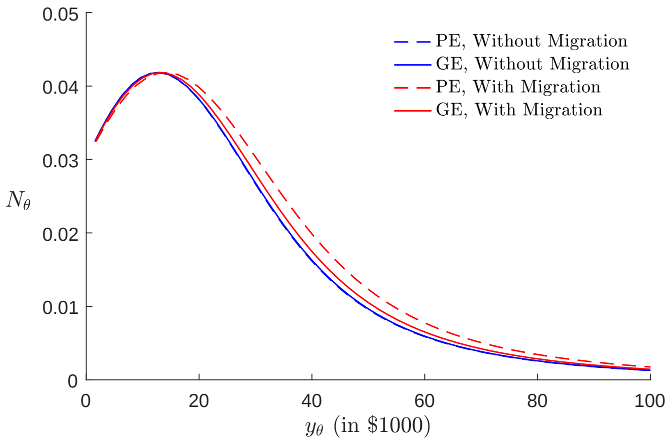
general equilibrium responses is thus most important from a policy perspective in the case in which the overall migration elasticity is the smallest.

To shed further light on the underlying mechanisms, in Table 2.3.2, we decompose the effects at the optimal progressivity rate with migration and general equilibrium effects. As theoretically expected, the effect of endogenous wages absent of migration ($\mathscr{WE}_i$) is small.[15] However, their interaction with migration responses ($\mathscr{MEW}_i$) makes general equilibrium effects ($\mathscr{GE}_i = \mathscr{WE}_i + \mathscr{MEW}_i$) non-negligible. As argued, the sign of the migration effect on wages is positive. The magnitude appears unrelated to the shape of the migration semi-elasticity. In all three specifications, the wage effect on migration ($\mathscr{WEM}_i$) is quantitatively more important than the other general equilibrium effects. Thus, the wage effect on migration is the main driver offsetting the migration-induced reduction in tax progressivity. It is weaker for a decreasing than for a constant or increasing migration semi-elasticity, as assumed in the N-type model of Section 2.3.

Finally, we quantify the implications for income inequality. In this exercise, we consider an increasing semi-elasticity of migration. In Figure 2.3.2, we compare the income distributions resulting from the optimal tax progressivities chosen in each policy regime. Both the level and the variance in incomes with migration and general equilibrium effects lie, as the optimal tax progressivity, in between the Lehmann, Simula, and Trannoy (2014) specification and the Mirrlees (1971) benchmark. One can make a similar statement about marginal and average taxes. For example, in partial equilibrium, migration reduces the average tax rate in the population by ten percentage points. However, the respective reduction in general equilibrium is only five per-

[15] As in Sachs, Tsyvinski, and Werquin (2020), the sign is positive, starting from an initially progressive tax code.

Figure 2.3.2: Income Distribution under the Four Regimes



Note: Case with Increasing Migration Semi-Elasticity.

centage points. Altogether, depending on the shape of the migration semi-elasticity, the common concern of a migration threat limiting a government's scope for redistribution appears overblown.

2.4 Conclusion

In this chapter, we introduce migration into the nonlinear taxation in general equilibrium. By adding an extensive margin in our N-type model and our quantitative framework, we make the canonical Stiglitz (1982) and Mirrlees (1971) models with endogenous labor supply and wages more realistic. As we have shown, contrary to conventional wisdom, migration leads to a more progressive tax code in terms of marginal tax rates. This finding is at odds with Lehmann, Simula, and Trannoy (2014), who conclude that introducing migration in partial equilibrium reduces marginal tax rates. By weakening general equilibrium trickle-down forces, migration responses move optimal marginal tax rates closer to the partial equilibrium optimum. We explore the underlying mechanisms in our quantitative framework, discovering a novel wage effect on migration as well as a migration effect on wages. Depending on the shape of the migration semi-elasticity, general equilibrium wage effects offset almost half of the migration-reduced decline in optimal progressivity. Thus, the threat of migration appears exaggerated, and the canonical Mirrlees (1971) model with fixed wages provides a more realistic benchmark than previously expected.

Furthermore, our theoretical results suggests an alternative explanation of the recent empirical literature on the effects of globalization on redistribution and inequality. Most prominently, Egger, Nigai, and Strecker (2019) demonstrate that increases in both international trade and migration in OECD countries in the 1980s and early 1990s led to higher average tax burdens (but less in the following years). A well-known explanation for this finding is that redistribution, as well as government size (e.g., Rodrik (1998)), compensates for the adverse effects of globalization on workers from lower parts of the income distribution (see Autor, Dorn, and Hanson (2013)). Our finding does not call into question this widespread view. Instead, it offers an alternative explanation for why globalization may lead to higher tax rates along with a rise in wage inequality. The main difference is that, in the former view, globalization directly amplifies pre-existing inequities, whereas, in our framework, the policy response to international mobility induces the rise in inequality.

Appendix

2.A Proof of Propositions 7 and 8

The optimal tax code can be implicitly described by the households' first-order condition (2.2). In the following, we, firstly, characterize the solution to the "inner" problem. That is, we solve for the optimal allocation $\left\{c_{i,\theta}, l_{i,\theta}\right\}_{\theta\in\Theta}$ for a given population $\left\{N_{i,\theta}\right\}_{\theta\in\Theta}$. Secondly, we maximize welfare by choosing $\left\{N_{i,\theta}\right\}_{\theta\in\Theta}$, which is the "outer" problem.

Inner problem. The Lagrangian function of the benevolent social planner in country i is defined by

$$\begin{aligned}\mathscr{L}_i\left(\{N_{i,\theta}\}_{\theta\in\Theta}\right) \equiv\, & u\left(c_{i,1}, l_{i,1}\right) + \sum_{\theta\in\{2,\ldots,N\}} \mu_{i,\theta}\left[u\left(c_{i,\theta}, l_{i,\theta}\right) - u\left(c_{i,\theta-1}, \frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)\right] \\ & + \xi_i\left[F_i\left(\{N_{i,\theta}l_{i,\theta}\}_{\theta\in\Theta}\right) - \sum_{\theta\in\Theta} N_{i,\theta}c_{i,\theta}\right] + \sum_{\theta\in\Theta}\lambda_{i,\theta}\left[N_{i,\theta} - \rho_i\left(\Delta_i;\theta\right)\right]\end{aligned}$$

for a given population. Assuming that the optimization problem is convex and using the definitions of wages, wage elasticities, and migration semi-elasticities, the following first-order conditions describe the unique optimum

$$\begin{aligned}[c_{i,\theta}]: 0 = \, & \mathbb{1}\left[\theta = 1\right] u_c\left(c_{i,\theta}, l_{i,\theta}\right) - N_{i,\theta}\xi_i + \mathbb{1}\left[\theta > 1\right]\mu_{i,\theta}u_c\left(c_{i,\theta}, l_{i,\theta}\right) - \mathbb{1}\left[\theta < N\right]\mu_{i,\theta+1}u_c\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right) \\ & - \lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}u_c\left(c_{i,\theta}, l_{i,\theta}\right) \end{aligned} \tag{2.20}$$

$$\begin{aligned}[l_{i,\theta}]: 0 = \, & \mathbb{1}\left[\theta = 1\right] u_l\left(c_{i,\theta}, l_{i,\theta}\right) + \mathbb{1}\left[\theta > 1\right]\mu_{i,\theta}u_l\left(c_{i,\theta}, l_{i,\theta}\right) - \mathbb{1}\left[\theta < N\right]\mu_{i,\theta+1}u_l\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)\frac{w_{i,\theta}}{w_{i,\theta+1}} \\ & + \xi_i w_{i,\theta}N_{i,\theta} - \lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}u_l\left(c_{i,\theta}, l_{i,\theta}\right) \\ & - \sum_{k\in\{2,\ldots,N\}}\mu_{i,k}u_l\left(c_{i,k-1}, \frac{y_{i,k-1}}{w_{i,k}}\right)\frac{y_{i,k-1}}{l_{i,\theta}w_{i,k}}\left(\gamma_{i,k-1,\theta} - \gamma_{i,k,\theta}\right), \end{aligned} \tag{2.21}$$

for $\theta \in \Theta$, where $\mathbb{1}[\cdot]$ is the indicator function. Inserting Equation (2.20) into (2.21) and making use of the high-skilled worker's first-order condition, the marginal tax rate of a worker θ can be written as

$$T_i'(y_{i,\theta}) = \frac{\mathbb{1}[\theta < N] \frac{\mu_{i,\theta+1}}{\xi_i} \frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta} w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}} \left[1 + \frac{u_l\left(c_{i,\theta}, \frac{l_{i,\theta} w_{i,\theta}}{w_{i,\theta+1}}\right)}{w_{i,\theta+1} u_c\left(c_{i,\theta}, l_{i,\theta}\right)}\right]}{1 + \mathbb{1}[\theta < N] \frac{\mu_{i,\theta+1}}{\xi_i} \frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta} w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}} + \frac{\frac{1}{y_{i,\theta} N_{i,\theta}} \sum_{k \in \{2,\ldots,N\}} \frac{\mu_{i,k}}{\xi_i} u_l\left(c_{i,k-1}, \frac{l_{i,k-1} w_{i,k-1}}{w_{i,k}}\right) \frac{y_{i,k-1}}{w_{i,k}} \left(\gamma_{i,k-1,\theta} - \gamma_{i,k,\theta}\right)}{1 + \mathbb{1}[\theta < N] \frac{\mu_{i,\theta+1}}{\xi_i} \frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta} w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}}.$$

In their Proposition (1), Ales, Kurnaz, and Sleet (2015) show that one can decompose the formula for the optimal tax rate in the N-type Stiglitz (1982) setting without migration ($\eta_{i,\theta} = 0$, $\forall \theta \in \Theta$) into a Mirrleesian and a wage compression term. A similar decomposition applies here with the difference that one needs to account for migration responses ($\eta_{i,\theta} > 0$).

On the one hand, the Mirrleesian term is augmented by the direct partial equilibrium impact of migration on the objective function, which Lehmann, Simula, and Trannoy (2014) label as the "migration threat." On the other hand, migration interacts with the wage compression term. Thus, this setting nests the well-known partial equilibrium effect of migration on the optimal taxation and adds general equilibrium moderation effects. In the following, we derive conditions under which the classical partial equilibrium downward force of migration on taxes is offset by our novel general equilibrium moderation effects.

Under a CES production function, this expression for the optimal marginal tax rate simplifies to

$$T_i'(y_{i,\theta}) = \frac{\mathbb{1}[\theta < N] \frac{\mu_{i,\theta+1}}{\xi_i} \frac{u_c\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}} \left[1 + \frac{u_l\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)}{w_{i,\theta+1} u_c\left(c_{i,\theta}, l_{i,\theta}\right)} \frac{\sigma - 1}{\sigma}\right] + \mathbb{1}[\theta > 1] \frac{\mu_{i,\theta}}{\xi_i} \frac{y_{i,\theta-1}}{y_{i,\theta} N_{i,\theta}} \frac{1}{\sigma} \frac{u_l\left(c_{i,\theta-1}, \frac{y_{i,\theta-1}}{w_{i,\theta}}\right)}{w_{i,\theta}}}{1 + \mathbb{1}[\theta < N] \frac{\mu_{i,\theta+1}}{\xi_i} \frac{u_c\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}}.$$

Therefore, setting $\theta = 1$ and $\theta = N$, the bottom and top tax rates are characterized by

$$T_i'(y_{i,1}) = \frac{\frac{\mu_{i,2}}{\xi_i} \frac{u_c\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{N_{i,1}}}{1 + \frac{\mu_{i,2}}{\xi_i} \frac{u_c\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{N_{i,1}}} \left[1 + \frac{u_l\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{w_{i,2} u_c\left(c_{i,1}, l_{i,1}\right)} \frac{\sigma - 1}{\sigma}\right]$$

and

$$T_i'(y_{i,N}) = \frac{\mu_{i,N}}{\xi_i} \frac{y_{i,N-1}}{y_{i,N} N_{i,N}} \frac{1}{\sigma} \frac{u_l\left(c_{i,N-1}, \frac{y_{i,N-1}}{w_{i,N}}\right)}{w_{i,N}}$$

respectively. These expressions depend on the (relative) Lagrange multipliers $\frac{\mu_{i,\theta}}{\xi_i}$.

One can obtain the shadow value of public funds by summing up Equation (2.20) over all types and using the separability of consumption and leisure, which yields $\xi_i = \frac{1-\sum_{\theta\in\Theta}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}}{\sum_{\theta\in\Theta}N_{i,\theta}/u_c(c_{i,\theta},l_{i,\theta})}$. Plugging this value back into (2.20), the normalized multiplier on the incentive constraint for $k \in \{2,...,N\}$ reads as

$$\frac{\mu_{i,k}}{\xi_i} = \frac{\sum_{l=k}^{N} N_{i,l}/u_c(c_{i,l},l_{i,l})}{1-\sum_{\theta\in\Theta}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}} \left(1 + \frac{\sum_{l=1}^{k-1} N_{i,l}/u_c(c_{i,l},l_{i,l})}{\sum_{l=k}^{N} N_{i,l}/u_c(c_{i,l},l_{i,l})} \sum_{l=k}^{N} \lambda_{i,l}\eta_{i,l}N_{i,l} - \sum_{l=1}^{k-1} \lambda_{i,l}\eta_{i,l}N_{i,l}\right) \equiv \frac{\mu_{i,k}^{ex}}{\xi_i^{ex}} \Delta_{i,k},$$

where we define the normalized multiplier without migration responses ($\eta_{i,\theta} = 0$, $\forall\theta \in \Theta$) as

$$\frac{\mu_{i,k}^{ex}}{\xi_i^{ex}} \equiv \sum_{l=k}^{N} N_{i,l}/u_c(c_{i,l},l_{i,l})$$

and a scaling factor as

$$\Delta_{i,k} \equiv \frac{1 + \frac{\sum_{l=1}^{k-1} N_{i,l}/u_c(c_{i,l},l_{i,l})}{\sum_{l=k}^{N} N_{i,l}/u_c(c_{i,l},l_{i,l})} \sum_{l=k}^{N} \lambda_{i,l}\eta_{i,l}N_{i,l} - \sum_{l=1}^{k-1} \lambda_{i,l}\eta_{i,l}N_{i,l}}{1-\sum_{\theta\in\Theta}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}}.$$

This procedure gives us the exogenous technology planner's optimal bottom and top tax rates

$$T_i^{ex\prime}(y_{i,1}) \equiv \frac{\frac{\mu_{i,2}^{ex}}{\xi_i^{ex}} \frac{u_c\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{N_{i,1}}}{1 + \frac{\mu_{i,2}^{ex}}{\xi_i^{ex}} \frac{u_c\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{N_{i,1}}} \left[1 + \frac{u_l\left(c_{i,1}, \frac{y_{i,1}}{w_{i,2}}\right)}{w_{i,2}u_c(c_{i,1},l_{i,1})} \frac{\sigma-1}{\sigma}\right] > 0$$

and

$$T_i^{ex\prime}(y_{i,N}) \equiv \frac{\mu_{i,N}^{ex}}{\xi_i^{ex}} \frac{y_{i,N-1}}{y_{i,N} N_{i,N}} \frac{1}{\sigma} \frac{u_l\left(c_{i,N-1}, \frac{y_{i,N-1}}{w_{i,N}}\right)}{w_{i,N}} < 0,$$

to which we can now compare the optimal Nash equilibrium tax rates. The comparison between $T_i'(y_{i,\theta})$ and $T_i^{ex\prime}(y_{i,\theta})$ depends on the adjustment in the Lagrange multipliers measured by the scaling factor $\Delta_{i,k}$ (i.e., $\frac{\mu_{i,\theta}}{\xi_i}$ vs. $\frac{\mu_{i,\theta}^{ex}}{\xi_i^{ex}}$).[16] More precisely, for Propositions 7 and 8, we need to

[16]Notice that the right-hand side of $T_i^{ex\prime}(y_{i,\theta})$ depends on the equilibrium allocation that may be endogenous to tax policy. For simplicity, we evaluate, in the following comparison, the right-hand side at the allocation chosen in the Nash equilibrium. In Appendix 2.C, we provide conditions under which the right-hand side is given in closed form and, thus, independent from the equilibrium allocation.

show that $\Delta_{i,N} < 1$ and $\Delta_{i,2} < 1$, which holds for $\lambda_{i,N}\eta_{i,N}N_{i,N} < 0$ and $\sum_{l=2}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} < 0$, respectively.

Outer problem. To prove these statements, we now derive the first-order conditions with respect to the population masses

$$[N_{i,\theta}] : 0 = -\sum_{k\in\{2,\dots,N\}} \mu_{i,k}u_l\left(c_{i,\theta-1}, \frac{l_{i,k-1}w_{i,k-1}}{w_{i,k}}\right)\frac{l_{i,k-1}w_{i,k-1}}{N_{i,\theta}w_{i,k}}\left(\gamma_{i,k-1,\theta}-\gamma_{i,k,\theta}\right)$$
$$+\xi_i\left(l_{i,\theta}w_{i,\theta}-c_{i,\theta}\right)+\lambda_{i,\theta}. \tag{2.22}$$

For a constant elasticity of substitution production function, Equation (2.22) simplifies to

$$-\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} = \xi_i T_i\left(y_{i,\theta}\right)N_{i,\theta}\eta_{i,\theta} + \frac{1}{\sigma}\eta_{i,\theta}\left[\mu_{i,\theta+1}u_l\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)\frac{y_{i,\theta}}{w_{i,\theta+1}} - \mu_{i,\theta}u_l\left(c_{i,\theta-1}, \frac{y_{i,\theta-1}}{w_{i,\theta}}\right)\frac{y_{i,\theta-1}}{w_{i,\theta}}\right].$$

Noting that $T_i\left(y_{i,N}\right) \geq 0$, since $T_i\left(y_{i,\theta}\right) \geq T_i\left(y_{i,\theta-1}\right)$ and $\sum_{\theta\in\Theta}T_i\left(y_{i,\theta}\right)N_{i,\theta} \geq 0$, we conclude that

$$-\lambda_{i,N}\eta_{i,N}N_{i,N} = \xi_i T_i\left(y_{i,N}\right)N_{i,N}\eta_{i,N} - \frac{1}{\sigma}\eta_{i,N}\mu_{i,N}u_l\left(c_{i,N-1}, \frac{y_{i,N-1}}{w_{i,N}}\right)\frac{y_{i,N-1}}{w_{i,N}} > 0.$$

Thus, do not rely on any assumption about the migration semi-elasticity for the result about top tax rate (Proposition 7).

As mentioned above, for the decline in bottom tax rate postulated in Proposition 8, we need to show that

$$-\sum_{\theta=2}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} = -\sum_{\theta=1}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} + \lambda_{i,1}\eta_{i,1}N_{i,1} > 0.$$

Let Assumption 1 hold. Then, in the symmetric Nash equilibrium

$$-\sum_{\theta=1}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} = \xi_i\sum_{\theta=1}^{N}T_i\left(y_{i,\theta}\right)n_{i,\theta}\eta_{i,\theta}$$
$$+\frac{1}{\sigma}\sum_{\theta=1}^{N}\eta_{i,\theta}\left[\mu_{i,\theta+1}u_l\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)\frac{y_{i,\theta}}{w_{i,\theta+1}} - \mu_{i,\theta}u_l\left(c_{i,\theta-1}, \frac{y_{i,\theta-1}}{w_{i,\theta}}\right)\frac{y_{i,\theta-1}}{w_{i,\theta}}\right]$$

The second summand is positive for $\eta_{i,\theta} \geq \eta_{i,\theta-1}$. For a constant population size ($n_{i,\theta} = n_{i,\theta-1}$), the first summand can be written as

$$\xi_i n_{i,\theta}\sum_{\theta=1}^{N}T_i\left(y_{i,\theta}\right)\eta_{i,\theta} \geq \left(\frac{1}{N}\sum_{\theta=1}^{N}\eta_{i,\theta}\right)\xi_i\sum_{\theta=1}^{N}T_i\left(y_{i,\theta}\right)n_{i,\theta} = 0$$

where the first inequality is Chebyshev's sum inequality for $\eta_{i,\theta} \geq \eta_{i,\theta-1}$ and $T_i\left(y_{i,\theta}\right) \geq T_i\left(y_{i,\theta-1}\right)$ and the second inequality follows from the government's budget constraint. Therefore, $-\sum_{\theta=1}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} \geq$

0. To conclude that $-\sum_{\theta=2}^{N}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}>0$, notice that

$$\lambda_{i,1}\eta_{i,1}N_{i,1}=-\xi_i T_i\left(y_{i,1}\right)n_{i,1}\eta_{i,1}-\frac{1}{\sigma}\eta_{i,1}\mu_{i,2}u_l\left(c_{i,1},\frac{y_{i,1}}{w_{i,2}}\right)\frac{y_{i,1}}{w_{i,2}}>0$$

since $T_i\left(y_{i,1}\right)\leq 0$, for $\sum_{\theta=1}^{N}T_i\left(y_{i,\theta}\right)N_{i,\theta}\geq 0$ and $T_i\left(y_{i,\theta}\right)\geq T_i\left(y_{i,\theta-1}\right)$, and $u_l\left(c_{i,1},\frac{y_{i,1}}{w_{i,2}}\right)<0$.

2.B Proof of Proposition 9

Recall that the optimal marginal tax rate of a middle type $\theta\in\{2,...,N-1\}$ is given by

$$T_i'\left(y_{i,\theta}\right)=\frac{\Delta_{i,\theta+1}\frac{\mu_{i,\theta+1}^{ex}}{\xi_i^{ex}}\frac{u_c\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}\left[1+\frac{u_l\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{w_{i,\theta+1}u_c\left(c_{i,\theta},l_{i,\theta}\right)}\frac{\sigma-1}{\sigma}\right]+\Delta_{i,\theta}\frac{\mu_{i,\theta}^{ex}}{\xi_i^{ex}}\frac{y_{i,\theta-1}}{y_{i,\theta}N_{i,\theta}}\frac{1}{\sigma}\frac{u_l\left(c_{i,\theta-1},\frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)}{w_{i,\theta}}}{1+\Delta_{i,\theta+1}\frac{\mu_{i,\theta+1}^{ex}}{\xi_i^{ex}}\frac{u_c\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}},$$

whereas the exogenous technology planner tax rate reads as

$$T_i^{ex\prime}\left(y_{i,\theta}\right)\equiv\frac{\frac{\mu_{i,\theta+1}^{ex}}{\xi_i^{ex}}\frac{u_c\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}\left[1+\frac{u_l\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{w_{i,\theta+1}u_c\left(c_{i,\theta},l_{i,\theta}\right)}\frac{\sigma-1}{\sigma}\right]+\frac{\mu_{i,\theta}^{ex}}{\xi_i^{ex}}\frac{y_{i,\theta-1}}{y_{i,\theta}N_{i,\theta}}\frac{1}{\sigma}\frac{u_l\left(c_{i,\theta-1},\frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)}{w_{i,\theta}}}{1+\frac{\mu_{i,\theta+1}^{ex}}{\xi_i^{ex}}\frac{u_c\left(c_{i,\theta},\frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}}.$$

Then, the presence of migration responses raises θ-workers' optimal tax rate, i.e., $T_i\left(y_{i,\theta}\right)>T_i^{ex\prime}\left(y_{i,\theta}\right)$, if and only if

$$T_i^{ex\prime}\left(y_{i,\theta}\right)<\frac{\Delta_{i,\theta+1}-\Delta_{i,\theta}}{1-\Delta_{i,\theta+1}}\frac{\mu_{i,\theta}^{ex}}{\xi_i^{ex}}\frac{y_{i,\theta-1}}{y_{i,\theta}N_{i,\theta}}\frac{1}{\sigma}\frac{-u_l\left(c_{i,\theta-1},\frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)}{w_{i,\theta}}.$$

However, under Assumption 2, the right-hand side of the expression converges to zero, and Proposition 9 follows. Observe that we do not need to impose any assumption on the shape of the migration semi-elasticity or symmetry assumptions.

2.C A Closed-Form Example

The purpose of this section is to provide an example in which one obtains closed-form expressions for the optimal tax rates chosen by the exogenous technology planner. Let $N=2$

and label L and H as the low- and high-skill worker type. Suppose that $F_i(N_{i,L}l_{i,L}, N_{i,H}l_{i,H}) = A_i(N_{i,L}l_{i,L})^{\alpha}(N_{i,H}l_{i,H})^{1-\alpha}$ for $\alpha \in (0,1)$. Then, the own- and cross-wage elasticities are given by $\gamma_{i,L,L} = -(1-\alpha)$, $\gamma_{i,H,H} = -\alpha$, $\gamma_{i,L,H} = 1-\alpha$, and $\gamma_{i,H,L} = \alpha$. The income share of the unskilled relative to the skilled workers reads as $\frac{N_{i,L}l_{i,L}w_{i,L}}{N_{i,H}l_{i,H}w_{i,H}} = \frac{\alpha}{1-\alpha}$. Moreover, let the consumption utility be linear $u(c) = c$, and assume an isoelastic disutility from labor $v(l) = \frac{l^{1+1/\varepsilon}}{1+1/\varepsilon}$ with ε denoting the Frisch elasticity of labor supply. Finally, consider a setup with symmetric countries – thus, the symmetric Nash equilibrium in which no mobility occurs ($N_{i,\theta} = n_{i,\theta}$ for $i = A,B$ and $\theta = L,H$).

Then, one can write the exogenous technology planner's marginal tax rate for the high-skilled workers as $T_i^{ex\prime}(y_{i,H}) = -\left(\frac{\alpha}{1-\alpha}\frac{n_{i,H}}{n_{i,L}}\right)^{1+1/\varepsilon}\frac{l_{i,H}^{1/\varepsilon}}{w_{i,H}}$. Using the workers' first-order condition, the marginal tax rate at the top simplifies to

$$\frac{T_i^{ex\prime}(y_{i,H})}{1-T_i^{ex\prime}(y_{i,H})} = -\left(\frac{\alpha}{1-\alpha}\frac{n_{i,H}}{n_{i,L}}\right)^{1+1/\varepsilon} < 0.$$

Applying similar steps, the marginal tax rate for low-skilled workers reads as

$$\frac{T_i^{ex\prime}(y_{i,L})}{1-T_i^{ex\prime}(y_{i,L})} = \frac{n_{i,H}}{n_{i,L}} > 0.$$

Now, we show that, in this parametrization, there is a negative reduced-form relationship between high-skilled workers' gross income and their marginal tax rate. Notice that this exercise is non-trivial in our setup since one needs to consider general equilibrium wage effects from labor supply and migration. By the high-skilled workers' first-order condition $(l_{i,H})^{1/\varepsilon} = w_{i,H}[1-T'(y_{i,H})]$, their income response to a cut in the top tax rate depends on a direct labor supply and an indirect wage response $\frac{dlog(y_{i,H})}{dlog[1-T'(y_{i,H})]} = (1+\varepsilon)\frac{dlog(w_{i,H})}{dlog[1-T'(y_{i,H})]} + \varepsilon$, whose overall sign is not clear a priori. To calculate the indirect wage response, first derive low-skilled workers' consumption, income, and wage responses

$$\frac{dc_{i,L}}{d[1-T'(y_{i,H})]} = \frac{1-T'(y_{i,L})}{1-T'(y_{i,H})}y_{i,L}(1+\varepsilon)\frac{dlog(w_{i,L})}{dlog[1-T'(y_{i,H})]},$$

$$\frac{dl_{i,L}}{d[1-T'(y_{i,H})]} = \frac{1-T'(y_{i,L})}{1-T'(y_{i,H})}l_{i,L}\varepsilon\frac{dlog(w_{i,L})}{dlog[1-T'(y_{i,H})]},$$

and

$$\frac{dlog(w_{i,L})}{dlog[1-T'(y_{i,H})]} = \frac{dlog\left[\alpha A_i(N_{i,L}l_{i,L})^{\alpha-1}(N_{i,H}l_{i,H})^{1-\alpha}\right]}{dlog[1-T'(y_{i,H})]} = -\frac{1-\alpha}{\alpha}\frac{dlog(w_{i,H})}{dlog[1-T'(y_{i,H})]}.$$

Accordingly, high-skilled workers' wages change as follows

$$\frac{dlog\left(w_{i,H}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}=\frac{\alpha}{1+\alpha\varepsilon}\eta_{i,L}\left[1-T'\left(y_{i,H}\right)\right]y_{i,L}\left(1+\varepsilon\right)\frac{dlog\left(w_{i,L}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}+\frac{\alpha\varepsilon}{1+\alpha\varepsilon}\frac{dlog\left(w_{i,L}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}$$
$$-\frac{\alpha}{1+\alpha\varepsilon}\eta_{i,H}\left[1-T'\left(y_{i,H}\right)\right]y_{i,H}\left[\left(1+\varepsilon\right)\frac{dlog\left(w_{i,H}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}+\varepsilon\right]-\frac{\alpha\varepsilon}{1+\alpha\varepsilon}.$$

Using the expression for low-skilled workers' wage response, one can rewrite high-skilled workers' wage change as

$$\frac{dlog\left(w_{i,H}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}=-\frac{\alpha\varepsilon}{1+\varepsilon}\frac{1+\eta_{i,H}\left[1-T'\left(y_{i,H}\right)\right]y_{i,H}}{1+\left(1-\alpha\right)\eta_{i,L}\left[1-T'\left(y_{i,H}\right)\right]y_{i,L}+\alpha\eta_{i,H}\left[1-T'\left(y_{i,H}\right)\right]y_{i,H}}.$$

Therefore, recalling that $\alpha<1$, the relationship between high-skilled workers' income and the retention rate of the top tax rate is positive

$$\frac{dlog\left(y_{i,H}\right)}{dlog\left[1-T'\left(y_{i,H}\right)\right]}=-\alpha\varepsilon\frac{1+\eta_{i,H}\left[1-T'\left(y_{i,H}\right)\right]y_{i,H}}{1+\left(1-\alpha\right)\eta_{i,L}\left[1-T'\left(y_{i,H}\right)\right]y_{i,L}+\alpha\eta_{i,H}\left[1-T'\left(y_{i,H}\right)\right]y_{i,H}}+\varepsilon>0.$$

2.D Coordinated Tax Policy

We consider a situation in which the two governments can jointly set their country-specific tax schedules to maximize world welfare. Then, the world social planner chooses $\left\{c_{i,\theta},l_{i,\theta},N_{i,\theta}\right\}_{\theta\in\Theta,i=A,B}$ to maximize

$$\sum_{i=A,B}u\left(c_{i,1},l_{i,1}\right) \tag{2.23}$$

subject to the high-skilled workers' incentive constraints (Equation (2.5)), each country's resource constraint (Equation (2.6)), the endogeneity of wages (Equation (2.1)), and the equilibrium population (Equation (2.3)).

