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PARENTAL TIME INVESTMENT AND INTERGENERATIONAL MOBILITY*

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This article constructs an overlapping generations general equilibrium model to explore the extent to which heterogeneity in time investment shapes intergenerational mobility of lifetime income. The calibrated model successfully accounts for untargeted distributional aspects of income mobility. Counterfactual exercises show that removing heterogeneity in parental time investment reduces intergenerational persistence by around 7–8% for early childhood but only marginally in later childhood. Policy experiments find that an asset-tested subsidy for parental monetary investments in early childhood can raise intergenerational mobility in a cost-effective way, though it reduces mobility substantially if given to parents with older school-aged children.

1. INTRODUCTION

Empirical evidence shows that more educated and richer parents spend more time with their children (Guryan et al., 2008; Ramey and Ramey, 2010). Parents with more economic resources naturally make greater monetary investments, yet why, given the same time endowment, do they also invest more time in their children? Given the large gap in the financial resources available to families from different backgrounds in a context of high-income inequality, does the difference in parental time investment matter when it comes to intergenerational mobility? In this article, I develop an overlapping generations model to quantitatively investigate the implications of heterogeneity in parental time investment on how lifetime income persists across generations.

The model economy builds on a standard heterogeneous-agent incomplete-markets framework (Huggett, 1993; Aiyagari, 1994). Following (Becker and Tomes, 1986), altruistic parents care about their descendants' utility. Households are heterogeneous in multiple dimensions such as human capital, assets, education, and age. Young parents, who face additional state variables for their child, such as their human capital and learning ability, choose how much time and money to invest in their children in addition to standard consumption-savings and labor supply decisions. Children's human capital evolves according to a multiple-period production technology featuring dynamic complementarity and self-productivity, as highlighted by Cunha and Heckman (2007).[1] Moreover, the technology allows for flexible degrees of

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[1] Dynamic complementarity denotes a higher productivity of investments with a greater current stock of human capital, and self-productivity refers to positive effects of human capital in a given period on human capital in the following period.

complementarity between parental time and monetary investments. This complementarity, which is allowed to vary with the age of the child, and the amount of monetary investments endogenously captures the role of quality time in producing children's human capital.

When children become young adults, they make a college decision that affects their future life-cycle wage profiles. Parents can affect this decision indirectly through their parental investments and inter vivos transfers to their children, since college decisions are affected by precollege human capital and assets. College wage premiums are endogenously determined in general equilibrium (GE) with aggregate production technology. Adult human capital is subject to idiosyncratic shocks, which cannot be fully insured since households have access to nonstate-contingent assets. Households face borrowing limits in each period as well as across generations because parents are not allowed to borrow against their descendants' income.

The model economy is calibrated to U.S. data by matching relevant target statistics. In particular, my calibration strategy requires the model economy to deliver positive educational gradients in parental time investment that are empirically consistent with those observed in the American Time Use Survey (ATUS) data. This is achieved with the help of the degrees of complementarity between parental time and monetary investments as well as dispersion of parental income. In general, higher complementarity tends to give richer parents a stronger incentive to invest more time. At the same time, lower complementarity between money and time is needed to match the observed educational gradients in parental time with older children, since income gaps tend to increase as parents become older.

I evaluate the calibrated model as a quantitative theory of intergenerational mobility by confronting it with the empirical income quintile transition matrix, so as to establish its success in explaining the disaggregated moments in the latter.[2] My model successfully replicates the quintile income transition matrix in U.S. data, although the calibration targets only overall intergenerational mobility statistics (i.e., correlations between the percentile rank of parents' income and that of children's income and earnings). In particular, the upward mobility rate—the probability of the children of parents in the bottom income quintile moving up to the first income quintile—calculated by the model (7.1%) is very close to its counterpart (7.5%) in U.S. data (Chetty et al., 2014a).

Using the model economy, I conduct counterfactual exercises to investigate how differences in parental investments across households at different stages of childhood shape the intergenerational persistence of lifetime income. First, the model implies that removing heterogeneity in parental monetary investment generally leads to significant increases in intergenerational mobility, in line with the literature highlighting the importance of parental monetary investment gaps (Restuccia and Urrutia, 2004; Holter, 2015; Caucutt and Lochner, 2020). On top of that, I find that equalizing parental time investment in the first five years of childhood decreases the intergenerational elasticity or rank correlation of lifetime income by around 7–8%, while removing it in later childhood results in a less pronounced reduction in the intergenerational persistence of lifetime income. My baseline calibration result implies that parental time and monetary investments are poor substitutes for human capital development in early childhood, while they are much more substitutable in later childhood.[3] Since it is very hard to substitute the high demands on parental time investment in early childhood with monetary investments, equalizing time investments during this early period generates significant impacts on intergenerational mobility. By contrast, I find that shutting down heterogeneity in inter vivos transfers induces parents to rely more heavily on childhood human capital production to transmit their economic status, leading to a greater persistence of lifetime income across generations.