Observe that, as before, the set of constraints needs to hold at a country level.[17] Then, the Lagrangian of the outer problem reads as

$$\mathscr{L}\left(\left\{N_{i,\theta}\right\}_{\theta\in\Theta,i=A,B}\right)\equiv\sum_{i=A,B}u\left(c_{i,1},l_{i,1}\right)+\sum_{i=A,B}\sum_{\theta\in\{2,\ldots,N\}}\mu_{i,\theta}\left[u\left(c_{i,\theta},l_{i,\theta}\right)-u\left(c_{i,\theta-1},\frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)\right]$$
$$+\sum_{i=A,B}\xi_i\left[F_i\left(\left\{N_{i,\theta}l_{i,\theta}\right\}_{\theta\in\Theta}\right)-\sum_{\theta\in\Theta}N_{i,\theta}c_{i,\theta}\right]+\sum_{i=A,B}\sum_{\theta\in\Theta}\lambda_{i,\theta}\left[N_{i,\theta}-\rho_i\left(\Delta_i;\theta\right)\right],$$

[17] Alternatively, one could consider a planner problem where the aggregate resource constraint (2.6) only has to hold worldwide, allowing governments to achieve cross-country redistribution by trading consumption levels. Although straightforward to consider, we disregard such incentives for the sake of comparability and due to their limited feasibility.

which yields the following first-order conditions

$$[c_{i,\theta}]: 0 = \mathbb{1}[\theta=1]\, u_c(c_{i,\theta}, l_{i,\theta}) - N_{i,\theta}\xi_i + \mathbb{1}[\theta>1]\,\mu_{i,\theta} u_c(c_{i,\theta}, l_{i,\theta}) - \mathbb{1}[\theta<N]\,\mu_{i,\theta+1} u_c\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)$$
$$- \lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}u_c(c_{i,\theta}, l_{i,\theta}) + \lambda_{j,\theta}\eta_{j,\theta}N_{j,\theta}u_c(c_{j,\theta}, l_{j,\theta}) \tag{2.24}$$

$$[l_{i,\theta}]: 0 = \mathbb{1}[\theta=1]\, u_l(c_{i,\theta}, l_{i,\theta}) + \mathbb{1}[\theta>1]\,\mu_{i,\theta} u_l(c_{i,\theta}, l_{i,\theta}) - \mathbb{1}[\theta<N]\,\mu_{i,\theta+1} u_l\left(c_{i,\theta}, \frac{y_{i,\theta}}{w_{i,\theta+1}}\right)\frac{w_{i,\theta}}{w_{i,\theta+1}}$$
$$+ \xi_i w_{i,\theta}N_{i,\theta} - \lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta}u_l(c_{i,\theta}, l_{i,\theta}) + \lambda_{j,\theta}\eta_{j,\theta}N_{j,\theta}u_l(c_{j,\theta}, l_{j,\theta})$$
$$- \sum_{k\in\{2,\dots,N\}} \mu_{i,k}u_l\left(c_{i,k-1}, \frac{y_{i,k-1}}{w_{i,k}}\right)\frac{y_{i,k-1}}{y_{i,\theta}}(\gamma_{i,k-1,\theta} - \gamma_{i,k,\theta}), \tag{2.25}$$

for $i = A, B$ and $\theta \in \Theta$. Observe that the world social planner now takes into account cross-country externalities from international migration.

As before, plug (2.24) into (2.25) and use the workers' first-order condition as well as the CES production function to get the worker's marginal tax rate

$$T_i^{co\prime}(y_{i,\theta}) = \frac{\mathbb{1}[\theta<N]\frac{\mu^{co}_{i,\theta+1}}{\xi^{co}_i}\frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}\left[1 + \frac{u_l\left(c_{i,\theta}, \frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{w_{i,\theta+1}u_c(c_{i,\theta}, l_{i,\theta})}\frac{\sigma-1}{\sigma}\right]}{1 + \mathbb{1}[\theta<N]\frac{\mu^{co}_{i,\theta+1}}{\xi^{co}_i}\frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}}$$
$$+ \frac{\mathbb{1}[\theta>1]\frac{\mu^{co}_{i,\theta}}{\xi^{co}_i}\frac{y_{i,\theta-1}}{y_{i,\theta}N_{i,\theta}}\frac{1}{\sigma}\frac{u_l\left(c_{i,\theta-1}, \frac{l_{i,\theta-1}w_{i,\theta-1}}{w_{i,\theta}}\right)}{w_{i,\theta}}}{1 + \mathbb{1}[\theta<N]\frac{\mu^{co}_{i,\theta+1}}{\xi^{co}_i}\frac{u_c\left(c_{i,\theta}, \frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right)}{N_{i,\theta}}},$$

where $\frac{\mu^{co}_{i,\theta}}{\xi^{co}_i}$ denote the relative Lagrangian multipliers under tax coordination. Then, sum up Equations (2.24) over all types and plug the resulting expression for

$$\xi^{co}_i = \frac{1 - \sum_{\theta\in\Theta}\lambda_{i,\theta}\eta_{i,\theta}N_{i,\theta} + \sum_{\theta\in\Theta}\lambda_{j,\theta}\eta_{j,\theta}N_{j,\theta}\frac{u_c(c_{j,\theta}, l_{j,\theta})}{u_c(c_{i,\theta}, l_{i,\theta})}}{\sum_{\theta\in\Theta}N_{i,\theta}/u_c(c_{i,\theta}, l_{i,\theta})}$$

back into (2.24) to solve for $\mu^{co}_{i,\theta}$.

However, under cross-country symmetry, the marginal value of public funds is the same as the one of the exogenous technology planner, $\xi^{co}_i = \xi^{ex}_i$. Moreover, Equation (2.24) simplifies to

$$0 = \mathbb{1}[\theta=1]\, u_c(c_{i,\theta}, l_{i,\theta}) - N_{i,\theta}\xi^{ex}_i + \mathbb{1}[\theta>1]\,\mu^{co}_{i,\theta} u_c(c_{i,\theta}, l_{i,\theta}) - \mathbb{1}[\theta<N]\,\mu^{co}_{i,\theta+1} u_c\left(c_{i,\theta}, \frac{l_{i,\theta}w_{i,\theta}}{w_{i,\theta+1}}\right).$$

This implies that the other Lagrange multipliers coincide as well, $\mu_{i,\theta}^{co} = \mu_{i,\theta}^{ex}$, and the coordination solution is equivalent to the autarky allocation: $T_i^{co\prime}\left(y_{i,\theta}\right) = T_i^{ex\prime}\left(y_{i,\theta}\right)$.

Chapter 3

A Theory of Economic Disintegration

Joint with Eckhard Janeba.

3.1 Introduction

After decades of international integration, recent unilateral movements towards economic disintegration have emerged. The United Kingdom's decision to leave the European Union is a prominent example of such protective policy measures. Similarly, this is the case for the renegotiation of NAFTA and the failure to finalize trade agreements like TPP and TTIP.[1]

The emergence of protectionism and deglobalization alters nations' economic structure along various dimensions, such as trade costs, production standards, business regulations, and migration opportunities. Thereby, economic disintegration affects consumers' and firms' choices, as well as national tax and international trade policies non-trivially. In this chapter, we investigate the policy implications of deglobalization, particularly of the unilateral kind, in which one country disintegrates from a set of other countries, as in the Brexit case. While we frequently speak about disintegration, our model speaks both to unilateral integration and disintegration.

National policies. We study the impact of unilateral economic disintegration on domestic policies worldwide. We focus on national *tax policies* in the presence of firm relocation, which appear to be the most relevant margin of adjustment available to governments to respond to economic disintegration.[2] However, one can broadly understand our economic insights in the

[1] This recent spread of protectionism has launched a considerable line of structural and empirical research (see, for instance, Barattieri, Cacciatore, and Ghironi (2021), Amiti, Redding, and Weinstein (2019), Fajgelbaum et al. (2020), and Li and Whalley (2021)).

[2] A significant body of theoretical and empirical research suggests that countries use their taxes to attract internationally mobile capital, labor, and foreign direct investment. The ongoing globalization of the world economy is known

context of other domestic policy instruments influencing the spatial distribution of economic activity.

We introduce international firm relocation into the classical multi-country, multi-sector general equilibrium trade model of Melitz and Ottaviano (2008) in a highly tractable way. We overcome critical challenges in the economic geography literature (see below) by reducing the dimensionality of the individual relocation decisions without losing generality at an aggregate level. We allow for firm heterogeneity in relocation costs, but assume that industries differ in which subset of countries the mobile firms are able to produce. The parsimony in the modeling of mobility allows us to derive each country's Nash equilibrium business tax policy in closed form as a function of country-pair specific trade costs, firm-location fixed cost distributions, country sizes, and consumers' preferences. We characterize economic disintegration along several model dimensions: The level of tariffs, non-tariff trade costs, business frictions, the deharmonization of production standards and business regulations, and changes in market sizes induced by household mobility.

Moreover, we highlight significant differences between unilateral economic disintegration and reverse multilateral integration. Thus, existing models of multilateral (dis-)integration lack critical insights when applying them to the effects of unilateral economic disintegration on national policies. Our workhorse example for disintegration is a country's departure from an economic union, as in the Brexit case. However, the effects on national policies we derive extend, similar to those on international policies, more broadly to any economic disintegration, such as the exit from a free-trade area, a trade agreement, or another international treaty like the Paris Agreement by the United Nations.

Specific results (1). When a country's departure (e.g., from an economic union) raises bilateral trade costs (*trade-cost effect*), the leaving country's tax will decline. The trade-cost effect on the business taxes set by the remaining union member countries depends on the union size. Under considerable asymmetries in member countries' size, tax policy reactions within the union point in opposite directions and lead to a convergence of taxes inside the union. Since third countries outside the economic union become more attractive as a business location relative to the other countries, their ability to tax improves. These insights hold for both tariffs and non-tariff barriers to trade.

Furthermore, relocating firms face higher mobility costs (captured by a mean-preserving spread in the cost relocation distributions) when production standards and business regulations

to increase the mobility of production factors and firms across space. As a result, it has led to less progressive income tax schedules (Egger, Nigai, and Strecker (2019)) and lower taxes on corporations (Dyreng, Hanlon, Maydew, and Thornock (2017)), which fuels fears of a "race to the bottom" of taxes.

diverge across nations after a country's disintegration (*de-harmonization effect*). Thus, in the short run, when firms do not anticipate this cost change, they may become less mobile across countries, which tends to raise taxes in our model. In the long run, economic disintegration discourages investment in the leaving country because it raises setup costs in that country (*business-friction effect*). We capture this effect, in our static model, as a shift in the relocation cost function. Hence, we highlight substantial differences in the domestic policy responses depending on whether or not firms anticipate the economic disintegration. Although the de-harmonization and the business-friction effect are at first glance tailor-made to the case of a country's exit from an integrated area (e.g., Brexit), they may also occur in other situations of economic disintegration (e.g., the departure of the US from a multilateral institution). We also document a *migration effect* that accounts for disintegration-induced household emigration from the leaving country and resembles in its consequences the business-friction effect. Finally, we identify a *union-size effect* that is similar to the trade-cost effect.

International policies. In addition to studying domestic policies, we develop a novel approach to deal with the impact of unilateral disintegration on international policies, i.e., the readjustment of cooperative and non-cooperative *trade policies* worldwide. We show that the disintegration of one country from an agreement has global repercussions for existing international agreements. Consequently, the welfare implications of unilateral economic disintegration become less straightforward compared to those of a reverse multilateral integration.[3]

Specific results (2). We demonstrate that both the leaving country and the remaining members intensify existing trade agreements with third countries and reduce protectionism, rejecting the hypothesis that a country's disintegration triggers a domino effect. We focus on two important cases: the departure from an integrated area or economic union (case 1), where countries coordinate their internal non-tariff trade policies, and the exit from a customs union (case 2), where only the external trade policies are jointly set. In the first case, we predict that the countries inside the union integrate more with each other. They lower their internal non-tariff barriers to trade. In the second case, the remaining customs union member countries negotiate lower tariffs with third countries in regional trade agreements. The leaving country also intensifies trade agreements with third countries. Similarly, non-cooperative trade policies by the union members, as well as by the leaving country, become less protective. Overall, our results suggest a counterforce to deglobalization.

[3]Contributors to the modern trade policy literature, initiated by Bagwell and Staiger (1999), highlight the advantages of forming international trade agreements to overcome the Prisoner's Dilemma of terms-of-trade manipulation. Ceteris paribus, in a state of multilateral economic disintegration, countries are worse off than under free trade.

The intuition behind the further integration efforts, for instance, among the remaining union members is that, before the departure, the existing policies were optimal given trade costs between all countries. However, upon the departure of a country, the old trade policies are no longer optimal for the reduced set of remaining countries. We show that the adjustment of the policies goes in the direction of further integration if this change makes third countries, including the leaving country, worse off. This holds in our model when trade costs between countries are not too different and initial taxes are positive.

Policy implications. Our model allows us to speak to the likely *domestic policy* consequences of the UK leaving the EU. The results suggest that the UK lowers taxes after Brexit. We predict business taxes in the remainder of the EU to converge. Third countries, e.g., the US, can tax more after Brexit. At the same time, our model applies beyond the case of Brexit. A similar argument applies to countries that engage in a trade war or consider leaving the WTO. When the US pursues a trade war with China, our model predicts that the US would need to lower business taxes to compensate for the loss in attractiveness as a business location.

Naturally, these predictions are somewhat speculative, as Brexit happened only recently and overlapped with the Corona pandemic, making it difficult to separate the different motivations behind policy changes. In retrospect, we consider the 2004 and 2007 Eastern enlargement of the European Union as an example of unilateral economic integration (in the sense of a subset of countries joining an economic union, while other countries remained outsiders to the union). Overall, corporate tax rates have been on a declining trend in most countries long before and after enlargement, reflecting a general race-to-the-bottom situation. Our model can explain this globalization-induced trend by countries' intensified exposure to an increased number of competing markets.[4] Beyond this overall trend, there are differences in the extent of downward adjustment that can be related to our model predictions.

For example, while only a few of the ten countries that joined the EU in 2004 lowered corporate tax rates (Czech Republic, Estonia, Slovenia), most new member countries kept their business taxes roughly constant in the following years, reflecting a relative increase.[5] Our model predicts this relative rise in business taxes of the new members. Inside the European Union, a diversion of tax rates should be expected, according to our model, with smaller countries lowering their rates and large countries raising theirs. Interestingly, Austria and other smaller countries, such as the Netherlands, Finland, Denmark, and Greece, decreased their corporate tax rates,

[4]However, our model does not capture various other motives for this trend, e.g., profit shifting by multinational corporations and distributional objectives.

[5]The statements in this paragraph about business taxes draw on the statutory corporate tax data from the OECD tax database.

while France kept them roughly constant. Germany reduced its business tax substantially from high levels, contrary to our described trade-cost effect following the 2004 enlargement of the EU. However, within our framework, the decline is consistent with Germany's increased integration with large Asian markets, for instance, China.

Besides domestic tax policies, our model also speaks to the policy implications regarding *international trade policies*. In the context of Brexit, the remaining EU members integrate more with each other and reconsider protectionist policies toward third countries. The UK compensates for the rise in trade frictions vis-à-vis the EU by deepening trade relations with third countries. Consequently, the welfare implications from Brexit become ambiguous. It may well be the case that the UK and the remaining European Union are adversely affected, as the conditions under which these countries trade with each other worsen due to Brexit. At the same time, both the UK and the EU can now renegotiate trade agreements with other countries (e.g., the US, India, and China) without the need to consider each other. Thus, when reevaluating trade policies towards these countries, the EU's objective function changes as the UK no longer sits at the negotiating table. Similarly, the UK now sets its policies towards China, India, and the US solely in its own interest. In turn, cooperative and non-cooperative trade policies towards these countries readjust, making the welfare implications of disintegration non-trivial.

Related literature. This chapter relates to two strands of literature. Firstly, we add to the debate on domestic policy in the presence of economic mobility. Usually, in this literature, there are locally separated regions whose economic outcomes are linked to each other through the mobility of capital (Zodrow and Mieszkowski (1986) and Wilson (1986)), labor (Lehmann, Simula, and Trannoy (2014)), or foreign direct investment (Haufler and Wooton (1999) and Haufler and Wooton (2006)). Location rents incentivize governments to modify their domestic policy instruments, such as taxes, to attract these factors. As in our model, some of the authors, for instance, Bucovetsky (1991) and Haufler and Wooton (1999), address cross-country asymmetries. We show that, besides the relative size of a given market, as highlighted in previous work, the world economy's institutional structure profoundly affects domestic policy differentials.

We investigate the relationship between regional taxes and trade costs, as Ottaviano and Van Ypersele (2005) and Haufler and Wooton (2010). In their two-country settings, a reduction in trade barriers makes it less critical for a firm to set up an FDI platform in the larger market. Export costs to this market are then low, and the firm can easily access both markets irrespective of its location. Vice versa, if trade costs are high, firms would like to locate in the large market regardless of the business tax differential until the location rents in the large market are absorbed by an increased degree of regional competition. Although some of the literature has addressed this link, no work endogenizes national and international policies in a model with more than two geographically linked regions.[6] Whereas the two-country and the partial three-country settings may conceptually address the impact of an integration that countries accomplish multilaterally, these models cannot examine a country's unilateral decision to integrate or disintegrate from a set of other countries. As we show in our model, it is misleading to reverse the sign of existing conclusions about multilateral economic (dis-)integration to speak to the effects of unilateral (dis-)integration. Similarly, as we show, it is misleading to consider only a subset of disintegration dimensions.

Two key challenges have, so far, prevented progress to more realistic multi-country models. The first one is that, in a multi-country setting, firm relocation is a multinomial choice problem. The equilibrium distribution of firms across regions is a function of relative location rents, which are, in turn, endogenous to the distribution of firms. As a result, it is hard to derive the objective function of the government in each country. Secondly, each country's tax is the best response to all the other countries' taxes. Therefore, the optimal tax in a country is a general equilibrium object. We overcome both of these issues by reducing the dimensionality of the firm-level relocation problem. Simultaneously, on an aggregate level, the firm distribution is a high-dimensional object that is still tractable enough to solve for general equilibrium tax policies. While our setup of firm mobility is in itself of theoretical interest, we expect it to be helpful in quantitative models that would be otherwise computationally too intense, for instance, when they involve many layers of optimization.

The second strand is the literature on trade policy. As in Ossa (2011) and Bagwell and Staiger (2012), we deal with the effects of trade policies under firm relocation. However, these authors do not consider domestic policies, which is a focus of this chapter. Furthermore, we augment the classical debate on optimal tariffs, started by Bagwell and Staiger (1999), by two dimensions. Firstly, instead of explicitly deriving globally optimal trade policies, we study the incidence of economic disintegration on trade policies worldwide, taking existing imperfections of trade agreements as given. One can apply this approach to various other contracting situations, beyond trade policies, where agents renegotiate preexisting arrangements after one party leaves an agreement. Secondly, we examine other components of trade policy, i.e., non-tariff trade barriers.

[6]For example, in the three-country models of Raff (2004) and Cook and Wilson (2013), one country's government is presumed to be completely inactive. Darby, Ferrett, and Wooton (2014) consider a three-country model of tax policy and trade, but two of the three markets are connected only through a hub region. Most recently, Fuest and Sultan (2019) assume partial mobility of capital and examine tax policies in a three-country model but ignore trade costs. Complementary to this, there are more recent papers in which contributors estimate the effects of tax or subsidy competition in quantitative economic geography models, such as Ossa (2015). So far, this quantitative literature has not addressed the link to economic integration in further detail.

Contrary to tariffs, these non-tariff policy dimensions embrace no government revenue collection motive while still affecting the terms of trade and the spatial distribution of economic activity. Thus, this chapter adds to the growing literature on the economics of deep integration moving beyond the notion of tariff-oriented trade agreements (for example, Grossman, McCalman, and Staiger (2021) and Staiger and Sykes (2021)).

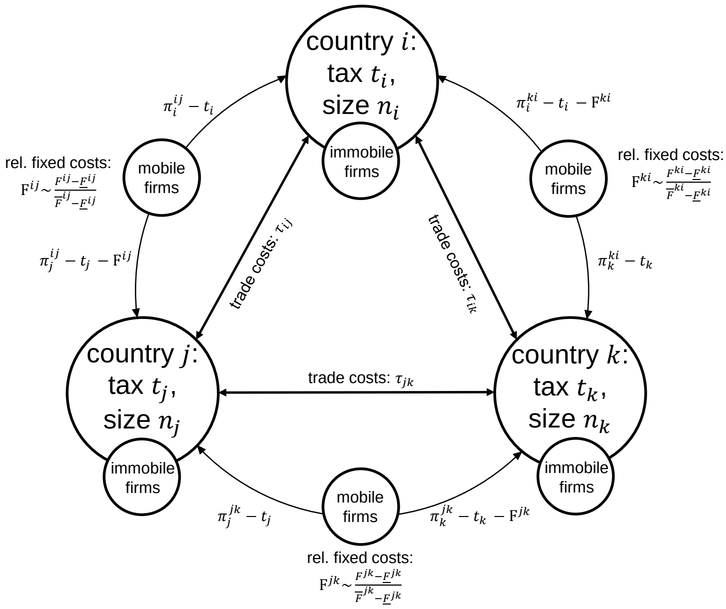
Instead of interpreting our results in the context of unilateral economic disintegration, one can also relate them to the large literature on the gains from trade (see Costinot and Rodriguez-Clare (2014) and Ossa (2016) for two notable reviews). In this literature, contributors quantitatively investigate the effects of trade openness in multi-country, multi-sector general equilibrium trade models. A primary focus is on the quantitative effects of trade openness on welfare and optimal tariffs. In this chapter, we depart from this by highlighting other policy margins, for example business taxation and non-tariff trade barriers.

Outline of the chapter. This chapter is structured as follows. In Section 3.2.1, we first develop a multi-country, multi-sector general equilibrium trade model with firm mobility and non-cooperative business taxation. Then, we derive the effects of economic disintegration along several dimensions in a three-country (Section 3.2.2) and a K-country setup (Section 3.2.3), and consider various model extensions (Section 3.2.4). In Section 3.2.5, we endogenize trade policies to study the readjustment of tariff and non-tariff trade policies worldwide in reaction to economic disintegration. Section 3.3 concludes. We relegate all relevant proofs to the Appendix.

3.2 The Impact of Unilateral Economic Disintegration on National and International Policies

We start with an analysis of the impact of economic disintegration on tax policies. We refer to economic disintegration as the departure of one country from a trade agreement formed by a set of countries (in the following, called an "economic union"). We introduce firm mobility and tax policy into a three-country version of the Melitz and Ottaviano (2008) multi-sector general equilibrium trade model. Our approach allows us to derive each country's optimal Nash equilibrium tax policies. We then identify several model dimensions of economic disintegration and analyze their effects on tax and trade policies.

Figure 3.2.1: The Three-Country Model



3.2.1 The Three-Country Model

Timing. We build a five-stage economy, $\mathscr{E}$, of trade and tax policies which we solve by backward induction. In the initial stage (Stage 0), countries choose their cooperative and non-cooperative trade policies (Section 3.2.5). For the moment, if not stated otherwise, we hold all trade policies fixed. Stages 1-4 feature, for a given set of trade policies, a game of fiscal competition with initially three countries. Let $\mathscr{K}$ denote the non-empty set of countries and $K := |\mathscr{K}| \in \mathbb{Z}^+$ its cardinality. Accordingly, in this section, we consider $K = 3$, but in Section 3.2.3, we extend the model and the main economic insights to $K > 3$. Figure 3.2.1 illustrates the three-country economy.

In the first stage, taking trade policies as given, each government non-cooperatively chooses a business tax, t_i, to maximize national welfare consisting of consumer surplus and tax revenues. For given tax and trade policies, a continuum of mobile firms selects into countries in the second stage. In the third stage, each mobile firm competes in an oligopolistic industry with two other immobile firms in general equilibrium. All firms are single-product businesses and trade their products worldwide. To achieve tractability, we assume that, in each industry, firms can produce in only two out of multiple countries. Industries differ in the pair of countries in which firms produce and the country-specific fixed costs of setting up a firm. In the fourth stage, households optimally choose their consumption of varieties. To fix ideas, we define the subgame-perfect Nash equilibrium of this game (Stages 1-4).

Definition 1. Consider economy $\mathscr{E}$. For given trade policies, the set of tax policies, $\{t_i\}_{i \in \mathscr{K}}$, location and output choices form a subgame-perfect Nash equilibrium, if

1. consumers choose their consumption bundle to maximize utility, taking prices as given,
2. oligopolistic firms maximize their profits over quantities, taking location decisions of all firms and taxes of all countries as given,
3. mobile firms choose their location optimally, taking taxes as given and anticipating how firms and consumers react optimally in their output and consumption decisions, and
4. governments maximize national welfare over taxes, taking the other countries' taxes as given and anticipating the behavior of firms and consumers as described in Stages 1-3.

Unilateral economic disintegration. We analyze unilateral economic disintegration by carrying out comparative statics of this subgame-perfect Nash equilibrium. Specifically, the trade costs between any pair of countries depend on the level of economic integration between these two countries and may differ across country pairs. An increase in the trade costs of respective country pairs captures economic disintegration. We label the resulting impact on tax policies as a trade-cost effect. Moreover, we consider country-pair-specific distributions of the fixed cost to set up a firm that will allow us to derive a de-harmonization effect and a business-friction effect. Finally, we deal with household migration between countries as a simultaneous offsetting change in the population between country pairs (migration effect). There are two main advantages of our approach. Firstly, we impose no a priori assumption on the specific type of economic disintegration. Secondly, all the effects are equally applicable to small and large policy changes.

Households

Preferences. In each country $i \in \mathscr{K}$, a number n_i of identical households consumes a continuum of differentiated varieties, which oligopolistic firms produce, and a numéraire commodity, z_i, produced under perfect competition. Differentiated varieties, $x_i(\mu)$, are indexed by $\mu \in \Omega := [0,1]$. Each variety is produced in an oligopolistic industry, which consists of three firms.[7] Households

[7] All the results carry over when one leaves out the immobile firms and considers only a single mobile firm that produces a given variety, which mimics the firm structure in Melitz and Ottaviano (2008) (but now with endogenous location choice). To endogenize the degree of local competition to firm relocation, we decide to conduct our baseline analysis under an oligopolistic market structure.

derive the following utility

$$u_i := z_i + \alpha \int_{\mu \in \Omega} x_i(\mu)\, d\mu - \frac{\beta}{2} \int_{\mu \in \Omega} x_i(\mu)^2\, d\mu - \frac{\eta}{2} \left(\int_{\mu \in \Omega} x_i(\mu)\, d\mu \right)^2 \tag{3.1}$$

from the consumption of products manufactured by the numéraire and the oligopolistic industries with $\alpha, \beta > 0$ and, in the base version of our model, $\eta = 0$. These preferences are a particular case of those in Melitz and Ottaviano (2008). In Section 3.2.4, we deal with cross-price effects ($\eta > 0$). Household income comes from the business taxes the government rebates in lump-sum fashion and from inelastically supplying labor. Under the assumption that the production of the numéraire good takes place in every country, the numéraire industry pins down a wage rate w which equalizes across countries.

Utility maximization (stage 4). The quadratic utility function generates a system of linear aggregate demand functions

$$X_i(\mu) = \frac{n_i(\alpha - p_i(\mu))}{\beta} \tag{3.2}$$

for each country and industry, where $p_i(\mu)$ denotes the industry-specific local consumer price. Below, we state conditions under which solutions are interior.

Firms

Production and trade. Each firm in the oligopolistic industries faces a linear production function with labor as the only input. Exporting one unit of the consumption good from country j to i costs τ_{ij}, where $\tau_{ij} = \tau_{ji} \in \mathbb{R}^+$ and $\tau_{ii} = 0$, such that the marginal costs of production read as $w + \tau_{ij}$.[8] We interpret trade costs in a broader sense as the degree of economic integration. These refer to all non-tariff barriers to trade of goods and services such as consumer protection, quality requirements, health standards, and environmental protection. Moreover, our definition of trade costs includes transport cost differentials arising from geographical characteristics and tariffs. Altogether, trade costs raise the unit costs of producing for a foreign market. When introducing firm heterogeneity, we also address non-tariff barriers that affect firms' setup costs. For the time being, we assume trade costs to be exogenous, although subject to change with disintegration, and we abstract from revenue effects of trade taxes/subsidies. In Section 3.2.4, we deal with revenue effects and show that our results carry over. Moreover, we endogenize tariff and non-tariff trade policies (Section 3.2.5).

[8] In Section (1.7) of the Supplementary Online Appendix, we relax the assumption that trade costs are symmetric across firms and industries (see https://www.vwl.uni-mannheim.de/(...)/Janeba/Supplementary_Online_Appendix_for_A_Theory_of_Economic_Disintegration_22102021.pdf).

In order to avoid corner solutions, assume that $\tau_{ij} \leq \frac{\alpha - w}{3}$, for all i, j, so that trade flows are weakly positive in equilibrium. As Haufler and Wooton (2010), we assume that firm profits do not accrue to residents in $\mathcal{K}$. As we describe in Section 3.2.4, our results are robust to the accrual of profits in residents' incomes.

Firm heterogeneity. Inspired by Melitz (2003), we introduce firm heterogeneity as follows. In each industry, there are three firms.[9] One immobile firm produces in each of the two countries, say countries i and j. A third, mobile firm can decide in which of these two countries it locates. In the third country, say country k, the production of that specific good is not possible, perhaps due to technological, regulatory, or geographical frictions (and consumption takes place through imports). This location structure is in line with the Ricardian idea of international specialization. However, industries differ in which two of the three countries they can produce. Specifically, there are three types of industries. In an ij-industry, firms are active either in country i or j. jk- and ki-industries are defined accordingly. Throughout the analysis, superscripts will indicate the particular industry type. To rule out asymmetries in initial conditions, let the mass of potential firms be ex ante equal across countries. That is, we partition the set of industries Ω into K equally sized intervals.

Our use of the term "industry" should be explained. An industry is a collection of firms producing a specific variety. For $\eta = 0$, there are two interpretations of this firm structure. On the one hand, there may be a set of industries with three firms in each industry (e.g., the clothing sector and the car sector). Each industry differs in the countries that serve as a (potential) production location. On the other hand, the setting could refer to a continuum of varieties (e.g., in the food sector). Three firms produce a specific variety (e.g., apples and bananas). Varieties differ in the countries where firms can produce them.

Industries differ in a relative fixed cost F^{ij} that the mobile firm pays when comparing the two possible locations – i.e., a firm pays F^{ij} more in country j than in i. One can, therefore, interpret this fixed cost as the cost of relocating from country i to j.[10] We assume that F^{ij} has policy and non-policy components. The policy components are given by the country-specific level of frictions when setting up a business, v^i and v^j, which are determined by factors such as bureaucracy, regulatory complexity, access to infrastructure, and the availability of land. Another policy component is the degree of harmonization in production standards and business regulations between two countries, ε^{ij}. The former affects the level of relative relocation costs, whereas the latter

[9] In Section 3.2.4, we relax this assumption.

[10] This is the main difference to Melitz and Ottaviano (2008). In their setting, firms vary by their marginal cost draw, giving rise to endogenous firm exit and entry. Here, firm heterogeneity comes from relocation cost draws, which leads to endogenous firm emigration and immigration.

alters their variance. An idiosyncratic location preference shock, ε, pins down the non-policy component.

Formally, let $F^{ij} := \nu^j - \nu^i + \varepsilon^{ij} + \varepsilon$ where $\varepsilon^{ij} + \varepsilon \in \left[\underline{\varepsilon}_{ij} + \underline{\varepsilon}, \overline{\varepsilon}^{ij} + \overline{\varepsilon}\right]$ is drawn from a uniform cumulative distribution function with zero mean. Therefore, F^{ij} is also uniformly distributed with a CDF $G^{ij}\left(F^{ij}\right) = \frac{F^{ij} - \underline{F}^{ij}}{\overline{F}^{ij} - \underline{F}^{ij}}$, where $\underline{F}^{ij} := \nu^j - \nu^i + \underline{\varepsilon}_{ij} + \underline{\varepsilon}$ and $\overline{F}^{ij} := \nu^j - \nu^i + \overline{\varepsilon}^{ij} + \overline{\varepsilon}$. In this section, we impose, for simplicity, symmetry in relocation cost distributions across country pairs. That is, we assume $G^{ij}\left(F^{ij}\right) = G\left(F^{ij}\right) = \frac{F^{ij} - \underline{F}}{\overline{F} - \underline{F}}$. In Section 3.2.2, we explicitly deal with the effects of the country- and country-pair-specific policy components that alter the mean and the variance of relocation costs ($\nu^j - \nu^i$ and $\overline{\varepsilon}^{ij}$, respectively). Altogether, each mobile firm pays different fixed costs of production, giving rise to an extensive margin of firm relocation, which affects local prices and production quantities.

Profit maximization (stage 3). A firm producing in country i and belonging to industry ij maximizes profits by choosing the sales in the home market, x_{ii}, and exports to j and k, x_{ji} and x_{ki}. The maximization problem in the third stage is, therefore, defined as

$$\pi_i^{ij}(\mu) := \max_{x_{ii}(\mu), x_{ji}(\mu), x_{ki}(\mu)} [p_i(\mu) - w] x_{ii}(\mu) + [p_j(\mu) - w - \tau_{ij}] x_{ji}(\mu) + [p_k(\mu) - w - \tau_{ik}] x_{ki}(\mu) \quad (3.3)$$

subject to the oligopolistic market structure. Then, pre-tax variable profits of a firm located in country i read as

$$\pi_i^{ij}(\mu) = \begin{cases} \frac{n_i(\alpha - w + \tau_{ij})^2}{16\beta} + \frac{n_j(\alpha - w - 2\tau_{ij})^2}{16\beta} + \frac{n_k(\alpha - w - 2\tau_{ik} + \tau_{jk})^2}{16\beta} & if\ mobile\ firm\ locates\ in\ i \\ \frac{n_i(\alpha - w + 2\tau_{ij})^2}{16\beta} + \frac{n_j(\alpha - w - 3\tau_{ij})^2}{16\beta} + \frac{n_k(\alpha - w - 3\tau_{ik} + 2\tau_{jk})^2}{16\beta} & if\ mobile\ firm\ locates\ in\ j. \end{cases} \quad (3.4)$$

Thus, prices and mark-ups are endogenous to the location decisions of firms. The asymmetry in profits from markets j and k is the consequence of our assumption that in an ij-industry there is an immobile firm present in country j that faces no trade cost in serving its home market, whereas in country k there is no domestic firm active by assumption.[11] In each country i, firms are taxed lump-sum with t_i.

Firm relocation (stage 2). We now turn to the second stage: the location decision of mobile firms. The mobile firm in industry ij produces in country i as long as after-tax profits[12] are larger

[11] One may easily relax this assumption, as long as this additional firm in country k is immobile.

[12] While pre-tax variable profits (4) are non-negative, we cannot guarantee directly that net profits (after tax and fixed cost) are as well. In simulations, we were able to verify for various parameter value combinations that there exist subgame-perfect equilibria in which the profits of all firms were non-negative. The requirement seems to hold more easily when the range of fixed costs is not too broad. In the following, we assume throughout that net profits are non-negative.

in i than in j:

$$\pi_i^{ij}(\mu) - t_i \geq \pi_j^{ij}(\mu) - t_j - F^{ij}. \tag{3.5}$$

In other words, a firm prefers country i if the advantage in gross profits exceeds the tax differential corrected by the relative fixed cost. Since we have a continuum of industries that differ in fixed costs, we can now characterize the mass of industries and firms in a country. For this, we define the following threshold industries in which the mobile firm is indifferent between the two countries

$$\gamma^{ij} := \pi_j^{ij}(\mu) - t_j - \left[\pi_i^{ij}(\mu) - t_i\right], \quad \gamma^{ki} := \pi_i^{ki}(\mu) - t_i - \left[\pi_k^{ki}(\mu) - t_k\right]. \tag{3.6}$$

In country i, the mass of industries with one regional firm (i.e., one immobile firm) is given by

$$G\left(\gamma^{ij}\right) + \left[1 - G\left(\gamma^{ki}\right)\right], \tag{3.7}$$

where the first term refers to the industries with low fixed costs in country j relative to i, and similar for the second term, where fixed costs measure the set-up cost in country i relative to k. The mass of industries with two regional firms (i.e., one mobile and one immobile firm) in i reads as

$$\left[1 - G\left(\gamma^{ij}\right)\right] + G\left(\gamma^{ki}\right). \tag{3.8}$$

Notice that households in country i consume goods produced by jk-industries, but there is no production in or relocation towards i, which significantly simplifies the analysis. Mobility between more than two countries would make necessary extensive numerical simulations, as in Ossa (2015). The advantage of our model is that, although the firm-level location decision is binary, the equilibrium firm distribution is a high-dimensional object that is tractable enough to study policy implications. Our concept of mobility allows us to write the threshold industry level in closed form as a function of the model parameters. In particular, it is linear in the tax differential

$$\gamma^{ij} = \tau_{ij}\left(n_j - n_i\right)\frac{6(\alpha - w) - 3\tau_{ij}}{16\beta} + n_k\left(\tau_{ik} - \tau_{jk}\right)\frac{6(\alpha - w) - 3\left(\tau_{ik} + \tau_{jk}\right)}{16\beta} + t_i - t_j. \tag{3.9}$$

Comparative statics. The partial equilibrium comparative statics are intuitive. The higher the tax in country i relative to j and k, the more firms move out of that country (γ^{ij} increases and γ^{ki} decreases, respectively). Observing that the sign of $\frac{\partial \gamma^{ij}}{\partial \tau_{ij}}$ depends on the country's relative size, one may recognize a partial equilibrium feature of economic disintegration discovered in earlier work: As in Ottaviano and Van Ypersele (2005) and Haufler and Wooton (2010), a rise in trade costs pushes firms to move to larger countries. In this case, market access considerations become more important compared to business tax differentials for mobile firms. Moreover, if trade becomes more costly for firms located abroad, firms move to country i ($\frac{\partial \gamma^{ij}}{\partial \tau_{ik}} > 0$ and $\frac{\partial \gamma^{ij}}{\partial \tau_{jk}} < 0$).

Governments

Non-cooperative tax policies (stage 1). In this section, we consider the first stage of our economy. That is, for a given level of trade costs, we derive Nash equilibrium taxes set by benevolent social planners in each country, who take the effect of taxes on households' consumption choices and location and output decisions of all firms and industries into account.

Consider country i. We compute the total number of firms (as opposed to the mass of industries) by adding Equation (3.7) and two times Equation (3.8). Hence, tax revenues read as $T_i := t_i \left[3 - G\left(\gamma^{ij}\right) + G\left(\gamma^{ki}\right)\right]$. Moreover, the Appendix shows that consumer surplus is given by

$$S_i := G\left(\gamma^{ij}\right)\Delta_i^{ij} + G\left(\gamma^{jk}\right)\Delta_i^{jk} + G\left(\gamma^{ki}\right)\Delta_i^{ki} + \delta_i^{ij} + \delta_i^{jk} + \delta_i^{ki}, \tag{3.10}$$

where Δ_i^{ij}, Δ_i^{jk},Δ_i^{ki}, δ_i^{ij}, δ_i^{jk}, and δ_i^{ki} are defined as functions of the model's primitives

$$\Theta := \left(\alpha, \beta, w, (n_i)_{i\in\mathscr{K}}, (\tau_{ij})_{i,j\in\mathscr{K}}, \underline{F}, \overline{F}\right).$$

The benevolent social planner in country i maximizes the sum of consumer surplus and tax revenues (recall that profits go to absentee owners) and, therefore, solves the following optimization problem

$$W_i := \max_{t_i} S_i + T_i + n_i w, \tag{3.11}$$

taking t_j and t_k as given. Similarly, welfare is maximized in countries j and k over t_j and t_k, respectively.

The first-order condition of the social planner problem yields a reaction function $t_i\left(t_j, t_k, \Theta\right)$ for each country i. As we show in the Appendix, the reaction functions are linear in taxes and there is a unique intersection of the reaction functions, $t_i(\Theta)$ for $i \in \mathscr{K}$, forming the solution to the tax competition game. In the following, we consider the equilibrium of this game with three countries.

Nash equilibrium comparative statics. Lemma 6 verbally summarizes comparative statics of Nash equilibrium taxes with respect to trade costs and country population sizes (without offsetting population changes elsewhere). For a more technical statement, we refer to the Appendix.

Lemma 6 (trade cost and population changes)**.** *In the subgame-perfect Nash equilibrium of economy $\mathscr{E}$,*

(a) *a rise in country i's population size, n_i, increases that country's business tax, whereas an increase in another country's population, n_j, reduces country i's tax, as long as trade between these countries is not too cheap relative to the one between other countries ($\tau_{ij} \not\ll \tau_{jk}$), and*

(b) a rise in country i's trade costs vis-à-vis another country j, τ_{ij}, decreases country i's business tax, as long as it is not too large relative to the other country ($n_i \ngtr\!\!\!\!\gg n_j$). An increase in the trade costs of other countries, τ_{jk}, raises country i's business tax.

Proof. See Appendix 3.A.2. □

Lemma 6 (a) shows that an increase in absolute market size, for instance, induced by population growth in a country, improves that country's ability to tax. Therefore, larger countries tend to tax more. The effect of a growing population in another country is less clear. The relationship between t_i and n_j is positive if the trade of country j with k is very costly compared to the one with country i. On the other hand, $\frac{dt_i}{dn_j} < 0$ if τ_{ij} and τ_{jk} are sufficiently similar. The same arguments apply to the effects of n_k on t_i. When i and j form an economic union (i.e., $\tau_{ik} = \tau_{jk} > \tau_{ij}$), an enlargement of market k reduces taxes inside the union.

Moreover, higher trade costs between countries j and k unambiguously lead to an increase in the tax in country i. Intuitively, countries j and k lose attractiveness when their trade costs rise, which puts country i in the position to tax more. Moreover, provided that country i is not too large, higher trade costs for firms in i put pressure on i's government to lower the tax to attract firms. If country i is very large relative to j, $\frac{dt_i}{d\tau_{ij}}$ can be positive. Then, an increase in τ_{ij} makes tax savings motives less relevant for the location choice of firms because these just want to have low-cost access to the huge market. In other words, the tax base of country i becomes less elastic in response to a rise in τ_{ij}. However, one should note that the taxes in i and j cannot increase simultaneously. That is, there will always be a country that has to lower its tax.

Having dealt with these comparative statics, in Corollary 1 in the Appendix, we consider comparative statics of the (unweighted) average taxes with respect to trade costs. When bilateral trade costs between i and j increase, the average tax in these countries falls. The same holds for the average tax worldwide. A rise in τ_{ij} reduces economic activity worldwide, and attracting firms to improve domestic prices becomes more important.

3.2.2 The Impact of Economic Disintegration on Tax Policies

In the following, we consider several channels through which economic disintegration affects tax policy. First, the costs of bilateral trade between countries change (trade-cost effect). Moreover, economic disintegration alters the international mobility of firms via location fixed costs (deharmonization effect and business-friction effect). Finally, we deal with the possible migration of households (migration effect). As already mentioned, we do not impose any assumption on the underlying institutional structure. Our leading example is the exit from an economic union, as in the Brexit case. However, the main insights carry over to other forms of disintegration.

Trade-Cost Effect

Suppose, for instance, that countries i and j are in an economic union (e.g., the EU) and have similar trade costs. What happens to taxes when trade between country k (e.g., the UK) and the economic union becomes more (or less) costly? As Proposition 10 shows, the answer depends on the relative sizes of the three markets. The proposition follows from Lemma 6. Again, we relegate a more technical formulation of Proposition 10 to the Online Appendix.

Proposition 10 (trade-cost effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$. Let trade costs between the leaving and the remaining countries be sufficiently similar initially. Then, the disintegration of country k via a rise in bilateral trade costs with countries i and j*

(a) *reduces the leaving country's business tax, as long as its population is not too large relative to the other countries, and*

(b) *reduces taxes in the other countries, as long as these are not too large in terms of population relative to the leaving country. Under considerable asymmetries in population sizes, business taxes in countries i and j converge.*

(c) *Under symmetric population sizes of all three countries, the disintegration reduces taxes in all countries.*

Proof. See Appendix 3.A.2. □

When countries have the same population size ($n_i = n_j = n_k$), the tax in the leaving country always declines. The same holds if it is not too large relative to the other countries (the economic union). The market access argument described above drives this result.

If market sizes are equal, taxes in the remaining economic union decrease. If the leaving country is huge (small) relative to the economic union, taxes in the union will decline (rise). Notice that the reaction of taxes inside the economic union can be asymmetric depending on the relative size of the two markets. Let j be the largest of the three markets. Observe that the increase in trade costs with country k may help the smaller country i to tax more, whereas the larger country j needs to lower its tax. Country j still taxes more than i, but taxes converge as a reaction to the disintegration of k.

By comparing Proposition 10 to Lemma 6, one can easily see how a two-country setting, as studied in the previous literature, fails to capture the effects of a country's economic disintegration. In Lemma 6, we show that firms move to the larger market in response to a rise in trade costs (e.g., the economic union). This reaction would lower business taxes in the smaller leaving country but increases taxes in the larger market (the economic union). According to Proposition 10, however, business taxes may decline everywhere. Moreover, a two-country setting cannot address the potentially asymmetric reactions among the remaining member countries.

As we show in the Appendix, the assumption that trade costs are initially similar can easily be relaxed. We demonstrate how to adjust the proposition when trade costs differ. The size of the additional term is relatively small and does not alter the main insights concerning relative market sizes. Moreover, the magnitude of the adjustment is decreasing in the number of competing countries K (see Section 3.2.3).

Proposition 10 is our first main result. It speaks to the hypothesis that, after Brexit, the UK lowers its tax, and this, in turn, puts pressure on the tax policies of countries inside the union. Taking the populations of the UK and France (which is very similar at 66 and 67 million) and Germany at 83 million, a UK departure from a union among these three countries would lead to lower taxes in all countries according to this simple three-country setup. The hypothetical exit of a somewhat smaller country like Spain (47 million) from a joint union with France and Germany, however, would lead to an increase in the business tax in France (whereas still lowering taxes in the other two countries).

De-Harmonization Effect and Business-Friction Effect

De-harmonization effect. So far, we have considered asymmetries which directly affect production choices by firms, that is, the intensive margin of firm decisions. Through pre-tax profit differentials, these asymmetries also change cutoff industries, which determine the relative number of firms. By contrast, we now consider the direct effects of economic disintegration on firm relocation. Recall from Equation (3.5) that a firm in industry ij locates in country i only if $\pi_i^{ij}(\mu) - t_i \geq \pi_j^{ij}(\mu) - t_j - F^{ij}$. That is, the firm has to cover a location cost drawn from a cost distribution. This cost distribution may differ between country pairs. Note that these cost distributions influence relocation elasticities, which vary origin-destination-wise. Relocation within the union is typically cheaper than from the inside of the union to the outside. Thus, the relocation-cost differential is another dimension of economic integration. It describes the degree of harmonization or mutual acceptance of production standards and other business regulations a country pair has reached. One should note that, through this channel, economic integration tends to intensify tax competition, as it simplifies firm relocation and, hence, makes tax bases more elastic. Contributors to the tax competition literature have extensively studied this mechanism. However, the literature is silent about what happens to taxes when one country disintegrates from a set of other countries by de-harmonizing and, as a result, faces a less elastic tax base. This de-harmonization effect is intuitive in the case of an exit from an economic union. However, it applies more broadly to disintegration whenever governments reduce their efforts to reach similar standards and regulations by multilateral agreements, such as in health and environmental protection.

We operationalize this channel as follows. Recall that $F^{ij} \in \left[\underline{F}_{ij}, \overline{F}^{ij}\right]$ is drawn from a uniform distribution $G^{ij}\left(F^{ij}\right) = \frac{F^{ij} - \underline{F}^{ij}}{\overline{F}^{ij} - \underline{F}^{ij}}$. Suppose for now that both countries have the same level of business frictions ($\nu^i = \nu^j$) such that $-\underline{F}_{ij} = \overline{F}^{ij}$. Now we can directly interpret $\overline{\varepsilon}^{ij}$ and, hence, $\overline{F}^{ij} = \overline{\varepsilon}^{ij} + \overline{\varepsilon}$ as the degree of harmonization between i and j. Therefore, economic disintegration induces a mean-preserving spread in the distribution of relative fixed costs. The higher $\overline{\varepsilon}^{ij}$ (and, accordingly, $\overline{F}^{ij} = -\underline{F}_{ij}$), the more firms, and in this setting also industries, are attached to a particular country, and the less should business tax differentials matter for location decisions. When country k disintegrates from i and j, $\overline{\varepsilon}^{jk}$ and $\overline{\varepsilon}^{ki}$ rise in our model.

To dissect this effect, let us for now assume full country symmetry in all primitives of the model other than the distribution of fixed costs between any two countries. Then, we can derive each country's equilibrium tax as a function of $\left(\overline{\varepsilon}^{ij}\right)_{i,j \in \mathscr{K}}$. For a detailed exposition, we refer to the Appendix. We can now state Proposition 11.

Proposition 11 (de-harmonization effect). *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ and suppose that trade costs and country sizes are identical. Let the degree of harmonization in business regulations across countries be sufficiently similar initially. Then, a rise in the degree of harmonization between two countries reduces all country's business taxes. Hence, the disintegration of country k via a de-harmonization between countries raises taxes everywhere.*

Proof. See Appendix 3.B.1. □

The mechanism behind this finding is known from the literature. By construction of our model, a rise in $\overline{\varepsilon}^{jk}$ makes tax bases in the countries j and k less elastic, which tends to increase taxes in these countries. In the Nash equilibrium, this spills over to the tax of country i. Due to the strategic complementarity of tax policies, t_i increases.

In most cases and in particular for similar initial conditions, the tax of a country goes up when the fixed cost distribution widens between that country and another one, that is, t_i increases in $\overline{\varepsilon}^{ij}$. As we show in the Appendix, however, there may be cases in which the tax falls ($\frac{dt_i}{d\overline{\varepsilon}^{ij}} < 0$).

With regard to economic disintegration, the proposition describes another potential effect of the disintegration of country k from i and j, which we label as a de-harmonization effect. When $\overline{\varepsilon}^{jk}$ and $\overline{\varepsilon}^{ki}$ increase simultaneously, tax bases become less elastic between the economic union and the exiting country k. The lower mobility of firms causes taxes to rise everywhere. Note that the response in tax rates from the de-harmonization effect is the opposite of the one from the trade-cost effect (in the case of symmetric countries). So far, we have described origin-destination-specific asymmetries in the firm relocation costs and analyzed the impact of a drop in the mobility of firms between countries. Our second main result suggests that business taxes will increase everywhere if economic disintegration occurs only in the form of more firm attachment to their countries. When interpreting the reduction in firm mobility as a feature of economic disintegration, two notes of caution are indicated, however.

First, the rise in $\overline{\varepsilon}^{jk}$ and $\overline{\varepsilon}^{ki}$ characterizes the economic disintegration of country k only in the short run, as it regards those firms which already exist and decide to relocate after the disintegration of k. For example, when firms anticipate the exit of country k from the economic union, the country's disintegration may discourage prospective entrepreneurs from investing in a firm located in k initially. To summarize, in the long run, the mass of potential firms is endogenous to the degree of economic integration. Accordingly, one of our extensions in Section 3.2.4 regards the effects of changing the ex-ante distribution of firms.

Business-friction effect. Second, we have assumed that economic disintegration triggers a mean-preserving spread in the relocation cost distribution. Therefore, a rise in $\overline{\varepsilon}^{jk}$ affects countries j and k in the same way, which seems reasonable in the context of production standards and harmonization of regulations. However, regarding the effects of the disintegration of country k from j, it might be that production frictions in country k increase such that firm relocation from j to k becomes more costly than vice versa.

Therefore, we now consider the case where the disintegration causes firm relocation cost distributions to shift. As before, $F^{ij} \in \left[\underline{F}_{ij}, \overline{F}^{ij}\right]$ is drawn from a uniform distribution $G^{ij}\left(F^{ij}\right) = \frac{F^{ij}-\underline{F}^{ij}}{\overline{F}^{ij}-\underline{F}^{ij}}$ where $\overline{F}^{ij} - \underline{F}^{ij} = \overline{F}^{jk} - \underline{F}^{jk} = \overline{F}^{ki} - \underline{F}^{ki}$. However, now the relocation cost distributions are allowed to have a different mean: $v^{ij} := v^j - v^i \gtreqless v^{jk} := v^k - v^j \gtreqless v^{ki} := v^i - v^k$.

By considering comparative statics of taxes with respect to these means, we can study the effects of a shift in the relocation cost distributions. In particular, we are interested in the case where locating in the leaving country becomes more costly relative to setting up a business in the other countries (e.g, the economic union). In Proposition 12, we show that the effects point in intuitive directions. We prove the statement in the Appendix.

Proposition 12 (business-friction effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$. An increase in the average cost of setting up a business in a country relative to another country induces a lower tax in the former country and increases the tax in the latter one. Hence, the disintegration of country k via a rise in business frictions lowers the business tax in the leaving country and increases taxes elsewhere.*

Proof. See Appendix 3.B.2. □

When v^{ij} increases, the cost of locating in country j relative to country i goes up on average. As a consequence, country i gains market shares. Vice versa, country i loses industries after a rise in v^{ki}. In the former case, country i's ability to tax improves. In the latter case, country i has to lower its business tax. A change in v^{jk} does not affect t_i because the reduction in t_k just offsets the rise in t_j.

Consider again the situation in which country k disintegrates from an economic union formed by i and j. When this disintegration makes it relatively more costly to set up a business in country k than inside the economic union, v^{ki} decreases and v^{jk} rises. By Proposition 12, country k has to lower its business tax. Members of the economic union tax more.

Migration Effect

So far, we have dealt with changes in parameters that directly affect the production side. Now, we deal with economic disintegration as a trigger of household migration.[13] Migration flows are particularly relevant if a country leaves an economic union that guarantees the free movement of labor. To provide an example, after Brexit, some EU citizens in the UK may return to their home countries or other countries in the union. However, also other forms of economic disintegration induce household migration. The reason is that economic disintegration affects local prices and, therefore, utility levels of households in a given country. When households are internationally mobile just like firms, they will migrate from one jurisdiction to another as long as the difference in utilities exceeds the migration cost.

In the following, we deal with the effects of exogenously driven migration on taxes. Unlike Lemma 6, we now assume that the world population stays constant and consider only population shifts between countries. Moreover, we return to the case where fixed cost distributions are symmetric, $\overline{F}^{ij} = \overline{F}$ for any i, j. Proposition 13 follows from the comparative statics of Lemma 6. For a more detailed statement, we refer to the Appendix.