[2] This exercise is not commonly done in the literature. An early example of the model-generated quartile transition matrix in Fernandez and Rogerson (1998) shows that this is not a trivial task.

[3] Caucutt et al. (2020) find strong complementarity between parental time and monetary investments using the sample of children aged between 0 and 12.

I also use my model to conduct several cost-neutral policy experiments.[4] Among the set of various policy instruments intended to raise intergenerational mobility, I find that an asset-tested subsidy to parental monetary investments in early childhood is most cost-effective with the parental time investment channel amplifying the effects of such monetary subsidies. This effectively induces poor parents to spend more time with their children through the high complementarity between money and time, thereby benefiting able children born into poor families. Therefore, this policy not only increases mobility but also has large aggregate efficiency and welfare gains as a by-product. These gains are comparable to the expansion of primary and secondary schools—called Great Equalizers (Downey et al., 2004)—which can moderately increase intergenerational mobility. By contrast, policies that facilitate access to college by subsidizing college costs are found to be less effective in raising mobility since college decisions are largely self-selected based on precollege human capital that is already formed during childhood. Finally, I find that there are limited mobility effects of providing lump-sum time investments by nonparents to children from poor families since they crowd out parental time investment while generating efficiency losses due to the distortionary taxes required to finance the cost.

A growing empirical literature examining sources of such low mobility, as reviewed in Black and Devereux (2011), suggests that family background is a key determinant of intergenerational mobility in the United States. However, the specific family factors that are quantitatively relevant for low mobility remain unexplored, as do the mechanisms through which such factors shape the intergenerational persistence of lifetime income. The answers to these questions are essential for designing policies to increase intergenerational mobility. In this regard, my article contributes to understanding the mechanisms underlying the intergenerational mobility of lifetime income.

In particular, my article builds on the literature on intergenerational economic persistence in quantitative dynamic equilibrium models with heterogeneous households, where the distribution of income evolves over time endogenously. Following a seminal study by Restuccia and Urrutia (2004)—which presents a model that abstracts from potentially important features such as capital accumulation, valued leisure, idiosyncratic labor market shocks, and multistage parental investments—recent papers have increasingly considered models with richer environments (e.g., Holter, 2015; Rauh, 2017; Lee and Seshadri, 2019; Daruich, 2020). My study is novel in its explicit focus on the channel of parental time investment, which has thus far received scant attention in this literature. Although models that endogenize parental time investments in a rich incomplete market environments do exist (e.g., Lee and Seshadri, 2019), my quantitative exercises focus on the role of heterogeneity in parental time investment, which has yet to be explored.[5]

Another body of work uses structural models that abstract from early childhood development. Here, the initial conditions of adult human capital around the early 20s are found to be crucial in accounting for lifetime income inequality (e.g., Keane and Wolpin, 1997; Huggett

[4] The policy experiments in this article are conducted from a positive perspective. Specifically, I focus on whether policies that are often considered as instruments to raise mobility do, in fact, increase the intergenerational mobility of lifetime income, while taking into account their ramifications for aggregate efficiency and average welfare. Therefore, this article does not speak to the question of whether the current mobility is too low (or too high) from a normative perspective. The optimal degree of intergenerational mobility is an important question that goes beyond the scope of this article and is left for future work.

[5] See, for example, Erosa and Koreshkova (2007) and Zhu and Vural (2013), who also present a model with endogenous parental time in a single childhood period. See also Morchio (2018), Youderian (2019), and Daruich (2020). Youderian (2019) also conducts policy experiments that mostly focus on overall human capital achievement. Her set of thought experiments differs from mine in that she does not model a college education choice, multiperiod parental time investments, or inter vivos transfers, among others. Moreover, as my model incorporates GE unlike (Youderian, 2019), my paper is better able to speak to aggregate and distributional effects of policies in an environment where returns to human capital investments and those to inter vivos transfers are endogenously determined. Finally, my benchmark mobility measure is based on a complete measure of lifetime income that includes labor and capital income, whereas she focuses solely on labor income.

TABLE 1
TIMELINE OF LIFE-CYCLE EVENTS

	Parent's Age									
	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65–79
Model age j	1	2	3	4	5	6	7	8	9	10–12
Key decisions	← – – – – – – – – – – – – – – – Consumption-savings – – – – – – – – – – – – – →									
	← – – – – – – – – – – – – Labor supply– – – – – – – – – – – →									Retired
	College		← – Parental – → Investments			Inter-vivos				
Child's model age			← ––Childhood –– →				1	2	3	4–6

et al., 2011). This result naturally implies that studying the conditions preceding the early 20s is essential for understanding the degree of lifetime income mobility over generations. Therefore, my model endogenizes various channels before adulthood to examine how lifetime income persistence is shaped by various forces before adulthood.

Finally, this article is also related to the literature that uses equilibrium models of human capital investment across generations to study policies designed to raise the human capital of children from disadvantaged families (e.g., Fernandez and Rogerson, 1998; Caucutt and Lochner, 2020). To date, this literature has tended to focus on parents' inadequate financial investments in children's human capital. In contrast, my paper highlights, in the presence of parental influences through financial resources, the separate role of parental time investments in improving human capital of children from disadvantaged families.

The article is organized as follows: Section 2 describes the model environment. Section 3 explains how the parameters of the baseline model economy are calibrated. Section 4 evaluates the baseline model economy as a quantitative theory of intergenerational mobility through nontargeted mobility statistics. Section 5 presents counterfactual exercises to investigate the quantitative role of heterogeneity in parental time investment on intergenerational mobility, and Section 6 explores a series of cost-neutral policy experiments that are meant to increase intergenerational mobility. Section 7 concludes.

2. MODEL

The model builds on a standard incomplete-markets GE framework where the economy consists of heterogeneous households, the representative firm, and government.

2.1. *Households.* The economy is populated by overlapping generations of a continuum of households. A household is composed of an adult who lives with a child until the child grows up. One model period corresponds to five years, and an adult lives for 12 model periods (age 20–79) as an economic decision maker. In Table 1, I summarize the timeline of life-cycle events for a sample parent for illustration. The adult agent supplies labor beginning at period $j = 1$ (age 20) until retirement at the beginning of $j = 10$ (age 65). The agent then lives for three periods after retirement and dies at the end of period $j = 12$. In all periods, the agent makes a consumption-savings choice. The next generation is born when the agent enters the period $j = 3$. The parent then invests time and money in their children in periods $j = 3, 4, 5$. Before the child becomes independent, the parent decides on inter vivos transfers ($j = 6$). The newly formed household faces the same lifetime structure as described above.

All households have identical preferences over consumption c and hours worked n, represented by a standard separable utility function

$$\frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} \tag{1}$$

with the disutility constant $b > 0$.

In each period of a person's working life, earnings y are subject to progressive taxation following the parametric form of Benabou (2002) and Heathcote et al. (2014). Specifically, after-tax earnings for those who earns y is given by

$$\lambda_j (y/\bar{y})^{-\tau_j} y, \tag{2}$$

where τ_j shapes the degree of progressivity, λ_j captures the scale of taxation, and $\bar{y}$ denotes average earnings. Note that τ_j and λ_j are indexed by age to allow labor taxation to depend on family structure, consistent with U.S. data (e.g., Guner et al., 2014; Holter et al., 2019).

In all periods, capital income is taxed at the rate of τ_k unless the net worth is nonpositive, and consumption is taxed at the rate of τ_c. Households receive transfers T and face an exogenous borrowing limit $\underline{a} \leq 0$ (Aiyagari, 1994).

This article considers stationary environments in which market-clearing prices and aggregate quantities are constant over time. Therefore, the time index for the variables is omitted and a variable with a prime denotes its value in the next period. I now present the household's decision problems starting from period 1.

Period 1: Young adult with college education choice. A child becomes an independent economic decision maker at model age $j = 1$ (20 years old) with three state variables in addition to j: a human capital stock of h, a level of asset holdings a, and the childhood learning ability ϕ. As discussed below, the first two state variables, h and a, are endogenously determined by their parents. Although childhood learning ability is not directly relevant to those who have already become adults, it is still a state variable because it affects the learning ability of their child, who will be born in period $j = 3$. An important decision to be made at the beginning of $j = 1$ is whether to attain college education or not.[6] Given the discrete nature of this choice, it is convenient to define the value of not completing college and that of completing college separately.