[13] See, for example, Caliendo, Parro, et al. (2021) who document the migration flows caused by the EU enlargement and quantitatively assess the resulting welfare effects.

Proposition 13 (migration effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ and suppose that trade costs are sufficiently similar initially. Then, household migration from country i to j decreases country i's tax and increases the tax in j. The reaction in country k's tax is positive if and only if trade with country j is cheaper than with i ($\tau_{jk} < \tau_{ik}$). Hence, the migration into an integrated area triggered by the disintegration of country k lowers the leaving country's business tax and increases taxes inside the integrated area.*

Proof. See Appendix 3.B.3. □

The effects of migration (i.e., a change in the size of countries while holding $\sum_{i \in \mathscr{K}} n_i$ fixed) on taxes depend on the origin and the destination of migration flows. Migration from the leaving country into another country reduces the leaving country's tax and allows the destination country to tax more. The tax in the third country, which is not directly affected by migration, rises as well if trade with the destination country is cheap. Hence, migration into an integrated area, such as an economic union, a customs union, or a free-trade area, in which trade is cheaper than outside, increases taxes in the integrated area. The intuition is that the integrated area grows as a whole such that member countries become more attractive to mobile firms irrespective of whereto migrants precisely move.

What is the average effect of a population shift from the leaving country towards a member country? One can see from Corollary (3) in the Appendix that the average tax of these two countries declines. In other words, the leaving country reduces its tax by more than the member country can raise its tax. The average tax of the world increases. As described above, the population shift improves the other member country's ability to tax. In sum, taxes in the integrated area increase. This rise outweighs the reduction in the tax of the leaving country, such that the effect on the average tax of the world is positive.

Altogether, referring to our leading example of an economic union, migration from outside to inside the union increases taxes inside the union and reduces the leaving country's tax. This migration effect is the third central insight from our model.

3.2.3 The K-Country Model

Having described the three-country model, extending our economy $\mathscr{E}$ to an arbitrary number of K countries is straightforward and, at the same time worthwhile, because it allows us to analyze the effects of disintegration on third countries that are not directly affected. Let $\mathscr{K}_{EU} \subseteq \mathscr{K}$ denote the non-empty set of countries from which the leaving country disintegrates and $K_{EU} := |\mathscr{K}_{EU}| \in \mathbb{Z}^+$ its cardinality. For example, this can be a customs union, a free-trade area, or a set of countries in a trade agreement. Therefore, in the following, we refer to a country $m \in \mathscr{K}_{EU}$ to as a "member country." Note that $1 \leq K_{EU} \leq K$. For simplicity, let us consider the case where $\overline{F} = -\underline{F} > 0$. As we have seen, we can readily relax this assumption. However, in this section, we want to focus on two additional dimensions of economic disintegration, which the three-country model is unable to address. First, we show the effect of a rise in trade costs between a country leaving the economic union and the remaining member countries on third countries' tax policy. In the Brexit case, these are countries that were already outside the union before the exit (like the US, India, or China), which occurs when $K_{EU} < K$. Secondly, we impose some symmetry assumptions and derive the tax policy of each country as a function of K_{EU}. These assumptions allow us to model economic disintegration purely as a change in K_{EU}. For a detailed derivation of the K-country model, we refer to the Appendix.

Trade-Cost Effect

We now state Proposition 14, which is the K-country counterpart to Proposition 10.[14] It is useful to define the average population of the member countries as $\bar{n}_{EU} = \frac{1}{K_{EU}} \sum_{m \in \mathscr{K}_{EU}} n_m$. We relegate the proof and a more technical statement of the proposition to the Appendix.

Proposition 14 (trade-cost effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$. Let trade costs between the leaving and the remaining countries be sufficiently similar initially. Suppose that country $l \in \mathscr{K} \backslash \mathscr{K}_{EU}$ disintegrates from the member countries $m \in \mathscr{K}_{EU}$. Then, the disintegration of country l via a rise in trade costs*

(a) *decreases the leaving country's business tax unless its population is very large relative to $\overline{n}_{EU}$,*

(b) *decreases taxes in the remaining member countries under symmetric population sizes, if the union faces many competing markets, and can have asymmetric effects under considerable asymmetries in market sizes, and*

(c) *raises taxes in third countries $\mathscr{K} \setminus (\mathscr{K}_{EU} \cup l)$.*

Proof. See Appendix 3.C.2. □

Trade disintegration between l and $\mathscr{K}_{EU}$ makes third countries relatively more attractive, which allows them to tax more (part (c)). As for the three-country case already described, the

[14] Observe that we only consider direct effects of economic disintegration, i.e. changes in the trade relations of the leaving country with $\mathscr{K}_{EU}$. In particular, we hold trade relations with third countries fixed which is plausible in the Brexit case since the UK remains part of the WTO. Moreover, it ignores the possibility that the UK might form new trade agreements, e.g. with the US. In Section 3.2.5, we study, however, the readjustment of trade policies for a given set of trade agreements.

tax of country l will decrease in the aftermath of its disintegration (e.g., from the economic union) provided that it is not too large relative to the average member country.

The reaction of taxes in member countries is case-specific. It depends on the size of the leaving country, the respective member country, as well as the average member country. In general, the effect in a member country is positive if the average market size is large enough relative to the respective member country's market and the one of the leaving country, revealing a similar convergence result as in Proposition 10.

After imposing cross-country symmetry in market size ($n := n_m = n_l$), the derivative in (b) reduces to

$$\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} = \frac{4K_{EU} - 2K - 1}{2K - 1} \frac{3n(\alpha - w - \tau)}{16\beta} \begin{cases} > 0 & for\, 4K_{EU} > 2K + 1 \\ < 0 & for\, 4K_{EU} < 2K + 1 \end{cases}. \tag{3.12}$$

Referring to the departure from an economic union, taxes inside the union rise when it has many member countries. In our setting, this corresponds to a particularly strong internal market, which covers most of the demand for tradeable goods and services. Furthermore, one can observe the effects of globalization, here interpreted as the number of countries in the world that are related through trade and firm investment. The more competing countries the economic union faces (K), the more sensitive react members' tax bases and, hence, taxes to a member country's disintegration. Put differently, in a globalized world, the union is vulnerable to the fiscal consequences of economic disintegration. In the context of Brexit, the condition for members' taxes to rise, according to Equation (3.12), is clearly not given. The number of countries in the world is larger than twice the EU's 27 member states.

Equation (3.12) transparently reveals two respects in which reverse unilateral integration may differ from unilateral disintegration: Both the sign and the size of the effects depend on the initial level of economic integration. As described, whether the response of business taxes is positive in the union depends on the number of competing markets. Moreover, for initially high trade costs, the effect size is small. Thus, one should be careful when inferring the effects of disintegration (e.g., Brexit) from the years of partial integration (e.g., in the 1970s when the UK joined the union and the integration was at an early stage).

Similar to Proposition 10, there is no need to consider symmetric trade costs between the leaving and the member countries, as we show in the Appendix. Moreover, the statement regards any type of economic disintegration that leads to a rise in trade costs between the leaving country and the remaining members.

In Corollary 6 in the Appendix, we consider the impact on world, EU, and non-EU average taxes. The disintegration of country l increases on average taxes of third countries, but reduces the average tax worldwide. This result is robust and does not depend on country sizes or the number of countries in the union. The effect on the average tax in the remaining economic union is case-specific, as Equation (3.12) suggests. That is, the average tax inside the union rises in reaction to the disintegration when the remaining economic union size is considerable. Vice versa, at a late stage of globalization, the number of rival markets (i.e., K) is significant, and member countries need to lower their taxes to stay competitive on the world market after the exit of a union member.

Union-Size Effect

Another way to examine the consequences of economic disintegration for tax policy is to impose some symmetry assumptions across countries and to directly differentiate taxes with respect to K_{EU}, as if the number of member countries was defined on a continuous domain.[15] In particular, assume symmetry in country sizes as well as in internal and external trade costs as in a customs or an economic union.

Assumption 3. *Let $n := n_i = n_j$ for all $i, j \in \mathscr{K}$. Moreover, let internal and external trade costs be symmetric, $\tau^* := \tau_{ij} = \tau_{ik}$ for all $i, j, k \in \mathscr{K}_{EU}$ and $\tau := \tau_{lm} = \tau_{ln} > \tau^*$ for all $l \in \mathscr{K}$ and $m, n \in \mathscr{K} \setminus \mathscr{K}_{EU}$. Let $K_{EU} > 1$.*

In the Appendix, we show that under Assumption 3 the tax of member countries, t_m, and the one of non-member countries, t_n, are functions of a reduced set of model primitives $\widetilde{\Theta} := \left(\alpha, \beta, w, n, \tau^*, \tau, \overline{F}, K, K_{EU}\right)$. In Proposition 15, we summarize our main findings.

Proposition 15 (union-size effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K > 2$ countries. Let Assumption 3 hold and suppose that $K, K_{EU} \in \mathbb{R}^+$. Then,*

(a) business taxes inside the union are larger than outside,

(b) a rise in the number of member countries increases business taxes in member countries, and

(c) a rise in the number of member countries decreases business taxes in non-member countries.

Proof. See Appendix 3.C.3. □

[15]This procedure is in its flavor similar to the literature on the effects of federalism and government decentralization on private investment (e.g., Kessing, Konrad, and Kotsogiannis (2006)).

As implied by (a), being part of the economic or a customs union makes countries more attractive to firms, which lowers tax competition for these countries. Thus, ceteris paribus the tax of the country that leaves the economic or the customs union declines.

Moreover, when the union loses member countries, the taxes inside the union will fall, and those outside the union will rise. The latter mirrors Proposition 14 (c). The former, however, will only be in line with Proposition 14 (b) if the union is small compared to the rest of the world. This conflicting finding is not surprising since the analysis conducted in this section is, due to Assumption 3, much more gritty than the one in Section 3.2.3.

Regarding the effects of globalization on taxes, one may differentiate the average worldwide business tax with respect to K. In the Online Appendix, we show that the derivative is positive. That is, overall taxes decline as globalization increases the number of competing markets.

In this section, we have extended our model to any number of countries (K) with an arbitrary institutional structure (K_{EU}). As we have seen, the results and intuitions formed in the three-country world remain valid.

3.2.4 Extensions

In this section, we consider various extensions of the base model and summarize the main findings. In a Supplementary Online Appendix, we provide detailed derivations.[16]

Tariff revenues and subsidy expenditures. Firstly, we explicitly incorporate revenue effects from tariffs into our model. That is, aside from non-tariff trade barriers, we allow for the presence of import and export tariffs. Just as non-tariff trade barriers, trade taxes affect consumer surplus and revenues from taxing corporations. Besides, tariffs generate additional fiscal revenues. For non-negative import tariffs and export subsidies, the optimal business tax of a country is revised upwards. The intuition is as follows: when business taxes in a country rise, firms move away from that country. As a result, the government generates extra tariff revenues and saves expenditures on export subsidies.

Accordingly, the reaction of Nash equilibrium business taxes to a rise in non-tariff trade costs is downwards adjusted. The reason is that higher trade costs reduce trade volumes such that the extra gains in tariffs (expenditure savings) decline. Nonetheless, the key trade-offs, in particular concerning the above-described effects of economic disintegration, carry over.

[16]The Supplementary Online Appendix is available at https://www.vwl.uni-mannheim.de/(...)/Janeba/Supplementary_Online_Appendix_for_A_Theory_of_Economic_Disintegration_22102021.pdf.

Another remarkable feature is that the business tax of country i is U-shaped in foreign trade taxes. This pattern is similar to Proposition 1 in Haufler and Wooton (2010) but in our setting for trade policy instruments that have revenue effects.

Accrual of profits. Secondly, recall that, in our baseline economy, firm profits accrue to citizens in third countries or, at least, do not enter social welfare. This assumption is only reasonable for very wealthy investors and a government with a pronounced redistributive goal but not for smaller entrepreneurs or investors. Therefore, we now deal with the domestic accrual of profits.

We distinguish two polar cases of firm ownership. The first one considers internationally mobile entrepreneurs who only enter the social welfare of a country when they decide to locate their business there. Usually, this is the case for smaller businesses. In the second case, citizens directly hold a diversified portfolio of enterprises worldwide. This assumption is realistic for mid- and big-cap companies with shares traded on international financial markets. In both cases, the social marginal welfare weight of firm ownership slightly modifies the optimal business tax. Moreover, in the former case, taxes are revised downwards by the accrual of domestic profits and, in the Nash equilibrium, of foreign profits. In the latter case, taxes account for the accrual of international profit differentials. This distinction is intuitive, as, in the first case, social welfare is a function of national income. However, when citizens are shareholders of firms worldwide, they only care about the size but not about the location of accrued profits. For both cases, our main insights carry over, as shown in the Supplementary Online Appendix.

Industry size. We generalize our economy to an arbitrary number of immobile firms in each industry. Our results hold as long as the distribution of immobile firms is similar across countries. A rise in the number of immobile firms in one country has opposing effects on the optimal business tax there. On the one hand, more firms in the country mechanically raise the government's ability to tax. On the other hand, more firms increase the degree of local competition such that the country becomes less attractive as a business location to mobile firms. In the Nash equilibrium, these two effects point in the same direction for the taxes of the other countries. Using this model specification, we can shed light on the anticipatory effects of economic disintegration. Suppose that some previously immobile firms anticipate a country's disintegration and decide to initially locate in the economic union instead of the leaving country. This reallocation of firms mechanically lowers (improves) the disintegrating country's (member countries') ability to tax. At the same time, firms face more competition in the member countries, which lowers mark-ups there. Vice versa, in the leaving country, firms generate higher profits.

Cross-price effects. We generalize the model by allowing for cross-price effects in the demand for differentiated goods. For $\eta > 0$, the Nash equilibrium business taxes are revised upwards.

The substitutability between the differentiated varieties and the numéraire rises with η. Put differently, the presence of cross-price effects shifts down the demand for differentiated varieties, thereby reducing the welfare loss from firm emigration in the differentiated industries.

Relative to Lemma 6, there are two adjustments. The first one regards the marginal effect on the aforementioned reduction in welfare losses. The second adjustment captures that the consumer surplus loss from taxing businesses is endogenous to the average price level. For similar trade costs and market sizes, which rule out the Metzler paradox, the effects point in opposite directions. However, the central intuitions regarding the impact of disintegration (e.g., via a rise in trade costs) carry over.

Competition in regulations. We introduce competition in regulations into the first stage of our economy, in addition to the business tax as a policy instrument. That is, we endogenize each country's level of business frictions/regulations, ν^i, similar to the non-cooperative setting of business tax policies. Then, each government chooses the set of domestic policies $\left(t_i, \nu^i\right)$, taking all the other countries' business taxes and regulations as given. A rise in the level of regulations is welfare-detrimental as it triggers firm emigration, which reduces consumer surplus and tax revenues. Therefore, to obtain interior solutions, we introduce a country-specific reduced form regulation surplus $V_i\left(\nu^i\right)$ that is assumed to be increasing, concave, and, for simplicity, independent from taxes. In the context of environmental protection, this surplus could measure the value of clean air. Even without cross-country complementarities in this surplus function ($\frac{dV_i}{d\nu^j} = 0$), the optimal level of regulations is inefficiently low since a country's government does not consider the positive externality of business regulations on other countries' welfare. Thus, just as in the tax competition game, countries would gain from the international coordination of business regulations.

We demonstrate that the domestic policies interact: The optimal business tax is not only affected by the level of regulations, as in Proposition 12, but also vice versa. Interestingly, their (partial equilibrium) comparative statics may point in opposite directions. For example, whereas a rise in τ_{jk} improves country i's ability to tax, it amplifies the size of lost tax revenues and, hence, the welfare costs of ν^i. Accordingly, country i's optimal level of business regulations declines. Altogether, the impact of economic disintegration on the other domestic policies may significantly differ from those on business taxes, even if the domestic policy closely resembles a business tax from mobile firms' perspective as it is the case for business regulations.

Harmonization of business taxes. We also consider the scenario of partial harmonization (e.g., Conconi, Perroni, and Riezman (2008)), where a subset of countries in a harmonized area, $\mathscr{K}_H \subset \mathscr{K}$, coordinates their level of business taxes to maximize their joint welfare. Again, there exists a unique Nash equilibrium in taxes set by the subset of countries and all other countries, which can be derived from the government's (modified) reaction functions. The formulas for the non-cooperative tax policies of countries outside the harmonized area are unaltered relative to the case without tax harmonization. The reaction function in the harmonized area, t_H, accounts for average effects on consumer surplus and tax revenues.

The coordination of business taxes among some countries reduces ceteris paribus the degree of tax competition relative to the setting without harmonization. Conceptually, the harmonized area behaves in its setting of business taxes similar to a large country. Therefore, the impact of economic disintegration on the coordinated business tax resembles the one on a large country's tax policy.

To further shed light on the economic disintegration, we impose cross-country symmetry in market sizes and trade costs. This assumption yields a symmetric tax outside of $\mathscr{K}_H$ in addition to the one inside. In line with the intuition that the harmonized area acts as a large market and is more attractive as a business location than the other isolated markets, the business tax inside the area is higher than outside. Similar to Proposition 15, we differentiate business taxes with respect to the number of members in the harmonized area, K_H, as if it was defined on a continuous domain. Both inside and outside the area, business taxes are positively associated with K_H. Hence, a country's departure from the set of countries that coordinates their business tax policy decreases taxes worldwide. The reason is that the according reduction in K_H is equivalent to creating a new player in the tax competition game and, as a result, amplifies the degree of competition.

Richer labor market. In the following, we shortly describe how the presence of a richer (more carefully modeled) labor market affects our insights.[17] In our economy, free trade in the numéraire commodity equalizes the wage rate across countries and labor supply is inelastic. Suppose that trade in the numéraire commodity is not possible, and elastic labor supply (via an additively separable disutility from labor) and labor demand determine a country's wage rate on the labor market. As households' utility is linear in the consumption of the numéraire, a change in tax revenues that the government rebates to households in lump-sum fashion has no income effects on labor supply. However, due to endogenous firm migration, a change in business taxes affects labor demand. The lower a country's business tax, the more mobile firms move into that country, increasing labor demand. The equilibrium wage rate and, thus, welfare in the country rises. Therefore, a richer labor market gives a country's government an additional incentive

[17] The quantitative importance of trade shocks on labor market outcomes is demonstrated by Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014), and others.

(aside from lower consumer prices and higher tax revenues) to reduce business taxes to attract mobile firms (more tax competition). Altogether, this extra wage channel strengthens our main results.

Proportional tax on profits. Furthermore, one may replace the lump-sum tax with a proportional tax on profits $\tilde{t}_i(\mu)$. Observe that the latter tax is equivalent to the former one for $\tilde{t}_i(\mu) = t_i/\pi_i^{ij}(\mu)$ in a given industry $\mu \in [0,1]$. A rise in the lump-sum tax is associated with a higher proportional tax. Accordingly, our analysis above addresses the level of proportional taxes. The proportional tax affects firm relocation (threshold industries γ^{ij}) in the same way as the lump-sum tax. Country i's tax rate $\tilde{t}_i(\mu)$ is the same for all industries with the same firm mobility outcomes and, thus, with the same profit level $\pi_i^{ij}(\mu)$ (e.g., for all $F^{ij} < \gamma^{ij}$). However, it declines in the industry's profit level $\pi_i^{ij}(\mu)$. Domestic firms in sectors with less competing firms in their home market (e.g., $F^{ij} < \gamma^{ij}$ in country i) realize high profits, whereas firms with more local competition (e.g., $F^{ij} \geq \gamma^{ij}$ in country i) have low profits. Thus, a country i's government gives a tax discount on high-profit industries. These are sectors in which the government tries to attract firms that choose the other country. The government levies a higher tax on more competitive/low-profit industries where the government can attract firms in any case.

Firm relocation across multiple countries. Finally, one may relax the assumption of binary firm relocation choices. To achieve the degree of tractability necessary to solve explicitly for the Nash equilibrium business tax policies, we have restricted the analysis to a firm's location choice between two countries in a given industry. If, by contrast, firm location were a multinomial choice problem, mobile firms would relocate across multiple countries. This additional firm mobility would intensify tax competition as it scales up each country's elasticity of relocation: Since each mobile firm can relocate to any other country instead of one specific country that may be relatively unattractive as a business location, a rise in a country's trade costs would induce stronger firm emigration responses. Vice versa, a decline in a country's trade costs triggers additional firm immigration because firms from all industries (also those where the country is not part of the relocation choice set in our model) can move into the country. Therefore, we expect that firm relocation across multiple countries strengthens our findings.

3.2.5 The Impact of Economic Disintegration on Trade Policies

In this section, we consider another dimension of economic disintegration: Trade policies around the world endogenously react to economic disintegration. Referring to our model described in Section 3.2.1, the setting of cooperative and non-cooperative trade policies can be modeled as the initial stage of our economy (Stage 0).[18] Taking an arbitrary, previously determined set of trade agreements as given, we develop a novel approach for studying the readjustment of trade policies worldwide triggered by economic disintegration.

We consider unilateral economic disintegration as the departure of one country from an economic union (e.g., "soft Brexit") or a customs union (e.g., "hard Brexit"). We refer to an economic union as a set of countries that form a customs union and cooperatively set their internal non-tariff trade policies. A customs union is defined, as usual, by a set of countries that jointly negotiate their common external tariffs.

We analyze the effects on trade policies around the world in response to disintegration: How do (non-tariff) trade policies inside the union change, and how do these affect endogenous tax policies in turn? How are regional trade agreements between the economic union and third countries affected? What are the effects on trade agreements between the leaving country and third countries?

Readjustment of trade policies (stage 0). To answer these questions, we develop a novel approach for the study of trade policies. Trade costs between two countries $\tilde{\tau}_{ij} = t_{ij} + \tau_{ij}$ include tariffs t_{ij} (trade taxes) and non-tariff trade costs τ_{ij}.[19] Note the difference in notation between tariffs (trade taxes) t_{ij} and business taxes t_i. As in the previous section, non-tariff trade costs τ_{ij} entail local characteristics (such as geographical frictions) and non-tariff trade policies (such as environmental protection and product standards) that do not have government revenue effects, unlike tariffs t_{ij}. Nevertheless, similar to tariffs, governments in an economic union can negotiate non-tariff trade policies to a certain extent. We draw on the idea that cooperative trade policies result from efficient bargaining (see Grossman and Helpman (1995) and subsequent literature). Then, under the transferability of utilities, efficient cooperative trade policies maximize the respective sum of welfare, as described below. Our approach considers trade policies before (labeled as "old" optimum) and after the disintegration ("new" optimum). However, before presenting our approach in more detail, we state the following result.

[18] At first glance, studying cooperative trade policies seems contradictory to the non-cooperative approach we have adopted in the context of tax policies. However, it fits well the situation of the EU, in which member countries have jointly introduced projects like the Single European Act (SEA) of 1987 to facilitate trade and commerce in the union, whereas the setting of business tax policies has so far been independent (due to unanimity requirements in tax matters at the EU level). The SEA and the free flow of goods, factors, and services in the EU have taken precedence over tax policies and, therefore, justify our timing assumptions: Countries choose trade policies simultaneously before tax policies.

[19] This definition of trade costs also allows us to incorporate tariffs (that affect government revenues) into our model of Section 3.2 (see Section (1.1) in the Supplementary Online Appendix available at https://www.vwl.uni-mannheim.de/(...)/Janeba/Supplementary_Online_Appendix_for_A_Theory_of_Economic_Disintegration_22102021.pdf).

Lemma 7. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $\overline{F}^{ij} = -\underline{F}_{ij}$ for all i, j. For positive business taxes, similar trade costs, and small tariffs, a rise in bilateral trade costs between two countries raises welfare in third countries: $\frac{dW_k}{d\tau_{ij}} > 0$ and $\frac{dW_k}{dt_{ij}} > 0$.*

In the Supplementary Online Appendix (Section (2.1)), we prove Lemma 7 in our model. The proof employs the optimality of a country's business taxes and the Nash equilibrium comparative statics to capture the impact of other countries' adjustment in tax policies on a country's welfare.

Lemma 7 has an intuitive appeal. It means that any protective measure (i.e., tariffs t_{ij} as well as non-tariff barriers summarized in τ_{ij}) between two countries proves beneficial to third countries (positive gradient of the welfare function). The reason is that the third country becomes more attractive to businesses as trade costs between the two other countries rise. Not even the reduction in the latter countries' business taxes resulting from the rise in trade costs can compensate for this: Firms move to the third country, and prices decline there. This price effect raises welfare.

The assertion that third countries benefit from a rise in trade costs between two other countries is more general and well-known in the literature on trade policy. Usually, contributors to this literature refer to it as the terms-of-trade effect of bilateral trade costs (in particular tariffs) on the world price and, in turn, on a third countries' welfare. It may result in bilateral opportunism (as in Bagwell and Staiger (2004)). We now present our approach.

The approach. Assume that each government optimization problem is concave and solutions are interior.[20] Moreover, suppose that trade policy changes are small. Then, we can describe our approach as a four-step procedure:

1. Approximate the respective objective function (e.g., countries' joint welfare in a trade agreement) in the new optimum around the old optimum.

2. Use the optimality of the old and new trade policy choices.

3. Impose the first-order conditions of the old optimum.

4. Relate the sign of the gradient of welfare to the change in trade policies.

Our main observation is that the objective function of the economic union (the customs union, respectively) changes when one member country leaves. As a consequence, the optimal choice

[20] To gain an intuition for why solutions are interior, consider, for instance, the multilateral negotiation of bilateral non-tariff trade costs, τ_{mn}, inside a union. On the one hand, a rise in τ_{mn} may reduce welfare in countries m and n. On the other hand, other member countries inside the union benefit from a higher τ_{mn} (Lemma 7). As a result, there is a trade-off when choosing τ_{mn} to maximize joint welfare.

of the internal non-tariff, as well as external trade policies, is affected. External trade policies include, in particular, tariffs. These form within the framework of regional trade agreements with other markets as customary in the WTO or countries set them non-cooperatively. Moreover, one should note that the described economic disintegration means effectively, although not legally, the creation of a new trading partner for all countries worldwide, with whom they can form new trade agreements.

Using the described four-step procedure, we compare cooperative and non-cooperatively chosen trade policies in the old optimum to those in the new optimum. We summarize the insights from our approach in Proposition 16. For a more detailed exposition, we refer to the Appendix.

Proposition 16 (endogenous trade policy responses to disintegration)**.** *Let the assumptions of Lemma 7 hold. Assume that each optimization problem is concave and solutions are interior and let trade policy changes be small.*

(a) *Suppose countries l and $\mathscr{K}_{EU}$ initially form an economic union (old optimum), where member countries bargain their internal non-tariff trade policies. When country l disintegrates from the economic union (new optimum), the remaining member countries integrate more with each other (lower non-tariff trade costs).*

(b) *Suppose countries l and $\mathscr{K}_{EU}$ initially form a customs union (old optimum). When country l leaves the customs union (new optimum), the leaving country lowers cooperative and non-cooperative tariffs toward third countries. Likewise, cooperative and non-cooperative tariffs by the customs union vis-à-vis third countries decline.*

Proof. See Appendix 3.D. □

In summary, the remaining member countries take efforts to lower their internal non-tariff barriers to trade (part (a)). Intuitively, changes in non-tariff trade barriers do not induce a first-order gain or loss on total welfare inside the economic union. However, for the old bargaining solution to be optimal, in the new optimum, the leaving country has to bear a welfare loss induced by the change in trade costs inside the union. Given Lemma 7, this can only be achieved by a reduction in trade costs. Hence, member countries integrate more with each other by reducing their internal non-tariff trade costs.

When the leaving country also exits the customs union, the union member countries lower cooperatively and non-cooperatively set trade barriers toward third countries (part (b)). For instance, the EU member countries and the US that are part of the WTO decrease their bilateral tariffs after Brexit. Moreover, the EU members implement lower tariffs toward non-WTO member countries. Similarly, trade barriers between the UK and the US decline after Brexit. The

UK also lowers tariffs toward non-WTO members. The intuition for these results is the same as the one for part (a). Therefore, the departure of a country from an economic union leads ceteris paribus to a deeper integration of multilaterally formed institutions around the world and less protectionism. This suggests a counterforce to deglobalization.

Repercussions on tax policies. Disintegration affects the formation of trade policies in the initial stage of our economy $\mathscr{E}$ (Stage 0). For instance, when a country leaves an economic union and stays in the customs union ("soft Brexit"), the trade-cost effect in Proposition 14 needs to be augmented by the readjustment of non-tariff trade costs as follows.[21]

The statement about the impact on the leaving country's (e.g., UK's) business tax remains qualitatively unchanged. However, the endogenous reduction in non-tariff trade costs inside the economic union puts additional downward pressure on the leaving country's business tax.