First, the household's value of not completing college ($\kappa = 1$) is given by

$$N(h, a, \phi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_2(h', a', \kappa, \phi) \right\} \tag{3}$$

subject to

$$\begin{aligned} (1+\tau_c)c + a' &\leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T \\ h' &= \exp(z')\gamma_{1,\kappa} h, \\ \kappa &= 1, \end{aligned} \tag{4}$$

where w_κ is the rental price of human capital for skill type κ per unit hours of work, r is the interest rate, and a is the initial assets saved and transferred by parents (inter vivos transfers). Human capital evolves at the gross growth rate of $\gamma_{j,\kappa}$, which depends on age j and education κ to capture the empirical age-wage profile for different skilled workers, and is subject to the idiosyncratic shock (or market luck) z.[7] As in Huggett et al. (2011), I assume that z follows an

[6] Note that an adult agent enters the first period at age 20, which is near the end of college ages in practice. Therefore, this education choice problem is about college completion at the beginning of $j = 1$, and abstracts from time costs of college.

[7] Unlike Lee and Seshadri (2019) who allow adults to accumulate human capital endogenously (i.e., a Ben-Porath specification), my model chooses a parsimonious specification with exogenous growth rates of adult human capital (e.g., Caucutt and Lochner, 2020). As this might affect the degree of college selection in the model, my calibration

i.i.d. normal distribution with mean zero and the standard deviation of σ_z. Note that although z is drawn from an i.i.d. distribution, its effect persists for life because z is not a shock to earnings but rather a shock to human capital. Idiosyncratic shocks z cannot be fully insured because a is not a state-contingent asset. As h' is uncertain due to z', households take expectation on the next period value V_2.

To define the value of completing college, it is useful to discuss how a college education affects households in the model. On the one hand, college degree affects the agent's life-cycle wages in two ways. First, college education allows them to enter the skilled labor market (i.e., $\kappa = 2$), receiving w_2 over the life cycle. Second, college changes the life-cycle wage profile through $\{\gamma_{j,\kappa}\}_{j=1}^{8}$. On the other hand, college is costly and requires a stochastic fixed cost of $\psi(\xi, a)$ (e.g., see Caucutt and Lochner, 2020). Specifically, the college cost is defined as

$$\psi(\xi, a) = \max\left\{\exp(\xi) - \iota \exp(-a), 0\right\}, \tag{5}$$

where ξ is an exogenous source of stochastic fixed costs, following an i.i.d. normal distribution with a mean of μ_ξ and a standard deviation of σ_ξ. Given the positive degree parameter $\iota > 0$, the second component is designed to capture needs-based scholarships, with $\exp(-a)$ being positive and decreasing with a. The max operator makes sure that the college cost stays non-negative.

Thus, the value of completing college after the realization of ξ is given by

$$C(h, a, \phi, \xi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_2(h', a', \kappa, \phi) \right\} \tag{6}$$

subject to

$$\begin{aligned} (1+\tau_c)c + a' + \psi(\xi, a) &\leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T \\ h' &= \exp(z')\gamma_{1,\kappa} h, \\ \kappa &= 2, \end{aligned} \tag{7}$$

where additional elements reflect the benefits and costs of college education, as described above.

Households make a discrete choice regarding college education after observing a draw of ξ. The expected value at the beginning of $j = 1$ is then defined as

$$V_1(h, a, \phi) = \mathbb{E}_\xi \max\left\{N(h, a, \phi), C(h, a, \phi, \xi)\right\}. \tag{8}$$

Period 2: Young adult without children. In this period, households face a standard life-cycle problem. That is, households make consumption-savings and labor supply decisions.

$$V_2(h, a, \kappa, \phi) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z', \phi'|\phi} V_3(h', a', \kappa, \phi') \right\} \tag{9}$$

strategy ensures that the model generates a reasonable degree of selection in line with the empirical evidence. In addition, although my tractable specification is likely to overstate the persistence of wage relative to the data, it is able to easily generate an increasing dispersion of wages over the life cycle in line with the empirical observations (see Figure A.1). See further discussions in Section 3.

subject to

$$(10) \qquad (1+\tau_c)c + a' \leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T$$
$$h' = \exp(z')\gamma_{2,\kappa} h.$$

The only nonstandard element consists of taking expectation to the learning ability of the child to be born next period (i.e., ϕ') because each household is going to be endowed with a child whose ability is drawn stochastically at the beginning of period $j = 3$. I assume that it is correlated across generations, following an AR(1) process in logs

$$(11) \qquad \log \phi' = \rho_\phi \log \phi + \varepsilon_\phi,$$

where $\varepsilon_\phi \sim N(0, \sigma_\phi^2)$. The exogenous source of a positive correlation of human capital across generations—which is standard in the literature (e.g., Restuccia and Urrutia, 2004; Herrington, 2015; Holter, 2015; Rauh, 2017; Lee and Seshadri, 2019)—may capture both genetic transmission and any residual intergenerational persistence not explained by modeled elements.

Periods 3–5: Parental investments. At the beginning of $j = 3$, a child is born with the learning ability of ϕ. The child's human capital at the end of childhood is affected by parental inputs and government inputs in periods $j = 3, 4, 5$, and their learning ability. The human capital production technology captures how these affect the whole process. My modeling approach builds on the childhood skill formation literature (Cunha and Heckman, 2007) insofar as it holds that skill formation is a multistage process and that investments in different periods are complementary. In contrast to the standard approach, I consider flexible substitutabilities between parental time and monetary investments and between parental and public inputs.

Specifically, I first describe how parental inputs and government inputs are aggregated in each period. Investment inputs take the form of time and money. Let I_j denote the total investment inputs in period j, aggregated following the constant elasticity of substitution technology

$$(12) \qquad I_j = \left\{ \theta_j^p \left(\theta_j^x \left(\frac{h^\varsigma x_j}{\bar{h}^\varsigma \bar{x}} \right)^{\zeta_j} + \left(1 - \theta_j^x\right) \left(\frac{e_j}{\bar{e}} \right)^{\zeta_j} \right)^{\frac{\psi}{\zeta_j}} + \left(1 - \theta_j^p\right) \left(\frac{g_j}{\bar{g}} \right)^{\psi} \right\}^{\frac{1}{\psi}},$$

where x_j is parental time, e_j is private education spending, g_j denotes public education investment, $\theta_j^x \in (0, 1)$ captures the relative share of time investments, and $\theta_j^p \in (0, 1)$ denotes the relative share of private education inputs in period j.[8] Total parental time inputs $h^\varsigma x_j$ are allowed to reflect different productivity depending on the parent's human capital h and the degree of this exogenous human capital transmission $\varsigma \geq 0$. In addition, note that $\zeta_j \leq 1$ shapes the elasticity of substitution between time and money in period j, $1/(1 - \zeta_j)$, and is allowed to be general, as compared to unit elasticity in Lee and Seshadri (2019). This age-dependent complementarity, together with the different amount of monetary investments, could additionally capture the notion of quality time that differs across different socioeconomic backgrounds in producing children's human capital endogenously. Since these inputs have different units, each input is entered after being normalized by their corresponding unconditional means in the baseline economy, which is useful for calibrating ζ_j.[9] In contrast to the standard

[8] For notational convenience, the technology is indexed by the parent's age j, given that there is a one-to-one relationship between children's age and the parent's age in the model.

[9] As shown by Cantore and Levine (2012), normalization is necessary for the analysis of changing the elasticity of substitution parameter unless it is fixed at one (Cobb–Douglas).

assumption in the literature, private and public monetary investments are assumed to be substitutable in a flexible manner instead of adhering to the assumption of perfect substitutability (e.g., Restuccia and Urrutia, 2004; Holter, 2015).

Given the aggregated inputs in period j, the human capital developed at the end of period 5, $h_{c,6}$, is determined by the following technology:

$$h_{c,6} = \phi f(I_3, I_4, I_5), \tag{13}$$

where $\frac{\partial^2 f}{\partial I_i \partial I_j} > 0$, implying dynamic complementarity (Cunha and Heckman, 2007; Caucutt and Lochner, 2020)). As in Lee and Seshadri (2019), the technology features unit elasticity of substitution across periods and constant returns to scale. The following recursive formulation is convenient to capture this technology over the full period:

$$\begin{aligned} h_{c,j+1} &= \phi I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 5 \\ &= I_j^{\theta_j^I} h_{c,j}^{1-\theta_j^I} \quad \text{if } j = 3, 4, \end{aligned} \tag{14}$$

where $\theta_j^I \in (0, 1)$.[10]

I now describe the decision problem of parents, which incorporates the human capital investment choices described above. I assume that the child shares the household consumption c, according to the household equivalence scale q, and does not make time allocation decisions relevant to the household's economic status during childhood. The following functional equation summarizes a parent's decision problem for $j = 3$:

$$V_3(h, a, \kappa, \phi) = \max_{\substack{c,\, e \geq 0;\ a' \geq \underline{a} \\ x,\, n \in [0,1]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} - \varphi x + \beta \mathbb{E}_{z'} V_4(h', a', \kappa, h_c', \phi) \right\} \tag{15}$$