As before, the business taxes inside the remaining economic union (e.g., Germany, France,...) may react asymmetrically. Having said this, under symmetric population sizes, the response of taxes inside the union will be positive. The reason is that the endogenous decline in internal trade costs makes the economic union more attractive as a business location raising member countries' ability to tax.

Third countries (e.g., the US, China,...) may now experience a decline in their business taxes. On the one hand, trade barriers between the member countries and the leaving countries rise, which increases third countries' taxes (Proposition 14). On the other hand, the adjustment in member countries' trade policies lowers third countries' attractiveness as a business location. If the economic union is large enough relative to the leaving country, the latter effect dominates the former, leading to lower taxes in third countries.

Normative implications. As a byproduct of our above analysis, one can note that the normative implications of economic disintegration are generally ambiguous. The main reason for this insight is the fact that trade policies around the world change with the degree of economic integration between a subset of countries.

To give an example, consider the welfare in the country leaving an economic union. Several effects of trade policy changes add up. There are adverse effects since the remaining member countries in the economic union do not regard the leaving country's welfare when adjusting their cooperative and non-cooperative trade policies towards third countries as well as their internal degree of economic integration. Having said that, after the disintegration, the leaving country is free to set its non-cooperative external tariffs solely to its advantage. The renegotiation of existing trade agreements may be beneficial or detrimental to the leaving country. One can show

[21] For mathematical details, see Supplementary Online Appendix (Section (2.2)).

that the leaving country and the respective contractual partner improve their joint surplus after the disintegration. However, this does not mean that the leaving country is better off. It may well be the case that the presence of other countries in the trade agreement, here the member countries of the economic union, proves beneficial to the leaving country. As a consequence, the economic disintegration and the resulting absence of the member countries in the trade agreement are welfare-detrimental to the leaving country. By similar arguments, the normative effects on countries in the economic union and third countries are ambiguous.[22]

To summarize, in this section, we have endogenized different dimensions of trade policy, namely tariffs and non-tariff trade costs. Altogether, along these different dimensions of trade policy, the remaining countries of an economic union take further steps towards the economic integration of their internal market when being confronted with the disintegration of a former member. After the disintegration from a customs union, the leaving country, as well as the remaining economic union, intensify their trade relations with other countries.

These further steps of economic integration do, of course, not necessarily mean that economic disintegration stabilizes multilateral institutions. It is possible that leaving a union is beneficial from a unilateral perspective, although it is multilaterally detrimental. Moreover, each loss of a member country jeopardizes the credibility of these institutions and increases the uncertainty of economic policy (see Davis (2016), Steinberg (2019), and Caldara et al. (2020)). Also, note that these considerations assume a fixed set of trade agreements. It could be that, after disintegrating, the leaving country negotiates trade agreements with countries that do not form an agreement with member countries, and vice versa. Without imposing more structure on the underlying economy, it is a priori unclear whether countries breach existing or form new trade agreements.

3.3 Conclusion and Discussion

In this chapter, we study the policy implications of economic disintegration. We identify several empirically testable dimensions along which disintegration affects nations' economic environment and, in turn, national policies. We set up an analytically highly tractable multi-sector, multi-country general equilibrium trade model where a set of internationally mobile firms generates fiscal competition over business taxes. This particular policy is representative of any do-

[22] These findings hold under the economic conditions described in Bagwell and Staiger (1999) and the subsequent literature. In particular, the efficiency of global free trade remains valid in our approach. Our central insight is to take existing inefficiencies in trade policies as given. Based on this, trade policies react worldwide to economic disintegration. Therefore, its normative implications may be far from obvious, even if one considers only first-order effects, which we address in our approach.

mestic policy affecting the location of economic activity. Thereby, the firm relocation elasticity is a sufficient statistic for the optimal tax in a given country. This elasticity crucially depends not only on the economic conditions in that country but also on those worldwide. This observation even holds when a minimum of mobility is introduced, modeled as a bilateral location choice by one firm per industry. As a result, the whole economic structure influences domestic policies in each country. An important lesson from our approach is that a two-country analysis is potentially misleading when studying the effects of disintegration on national policies. By considering an arbitrary number of countries, our model takes such a broader perspective.

Economic disintegration may affect the underlying economic structure along very distinct dimensions. For example, it is not only associated with a change of the bilateral costs of trade (trade-cost effect). But disintegration also alters other economic parameters that affect nations' economic geography (de-harmonization effect and business-friction effect). As we show, these can have different policy implications. In sum, we make four policy predictions about uniltateral economic disintegration:

1. The leaving country reduces its business tax.
2. Business taxes in the remaining member countries converge.
3. Third countries' ability to tax improves.
4. Governments worldwide counter a country's economic disintegration by deepening their existing trade relations – a counterforce to deglobalization.

We recognize several limitations to our economic geography model. Firstly, we note the simplicity of the supply side in our model, such as the two-country industry structure that allowed us to obtain clear-cut policy predictions. For example, Caliendo and Parro (2015) demonstrate the quantitative importance of incorporating input-output linkages. However, putting an even more complex economic structure might cloud the relevant policy implications. Moreover, labor is an internationally mobile factor, as in Caliendo, Dvorkin, and Parro (2019). This feature holds especially true in the long run. Our comparative statics show that, even in the absence of wage effects, the number of residents strongly affects national policies and its connection to economic integration merely through market size (migration effect). Studying the interplay of national and international policies under the full mobility of firms, labor, and capital, we consider a promising area of future research.

Besides studying domestic policies, we show the impact of unilateral disintegration on trade policies. As we have argued, the counterforce-to-deglobalization result is robust to the underlying economic model. Moreover, beyond trade policies, it applies more broadly to any type of disintegration from a jointly formed agreement by economic agents. This readjustment of multilateral policies makes the welfare effects of economic disintegration very subtle. A limitation of our approach is that it can only address small policy changes. To consider large changes, one needs to know the sign and the size of the cross derivatives of welfare functions with respect to international policies. This requirement would make it necessary to impose more structure on the underlying economy.

Finally, in our analysis, we remain agnostic about the driving forces of economic disintegration. For instance, economic disintegration could reflect changes in a society's policy preferences. Thus, making these drivers of disintegration more explicit, e.g., by modeling political economy forces, is another interesting direction.

Appendix

3.A Proofs for Section 3.2.1

3.A.1 Derivation of Optimal Taxes

In order to derive consumer surplus, note that there are three continuums of industries. Depending on whether F^{ij} is less or greater than the threshold of an indifferent industry γ^{ij}, there are two distinct location outcomes per industry type such that we need to consider six different price levels. In the following, take country i's perspective. Use firms' optimal production quantities to show that the prices read as

$$p_i^{ij}(\mu) = \begin{cases} \frac{\alpha+3w+\tau_{ij}}{4} & if\, F^{ij} \geq \gamma^{ij} \\ \frac{\alpha+3w+2\tau_{ij}}{4} & if\, F^{ij} < \gamma^{ij} \end{cases}, \quad p_i^{jk}(\mu) = \begin{cases} \frac{\alpha+3w+2\tau_{ij}+\tau_{ik}}{4} & if\, F^{jk} \geq \gamma^{jk} \\ \frac{\alpha+3w+\tau_{ij}+2\tau_{ik}}{4} & if\, F^{jk} < \gamma^{jk} \end{cases},$$

and

$$p_i^{ki}(\mu) = \begin{cases} \frac{\alpha+3w+2\tau_{ik}}{4} & if\, F^{ki} \geq \gamma^{ki} \\ \frac{\alpha+3w+\tau_{ik}}{4} & if\, F^{ki} < \gamma^{ki} \end{cases}, \tag{3.13}$$

for any $j,k \in \mathscr{K} \setminus \{i\}$. In general, prices are lower in a country if a mobile firm locates there because trade costs are saved. Plug these prices into the demand functions $x_i^{ij}(\mu) = \frac{\alpha - p_i^{ij}(\mu)}{\beta}$, $x_i^{jk} = \frac{\alpha - p_i^{jk}(\mu)}{\beta}$, and $x_i^{ki}(\mu) = \frac{\alpha - p_i^{ki}(\mu)}{\beta}$ to obtain household consumer surplus. Multiply with the size of the market to obtain aggregate consumer surplus in country i

$$\begin{aligned}
S_i = & n_i \left(1 - G\left(\gamma^{ij}\right)\right) \left(\alpha x_i^{ij}(\mu) - \frac{\beta}{2}\left(x_i^{ij}(\mu)\right)^2 - p_i^{ij}(\mu) x_i^{ij}(\mu)\right)|_{F^{ij} \geq \gamma^{ij}} \\
& + n_i G\left(\gamma^{ij}\right) \left(\alpha x_i^{ij}(\mu) - \frac{\beta}{2}\left(x_i^{ij}(\mu)\right)^2 - p_i^{ij}(\mu) x_i^{ij}(\mu)\right)|_{F^{ij} < \gamma^{ij}} \\
& + n_i \left(1 - G\left(\gamma^{jk}\right)\right) \left(\alpha x_i^{jk}(\mu) - \frac{\beta}{2}\left(x_i^{jk}(\mu)\right)^2 - p_i^{jk}(\mu) x_i^{jk}(\mu)\right)|_{F^{jk} \geq \gamma^{jk}} \\
& + n_i G\left(\gamma^{jk}\right) \left(\alpha x_i^{jk}(\mu) - \frac{\beta}{2}\left(x_i^{jk}(\mu)\right)^2 - p_i^{jk}(\mu) x_i^{jk}(\mu)\right)|_{F^{jk} < \gamma^{jk}} \\
& + n_i \left(1 - G\left(\gamma^{ki}\right)\right) \left(\alpha x_i^{ki}(\mu) - \frac{\beta}{2}\left(x_i^{ki}(\mu)\right)^2 - p_i^{ki}(\mu) x_i^{ki}(\mu)\right)|_{F^{ki} \geq \gamma^{ki}} \\
& + n_i G\left(\gamma^{ki}\right) \left(\alpha x_i^{ki}(\mu) - \frac{\beta}{2}\left(x_i^{ki}(\mu)\right)^2 - p_i^{ki}(\mu) x_i^{ki}(\mu)\right)|_{F^{ki} < \gamma^{ki}}
\end{aligned}$$

which simplifies to $S_i = \delta_i^{ij} + G\left(\gamma^{ij}\right)\Delta_i^{ij} + \delta_i^{jk} + G\left(\gamma^{jk}\right)\Delta_i^{jk} + \delta_i^{ki} G\left(\gamma^{ki}\right)\Delta_i^{ki}$, where $\delta_i^{ij} := n_i \left(\frac{\left(3\alpha - 3w - \tau_{ij}\right)^2}{32\beta}\right)$, $\delta_i^{jk} := n_i \left(\frac{\left(3\alpha - 3w - 2\tau_{ij} - \tau_{ik}\right)^2}{32\beta}\right)$, $\delta_i^{ki} := n_i \left(\frac{(3\alpha - 3w - 2\tau_{ik})^2}{32\beta}\right)$, $\Delta_i^{ij} := n_i \left[\left(\frac{\left(3\alpha - 3w - 2\tau_{ij}\right)^2}{32\beta}\right) - \left(\frac{\left(3\alpha - 3w - \tau_{ij}\right)^2}{32\beta}\right)\right]$, $\Delta_i^{jk} := n_i \left[\left(\frac{\left(3\alpha - 3w - \tau_{ij} - 2\tau_{ik}\right)^2}{32\beta}\right) - \left(\frac{\left(3\alpha - 3w - 2\tau_{ij} - \tau_{ik}\right)^2}{32\beta}\right)\right]$, and $\Delta_i^{ki} := n_i \left[\left(\frac{(3\alpha - 3w - \tau_{ik})^2}{32\beta}\right) - \left(\frac{(3\alpha - 3w - 2\tau_{ik})^2}{32\beta}\right)\right]$.

The first-order condition with respect to the business tax (wage income is constant)

$$\frac{d\left(S_i + T_i\right)}{dt_i} = \frac{1}{\overline{F} - \underline{F}}\left(\Delta_i^{ij}\frac{d\gamma^{ij}}{dt_i} + \Delta_i^{ki}\frac{d\gamma^{ki}}{dt_i}\right) + 3 - G\left(\gamma^{ij}\right) + G\left(\gamma^{ki}\right) + t_i\frac{1}{\overline{F} - \underline{F}}\left(-\frac{d\gamma^{ij}}{dt_i} + \frac{d\gamma^{ki}}{dt_i}\right) = 0 \tag{3.14}$$

is a sufficient condition for a maximum by the concavity of welfare because

$$\frac{d^2\left(S_i + T_i\right)}{dt_i^2} = \frac{1}{\overline{F} - \underline{F}}\left(-\frac{d\gamma^{ij}}{dt_i} + \frac{d\gamma^{ki}}{dt_i}\right) + \frac{1}{\overline{F} - \underline{F}}\left(-\frac{d\gamma^{ij}}{dt_i} + \frac{d\gamma^{ki}}{dt_i}\right) = -\frac{4}{\overline{F} - \underline{F}} < 0.$$

Country i's reaction function is therefore given by

$$t_i = \frac{1}{4}\left(\Delta_i^{ij} - \Delta_i^{ki} + 3\overline{F} - 3\underline{F} + \pi_i^{ij} + \pi_i^{ki} - \pi_j^{ij} - \pi_k^{ki} + t_j + t_k\right). \tag{3.15}$$

Notice that t_i is linear in t_j and t_k. As standard in most of the tax competition literature, business taxes are strategic complements. Moreover, the slope of the reaction functions is less than 1. Hence, this system of equations exhibits a unique solution

$$t_i = \frac{3}{2}\left(\overline{F} - \underline{F}\right) + \frac{3}{10}\left(\Delta_i^{ij} - \Delta_i^{ki}\right) + \frac{1}{10}\left(\Delta_j^{jk} - \Delta_j^{ij}\right) + \frac{1}{10}\left(\Delta_k^{ki} - \Delta_k^{jk}\right) + \frac{1}{5}\left(\pi_i^{ij} + \pi_i^{ki} - \pi_j^{ij} - \pi_k^{ki}\right). \tag{3.16}$$

3.A.2 Proof of Lemma 6 and Proposition 10

By differentiating Equation 3.16[23], and using our assumption of non-negative trade flows, $\tau_{ij} \leq \frac{\alpha-w}{3}$ for all i, j, a technical statement of Lemma 6 follows.

Lemma 8. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K=3$ countries. For any $i, j, k \in \mathscr{K}$ the following Nash equilibrium comparative statics hold for t_i*

(a) *with respect to country sizes:* $\frac{dt_i}{dn_i} = 3\tau_{ij}\frac{2(\alpha-w)-\tau_{ij}}{320\beta} + 3\tau_{ik}\frac{2(\alpha-w)-\tau_{ik}}{320\beta}$ *and* $\frac{dt_i}{dn_j} = 9\tau_{jk}\frac{2(\alpha-w)-\tau_{jk}}{320\beta} - 27\tau_{ij}\frac{2(\alpha-w)-\tau_{ij}}{320\beta}$

(b) *with respect to trade costs:* $\frac{dt_i}{d\tau_{jk}} = 9\left(n_j+n_k\right)\frac{\alpha-w-\tau_{jk}}{160\beta} > 0$ *and* $\frac{dt_i}{d\tau_{ij}} = \left(3n_i - 27n_j\right)\frac{\alpha-w-\tau_{ij}}{160\beta}$.

By summing up the expressions in part (b) of Lemma 8, we obtain Corollary 2.

Corollary 2. *For any $i, j, k \in \mathscr{K}$,* $\frac{d\frac{1}{2}\left(t_i+t_j\right)}{d\tau_{ij}} = -12\left(n_i+n_j\right)\frac{\alpha-w-\tau_{ij}}{160\beta}$, $\frac{d\frac{1}{2}\left(t_i+t_k\right)}{d\tau_{ij}} = \left(6n_i - 9n_j\right)\frac{\alpha-w-\tau_{ij}}{160\beta}$, *and* $\frac{d\frac{1}{3}\sum_{k\in\mathscr{K}} t_k}{d\tau_{ij}} = -5\left(n_i+n_j\right)\frac{\alpha-w-\tau_{ij}}{160\beta}$.

The technical formulation of Proposition 10 also trivially follows from Lemma 8.

Proposition 17 (trade-cost effect)**.** *Consider the subgame-perfect equilibrium of economy $\mathscr{E}$ with $K=3$. Let $\tau_{ik} = \tau_{jk}$. Then, the disintegration of country k via a rise in bilateral trade costs with countries i and j has the following tax effects*

(a) $\frac{dt_i}{d\tau_{ik}} + \frac{dt_i}{d\tau_{jk}} = \left(3n_i + 9n_j - 18n_k\right)\frac{\alpha-w-\tau}{160\beta}$ *and*

(b) $\frac{dt_k}{d\tau_{ik}} + \frac{dt_k}{d\tau_{jk}} = \left(6n_k - 27n_i - 27n_j\right)\frac{\alpha-w-\tau}{160\beta}$.

Under symmetric population sizes of all three countries, the disintegration reduces taxes in all countries.

Observe that the assumption of identical trade costs, $\tau_{ik} = \tau_{jk}$, is not very restrictive. In particular, the insights about the role of market sizes remain unchanged. For $\tau_{ik} \neq \tau_{jk}$, the signs of the comparative statics are as follows $sign\left(\frac{dt_i}{d\tau_{ik}} + \frac{dt_i}{d\tau_{jk}}\right) = sign\left(3n_j + n_i - 6n_k + 3\left(n_j+n_k\right)\frac{\tau_{ik}-\tau_{jk}}{\alpha-w-\tau_{ik}}\right)$ and $sign\left(\frac{dt_k}{d\tau_{ik}} + \frac{dt_k}{d\tau_{jk}}\right) = sign\left(2n_k - 9n_i - 9n_j + \left(n_k - 9n_j\right)\frac{\tau_{ik}-\tau_{jk}}{\alpha-w-\tau_{ik}}\right)$.The correction term on the right side of the two previous lines adjusts for asymmetries in trade costs. Using the assumption on the primitives that ensure positive consumption choices, $\tau_{ij} \in \left[0, \frac{\alpha-w}{3}\right]$, one may evaluate the adjustment's magnitude: $\left|\frac{\tau_{ik}-\tau_{jk}}{\alpha-w-\tau_{ik}}\right| \in \left[0, \frac{1}{2}\right]$. Therefore, even for large asymmetries in trade costs, the adjustment term is comparably small. The central intuitions carry over.

[23] If not stated otherwise, the equation numbering relates to the equations in this Appendix and not to those in the main text.

3.B Proofs for Section 3.2.2

3.B.1 Proof of Proposition 11

First and similar to before, the first-order condition of the benevolent social planner in country i reads as

$$\frac{d\left(S_i+T_i\right)}{dt_i}=\Delta_i^{ij}\frac{d\gamma^{ij}}{dt_i}g^{ij}\left(\gamma^{ij}\right)+\Delta_i^{ki}\frac{d\gamma^{ki}}{dt_i}g^{ki}\left(\gamma^{ki}\right)+3-G^{ij}\left(\gamma^{ij}\right)+G^{ki}\left(\gamma^{ki}\right)+t_i\left(-g^{ij}\left(\gamma^{ij}\right)\frac{d\gamma^{ij}}{dt_i}+g^{ki}\left(\gamma^{ki}\right)\frac{d\gamma^{ki}}{dt_i}\right)=0 \tag{3.17}$$

which is necessary and sufficient by the second-order condition

$$\frac{d^2\left(S_i+T_i\right)}{dt_i^2}=-2g^{ij}\left(\gamma^{ij}\right)\frac{d\gamma^{ij}}{dt_i}+2g^{ki}\left(\gamma^{ki}\right)\frac{d\gamma^{ki}}{dt_i}=-\frac{1}{\overline{F}^{ij}}-\frac{1}{\overline{F}^{ki}}<0.$$

Under the symmetry assumptions (country sizes and trade costs), we can simplify the first-order condition to

$$\Delta\left(\frac{1}{2\overline{F}^{ij}}+\frac{1}{2\overline{F}^{ki}}\right)+3+t_j\frac{1}{2\overline{F}^{ij}}+t_k\frac{1}{2\overline{F}^{ki}}=t_i\left(\frac{1}{\overline{F}^{ij}}+\frac{1}{\overline{F}^{ki}}\right)$$

for every $i\in\mathscr{K}$ and $i\neq j,k$ where $\Delta:=n\left[\left(\frac{(3\alpha-3w-2\tau)^2}{32\beta}\right)-\left(\frac{(3\alpha-3w-\tau)^2}{32\beta}\right)\right]$. The intersection of the reaction functions delivers the following Nash equilibrium business tax

$$t_i=\frac{21\left(\overline{F}^{ij}\right)^2\overline{F}^{jk}\overline{F}^{ki}+24\overline{F}^{ij}\left(\overline{F}^{jk}\right)^2\overline{F}^{ki}+21\overline{F}^{ij}\overline{F}^{jk}\left(\overline{F}^{ki}\right)^2+9\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{ki}\right)^2}{3\left(\overline{F}^{ij}\right)^2\left[\overline{F}^{jk}+\overline{F}^{ki}\right]+3\left(\overline{F}^{jk}\right)^2\left[\overline{F}^{ij}+\overline{F}^{ki}\right]+3\left(\overline{F}^{ki}\right)^2\left[\overline{F}^{ij}+\overline{F}^{jk}\right]+7\overline{F}^{ij}\overline{F}^{jk}\overline{F}^{ki}}+\Delta. \tag{3.18}$$

Now, recalling $\overline{F}^{ij}=\overline{\varepsilon}^{ij}+\overline{\varepsilon}$, take derivatives

$$\begin{aligned}\frac{dt_i}{d\overline{\varepsilon}^{ij}}=\sigma^{-1}3\overline{F}^{ki}\Big(&-3\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{jk}\right)^3+13\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{jk}\right)^2\overline{F}^{ki}+21\left(\overline{F}^{ij}\right)^2\overline{F}^{jk}\left(\overline{F}^{ki}\right)^2+9\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{ki}\right)^3+42\overline{F}^{ij}\left(\overline{F}^{jk}\right)^3\overline{F}^{ki}\\&+60\overline{F}^{ij}\left(\overline{F}^{jk}\right)^2\left(\overline{F}^{ki}\right)^2+18\overline{F}^{ij}\overline{F}^{jk}\left(\overline{F}^{ki}\right)^3+24\left(\overline{F}^{jk}\right)^4\overline{F}^{ki}+45\left(\overline{F}^{jk}\right)^3\left(\overline{F}^{ki}\right)^2+21\left(\overline{F}^{jk}\right)^2\left(\overline{F}^{ki}\right)^3\Big)\end{aligned} \tag{3.19}$$

and

$$\begin{aligned}\frac{dt_i}{d\overline{\varepsilon}^{jk}}=\sigma^{-2}3\overline{F}^{ij}\overline{F}^{ki}\Big(&12\left(\overline{F}^{ij}\right)^3\overline{F}^{ki}+3\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{jk}\right)^2+30\left(\overline{F}^{ij}\right)^2\overline{F}^{jk}\overline{F}^{ki}+21\left(\overline{F}^{ij}\right)^2\left(\overline{F}^{ki}\right)^2\\&+14\overline{F}^{ij}\left(\overline{F}^{jk}\right)^2\overline{F}^{ki}+30\overline{F}^{ij}\overline{F}^{jk}\left(\overline{F}^{ki}\right)^2+12\overline{F}^{ij}\left(\overline{F}^{ki}\right)^3+3\left(\overline{F}^{jk}\right)^2\left(\overline{F}^{ki}\right)^2\Big)\end{aligned} \tag{3.20}$$

where $\sigma^{-2}>0$. A technical statement of Proposition 11 follows.

Proposition 18 (de-harmonization effect). *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K=3$ countries. Suppose that trade costs and country sizes are identical: $\tau := \tau_{ij} = \tau_{ik} = \tau_{jk}$ and $n := n_i = n_j = n_k$ for $i,j,k \in \mathscr{K}$.*

(a) Then, for any $i,j,k \in \mathscr{K}$ $\frac{dt_i}{d\bar{\varepsilon}^{jk}} > 0$. Moreover, $\frac{dt_i}{d\bar{\varepsilon}^{ij}} > 0$ for either $\overline{F}^{ij} \approx \overline{F}^{jk} \approx \overline{F}^{ki}$, or $\overline{F}^{ij} \approx 0$, or $\overline{F}^{jk} \approx 0$. However, if $\overline{F}^{ki} \approx 0$, $\frac{dt_i}{d\bar{\varepsilon}^{ij}} < 0$.

(b) Suppose that i and j form an economic union, i.e. $\overline{F}^{jk} = \overline{F}^{ki} \geq \overline{F}^{ij}$. Then, $\frac{dt_i}{d\bar{\varepsilon}^{jk}} + \frac{dt_i}{d\bar{\varepsilon}^{ki}} > 0$, $\frac{dt_j}{d\bar{\varepsilon}^{jk}} + \frac{dt_j}{d\bar{\varepsilon}^{ki}} > 0$, and $\frac{dt_k}{d\bar{\varepsilon}^{jk}} + \frac{dt_k}{d\bar{\varepsilon}^{ki}} > 0$. Hence, the disintegration of country k raises taxes everywhere.

Therefore, $\frac{dt_i}{d\bar{\varepsilon}^{jk}}$ is always positive. The sign of $\frac{dt_i}{d\bar{\varepsilon}^{ij}}$ (by a resembling argument, the sign of $\frac{dt_i}{d\bar{\varepsilon}^{ki}}$) depends on the relation between $\overline{F}^{ij}$, $\overline{F}^{jk}$, and $\overline{F}^{ki}$. Notice that for $\overline{F}^{ij} \approx \overline{F}^{jk} \approx \overline{F}^{ki}$, for $\overline{F}^{ij} \approx 0$, or for $\overline{F}^{jk} \approx 0$, $\frac{dt_i}{d\bar{\varepsilon}^{ij}} > 0$. Indeed, there exists is a set of weaker conditions sufficient for a positive sign, e.g. $4\overline{F}^{ki} > \overline{F}^{ji}$, $14\overline{F}^{ki} > \overline{F}^{ij}$, $6\overline{F}^{jk} > \overline{F}^{ij}$, or $\overline{F}^{jk} \approx \overline{F}^{ki}$. Notice, however, that for any $\overline{F}^{ki} > 0$ with $\overline{F}^{ki} \approx 0$, we can find a $\left(\overline{F}^{ij}\right)^2 \left(\overline{F}^{jk}\right)^3 > 0$ such that $\frac{dt_i}{d\bar{\varepsilon}^{ij}} < 0$.

Observe that $\frac{dt_i}{d\bar{\varepsilon}^{ij}} + \frac{dt_i}{d\bar{\varepsilon}^{ki}}$ is always positive. Suppose that i and j form an economic union (i.e., $\overline{F}^{jk} = \overline{F}^{ki} \geq \overline{F}^{ij}$) and that k disintegrates. Then, t_k increases because $\frac{dt_j}{d\bar{\varepsilon}^{jk}} + \frac{dt_j}{d\bar{\varepsilon}^{ki}} > 0$. It is easy to see that the business tax in any member country i increases as well, i.e., $\frac{dt_i}{d\bar{\varepsilon}^{jk}} + \frac{dt_i}{d\bar{\varepsilon}^{ki}} > 0$ for $\overline{F}^{jk} = \overline{F}^{ki}$.

Corollary 3 directly follows from the expressions derived for Proposition 18. As we can see, average taxes in any two or more countries are negatively associated with firm mobility.