subject to

$$\begin{aligned} (1+\tau_c)c + a' + e &\leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)\,a - \tau_k r \max\{a, 0\} + T \\ x + n &\leq 1 \\ h' &= \exp(z')\gamma_{3,\kappa} h \end{aligned}$$

$$h_c' = \left\{ \theta_3^p \left(\theta_3^x \left(\frac{h^\varsigma x}{\bar{h}^\varsigma \bar{x}} \right)^{\zeta_3} + (1-\theta_3^x) \left(\frac{e}{\bar{e}} \right)^{\zeta_3} \right)^{\frac{\psi}{\zeta_3}} + \left(1-\theta_3^p\right) \left(\frac{g_3}{\bar{g}} \right)^{\psi} \right\}^{\frac{\theta_3^I}{\psi}} h_c^{1-\theta_3^I}, \tag{16}$$

[10] One can easily recover f in (13) by

$$\begin{aligned} h_{c,6} &= \phi I_5^{\theta_5^I} h_{c,5}^{1-\theta_5^I} \\ &= \phi I_5^{\theta_5^I} \left(I_4^{\theta_4^I} h_{c,4}^{1-\theta_4^I} \right)^{1-\theta_5^I} \\ &= \phi I_5^{\theta_5^I} \left(I_4^{\theta_4^I} \left(I_3^{\theta_3^I} h_{c,3}^{1-\theta_3^I} \right)^{1-\theta_4^I} \right)^{1-\theta_5^I}. \end{aligned}$$

where $\varphi > 0$ captures the disutility of time investments and (16) is obtained by combining (12) and (14).[11] Note that parents have an incentive to invest their time x and money e in their children because these investments will lead to greater human capital at the end of childhood according to the production technology (13). On the other hand, these investments are costly: parental time reduces utility and private education spending reduces income available for consumption and savings.

For $j = 4, 5$, the decision problem is similarly defined as

$$V_j(h, a, \kappa, h_c, \phi) = \max_{\substack{c, e \geq 0;\ a' \geq \underline{a} \\ n, x \in [0,1]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} - \varphi x + \beta \mathbb{E}_{z'} V_{j+1}(h', a', \kappa, h_c', \phi) \right\} \tag{17}$$

subject to

$$(1+\tau_c)c + a' + e \leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T$$

$$n + x \leq 1$$

$$h' = \exp(z')\gamma_{j,\kappa} h$$

$$h_c' = \left\{ \theta_4^p \left(\theta_4^x \left(\frac{h^\varsigma x}{\bar{h}^\varsigma \bar{x}} \right)^{\zeta_4} + (1-\theta_4^x)\left(\frac{e}{\bar{e}}\right)^{\zeta_4} \right)^{\frac{\psi}{\zeta_4}} + \left(1-\theta_4^p\right)\left(\frac{g_4}{\bar{g}}\right)^{\psi} \right\}^{\frac{\theta_4^I}{\psi}} h_c^{1-\theta_4^I} \quad \text{if } j = 4 \tag{18}$$

$$= \phi \left\{ \theta_5^p \left(\theta_5^x \left(\frac{h^\varsigma x}{\bar{h}^\varsigma \bar{x}} \right)^{\zeta_5} + (1-\theta_5^x)\left(\frac{e}{\bar{e}}\right)^{\zeta_5} \right)^{\frac{\psi}{\zeta_5}} + \left(1-\theta_5^p\right)\left(\frac{g_5}{\bar{g}}\right)^{\psi} \right\}^{\frac{\theta_5^I}{\psi}} h_c^{1-\theta_5^I} \quad \text{if } j = 5, \tag{19}$$

where the state vector additionally includes the child's human capital at the beginning of the period, h_c. Recall that the state variable κ can take a value of either 1 (unskilled) or 2 (skilled), depending on the college decision made in the period $j = 1$.

Period 6: Inter vivos transfers. The decision problem in $j = 6$ includes a choice of inter vivos transfers a_c', which is transferred at the end of the period to the next generation as the latter enters $j = 1$ and forms a new household. This transfer could help their child's college decision financially and provide capital income flows over the life cycle. Specifically, at the beginning of $j = 6$, parents solve

$$V_6(h, a, \kappa, h_c, \phi) = \max_{a_c} \left\{ \tilde{V}_6(h, a - a_c, \kappa) + \eta\beta V_1(h_c', a_c', \phi) \right\} \tag{20}$$

$$a_c \in [0, a]$$

$$h_c' = \gamma_c h_c,$$

$$a_c' = (1+r)a_c,$$

where the continuation value includes the initial value function of the child, defined above in (8), weighted by the degree of altruism $\eta > 0$ and the standard discount factor β. Note that a_c cannot be negative, meaning that households are not allowed to borrow from their child's future income, and cannot be above their current asset holding a. As is clear

[11] Given the exogenous transmission of learning ability, the initial human capital when a child is just born is assumed to be homogeneous: $h_c = 1$ (see, e.g., Herrington, 2015; Lee and Seshadri, 2019).

in the continuation value term, the intergenerational link is modeled following a dynastic utility approach in the sense that parents care about their child's utility, which, in turn, depends on the next generation's utility, and so on. This recursive structure linked by altruism combines successive generations as a single dynasty as in Becker and Tomes (1986).

In the next stage of $j = 6$, parents with the asset net of the inter vivos transfers solve a standard consumption-savings and labor supply problem as follows:

$$\tilde{V}_6(h, a, \kappa) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{(c/q)^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_7(h', a', \kappa) \right\} \tag{21}$$

$$\text{subject to} \quad (1+\tau_c)c + a' \leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T$$
$$h' = \exp(z')\gamma_{6,\kappa} h.$$

Periods 7 onward: Without children. Once the child becomes an adult, the state variables do not include h_c and ϕ. The decision problems in the remaining periods are standard. Households make consumption-savings and labor supply decisions in periods $j = 7, 8, 9$ (age 50–64) until they retire in $j = 10$ (age 65). The household's problem in $j = 7, 8, 9$ is summarized by

$$V_j(h, a, \kappa) = \max_{\substack{c \geq 0;\ a' \geq \underline{a} \\ n \in [0,1]}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - b\frac{n^{1+\chi}}{1+\chi} + \beta \mathbb{E}_{z'} V_{j+1}(h', a', \kappa) \right\} \quad \text{if } j = 7, 8, 9 \tag{22}$$

subject to

$$(1+\tau_c)c + a' \leq \lambda_j (w_\kappa hn/\bar{y})^{-\tau_j} w_\kappa hn + (1+r)a - \tau_k r \max\{a, 0\} + T$$
$$h' = \exp(z')\gamma_{j,\kappa} h.$$

When households retire ($j = 10, 11, 12$), they receive social security pension payments Ω.[12] The value functions during the retirement stages are given by

$$V_j(h, a, \kappa) = \max_{c \geq 0;\ a' \geq \underline{a}} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \beta V_{j+1}(h, a', \kappa) \right\} \tag{23}$$

subject to

$$(1+\tau_c)c + a' \leq (1+r)a - \tau_k r \max\{a, 0\} + T + \Omega$$

and $V_{j=13}(\cdot) = 0$.

2.2. *Firm's problem and government.* A representative firm produces output with technology featuring constant returns to scale. The production function is assumed to be Cobb–Douglas

$$Y = K^\alpha H^{1-\alpha}, \tag{24}$$

[12] This assumption on the flat pension benefit is quite common (e.g., Abbott et al., 2019). I have considered a version of the model with a more realistic pension that increases with human capital in a concave manner. Given the nature and focus of this article, this change has little effect on the quantitative results.

where K is aggregate capital stock, H denotes the aggregate labor input, and $\alpha \in (0, 1)$. The aggregate labor input H is then defined as

$$H = \left[\nu H_1^{\rho} + (1-\nu)H_2^{\rho}\right]^{\frac{1}{\rho}}, \tag{25}$$

where $\rho < 1$ determines the elasticity of substitution, $1/(1-\rho)$, between skilled workers H_2 and unskilled workers H_1.

The representative firm in competitive markets solves the following profit maximization problem:

$$\max\left\{Y - w_1 H_1 - w_2 H_2 - (r+\delta)K\right\},$$

where δ is the capital depreciation rate. The first-order conditions are

$$[K] : \alpha K^{\alpha-1} H^{1-\alpha} = r + \delta, \tag{26}$$

$$[H_1] : (1-\alpha)K^{\alpha} H^{-\alpha}\frac{1}{\rho}\left[\nu H_1^{\rho} + (1-\nu)H_2^{\rho}\right]^{\frac{1}{\rho}-1}\nu\rho H_1^{\rho-1} = w_1, \tag{27}$$

$$[H_2] : (1-\alpha)K^{\alpha} H^{-\alpha}\frac{1}{\rho}\left[\nu H_1^{\rho} + (1-\nu)H_2^{\rho}\right]^{\frac{1}{\rho}-1}(1-\nu)\rho H_2^{\rho-1} = w_2. \tag{28}$$

Government tax revenues from labor income, capital income, and consumption are spent on four categories: (i) social security pension payments Ω to retirees; (ii) lump-sum transfers T to all households, (iii) public education for children $\{g_j\}_{j=3}^{5}$; and (iv) government spending G, which is not directly valued by households. Government balances its budget in each period.