Corollary 3. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K=3$ countries. Under the symmetry assumptions of Proposition 18, average taxes between any two and among all three countries increase with a reduction in harmonization, that is, for any $i,j,k \in \mathscr{K}$, $\frac{d\frac{1}{2}\left(t_i+t_j\right)}{d\bar{\varepsilon}^{ij}} > 0$, $\frac{d\frac{1}{2}(t_i+t_k)}{d\bar{\varepsilon}^{ij}} > 0$, and $\frac{d\frac{1}{3}\sum_{k\in\mathscr{K}} t_k}{d\bar{\varepsilon}^{ij}} > 0$.*

3.B.2 Proof of Proposition 12

Again, the first-order condition of the social planner in country i is described by Equation (3.17). Then, using $\overline{F}^{ij} - \underline{F}^{ij} = \overline{F}^{jk} - \underline{F}^{jk} = \overline{F}^{ki} - \underline{F}^{ki}$, the reaction function in country i reads as

$$t_i = \frac{1}{4}\left(\Delta_i^{ij} - \Delta_i^{ki} + 3\overline{F} - 3\underline{F} + \pi_i^{ij} + \pi_i^{ki} - \pi_j^{ij} - \pi_k^{ki} + t_j + t_k + \nu^{ij} - \nu^{ki}\right). \tag{3.21}$$

This set of reactions functions implies the equilibrium business tax in country i

$$t_i = \frac{3}{2}\left(\overline{F} - \underline{F}\right) + \frac{3}{10}\left(\Delta_i^{ij} - \Delta_i^{ki}\right) + \frac{1}{10}\left(\Delta_j^{jk} - \Delta_j^{ij}\right) + \frac{1}{10}\left(\Delta_k^{ki} - \Delta_k^{jk}\right) + \frac{1}{5}\left(\pi_i^{ij} + \pi_i^{ki} - \pi_j^{ij} - \pi_k^{ki} + \nu^{ij} - \nu^{ki}\right). \tag{3.22}$$

One can immediately observe that $\frac{dt_i}{d\nu^{ij}} = \frac{1}{5} > 0$, $\frac{dt_i}{d\nu^{ki}} = -\frac{1}{5} < 0$, and $\frac{dt_i}{d\nu^{jk}} = 0$. A formal statement of Proposition 12 follows.

Proposition 19 (business-friction effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K = 3$ countries. For any $i, j, k \in \mathscr{K}$ $\frac{dt_i}{d\nu^{ij}} > 0$, $\frac{dt_i}{d\nu^{ki}} < 0$, and $\frac{dt_i}{d\nu^{jk}} = 0$.*

3.B.3 Proof of Proposition 13

The formal version of Proposition 20 follows from the comparative statics of Lemma 8.

Proposition 20 (migration effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K = 3$ countries. For any $i, j, k \in \mathscr{K}$ one can derive the following Nash equilibrium comparative statics for t_i from disintegration induced population shifts*

(a) $\frac{dt_i}{dn_i} - \frac{dt_i}{dn_j} = 30\tau_{ij}\frac{2(\alpha-w)-\tau_{ij}}{320\beta} + 3\tau_{ik}\frac{2(\alpha-w)-\tau_{ik}}{320\beta} - 9\tau_{jk}\frac{2(\alpha-w)-\tau_{jk}}{320\beta}$ *and*

(b) $\frac{dt_i}{dn_j} - \frac{dt_i}{dn_k} = 27\left(\tau_{ik} - \tau_{ij}\right)\frac{2(\alpha-w)-\left(\tau_{ik}+\tau_{ij}\right)}{320\beta}$.

Migration into an integrated area raises taxes inside the area and lowers the tax outside.

Corollary 4 regards the effect of migration from country j to i on average taxes, holding $\sum_{l\in\mathscr{K}} n_l$ and n_k fixed.

Corollary 4. *For any $i, j, k \in \mathscr{K}$ and $i \neq j, k$, the effect of population shifts on average taxes are*

(a) $\frac{d\frac{1}{2}\left(t_i+t_j\right)}{dn_i} - \frac{d\frac{1}{2}\left(t_i+t_j\right)}{dn_j} = 3\left(\tau_{ik} - \tau_{jk}\right)\frac{2(\alpha-w)-\left(\tau_{ik}+\tau_{jk}\right)}{160\beta}$, *and*

(b) $\frac{d\frac{1}{3}\sum_{k\in\mathscr{K}} t_k}{dn_i} - \frac{d\frac{1}{3}\sum_{k\in\mathscr{K}} t_k}{dn_j} = 5\left(\tau_{jk} - \tau_{ik}\right)\frac{2(\alpha-w)-\left(\tau_{jk}+\tau_{ik}\right)}{320\beta}$.

3.C Proofs for Section 3.2.3

3.C.1 The K-Country Model in Section 2.3

Pre-tax profits in an ij-industry look very similar to those in the three-country case. Still, they depend on firm relocation in the following fashion

$$\pi_i^{ij}(\mu)=\begin{cases}\frac{n_i\left(\alpha-w+\tau_{ij}\right)^2}{16\beta}+\frac{n_j\left(\alpha-w-2\tau_{ij}\right)^2}{16\beta}+\sum_{l\in\mathscr{K}\setminus\{i,j\}}\frac{n_l\left(\alpha-w-2\tau_{il}+\tau_{jl}\right)^2}{16\beta} & if\ mobile\ firm\ locates\ in\ i\\ \frac{n_i\left(\alpha-w+2\tau_{ij}\right)^2}{16\beta}+\frac{n_j\left(\alpha-w-3\tau_{ij}\right)^2}{16\beta}+\sum_{l\in\mathscr{K}\setminus\{i,j\}}\frac{n_l\left(\alpha-w-3\tau_{il}+2\tau_{jl}\right)^2}{16\beta} & if\ mobile\ firm\ locates\ in\ j.\end{cases}\tag{3.23}$$

The mobile firm locates in country i if and only if $F^{ij}\geq\pi_j^{ij}(\mu)-t_j-\left(\pi_i^{ij}(\mu)-t_i\right):=\gamma^{ij}$. Again, simplifying the industry threshold becomes

$$\gamma^{ij}=(n_j-n_i)\frac{6\tau_{ij}(\alpha-w)-3\tau_{ij}^2}{16\beta}+\sum_{l\in\mathscr{K}\setminus\{i,j\}}n_l\left(\tau_{il}-\tau_{jl}\right)\frac{6(\alpha-w)-3\left(\tau_{il}+\tau_{jl}\right)}{16\beta}+t_i-t_j\tag{3.24}$$

and we derive partial equilibrium comparative statics as $\frac{d\gamma^{ij}}{dt_i}=1$, $\frac{d\gamma^{ij}}{dt_j}=-1$, $\frac{d\gamma^{ij}}{d\tau_{ij}}=(n_j-n_i)\frac{3(\alpha-w-\tau_{ij})}{8\beta}$, $\frac{d\gamma^{ij}}{d\tau_{il}}=n_l\frac{3(\alpha-w-\tau_{il})}{8\beta}$, and $\frac{d\gamma^{ij}}{d\tau_{jl}}=-n_l\frac{3(\alpha-w-\tau_{jl})}{8\beta}$for $j\neq l$.

Since $\gamma^{ij}=-\gamma^{ji}$ and $G()$ is symmetric with $\overline{F}=-\underline{F}$, Lemma 9 directly follows. It will prove convenient when deriving the objective function of the government.

Lemma 9. *Consider economy $\mathscr{E}$ with $K\geq 2$. Suppose that $\overline{F}=-\underline{F}$. Then, $G\left(\gamma^{ji}\right)=1-G\left(\gamma^{ij}\right)$. Moreover, the number of firms in country i is given by $k_i:=(K-1)+\frac{1}{2\overline{F}}\sum_{j\in\mathscr{K}\setminus i}\left(\overline{F}-\gamma^{ij}\right)$.*

Since there are K countries, one has to consider $\begin{pmatrix}K\\2\end{pmatrix}=\frac{K(K-1)}{2}$ continuums of industries yielding $K(K-1)$ different prices. These read as $p_i^{ij}(\mu)=\frac{\alpha+3w+k_j^*(\mu)\tau_{ij}}{4}$ for $k_j^*(\mu)\in\{1,2\}$ with $j\neq i$ and $p_i^{jl}(\mu)=\frac{\alpha+3w+k_j^*(\mu)\tau_{ij}+k_l^*(\mu)\tau_{il}}{4}$ for $\left(k_j^*(\mu),k_l^*(\mu)\right)\in\{(1,2),(2,1)\}$ with $j,l\neq i$. Plug into the demand functions $x_i^{ij}(\mu)=\frac{\alpha-p_i^{ij}(\mu)}{\beta}$ and $x_i^{jl}(\mu)=\frac{\alpha-p_i^{jl}(\mu)}{\beta}$ and sum over all households in a country. The aggregate surplus in country i derived from consumption of goods in ij- and jl-industries simplify to

$$S_i^{ij}(\mu)=n_i\left(\alpha x_i^{ij}(\mu)-\frac{\beta}{2}\left(x_i^{ij}(\mu)\right)^2-p_i^{ij}(\mu)x_i^{ij}(\mu)\right)=\begin{cases}n_i\frac{\left(3\alpha-3w-\tau_{ij}\right)^2}{32\beta} & w/\ prob\ \left(1-G\left(\gamma^{ij}\right)\right)\\ n_i\frac{\left(3\alpha-3w-2\tau_{ij}\right)^2}{32\beta} & w/\ prob\ G\left(\gamma^{ij}\right)\end{cases},\ and\tag{3.25}$$

$$S_i^{jl}(\mu) = n_i\left(\alpha x_i^{jl}(\mu) - \frac{\beta}{2}\left(x_i^{jl}(\mu)\right)^2 - p_i^{jl}(\mu)x_i^{jl}(\mu)\right) = \begin{cases} n_i\frac{\left(3\alpha-3w-2\tau_{ij}-\tau_{il}\right)^2}{32\beta} & w/\ prob\ \left(1-G\left(\gamma^{jl}\right)\right) \\ n_i\frac{\left(3\alpha-3w-\tau_{ij}-2\tau_{il}\right)^2}{32\beta} & w/\ prob\ G\left(\gamma^{jl}\right) \end{cases}. \tag{3.26}$$

Summing over industries gives the total surplus

$$\begin{aligned} S_i = & \sum_{j\in\mathscr{K}\setminus\{i\}}\left[\left(1-G\left(\gamma^{ij}\right)\right)n_i\frac{(3\alpha-3w-\tau_{ij})^2}{32\beta} + G\left(\gamma^{ij}\right)n_i\frac{(3\alpha-3w-2\tau_{ij})^2}{32\beta}\right] \\ & + \frac{1}{2}\sum_{j\in\mathscr{K}\setminus\{i\}}\ \sum_{l\in\mathscr{K}\setminus\{i,j\}}\left[\left(1-G\left(\gamma^{jl}\right)\right)n_i\frac{(3\alpha-3w-2\tau_{ij}-\tau_{il})^2}{32\beta} + G\left(\gamma^{jl}\right)n_i\frac{(3\alpha-3w-\tau_{ij}-2\tau_{il})^2}{32\beta}\right] \\ = & \sum_{j\in\mathscr{K}\setminus\{i\}}\left[\underbrace{n_i\frac{(3\alpha-3w-\tau_{ij})^2}{32\beta}}_{:=\delta_i^{ij}} + \frac{\gamma^{ij}-\underline{F}}{2\overline{F}}\underbrace{n_i\frac{(3\alpha-3w-2\tau_{ij})^2-(3\alpha-3w-\tau_{ij})^2}{32\beta}}_{:=\Delta_i^{ij}}\right] \\ & + \frac{1}{2}\sum_{j\in\mathscr{K}\setminus\{i\}}\ \sum_{l\in\mathscr{K}\setminus\{i,j\}}\left[\underbrace{n_i\frac{(3\alpha-3w-2\tau_{ij}-\tau_{il})^2}{32\beta}}_{:=\delta_i^{jl}}\right] \\ & + \frac{1}{2}\sum_{j\in\mathscr{K}\setminus\{i\}}\ \sum_{l\in\mathscr{K}\setminus\{i,j\}}\left[\frac{\gamma^{jl}-\underline{F}}{2\overline{F}}\underbrace{n_i\frac{(3\alpha-3w-\tau_{ij}-2\tau_{il})^2-(3\alpha-3w-2\tau_{ij}-\tau_{il})^2}{32\beta}}_{:=\Delta_i^{jl}}\right], \end{aligned}$$

where the factor $\frac{1}{2}$ is applied to avoid double count. Therefore, consumer surplus in country i can be written as

$$S_i = \sum_{j\in\mathscr{K}\setminus\{i\}}\left[\delta_i^{ij} + \frac{\gamma^{ij}-\underline{F}}{2\overline{F}}\Delta_i^{ij}\right] + \frac{1}{2}\sum_{j\in\mathscr{K}\setminus\{i\}}\ \sum_{l\in\mathscr{K}\setminus\{i,j\}}\left[\delta_i^{jl} + \frac{\gamma^{jl}-\underline{F}}{2\overline{F}}\Delta_i^{jl}\right] \tag{3.27}$$

where Δ_i^{ij} , Δ_i^{jl}, δ_i^{ij} and δ_i^{jl} are functions of the model primitives Θ. Accordingly, the social planner in country i faces the following maximization problem

$$\max_{t_i} S_i + T_i + n_i w$$

where

$$T_i = t_i \left[(K-1) + \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\overline{F} - \gamma^{ij} \right) \right]. \tag{3.28}$$

The first-order condition is given by

$$\frac{d\left(S_i + T_i\right)}{dt_i} = \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \frac{d\gamma^{ij}}{dt_i} \Delta_i^{ij} + (K-1) + \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\overline{F} - \gamma^{ij} \right) + t_i \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \left(-\frac{d\gamma^{ij}}{dt_i} \right) = 0 \tag{3.29}$$

which is sufficient by the second-order condition

$$\frac{d^2\left(S_i + T_i\right)}{dt_i^2} = \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \left(-\frac{d\gamma^{ij}}{dt_i} \right) + \frac{1}{2\overline{F}} \sum_{j \in \mathscr{K} \setminus \{i\}} \left(-\frac{d\gamma^{ij}}{dt_i} \right) = -\frac{(K-1)}{\overline{F}} < 0.$$

The reaction function of country i can be simplified to

$$t_i = \frac{1}{2(K-1)} \left(\sum_{j \in \mathscr{K} \setminus \{i\}} \Delta_i^{ij} + 3\overline{F}(K-1) + \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\pi_i^{ij} - \pi_j^{ij} \right) + \sum_{j \in \mathscr{K} \setminus \{i\}} t_j \right). \tag{3.30}$$

Again, business taxes are strategic complements, the relation is linear, and the slope is less than 1. Thus, there will be a unique interior intersection of reaction functions in this tax competition game. In the following, we derive this intersection. First of all, plug

$$\begin{aligned}
t_i - t_l = \frac{1}{K-1} & \left(\sum_{j \in \mathscr{K} \setminus \{i\}} \Delta_i^{ij} + 3\overline{F}(K-1) - \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\pi_j^{ij} - \pi_i^{ij} + t_i - t_j \right) \right. \\
& \left. - \sum_{j \in \mathscr{K} \setminus \{l\}} \Delta_l^{lj} - 3\overline{F}(K-1) + \sum_{j \in \mathscr{K} \setminus \{l\}} \left(\pi_j^{lj} - \pi_l^{lj} + t_l - t_j \right) \right) \\
= \frac{1}{K-1} & \left(\sum_{j \in \mathscr{K} \setminus \{i\}} \Delta_i^{ij} - \sum_{j \in \mathscr{K} \setminus \{l\}} \Delta_l^{lj} + \sum_{j \in \mathscr{K} \setminus \{l\}} \left(\pi_j^{lj} - \pi_l^{lj} \right) - \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\pi_j^{ij} - \pi_i^{ij} \right) \right. \\
& \left. + \sum_{j \in \mathscr{K}} (t_l - t_j) - (t_l - t_l) + \sum_{j \in \mathscr{K}} (t_j - t_i) - (t_i - t_i) \right) \\
= \frac{1}{K-1} & \left(\sum_{j \in \mathscr{K} \setminus \{i\}} \Delta_i^{ij} - \sum_{j \in \mathscr{K} \setminus \{l\}} \Delta_l^{lj} + \sum_{j \in \mathscr{K} \setminus \{l\}} \left(\pi_j^{lj} - \pi_l^{lj} \right) - \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\pi_j^{ij} - \pi_i^{ij} \right) + K(t_l - t_i) \right) \\
= \frac{1}{2K-1} & \left(\sum_{j \in \mathscr{K} \setminus \{i\}} \Delta_i^{ij} - \sum_{j \in \mathscr{K} \setminus \{l\}} \Delta_l^{lj} + \sum_{j \in \mathscr{K} \setminus \{l\}} \left(\pi_j^{lj} - \pi_l^{lj} \right) - \sum_{j \in \mathscr{K} \setminus \{i\}} \left(\pi_j^{ij} - \pi_i^{ij} \right) \right)
\end{aligned}$$

into

$$\begin{aligned}
t_i &= \frac{1}{K-1}\left(\sum_{j\in\mathscr{K}\setminus\{i\}}\Delta_i^{ij} + 3\overline{F}(K-1) - \sum_{j\in\mathscr{K}\setminus\{i\}}\left(\pi_j^{ij}-\pi_i^{ij}\right) - \sum_{j\in\mathscr{K}\setminus\{i\}}(t_i - t_j)\right)\\
&= 3\overline{F} + \frac{K}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\setminus\{i\}}\Delta_i^{ij} + \frac{K}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\setminus\{i\}}\left(\pi_i^{ij}-\pi_j^{ij}\right)\\
&+ \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}}\sum_{m\in\mathscr{K}\setminus\{j\}}\Delta_j^{jm} - \frac{1}{(K-1)(2K-1)}\sum_{m\in\mathscr{K}\setminus\{i\}}\Delta_i^{im}\\
&- \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}}\sum_{m\in\mathscr{K}\setminus\{j\}}\left(\pi_m^{jm}-\pi_j^{jm}\right) + \frac{1}{(K-1)(2K-1)}\sum_{m\in\mathscr{K}\setminus\{i\}}\left(\pi_m^{im}-\pi_i^{im}\right)\\
&= 3\overline{F} + \frac{1}{2K-1}\sum_{j\in\mathscr{K}\setminus\{i\}}\Delta_i^{ij} + \frac{1}{2K-1}\sum_{j\in\mathscr{K}\setminus\{i\}}\left(\pi_i^{ij}-\pi_j^{ij}\right)\\
&+ \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}}\sum_{m\in\mathscr{K}\setminus\{j\}}\Delta_j^{jm} - \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}}\sum_{m\in\mathscr{K}\setminus\{j\}}\left(\pi_m^{jm}-\pi_j^{jm}\right).
\end{aligned}$$

Then, notice that

$$\sum_{j\in\mathscr{K}}\sum_{m\in\mathscr{K}\setminus\{j\}}\left(\pi_m^{jm}-\pi_j^{jm}\right) = \sum_j\sum_{m>j}\left(\pi_m^{jm}-\pi_j^{jm}\right) - \sum_j\sum_{m>j}\left(\pi_m^{jm}-\pi_j^{jm}\right) = 0 \tag{3.31}$$

to obtain Lemma 10.

Lemma 10. *Consider economy $\mathscr{E}$ with K countries. Suppose that $\overline{F} = -\underline{F}$. Then, the subgame-perfect Nash equilibrium of the tax competition game is given by*

$$t_i = 3\overline{F} + \frac{1}{2K-1}\sum_{j\in\mathscr{K}\setminus\{i\}}\Delta_i^{ij} + \frac{1}{2K-1}\sum_{j\in\mathscr{K}\setminus\{i\}}\left(\pi_i^{ij}-\pi_j^{ij}\right) + \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}}\sum_{l\in\mathscr{K}\setminus\{j\}}\Delta_j^{jl}$$

for any $i\in\mathscr{K}$.

One can immediately see that $\frac{dt_i}{d\overline{F}} > 0$. This statement is a standard result from the literature on tax competition. A rise in $\overline{F}$ widens the range of relative fixed costs. Some industries will choose to stay in country i no matter how large the tax differential is.

We now derive further comparative statics. Since

$$\pi_i^{ij} - \pi_j^{ij} = (n_i - n_j)\frac{6\tau_{ij}(\alpha - w) - 3\tau_{ij}^2}{16\beta} - \sum_{l\in\mathscr{K}\setminus\{i,j\}} n_l \frac{6(\alpha - w)(\tau_{il} - \tau_{jl}) - 3\left(\tau_{il}^2 - \tau_{jl}^2\right)}{16\beta}, \tag{3.32}$$

differentiation with respect to trade costs yields $\frac{d\left(\pi_i^{ij}-\pi_j^{ij}\right)}{d\tau_{ij}} = 6\left(n_i - n_j\right)\frac{\alpha-w-\tau_{ij}}{16\beta}$, $\frac{d\left(\pi_i^{ij}-\pi_j^{ij}\right)}{d\tau_{il}} = -6n_l\frac{\alpha-w-\tau_{il}}{16\beta}$, $\frac{d\left(\pi_i^{ij}-\pi_j^{ij}\right)}{d\tau_{jl}} = 6n_l\frac{\alpha-w-\tau_{jl}}{16\beta}$, $\frac{d\left(\pi_i^{il}-\pi_l^{il}\right)}{d\tau_{il}} = 6\left(n_i - n_l\right)\frac{\alpha-w-\tau_{il}}{16\beta}$, $\frac{d\left(\pi_i^{il}-\pi_l^{il}\right)}{d\tau_{ij}} = -6n_j\frac{\alpha-w-\tau_{ij}}{16\beta}$, and $\frac{d\left(\pi_i^{il}-\pi_l^{il}\right)}{d\tau_{lj}} = 6n_j\frac{\alpha-w-\tau_{lj}}{16\beta}$. It is more convenient to write t_i as

$$t_i = 3\overline{F} + \frac{K}{(K-1)(2K-1)}\sum_{l\in\mathscr{K}\backslash\{i\}}\Delta_i^{il} + \frac{1}{2K-1}\sum_{l\in\mathscr{K}\backslash\{i\}}\left(\pi_i^{il}-\pi_l^{il}\right) + \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\backslash\{i\}}\sum_{l\in\mathscr{K}\backslash\{j\}}\Delta_j^{jl} \tag{3.33}$$

such that

$$\begin{aligned}\frac{dt_i}{d\tau_{ij}} &= \frac{K}{(K-1)(2K-1)}\left(-3n_i\frac{\alpha-w-\tau_{ij}}{16\beta}\right) + \frac{1}{2K-1}6\left(n_i-n_j\right)\frac{\alpha-w-\tau_{ij}}{16\beta}\\ &\quad + \frac{1}{2K-1}\sum_{l\in\mathscr{K}\backslash\{i,j\}}\left(-6n_j\frac{\alpha-w-\tau_{ij}}{16\beta}\right) + \frac{1}{(K-1)(2K-1)}\left(-3n_j\frac{\alpha-w-\tau_{ij}}{16\beta}\right)\end{aligned}$$

and

$$\begin{aligned}\frac{dt_i}{d\tau_{jk}} &= \frac{1}{2K-1}6n_j\frac{\alpha-w-\tau_{jk}}{16\beta} + \frac{1}{2K-1}6n_k\frac{\alpha-w-\tau_{jk}}{16\beta}\\ &\quad + \frac{1}{(K-1)(2K-1)}\left(-3n_j\frac{\alpha-w-\tau_{jk}}{16\beta}\right) + \frac{1}{(K-1)(2K-1)}\left(-3n_k\frac{\alpha-w-\tau_{jk}}{16\beta}\right).\end{aligned}$$

Furthermore, since

$$\begin{aligned}t_i &= 3\overline{F} + \frac{K}{(K-1)(2K-1)}3n_i\sum_{j\in\mathscr{K}\backslash\{i\}}\frac{\tau_{ij}^2-2\tau_{ij}(\alpha-w)}{32\beta}\\ &\quad + \frac{1}{2K-1}\sum_{j\neq i}\left(\left(n_i-n_j\right)\frac{6\tau_{ij}(\alpha-w)-3\tau_{ij}^2}{16\beta} + \sum_{l\in\mathscr{K}\backslash\{i,j\}}n_l\frac{6(\alpha-w)\left(\tau_{jl}-\tau_{il}\right)-3\left(\tau_{jl}^2-\tau_{il}^2\right)}{16\beta}\right)\\ &\quad + \frac{1}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\backslash\{i\}}\sum_{m\in\mathscr{K}\backslash\{j\}}3n_j\frac{\tau_{jm}^2-2\tau_{jm}(\alpha-w)}{32\beta},\end{aligned} \tag{3.34}$$

the comparative statics with respect to market size are

$$\begin{aligned}\frac{dt_i}{dn_i} &= \frac{K}{(K-1)(2K-1)}3\sum_{j\in\mathscr{K}\backslash\{i\}}\frac{\tau_{ij}^2-2\tau_{ij}(\alpha-w)}{32\beta} + \frac{1}{2K-1}\sum_{j\in\mathscr{K}\backslash\{i\}}\frac{6\tau_{ij}(\alpha-w)-3\tau_{ij}^2}{16\beta}\\ &= \frac{K-2}{(K-1)(2K-1)}3\sum_{j\in\mathscr{K}\backslash\{i\}}\tau_{ij}\frac{2(\alpha-w)-\tau_{ij}}{32\beta}\end{aligned} \tag{3.35}$$

and

$$\begin{aligned}\frac{dt_i}{dn_k} &= \frac{-1}{2K-1}\frac{6\tau_{ik}(\alpha-w)-3\tau_{ik}^2}{16\beta}+\frac{1}{2K-1}\sum_{j\in\mathscr{K}\setminus\{i,k\}}\frac{6(\alpha-w)\left(\tau_{jk}-\tau_{ik}\right)-3\left(\tau_{jk}^2-\tau_{ik}^2\right)}{16\beta}\\ &+\frac{1}{(K-1)(2K-1)}\sum_{m\in\mathscr{K}\setminus\{k\}}3\frac{\tau_{km}^2-2\tau_{km}(\alpha-w)}{32\beta}\\ &= -\frac{6(K-1)^2+3}{(K-1)(2K-1)}\frac{2\tau_{ik}(\alpha-w)-\tau_{ik}^2}{32\beta}+\frac{6(K-1)-3}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\setminus\{i,k\}}\frac{2(\alpha-w)\tau_{jk}-\tau_{jk}^2}{32\beta}.\end{aligned} \tag{3.36}$$

Simplify these expressions to obtain Lemma 11.

Lemma 11. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K\geq 2$ countries. Then, for any $i,j,k\in\mathscr{K}$ one can derive the following Nash equilibrium comparative statics for t_i*

(a) with respect to country sizes $\frac{dt_i}{dn_i}=\frac{3(K-2)}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\setminus\{i\}}\tau_{ij}\frac{2(\alpha-w)-\tau_{ij}}{32\beta}$ *and*

$$\frac{dt_i}{dn_k}=\frac{6(K-1)-3}{(K-1)(2K-1)}\sum_{j\in\mathscr{K}\setminus\{i,k\}}\frac{2(\alpha-w)\tau_{jk}-\tau_{jk}^2}{32\beta}-\frac{6(K-1)^2+3}{(K-1)(2K-1)}\frac{2\tau_{ik}(\alpha-w)-\tau_{ik}^2}{32\beta}$$

(b) with respect to trade costs $\frac{dt_i}{d\tau_{ij}}=\left(n_i(K-2)-2n_j\left[(K-1)^2+0.5\right]\right)\frac{3}{(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}$ *and* $\frac{dt_i}{d\tau_{jk}}=(n_j+n_k)\frac{3(2K-3)}{(K-1)(2K-1)}\frac{\alpha-w-\tau_{jk}}{16\beta}$.

To sum up, the intuitions from the three-country model hold. As already mentioned in the three-country setting, a country's size positively affects its ability to tax, whereas it is not clear how t_i reacts to an expansion of market k.

Furthermore, when trade costs between j and k rise, country i becomes relatively more attractive, which gives the latter country the leverage to tax more. Moreover, $\frac{dt_i}{d\tau_{ij}}$ will be negative if market i is not too large. Interestingly, the more countries there are, the larger market i has to be relative to j to have $\frac{dt_i}{d\tau_{ij}}>0$.

Similar to Corollary 2, we formulate Corollary 5.

Corollary 5. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K\geq 2$ countries. Define $\bar{t}:=\frac{1}{K}\sum_{k\in\mathscr{K}}t_k$, $\bar{t}_{EU}:=\frac{1}{K_{EU}}\sum_{k\in\mathscr{K}_{EU}}t_k$, and $\bar{t}_{nonEU}:=\frac{1}{K-K_{EU}}\sum_{k\in\mathscr{K}\setminus\mathscr{K}_{EU}}t_k$. Then,*

(a) for any $i,j,k\in\mathscr{K}$, $\frac{d\frac{1}{2}\left(t_i+t_j\right)}{d\tau_{ij}}=-\frac{3[(K-1)(2K-3)+2]\left(n_i+n_j\right)}{2(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}$,

$$\frac{d\frac{1}{2}(t_i+t_k)}{d\tau_{ij}}=\frac{3\left[n_i(3K-5)-n_j(2(K-1)(K-2)+2)\right]}{2(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}\text{ and }\frac{d\bar{t}}{d\tau_{ij}}=-\frac{3(n_i+n_j)}{K(K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta},$$

(b) *for* $i,j \in \mathscr{K}_{EU}$,

$$\frac{d\bar{t}_{EU}}{d\tau_{ij}} = -\frac{3\left[(K-K_{EU}+1)(2K-3)+2\right](n_i+n_j)}{K_{EU}(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta} \text{ and } \frac{d\bar{t}_{nonEU}}{d\tau_{ij}} = \frac{3(2K-3)(n_i+n_j)}{(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}.$$

(c) *for* $i \in \mathscr{K}_{EU}$ *and* $j \in \mathscr{K} \setminus \mathscr{K}_{EU}$,

$$\frac{d\bar{t}_{EU}}{d\tau_{ij}} = \frac{3\left(n_i\left[K-2+(K_{EU}-1)(2K-3)\right]-n_j\left[2(K-1)(K-K_{EU})+K_{EU}\right]\right)}{K_{EU}(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}$$

and

$$\frac{d\bar{t}_{nonEU}}{d\tau_{ij}} = \frac{3\left(n_j\left[K-2+(K-K_{EU}-1)(2K-3)\right]-n_i\left[2(K-1)K_{EU}+K-K_{EU}\right]\right)}{(K-K_{EU})(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}.$$

(d) *for* $i,j \in \mathscr{K} \setminus \mathscr{K}_{EU}$,

$$\frac{d\bar{t}_{EU}}{d\tau_{ij}} = \frac{3(2K-3)(n_i+n_j)}{(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta} \text{ and } \frac{d\bar{t}_{nonEU}}{d\tau_{ij}} = -\frac{3\left[(K_{EU}+1)(2K-3)+2\right](n_i+n_j)}{(K-K_{EU})(K-1)(2K-1)}\frac{\alpha-w-\tau_{ij}}{16\beta}$$

Part (a) of Corollary 5 is the K-country equivalent of Corollary 2. $(b)-(d)$ describe the effects of a rise in bilateral trade costs on average taxes. When trade between two member countries becomes more costly, members' taxes fall on average, whereas the average tax of non-member countries increases. On the contrary, the higher the bilateral trade costs for two non-member countries, the lower (higher) is the average tax of non-member (member) countries. Part (c) shows that the effects of a rise in trade costs between a member and a non-member country are unclear. They depend on relative sizes of the respective countries as well as the number of member countries.

3.C.2 Proof of Proposition 14

Similar to Proposition 17, we provide a formal statement of Proposition 14.

Proposition 21 (trade-cost effect)**.** *Consider the subgame-perfect Nash equilibrium of economy* $\mathscr{E}$ *with* $K \geq 2$ *countries. Suppose that trade costs between the leaving country* $l \in \mathscr{K} \setminus \mathscr{K}_{EU}$ *and countries* $m \in \mathscr{K}_{EU}$ *are the same,* $\tau = \tau_{ml}$, $\forall m \in \mathscr{K}_{EU}$, *and let country* l *disintegrate from the member countries. This triggers the following change in the tax of*

(a) *the leaving country* $l \in \mathscr{K} \setminus \mathscr{K}_{EU}$

$$\sum_{m \in \mathscr{K}_{EU}} \frac{dt_l}{d\tau_{ml}} = \frac{3K_{EU}(K-2)n_l - 3K_{EU}\left[2(K-1)^2+1\right]\bar{n}_{EU}}{(K-1)(2K-1)}\frac{\alpha-w-\tau}{16\beta}$$

(b) *the remaining member countries* $m \in \mathscr{K}_{EU}$

$$\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} = \frac{(K-1)\left[6K_{EU}\bar{n}_{EU} - 6n_l\left(K - K_{EU}\right) - 3n_m\right] + 3K_{EU}\left(n_l - \bar{n}_{EU}\right)}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}$$

and

(c) *third countries* $k \in \mathscr{K} \setminus \left(\mathscr{K}_{EU} \cup \{l\}\right)$

$$\sum_{j \in \mathscr{K}_{EU}} \frac{dt_k}{d\tau_{jl}} = \frac{3K_{EU}(2K-3)\left(\bar{n}_{EU} + n_l\right)}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}.$$

To show Proposition 21, we use Lemma 11. For part (a), take country l which is supposed to leave, in the sense that all bilateral trade costs between members and country l are going to increase, and sum $\frac{dt_l}{d\tau_{ml}}$ over all relevant country combinations (i.e., over the set $\mathscr{K}_{EU}$)

$$\begin{aligned}
\sum_{m \in \mathscr{K}_{EU}} \frac{dt_l}{d\tau_{ml}} &= \sum_{m \in \mathscr{K}_{EU}} \left(n_l(K-2) - 2n_m\left[(K-1)^2 + 0.5\right]\right) \frac{3}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} \\
&= \left(n_l K_{EU}(K-2) - \sum_{m \in \mathscr{K}_{EU}} n_m\left[2(K-1)^2 + 1\right]\right) \frac{3}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}. \qquad (3.37)
\end{aligned}$$

For $n := n_m = n_n$, we obtain a simpler expression $\sum_{m=1}^{K_{EU}} \frac{dt_n}{d\tau_{mn}} = \left(5K - 5 - 2K^2\right) \frac{3K_{EU}n}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} < 0$.

Proceed similarly to obtain the reaction of a member country $m \in \mathscr{K}_{EU}$ to the disintegration of l. It is important to note that two effects play a role here. First of all, there is a direct effect induced by the increase in bilateral trade costs between the countries m and l. At the same time, trade costs between l and the other member countries rise. Therefore, the overall effect on the business tax in country m reads as

$$\begin{aligned}
\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} &= \left(n_m(K-2) - 2n_l\left[(K-1)^2 + 0.5\right]\right) \frac{3}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} \\
&\quad + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \left(n_j + n_l\right) \frac{3(2K-3)}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} \\
&= \left((K-1)\left[2\sum_{j \in \mathscr{K}_{EU}} n_j - 2n_l\left(K - K_{EU}\right) - n_m\right]\right. \\
&\quad \left. + K_{EU}\left[n_l - \frac{1}{K_{EU}} \sum_{j \in \mathscr{K}_{EU}} n_j\right]\right) \frac{3}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}. \qquad (3.38)
\end{aligned}$$

Under symmetric market size $\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} = \left(4K_{EU} - 2K - 1\right) \frac{3n}{2K-1} \frac{\alpha - w - \tau}{16\beta}$.

For the proof of part (c) we only need to consider one set of effects, namely that the rise in trade costs considered here is a third country effect for non-member countries. That is, for any

$k \in \mathscr{K} \setminus (\mathscr{K}_{EU} \cup \{l\})$ the effect on business taxation is given by

$$\sum_{j \in \mathscr{K}_{EU}} \frac{dt_k}{d\tau_{jl}} = \sum_{j \in \mathscr{K}_{EU}} (n_j + n_l) \frac{3(2K-3)}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} = \left(\frac{1}{K_{EU}} \sum_{j \in \mathscr{K}_{EU}} n_j + n_l \right) \frac{3K_{EU}(2K-3)}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta} > 0. \tag{3.39}$$

As in Proposition 17, the main insights regarding market sizes carry over when dealing with asymmetries in trade costs. The positive effect on third countries' taxes (part (c)) is fully robust to the inclusion of differing trade costs. A correction term that accounts for the asymmetries adjusts the sign in part (a) as follows:

$$sign\left(\sum_{m \in \mathscr{K}_{EU}} \frac{dt_l}{d\tau_{ml}} \right) = sign\left(n_l - \frac{2(K-1)^2+1}{K-2} \bar{n}_{EU} \sum_{m \in \mathscr{K}_{EU}} \left(n_l - \frac{2(K-1)^2+1}{K-2} n_m \right) \frac{\tau - \tau_{ml}}{K_{EU}(\alpha - w - \tau)} \right) \tag{3.40}$$

Again, the adjustment is comparably small since $\left| \frac{\tau - \tau_{ml}}{K_{EU}(\alpha - w - \tau)} \right| \in \left[0, \frac{1}{2K_{EU}} \right]$. The larger the cardinality of the set of countries left by country l, the more negligible is this adjustment. The correction in part (b) is less straightforward

$$\begin{aligned} sign\left(\frac{dt_m}{d\tau_{ml}} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{dt_m}{d\tau_{jl}} \right) = sign\Bigg(& \frac{(2K-3)K_{EU}\bar{n}_{EU} - (K-1)n_m}{2(K-1)K - (2K-1)K_{EU}} - n_l + \frac{3(K-2)n_m - \left[4(K-1)^2+2\right] n_l}{6(K-1)K - (6K-3)K_{EU}} \\ & \cdot \frac{\tau - \tau_{ml}}{\alpha - w - \tau} + \sum_{j \in \mathscr{K}_{EU} \setminus \{m\}} \frac{(6K-9)(n_j + n_l)}{6(K-1)K - (6K-3)K_{EU}} \frac{\tau - \tau_{jl}}{\alpha - w - \tau} \Bigg). \end{aligned} \tag{3.41}$$

The adjustment term is bounded $\left| \frac{\tau - \tau_{ml}}{\alpha - w - \tau} \frac{1}{6(K-1)K - (6K-3)K_{EU}} \right| \in \left[0, \frac{1}{6(K-1)} \right]$ and decreases in the number of countries.

In Corollary 6, we consider average effects. For this we define the world, EU, and non-EU average taxes as follows: $\bar{t} := \frac{1}{K} \sum_{k \in \mathscr{K}} t_k$, $\bar{t}_{EU} := \frac{1}{K_{EU}} \sum_{k \in \mathscr{K}_{EU}} t_k$, and $\bar{t}_{nonEU} := \frac{1}{K - K_{EU} - 1} \sum_{k \in \mathscr{K} \setminus (\mathscr{K}_{EU} \cup \{l\})} t_k$.

Corollary 6. *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K \geq 2$ countries. Suppose that trade costs between the leaving country $l \in \mathscr{K} \setminus \mathscr{K}_{EU}$ and countries $m \in \mathscr{K}_{EU}$ are the same, $\tau = \tau_{ml}$, $\forall m \in \mathscr{K}_{EU}$, and let country l disintegrate from the member countries. This disintegration triggers the following change in the average tax of*
(a) the remaining member countries

$$\frac{d\bar{t}_{EU}}{d\tau} = \frac{[(2K-3)K_{EU} - (K-1)]\, 3\bar{n}_{EU} + [K_{EU} - 2(K-1)(K - K_{EU})]\, 3n_l}{(K-1)(2K-1)} \frac{\alpha - w - \tau}{16\beta}$$

(b) *third countries*

$$\frac{d\bar{t}_{nonEU}}{d\tau} = \frac{3K_{EU}(2K-3)(\bar{n}_{EU}+n_l)}{(K-1)(2K-1)}\frac{\alpha-w-\tau}{16\beta},$$

(c) *the world*

$$\frac{d\bar{t}}{d\tau} = -\frac{3K_{EU}(2K-1)\bar{n}_{EU}+3K_{EU}(K-K_{EU}-1)n_l}{K(K-1)(2K-1)}\frac{\alpha-w-\tau}{16\beta}.$$

3.C.3 Proof of Proposition 15

Assumption 4. *Let $n := n_i = n_j$ for all $i,j \in \mathscr{K}$. Moreover, let $\tau^* := \tau_{ij} = \tau_{ik}$ for all $i,j,k \in \mathscr{K}_{EU}$ with $j,k \neq i$ and $\tau := \tau_{lm} = \tau_{ln} > \tau^*$ for all $l \in \mathscr{K}$ and $m,n \in \mathscr{K} \setminus \mathscr{K}_{EU}$ with $m,n \neq l$. Let $K_{EU} > 1$.*

Under Assumption 4, the business tax of a member country $m \in \mathscr{K}_{EU}$ simplifies to

$$t_m = 3\overline{F} + 3n\frac{\tau^2-2\tau(\alpha-w)}{32\beta} + \frac{[(K-1)(2K-2K_{EU}+1)+K_{EU}](K_{EU}-1)}{(K-1)(2K-1)}3n(\tau-\tau^*)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta}, \tag{3.42}$$

whereas the tax in a non-member country $n \in \mathscr{K} \setminus \mathscr{K}_{EU}$ reads as

$$t_n = 3\overline{F} + 3n\frac{\tau^2-2\tau(\alpha-w)}{32\beta} + \frac{K_{EU}(K_{EU}-1)(2K-3)}{(K-1)(2K-1)}3n(\tau^*-\tau)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta}. \tag{3.43}$$

First of all, note that

$$t_n - t_m = \frac{K_{EU}(2K-3)+(K-1)(2K-2K_{EU}+1)+K_{EU}}{(K-1)(2K-1)}(K_{EU}-1)3n(\tau^*-\tau)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta}.$$

Hence, $t_n < t_m$ whenever $\tau^* < \tau$ and $K_{EU} > 1$. If $K_{EU} = 1$ or $\tau^* = \tau$ (which we rule out by assumption), then $t_n = t_m$. As we can see, the size of the business tax differential between member and non-member countries depends on the degree of economic integration in the world economy. Moreover, note that as the number of countries grows large, business taxes do not diverge

$$\lim_{K\to\infty} t_m = \lim_{K\to\infty} t_n + 3n(K_{EU}-1)(\tau-\tau^*)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta}, \tag{3.44}$$

where $\lim_{K\to\infty} t_n = 3\overline{F} + 3n\frac{\tau^2-2\tau(\alpha-w)}{32\beta}$.

For part (b) of the Proposition, differentiate t_m with respect to the number of member countries

$$\frac{dt_m}{dK_{EU}} = \frac{(K-1)[(2K-1)-4(K_{EU}-1)]+2K_{EU}-1}{(K-1)(2K-1)}3n\left(\frac{2(\alpha-w)(\tau-\tau^*)-\left(\tau^2-\tau^{*2}\right)}{32\beta}\right). \tag{3.45}$$

This expression is positive by the following argument. Firstly, note that the sign of $\frac{dt_m}{dK_{EU}}$ is the same as the sign of $\phi(K)$, where $\phi(K) := (K-1)[(2K-1)-4(K_{EU}-1)]+2K_{EU}-1$. $\phi(K)$ is positive, since $\phi(1)=2K_{EU}-1>0$ and $\phi'(K)=(4K-3)-4(K_{EU}-1)>4(K-1)-4(K_{EU}-1)\geq 0$, $\forall K\geq K_{EU}\geq 1$.

The other derivatives are also intuitive

$$\frac{dt_m}{d\tau^*}=-\frac{1}{(K-1)(2K-1)}6n_{K_{EU}}\left[(K-1)(2K-2K_{EU}+1)+K_{EU}\right](K_{EU}-1)\frac{\alpha-w-\tau^*}{32\beta}<0 \tag{3.46}$$

and

$$\begin{aligned}\frac{dt_m}{d\tau}&=6n_{K_{EU}}\frac{\tau-(\alpha-w)}{32\beta}+\frac{1}{(K-1)(2K-1)}6n_{K_{EU}}\left[(K-1)(2K-2K_{EU}+1)+K_{EU}\right](K_{EU}-1)\frac{\alpha-w-\tau}{32\beta}\\&=\frac{1}{(K-1)(2K-1)}6n_{K_{EU}}\left\{(K-1)\left[2K(K_{EU}-2)-2K_{EU}(K_{EU}-1)+3K_{EU}\right]+K_{EU}(K_{EU}-1)\right\}\frac{\alpha-w-\tau}{32\beta}\\&>\frac{1}{(K-1)(2K-1)}6n_{K_{EU}}\left\{(K-1)K_{EU}\left[2(K_{EU}-2)-2(K_{EU}-1)+3\right]+K_{EU}(K_{EU}-1)\right\}\frac{\alpha-w-\tau}{32\beta}\\&=\frac{1}{(K-1)(2K-1)}6n_{K_{EU}}\left\{(K-1)K_{EU}\left[-4+2+3\right]+K_{EU}(K_{EU}-1)\right\}\frac{\alpha-w-\tau}{32\beta}>0.\end{aligned} \tag{3.47}$$

The comparative statics in part (c) of Proposition 22 are given by

$$\frac{dt_n}{dK_{EU}}=\frac{(2K_{EU}-1)(2K-3)}{(K-1)(2K-1)}3n(\tau^*-\tau)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta}<0,$$

$$\frac{dt_n}{d\tau}=6n\frac{\tau-(\alpha-w)}{32\beta}+\frac{K_{EU}(K_{EU}-1)(2K-3)}{(K-1)(2K-1)}6n\frac{\tau-(\alpha-w)}{32\beta}<0,$$

and $\frac{dt_n}{d\tau^*}=\frac{K_{EU}(K_{EU}-1)(2K-3)}{(K-1)(2K-1)}6n\frac{\alpha-w-\tau^*}{32\beta}>0$. To summarize, we provide a technical version of Proposition 15.

Proposition 22 (union-size effect)**.** *Consider the subgame-perfect Nash equilibrium of economy $\mathscr{E}$ with $K>2$ countries. Let Assumption 4 hold and suppose that $K, K_{EU}\in\mathbb{R}^+$. Then, $\forall m\in\mathscr{K}_{EU}$ and $\forall n\in\mathscr{K}\setminus\mathscr{K}_{EU}$*

(a) $t_m>t_n$, (b) $\frac{dt_m}{dK_{EU}}>0, \frac{dt_m}{d\tau^*}<0$, $\frac{dt_m}{d\tau}>0$, and (c) $\frac{dt_n}{dK_{EU}}<0$, $\frac{dt_n}{d\tau^*}>0$, $\frac{dt_n}{d\tau}<0$.

The average worldwide business tax $\bar{t}=\frac{K_{EU}}{K}t_m+\frac{K-K_{EU}}{K}t_n$ can be written as

$$\bar{t}=3\overline{F}+3n\frac{\tau^2-2\tau(\alpha-w)}{32\beta}+\frac{K_{EU}(K_{EU}-1)}{K(K-1)}3n(\tau-\tau^*)\frac{2(\alpha-w)-(\tau+\tau^*)}{32\beta},$$

which is decreasing in the number of competing markets K.

3.D Proofs for Section 3.2.5

Define $\mathscr{K}_{TA}$ as the set and K_{TA} as the number of countries which participate in regional trade agreements (e.g., the WTO). Let $\boldsymbol{t}^{old}$ denote the vector of tariff policies before the disintegration of country l from the integrated area/economic union abbreviated EU. That is,

$$\boldsymbol{t}^{old} = \left(\boldsymbol{t}^{old}_{EU,EU}, \boldsymbol{t}^{old}_{EU,l}, \boldsymbol{t}^{old}_{EU,TA}, \boldsymbol{t}^{old}_{l,TA}, \boldsymbol{t}^{old}_{TA,TA}, \boldsymbol{t}^{old}_{Rest}\right) \tag{3.48}$$

is a vector of trade taxes consisting of (i) the null vector $\left(\boldsymbol{t}^{old}_{EU,EU}, \boldsymbol{t}^{old}_{EU,l}\right)$, which summarizes zero bilateral tariffs in the economic union, (ii) another vector $\left(\boldsymbol{t}^{old}_{EU,TA}, \boldsymbol{t}^{old}_{l,TA}, \boldsymbol{t}^{old}_{TA,TA}\right)$ which summarizes cooperatively chosen tariffs within the set of countries $\mathscr{K}_{TA}$, the leaving country, and the economic union, and (iii) another vector of tariffs which are set non-cooperatively

$$\boldsymbol{t}^{old}_{Rest} = \left(\boldsymbol{t}^{old}_{EU,Rest}, \boldsymbol{t}^{old}_{l,Rest}, \boldsymbol{t}^{old}_{TA,Rest}, \boldsymbol{t}^{old}_{,Rest,Rest}\right) \tag{3.49}$$

vis-à-vis countries from the rest of the world (e.g., Iran). Moreover, let

$$\boldsymbol{\tau}^{old} = \left(\boldsymbol{\tau}^{old}_{EU,EU}, \boldsymbol{\tau}^{old}_{EU,l}, \boldsymbol{\tau}^{old}_{EU,TA}, \boldsymbol{\tau}^{old}_{l,TA}, \boldsymbol{\tau}^{old}_{TA,TA}, \boldsymbol{\tau}^{old}_{Rest}\right) \tag{3.50}$$

denote the vector of bilateral non-tariff trade costs. A feature of an economic union is that member countries can cooperatively set these non-tariff trade costs. To begin with, we state the following lemma.

Lemma 12. *Suppose that business taxes are positive, trade taxes are small, and trade costs sufficiently similar ($\tilde{\tau}_{ml} = t_{ml} + \tau_{ml} \approx \tilde{\tau}_{np} = t_{np} + \tau_{np}$). Then, for any $i, j, k \in \mathscr{K}$, $\nabla_{t_{ij}} W_k(\boldsymbol{\tau}, \boldsymbol{t}) > 0$ and $\nabla_{\tau_{ij}} W_k(\boldsymbol{\tau}, \boldsymbol{t}) > 0$.*

Hence the cross-country welfare effects of higher trade costs are positive. In the Supplementary Online Appendix, we prove this statement.[24]

As mentioned above, countries inside the economic union choose non-tariff trade costs cooperatively. That is, $\left(\boldsymbol{\tau}_{EU,EU}, \boldsymbol{\tau}_{EU,l}\right)$ is the outcome of efficient Nash bargaining. Before the disintegration of country l, $\left(\boldsymbol{\tau}^{old}_{EU,EU}, \boldsymbol{\tau}^{old}_{EU,l}\right) := \underset{\left(\boldsymbol{\tau}_{EU,EU}, \boldsymbol{\tau}_{EU,l}\right)}{arg\,max} \sum_{m \in \mathscr{K}_{EU} \cup \{l\}} W_m(\cdot)$. After the disintegration, the remaining members negotiate their internal trade costs without consideration of country l's welfare $\left(\boldsymbol{\tau}^{new}_{EU,EU}\right) := \underset{\left(\boldsymbol{\tau}_{EU,EU}\right)}{arg\,max} \sum_{m \in \mathscr{K}_{EU}} W_m(\cdot)$.

Do the remaining member countries integrate more with each other after the disintegration of l? In other words, how do the vectors $\boldsymbol{\tau}^{old}_{EU,EU}$ and $\boldsymbol{\tau}^{new}_{EU,EU}$ compare with each other? Consider

[24] The Supplementary Online Appendix is available at https://www.vwl.uni-mannheim.de/(...)/Janeba/Supplementary_Online_Appendix_for_A_Theory_of_Economic_Disintegration_22102021.pdf.

the first-order Taylor approximation of members' welfare in the new optimum

$$\begin{aligned}\sum_{m\in\mathscr{K}_{EU}} W_m\left(\boldsymbol{\tau}^{new}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right) &= \sum_{m\in\mathscr{K}_{EU}} W_m\left(\boldsymbol{\tau}^{old}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right)\\ &\quad + \sum_{m\in\mathscr{K}_{EU}} \nabla_{\boldsymbol{\tau}_{EU,EU}} W_m\left(\boldsymbol{\tau}^{old}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right)\left(\boldsymbol{\tau}^{new}_{EU,EU}-\boldsymbol{\tau}^{old}_{EU,EU}\right)' + h.o.t.\\ &> \sum_{m\in\mathscr{K}_{EU}} W_m\left(\boldsymbol{\tau}^{old}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right) \end{aligned} \tag{3.51}$$

where the inequality holds by Lemma 12, and therefore implies

$$\sum_{m\in\mathscr{K}_{EU}} \nabla_{\boldsymbol{\tau}_{EU,EU}} W_m\left(\boldsymbol{\tau}^{old}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right)\left(\boldsymbol{\tau}^{new}_{EU,EU}-\boldsymbol{\tau}^{old}_{EU,EU}\right)' > 0. \tag{3.52}$$

By optimality of the old solution $\sum_{m\in\mathscr{K}_{EU}\cup\{l\}} \nabla_{\boldsymbol{\tau}_{EU,EU}} W_m\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right)=0$ and, accordingly,

$$\begin{aligned}&\sum_{m\in\mathscr{K}_{EU}\cup\{l\}} \nabla_{\boldsymbol{\tau}_{EU,EU}} W_m\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right)\left(\boldsymbol{\tau}^{new}_{EU,EU}-\boldsymbol{\tau}^{old}_{EU,EU}\right)'\\ &= \sum_{m\in\mathscr{K}_{EU}\cup\{l\}} \nabla_{\boldsymbol{\tau}_{EU,EU}} W_m\left(\boldsymbol{\tau}^{old}_{EU,EU},\boldsymbol{\tau}^{new}_{EU,l},\cdot\right)\left(\boldsymbol{\tau}^{new}_{EU,EU}-\boldsymbol{\tau}^{old}_{EU,EU}\right)' + h.o.t. = 0.\end{aligned}$$

Therefore, $-\nabla_{\boldsymbol{\tau}_{EU,EU}} W_l\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right)\left(\boldsymbol{\tau}^{new}_{EU,EU}-\boldsymbol{\tau}^{old}_{EU,EU}\right)' > 0$ and one can conclude that, whenever $\nabla_{\boldsymbol{\tau}_{EU,EU}} W_l\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right) > 0$ (i.e., the welfare of the leaving country is increasing in two member countries' trade costs as in Lemma 12), $\boldsymbol{\tau}^{new}_{EU,EU} < \boldsymbol{\tau}^{old}_{EU,EU}$.

By the construction of the economic union as a customs union trade taxes inside the union remain prohibited $\boldsymbol{t}^{old}_{EU,EU} = \boldsymbol{t}^{new}_{EU,EU} = 0$, whereas trade taxes between the leaving country and the economic union can be anything after the disintegration. That is, $\boldsymbol{t}^{old}_{EU,l} = 0$ and $\boldsymbol{t}^{new}_{EU,l} \gtreqless 0$. Observe that this includes the case where country l remains in the customs union.

Common external tariffs are an essential feature of the customs union. Therefore, when country l decides to remain a member of the customs union, there will be no first-order change in trade policies vis-à-vis third countries. To put it differently, the countries $\mathscr{K}_{EU}$ and l jointly decide on external trade taxes before and after the disintegration of l. Objective functions and instruments of tariff policies remain the same. Only non-tariff trade barriers inside the customs union change. This change, however, has no first-order effect on the other trade policies. To determine the exact sign of second-order effects, one needs to know about cross derivatives of welfare functions with respect to the respective trade policy instruments.

Now, suppose that country l departs from the customs union but stays within the set of countries that participate in regional trade agreements. Recall that before the disintegration member countries solve

$$\begin{aligned}\left(\boldsymbol{\tau}_{EU,EU}^{old},\boldsymbol{\tau}_{EU,l}^{old}\right) &:= \underset{\left(\boldsymbol{\tau}_{EU,EU},\boldsymbol{\tau}_{EU,l}\right)}{arg\ max} \sum_{m\in\mathscr{K}_{EU}\cup\{l\}} W_m(\cdot)\\ subject\ to\ \left(\boldsymbol{t}_{EU,EU}^{old},\boldsymbol{t}_{EU,l}^{old}\right) &= 0,\end{aligned} \tag{3.53}$$

but afterwards

$$\begin{aligned}\left(\boldsymbol{\tau}_{EU,l}^{new},\boldsymbol{t}_{EU,l}^{new}\right) &:= \underset{\left(\boldsymbol{\tau}_{EU,l},\boldsymbol{t}_{EU,l}\right)}{arg\ max} \sum_{m\in\mathscr{K}_{EU}\cup\{l\}} W_m(\cdot)\\ subject\ to\ \left(\boldsymbol{t}_{EU,EU}^{new}\right) &= 0\\ and\ \left(\boldsymbol{\tau}_{EU,EU}^{new}\right) &:= \underset{\left(\boldsymbol{\tau}_{EU,EU}\right)}{arg\ max} \sum_{m\in\mathscr{K}_{EU}} W_m(\cdot).\end{aligned} \tag{3.54}$$

Then, our approach delivers $\sum_{m\in\mathscr{K}_{EU}\cup\{l\}} \nabla_{\boldsymbol{t}_{EU,l}} W_m\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right)\left(\boldsymbol{t}_{EU,l}^{new}\right)' > 0$.

In principle, the sign of the relevant gradient and, therefore, the sign of post-disintegration trade taxes $\boldsymbol{t}_{EU,l}^{new}$ are ambiguous. In our model, for example, a domestic import tariff in country l would mean higher prices and a lower consumer surplus there. At the same time, ceteris paribus some marginal firms move to country l to gain low-cost market access, which means a rise in business tax revenues in l. Moreover, country l generates tariff revenues.

Given that we have dealt with the effects of economic disintegration on the trade policies between countries l and $\mathscr{K}_{EU}$, we can now speak to the impact on regional trade agreements of the economic union and the leaving country with third countries. Fix a country $TA \in \mathscr{K}_{TA}$. Once again, observe that the objective function and the trade policy instruments of the Nash bargaining change as follows:

$$\left(\boldsymbol{t}_{EU,TA}^{old},\boldsymbol{t}_{l,TA}^{old}\right) := \underset{\left(\boldsymbol{t}_{EU,TA},\boldsymbol{t}_{l,TA}\right)}{arg\ max} \sum_{m\in\mathscr{K}_{EU}\cup\{l,TA\}} W_m(\cdot) \tag{3.55}$$

and

$$\left(\boldsymbol{t}_{EU,TA}^{new}\right) := \underset{\left(\boldsymbol{t}_{EU,TA}\right)}{arg\ max} \sum_{m\in\mathscr{K}_{EU}\cup\{TA\}} W_m(\cdot)\ and\ \left(\boldsymbol{t}_{l,TA}^{new}\right) := \underset{\left(\boldsymbol{t}_{l,TA}\right)}{arg\ max}\, W_l(\cdot) + W_{TA}(\cdot). \tag{3.56}$$

Again, consider a first-order approximation of welfare in $\mathscr{K}_{EU}$ and TA in the new optimum and use the first-order conditions of the respective optimization to show that

$$-\nabla_{\boldsymbol{t}_{EU,TA}} W_l\left(\boldsymbol{\tau}^{old},\boldsymbol{t}^{old}\right)\left(\boldsymbol{t}_{EU,TA}^{new}-\boldsymbol{t}_{EU,TA}^{old}\right)' > 0,$$

which implies together with 12 $\boldsymbol{t}_{EU,TA}^{new} < \boldsymbol{t}_{EU,TA}^{old}$. By similar arguments,

$$-\sum_{m \in \mathscr{K}_{EU}} \nabla_{\boldsymbol{t}_{l,TA}} W_m \left(\boldsymbol{\tau}^{old}, \boldsymbol{t}^{old}\right) \left(\boldsymbol{t}_{l,TA}^{new} - \boldsymbol{t}_{l,TA}^{old}\right)' > 0.$$

Therefore, for $\sum_{m \in \mathscr{K}_{EU}} \nabla_{\boldsymbol{t}_{l,TA}} W_m \left(\boldsymbol{\tau}_{EU,EU}^{old}, \boldsymbol{t}_{EU,l}^{old}, \cdot\right) > 0$ (i.e., members of the economic union benefit from a trade war between l and TA), $\boldsymbol{t}_{l,TA}^{new} < \boldsymbol{t}_{l,TA}^{old}$. Hence, both country l and member countries of the economic union deepen their regional trade agreement with country TA by lowering trade taxes.

Consider, now, non-cooperative trade policies by the economic union vis-à-vis a country $Rest \in \mathscr{K} \setminus (\mathscr{K}_{TA} \cup \mathscr{K}_{EU} \cup \{l\})$. Use bold letters for trade policy instruments which are under the control of the respective government. Non-cooperative trade policies before and after the disintegration of l are given by

$$\left(\boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{old}}, \boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{old}}\right) := \underset{\left(\boldsymbol{t}_{\boldsymbol{EU},Rest}, \boldsymbol{t}_{\boldsymbol{l},Rest}\right)}{arg\ max} \sum_{m \in \mathscr{K}_{EU} \cup \{l\}} W_m(\cdot) \tag{3.57}$$

and

$$\left(\boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{new}}\right) := \underset{\left(\boldsymbol{t}_{\boldsymbol{EU},Rest}\right)}{arg\ max} \sum_{m \in \mathscr{K}_{EU}} W_m(\cdot) \ and \ \left(\boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{new}}\right) := \underset{\left(\boldsymbol{t}_{\boldsymbol{l},Rest}\right)}{arg\ max}\, W_l(\cdot). \tag{3.58}$$

Again, linearize welfare in the new optimum and use the optimality conditions to demonstrate that

$$-\nabla_{\boldsymbol{t}_{\boldsymbol{EU},Rest}} W_l \left(\boldsymbol{\tau}^{old}, \boldsymbol{t}^{old}\right) \left(\boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{new}} - \boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{old}}\right)' > 0 \ and \ -\sum_{m \in \mathscr{K}_{EU}} \nabla_{\boldsymbol{t}_{\boldsymbol{l},Rest}} W_m \left(\boldsymbol{\tau}^{old}, \boldsymbol{t}^{old}\right) \left(\boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{new}} - \boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{old}}\right)' > 0.$$

One can conclude that $\boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{new}} < \boldsymbol{t}_{\boldsymbol{EU},Rest}^{\boldsymbol{old}}$ and $\boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{new}} < \boldsymbol{t}_{\boldsymbol{l},Rest}^{\boldsymbol{old}}$. Therefore, the disintegration of l reduces not only cooperatively chosen tariffs but also non-cooperative tariffs.

The effects of the economic disintegration on regional TAs between countries, which are not part of the economic union, as well as non-cooperative trade policies by any third country, are of second order. The reason is that the objective functions and instruments of tariff policies remain the same. Therefore, policies are only indirectly altered. Cross derivatives of welfare functions measure the changes in these policies with respect to the respective trade policy instruments.

We summarize the insights formed in this section in Proposition 23.

Proposition 23 (endogenous trade policy responses to disintegration)**.** *Suppose that, initially, countries l and $\mathscr{K}_{EU}$ form an economic union (old optimum). In the new optimum, country l*

leaves the economic union. Moreover, suppose that business taxes are positive, trade taxes are small, and trade costs sufficiently similar. Then, $\tau^{new}_{EU,EU} < \tau^{old}_{EU,EU}$.

If country l also leaves the customs union, $\boldsymbol{t}^{new}_{EU,TA} < \boldsymbol{t}^{old}_{EU,TA}$, $\boldsymbol{t}^{new}_{l,TA} < \boldsymbol{t}^{old}_{l,TA}$, $\boldsymbol{t}^{\boldsymbol{new}}_{\boldsymbol{EU},Rest} < \boldsymbol{t}^{\boldsymbol{old}}_{\boldsymbol{EU},Rest}$, *and* $\boldsymbol{t}^{\boldsymbol{new}}_{\boldsymbol{l},Rest} < \boldsymbol{t}^{\boldsymbol{old}}_{\boldsymbol{l},Rest}$.

In summary, non-tariff barriers inside the economic union and cooperative (non-cooperative) trade taxes of $\mathscr{K}_{EU}$ and country l vis-à-vis $\mathscr{K}_{TA}$ ($\mathscr{K} \setminus (\mathscr{K}_{TA} \cup \mathscr{K}_{EU} \cup \{l\})$, respectively) decline. Therefore, the departure of a country from an economic union leads ceteris paribus to a deeper integration of multilaterally formed institutions around the world and less protectionism.

Bibliography

Aaberge, Rolf (2000). “Characterizations of Lorenz curves and income distributions.” In: *Social Choice and Welfare* 17.4, pp. 639–653.

Agersnap, Ole and Owen Zidar (2021). “The tax elasticity of capital gains and revenue-maximizing rates.” In: *American Economic Review: Insights* 3.4, pp. 399–416.

Aiyagari, S Rao (1994). “Uninsured idiosyncratic risk and aggregate saving.” In: *The Quarterly Journal of Economics* 109.3, pp. 659–684.

Ales, Laurence, Musab Kurnaz, and Christopher Sleet (2015). “Technical change, wage inequality, and taxes.” In: *American Economic Review* 105.10, pp. 3061–3101.

Amiti, Mary, Stephen J Redding, and David E Weinstein (2019). “The impact of the 2018 tariffs on prices and welfare.” In: *Journal of Economic Perspectives* 33.4, pp. 187–210.

Arrow, Kenneth J (1987). “The demand for information and the distribution of income.” In: *Probability in the Engineering and Informational Sciences* 1.1, pp. 3–13.

Artuç, Erhan, Shubham Chaudhuri, and John McLaren (2010). “Trade shocks and labor adjustment: A structural empirical approach.” In: *American Economic Review* 100.3, pp. 1008–45.

Atkinson, Anthony Barnes and Joseph E Stiglitz (1976). “The design of tax structure: direct versus indirect taxation.” In: *Journal of Public Economics* 6.1-2, pp. 55–75.

Autor, David, David Dorn, and Gordon H Hanson (2013). “The China syndrome: Local labor market effects of import competition in the United States.” In: *American Economic Review* 103.6, pp. 2121–68.

Bach, Laurent, Laurent E Calvet, and Paolo Sodini (2020). "Rich Pickings? Risk, Return, and Skill in Household Wealth." In: *American Economic Review* 110.9, pp. 2703–2747.

Bagwell, Kyle and Robert W Staiger (1999). "An economic theory of GATT." In: *American Economic Review* 89.1, pp. 215–248.

— (2004). "Multilateral trade negotiations, bilateral opportunism and the rules of GATT/WTO." In: *Journal of International Economics* 63.1, pp. 1–29.

— (2012). "The economics of trade agreements in the linear cournot delocation model." In: *Journal of International Economics* 88.1, pp. 32–46.

Bakija, Jon M and William M Gentry (2014). "Capital gains taxes and realizations: Evidence from a long panel of state-level data." In: *Unpublished Manuscript, Williams College*.

Barattieri, Alessandro, Matteo Cacciatore, and Fabio Ghironi (2021). "Protectionism and the business cycle." In: *Journal of International Economics* 129, p. 103417.

Benabou, Roland (2000). "Unequal societies: Income distribution and the social contract." In: *American Economic Review* 90.1, pp. 96–129.

Benhabib, Jess, Alberto Bisin, and Shenghao Zhu (2011). "The distribution of wealth and fiscal policy in economies with finitely lived agents." In: *Econometrica* 79.1, pp. 123–157.

Boadway, Robin and Kevin Spiritus (2021). "Optimal Taxation of Normal and Excess Returns to Risky Assets." In: *Tinbergen Institute Discussion Paper 2021-025/VI.*

Brülhart, Marius, Jonathan Gruber, Matthias Krapf, and Kurt Schmidheiny (2016). "Taxing Wealth: Evidence from Switzerland." In: *National Bureau of Economic Research Working Paper No. 22376*.

Bucovetsky, Sam (1991). "Asymmetric tax competition." In: *Journal of Urban Economics* 30.2, pp. 167–181.

Caldara, Dario, Matteo Iacoviello, Patrick Molligo, Andrea Prestipino, and Andrea Raffo (2020). "The economic effects of trade policy uncertainty." In: *Journal of Monetary Economics* 109, pp. 38–59.

Caliendo, Lorenzo, Maximiliano Dvorkin, and Fernando Parro (2019). “Trade and labor market dynamics: General equilibrium analysis of the china trade shock.” In: *Econometrica* 87.3, pp. 741–835.

Caliendo, Lorenzo and Fernando Parro (2015). “Estimates of the Trade and Welfare Effects of NAFTA.” In: *The Review of Economic Studies* 82.1, pp. 1–44.

Caliendo, Lorenzo, Fernando Parro, Luca David Opromolla, and Alessandro Sforza (2021). “Goods and factor market integration: a quantitative assessment of the EU enlargement.” In: *Journal of Political Economy* 129.12, pp. 3491–3545.

Card, David (2009). “Immigration and inequality.” In: *American Economic Review* 99.2, pp. 1–21.

Card, David and Thomas Lemieux (2001). “Can falling supply explain the rising return to college for younger men? A cohort-based analysis.” In: *The Quarterly Journal of Economics* 116.2, pp. 705–746.

Chamley, Christophe (1986). “Optimal taxation of capital income in general equilibrium with infinite lives.” In: *Econometrica*, pp. 607–622.

Chetty, Raj (2009). “Sufficient statistics for welfare analysis: A bridge between structural and reduced-form methods.” In: *Annual Review of Economics* 1.1, pp. 451–488.

— (2012). “Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply.” In: *Econometrica* 80.3, pp. 969–1018.

Conconi, Paola, Carlo Perroni, and Raymond Riezman (2008). “Is partial tax harmonization desirable?” In: *Journal of Public Economics* 92.1-2, pp. 254–267.

Cook, Nathaniel PS and John Douglas Wilson (2013). “Using trade policy to influence firm location.” In: *Economics Letters* 119.1, pp. 45–47.

Costinot, Arnaud and Andres Rodriguez-Clare (2014). “Trade theory with numbers: Quantifying the consequences of globalization.” In: *Handbook of International Economics*. Vol. 4. Elsevier, pp. 197–261.

Darby, Julia, Ben Ferrett, and Ian Wooton (2014). “Regional centrality and tax competition for FDI.” In: *Regional Science and Urban Economics* 49, pp. 84–92.

Davis, Steven J (2016). “An index of global economic policy uncertainty.” In: *National Bureau of Economic Research Working Paper No. 22740*.

Diamond, Peter A (1975). “A many-person Ramsey tax rule.” In: *Journal of Public Economics* 4.4, pp. 335–342.

— (1998). “Optimal income taxation: an example with a U-shaped pattern of optimal marginal tax rates.” In: *American Economic Review*, pp. 83–95.

Dix-Carneiro, Rafael (2014). “Trade liberalization and labor market dynamics.” In: *Econometrica* 82.3, pp. 825–885.

Dowd, Tim, Robert McClelland, and Athiphat Muthitacharoen (2015). “New evidence on the tax elasticity of capital gains.” In: *National Tax Journal* 68.3, p. 511.

Dustmann, Christian, Tommaso Frattini, and Ian P Preston (2013). “The effect of immigration along the distribution of wages.” In: *The Review of Economic Studies* 80.1, pp. 145–173.

Dyreng, Scott D, Michelle Hanlon, Edward L Maydew, and Jacob R Thornock (2017). “Changes in corporate effective tax rates over the past 25 years.” In: *Journal of Financial Economics* 124.3, pp. 441–463.

Egger, Peter H, Sergey Nigai, and Nora M Strecker (2019). “The taxing deed of globalization.” In: *American Economic Review* 109.2, pp. 353–90.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri (2020). “Heterogeneity and persistence in returns to wealth.” In: *Econometrica* 88.1, pp. 115–170.

Fajgelbaum, Pablo D, Pinelopi K Goldberg, Patrick J Kennedy, and Amit K Khandelwal (2020). “The return to protectionism.” In: *The Quarterly Journal of Economics* 135.1, pp. 1–55.

Farhi, Emmanuel and Iván Werning (2010). “Progressive estate taxation.” In: *The Quarterly Journal of Economics* 125.2, pp. 635–673.

Feldstein, Martin, Joel Slemrod, and Shlomo Yitzhaki (1980). "The effects of taxation on the selling of corporate stock and the realization of capital gains." In: *The Quarterly Journal of Economics* 94.4, pp. 777–791.

Feldstein, Martin S (1969). "The effects of taxation on risk taking." In: *Journal of Political Economy* 77.5, pp. 755–764.

Findeisen, Sebastian and Dominik Sachs (2016). "Education and optimal dynamic taxation: The role of income-contingent student loans." In: *Journal of Public Economics* 138, pp. 1–21.

Fuest, Clemens and Samina Sultan (2019). "How will Brexit affect tax competition and tax harmonization? The role of discriminatory taxation." In: *National Tax Journal* 72.1, pp. 111–138.

Gabaix, Xavier (2009). "Power laws in economics and finance." In: *Annu. Rev. Econ.* 1.1, pp. 255–294.

Gabaix, Xavier, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll (2016). "The dynamics of inequality." In: *Econometrica* 84.6, pp. 2071–2111.

Gahvari, Firouz and Luca Micheletto (2016). "Capital income taxation and the Atkinson–Stiglitz theorem." In: *Economics Letters* 147, pp. 86–89.

Gaillard, Alexandre and Philipp Wangner (2021). "Wealth, Returns, and Taxation: A Tale of Two Dependencies." In: *Working Paper*.

Gerritsen, Aart, Bas Jacobs, Alexandra V Rusu, and Kevin Spiritus (2020). "Optimal Taxation of Capital Income with Heterogeneous Rates of Return." In: *CESifo Working Paper No. 8395*.

Glazer, Amihai, Vesa Kanniainen, and Panu Poutvaara (2008). "Income taxes, property values, and migration." In: *Journal of Public Economics* 92.3-4, pp. 915–923.

Golosov, Mikhail, Narayana Kocherlakota, and Aleh Tsyvinski (2003). "Optimal indirect and capital taxation." In: *The Review of Economic Studies* 70.3, pp. 569–587.

Golosov, Mikhail, Aleh Tsyvinski, and Nicolas Werquin (2014). "A variational approach to the analysis of tax systems." In: *National Bureau of Economic Research Working Paper No. 20780*.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman (1988). “Investment, capacity utilization, and the real business cycle.” In: *American Economic Review*, pp. 402–417.

Grossman, Gene M and Elhanan Helpman (1995). “Trade wars and trade talks.” In: *Journal of Political Economy* 103.4, pp. 675–708.

Grossman, Gene M, Phillip McCalman, and Robert W Staiger (2021). “The “New” Economics of Trade Agreements: From Trade Liberalization to Regulatory Convergence?” In: *Econometrica* 89.1, pp. 215–249.

Grossman, Sanford J and Joseph E Stiglitz (1980). “On the impossibility of informationally efficient markets.” In: *American Economic Review* 70.3, pp. 393–408.

Guvenen, Fatih, Gueorgui Kambourov, Burhanettin Kuruscu, Sergio Ocampo-Diaz, and Daphne Chen (2019). “Use it or lose it: Efficiency gains from wealth taxation.” In: *National Bureau of Economic Research Working Paper No. 26284*.

Haufler, Andreas and Ian Wooton (1999). “Country size and tax competition for foreign direct investment.” In: *Journal of Public Economics* 71.1, pp. 121–139.

— (2006). “The effects of regional tax and subsidy coordination on foreign direct investment.” In: *European Economic Review* 50.2, pp. 285–305.

— (2010). “Competition for firms in an oligopolistic industry: The impact of economic integration.” In: *Journal of International Economics* 80.2, pp. 239–248.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante (2017). “Optimal tax progressivity: An analytical framework.” In: *The Quarterly Journal of Economics* 132.4, pp. 1693–1754.

Heathcote, Jonathan and Hitoshi Tsujiyama (2021a). “Optimal income taxation: Mirrlees meets Ramsey.” In: *Journal of Political Economy* 129.11, pp. 000–000.

— (2021b). “Practical Optimal Income Taxation.” In: *Working Paper*.

Hendren, Nathaniel (2016). “The policy elasticity.” In: *Tax Policy and the Economy* 30.1, pp. 51–89.

Internal Revenue Service (2020). *SOI Tax Stats - Private Foundations Harmonized Microdata Files (ASCII)*. https://www.irs.gov/statistics/soi-tax-stats-private-foundations-harmonized-microdata-files-ascii. Accessed: 2020-10-17.

Jakobsen, Katrine, Kristian Jakobsen, Henrik Kleven, and Gabriel Zucman (2020). "Wealth taxation and wealth accumulation: Theory and evidence from Denmark." In: *The Quarterly Journal of Economics* 135.1, pp. 329–388.

Judd, Kenneth L (1985). "Redistributive taxation in a simple perfect foresight model." In: *Journal of Public Economics* 28.1, pp. 59–83.

Kacperczyk, Marcin, Jaromir Nosal, and Luminita Stevens (2019). "Investor sophistication and capital income inequality." In: *Journal of Monetary Economics* 107, pp. 18–31.

Katz, Lawrence F and Kevin M Murphy (1992). "Changes in relative wages, 1963–1987: supply and demand factors." In: *The Quarterly Journal of Economics* 107.1, pp. 35–78.

Keen, Michael and Kai A Konrad (2013). "The theory of international tax competition and coordination." In: *Handbook of Public Economics* 5, pp. 257–328.

Kessing, Sebastian G, Kai A Konrad, and Christos Kotsogiannis (2006). "Federal tax autonomy and the limits of cooperation." In: *Journal of Urban Economics* 59.2, pp. 317–329.

Kleven, Henrik, Camille Landais, Mathilde Munoz, and Stefanie Stantcheva (2020). "Taxation and migration: Evidence and policy implications." In: *Journal of Economic Perspectives* 34.2, pp. 119–42.

Kleven, Henrik Jacobsen (2016). "Bunching." In: *Annual Review of Economics* 8, pp. 435–464.

Kline, Rex B (2015). *Principles and practice of structural equation modeling*. Guilford publications.

Krueger, Dirk and Alexander Ludwig (2013). "Optimal progressive labor income taxation and education subsidies when education decisions and intergenerational transfers are endogenous." In: *American Economic Review* 103.3, pp. 496–501.

Kuhn, Moritz, Moritz Schularick, and Ulrike I Steins (2020). “Income and wealth inequality in America, 1949–2016.” In: *Journal of Political Economy* 128.9, pp. 3469–3519.

Lehmann, Etienne, Laurent Simula, and Alain Trannoy (2014). “Tax me if you can! Optimal nonlinear income tax between competing governments.” In: *The Quarterly Journal of Economics* 129.4, pp. 1995–2030.

Li, Chunding and John Whalley (2021). “Trade protectionism and US manufacturing employment.” In: *Economic Modelling* 96, pp. 353–361.

Lusardi, Annamaria, Pierre-Carl Michaud, and Olivia S Mitchell (2017). “Optimal financial knowledge and wealth inequality.” In: *Journal of Political Economy* 125.2, pp. 431–477.

Markowitz, Harry (1952). “The utility of wealth.” In: *Journal of Political Economy* 60.2, pp. 151–158.

— (1959). “Portfolio selection.” In: *Investment under Uncertainty*.

Melitz, Marc J (2003). “The impact of trade on intra-industry reallocations and aggregate industry productivity.” In: *Econometrica* 71.6, pp. 1695–1725.

Melitz, Marc J and Gianmarco IP Ottaviano (2008). “Market size, trade, and productivity.” In: *The Review of Economic Studies* 75.1, pp. 295–316.

Milgrom, Paul and Chris Shannon (1994). “Monotone Comparative Statics.” In: *Econometrica*, pp. 157–180.

Mirrlees, James A (1971). “An exploration in the theory of optimum income taxation.” In: *The Review of Economic Studies* 38.2, pp. 175–208.

Ossa, Ralph (2011). “A ”new trade” theory of GATT/WTO negotiations.” In: *Journal of Political Economy* 119.1, pp. 122–152.

— (2015). “A quantitative analysis of subsidy competition in the US.” In: *National Bureau of Economic Research Working Paper No. 20975*.

— (2016). “Quantitative models of commercial policy.” In: *Handbook of Commercial Policy*. Vol. 1. Elsevier, pp. 207–259.

Ottaviano, Gianmarco IP and Tanguy Van Ypersele (2005). “Market size and tax competition.” In: *Journal of International Economics* 67.1, pp. 25–46.

Peress, Joel (2004). “Wealth, information acquisition, and portfolio choice.” In: *The Review of Financial Studies* 17.3, pp. 879–914.

Persson, Mats (1983). “The distribution of abilities and the progressive income tax.” In: *Journal of Public Economics* 22.1, pp. 73–88.

Piketty, Thomas (1997). “La redistribution fiscale face au chômage.” In: *Revue française d’économie* 12.1, pp. 157–201.

— (2014). *Capital in the 21st Century*. Harvard University Press Cambridge, MA.

Piketty, Thomas and Emmanuel Saez (2013). “A theory of optimal inheritance taxation.” In: *Econometrica* 81.5, pp. 1851–1886.

Piketty, Thomas, Emmanuel Saez, and Stefanie Stantcheva (2014). “Optimal taxation of top labor incomes: A tale of three elasticities.” In: *American Economic Journal: Economic Policy* 6.1, pp. 230–71.

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman (2018). “Distributional national accounts: methods and estimates for the United States.” In: *The Quarterly Journal of Economics* 133.2, pp. 553–609.

Raff, Horst (2004). “Preferential trade agreements and tax competition for foreign direct investment.” In: *Journal of Public Economics* 88.12, pp. 2745–2763.

Ramsey, Frank P (1927). “A Contribution to the Theory of Taxation.” In: *The Economic Journal* 37.145, pp. 47–61.

Rodrik, Dani (1998). “Why do more open economies have bigger governments?” In: *Journal of Political Economy* 106.5, pp. 997–1032.

Rothschild, Casey and Florian Scheuer (2013). “Redistributive taxation in the roy model.” In: *The Quarterly Journal of Economics* 128.2, pp. 623–668.

— (2016). “Optimal taxation with rent-seeking.” In: *The Review of Economic Studies* 83.3, pp. 1225–1262.

Sachs, Dominik, Aleh Tsyvinski, and Nicolas Werquin (2020). "Nonlinear tax incidence and optimal taxation in general equilibrium." In: *Econometrica* 88.2, pp. 469–493.

Saez, Emmanuel (2001). "Using elasticities to derive optimal income tax rates." In: *The Review of Economic Studies* 68.1, pp. 205–229.

— (2002). "The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes." In: *Journal of Public Economics* 83.2, pp. 217–230.

Saez, Emmanuel and Stefanie Stantcheva (2016). "Generalized social marginal welfare weights for optimal tax theory." In: *American Economic Review* 106.1, pp. 24–45.

— (2018). "A simpler theory of optimal capital taxation." In: *Journal of Public Economics* 162, pp. 120–142.

Saez, Emmanuel and Gabriel Zucman (2016). "Wealth inequality in the United States since 1913: Evidence from capitalized income tax data." In: *The Quarterly Journal of Economics* 131.2, pp. 519–578.

— (2019). *The triumph of injustice: How the rich dodge taxes and how to make them pay*. WW Norton & Company.

Samuelson, Paul Anthony (1947). *Foundations of economic analysis*. Harvard University Press Cambridge, MA.

Scheuer, Florian and Iván Werning (2017). "The taxation of superstars." In: *The Quarterly Journal of Economics* 132.1, pp. 211–270.

Seim, David (2017). "Behavioral responses to wealth taxes: Evidence from Sweden." In: *American Economic Journal: Economic Policy* 9.4, pp. 395–421.

Sen, Amartya (1974). "Informational bases of alternative welfare approaches: aggregation and income distribution." In: *Journal of Public Economics* 3.4, pp. 387–403.

Shourideh, Ali (2012). "Optimal taxation of wealthy individuals." In: *University of Pennsylvania Working Paper*.

Simula, Laurent and Alain Trannoy (2010). "Optimal income tax under the threat of migration by top-income earners." In: *Journal of Public Economics* 94.1-2, pp. 163–173.

Simula, Laurent and Alain Trannoy (2020). "Gini and optimal income taxation by rank." In: *CESifo Working Paper No. 8141*.

Staiger, Robert W and Alan O Sykes (2021). "The Economic Structure of International Trade-in-Services Agreements." In: *Journal of Political Economy* 129.4, pp. 1287–1317.

Stantcheva, Stefanie (2017). "Optimal taxation and human capital policies over the life cycle." In: *Journal of Political Economy* 125.6, pp. 1931–1990.

Steinberg, Joseph B (2019). "Brexit and the macroeconomic impact of trade policy uncertainty." In: *Journal of International Economics* 117, pp. 175–195.

Stiglitz, Joseph E (1982). "Self-selection and Pareto efficient taxation." In: *Journal of Public Economics* 17.2, pp. 213–240.

U.S. Department of the Treasury (2016). *Taxes Paid on Capital Gains for Returns with Positive Net Capital Gains: 1954-2014*. https://home.treasury.gov/policy-issues/tax-policy/office-of-tax-analysis. Accessed: 2021-04-11.

Verrecchia, Robert E (1982). "Information acquisition in a noisy rational expectations economy." In: *Econometrica*, pp. 1415–1430.

Wilson, John D (1986). "A theory of interregional tax competition." In: *Journal of Urban Economics* 19.3, pp. 296–315.

Yitzhaki, Shlomo (1987). "The relation between return and income." In: *The Quarterly Journal of Economics* 102.1, pp. 77–95.

Zemyan, Stephen M (2012). *The classical theory of integral equations: a concise treatment*. Springer Science & Business Media.

Zodrow, George R and Peter Mieszkowski (1986). "Pigou, Tiebout, property taxation, and the underprovision of local public goods." In: *Journal of Urban Economics* 19.3, pp. 356–370.

Zoutman, Floris T (2015). "The effect of capital taxation on households' portfolio composition and intertemporal choice." In: *Norwegian School of Economics Working Paper*.

Curriculum Vitae

2016–2022	University of Mannheim (Germany) *PhD in Economics*
2018–2019	Yale University (USA) *Visiting Scholar*
2016–2018	University of Mannheim (Germany) *MSc in Economics*
2015–2016	University of Zurich (Switzerland) *Visiting Student*
2013–2016	University of Mannheim (Germany) *BSc in Economics*