2.3. *Equilibrium.* Let $x_j \in X_j$ denote the age-specific state space defined according to the household's recursive problems in Subsection 2.1. A stationary recursive competitive equilibrium is a collection of factor prices w_1, w_2, r, the household's decision rules, value functions $V_j(x_j)$, government policies, and age-specific measures π_j over x_j such that

1. Given government policies and factor prices, household decision rules solve the household's life-cycle optimization problems defined in the previous subsection, and $V_j(x_j)$ are the associated value functions.
2. Factor prices w_1, w_2, and r are competitively determined according to (26)–(28).
3. Markets clear:

$$K = \sum_{j=1}^{12}\mu_j \int a_j d\pi_j,$$

$$H_s = \sum_{j=1}^{12}\mu_j \int h_j n_j(x_j) d\pi_j(\cdot|\kappa = s), \quad s = 1, 2.$$

4. Government budget is balanced: The sum of transfer payments, social security pension payments, public education spending, and the residual government spending $G(\geq 0)$ is equal to the sum of labor income tax revenues, capital income tax revenues, and consumption tax revenues;
5. The vector of age-specific household measures $\pi = (\pi_1, \pi_2, \ldots, \pi_{12})$ is the fixed point of $\pi(X) = P(X, \pi)$ where $P(X, \cdot)$ is a transition function determined by the household decision rules and the exogenous probability distributions, and X is the generic subset of the Borel σ-algebra B, defined over the state space $\mathbf{X} = \prod_{j=1}^{12} X_j$.

3. CALIBRATION

I calibrate the parameter values of the baseline model economy to match relevant U.S. statistics. As is standard, there are two sets of parameters. The first set of parameters is chosen externally without using model-generated data, whereas the second set of parameters is determined internally. I now describe them in detail.

3.1. *Parameters Calibrated Externally.* The two curvature parameters in the utility function, σ and χ, govern the household's willingness to substitute intertemporally. I set the value of σ equal to 1.5 so that the intertemporal elasticity of substitution for consumption is 2/3 and the value of χ is equal to 4/3, implying a Frisch elasticity of 0.75 (Chetty et al., 2013). As discussed in the previous section, when a parent lives with a child, consumption in the utility function is replaced by c/q. The value of q is set to 1.59 based on the OECD equivalence scale.

The gross growth rates of human capital during adulthood $\{\gamma_{j,\kappa}\}_{j=1}^{8}$ for each education level κ govern the age-wage profiles for high- and low-skilled workers. Table A.2 in Appendix A.2 reports these 16 values computed based on the estimates in the Panel Study of Income Dynamics (PSID) samples of Rupert and Zanella (2015). The key features captured by these estimates are that (i) growth rates are much higher in early adulthood and diminish with age; and (ii) college-educated households have significantly higher growth rates than those without a college degree. The parameter γ_c that maps childhood human capital to adulthood human capital is set to 33.9 so that the annual output in the baseline model is normalized to be one.

Several parameters in the childhood human capital production function are externally calibrated following the calibration strategy in Fuchs-Schündeln et al. (2020). First, the value of ψ is set to 0.588. This value yields an elasticity of substitution between private and public education of 2.43, implying that they are quite substitutable but are still far from being perfect substitutes. Second, the parameter for the relative share of private investments θ_p^j is set to 0.324 for $j = 4, 5$.[13] These parameter values are estimated in Kotera and Seshadri (2017) and are also used in Fuchs-Schündeln et al. (2020).

I now move on to the parameters related to government. As noted earlier, labor income taxation is progressive, with the degree of progressivity differing by household structure. Table A.3 in Appendix A.2 reports how these values are chosen for each j. A key feature to note is that progressivity is higher for households with a child. The tax rate for capital income τ_k is set to 0.36. Both labor and capital taxation parameters are based on the estimates in Holter et al. (2019). The consumption tax rate τ_c is set to 0.07 (McDaniel, 2007). To obtain the size of public education investments, I follow the approach by Restuccia and Urrutia (2004) and Holter (2015) insofar as I treat education spending by state and federal government as a public investment and education spending by local government as a private investment. This is motivated by the fact that early education in the United States is largely locally financed. Using the information from the 2016 edition of Education at Glance, published by the OECD, I obtain public investments in periods 3–5 relative to mean income of 0.060, 0.098, and 0.111, respectively.[14] Note that public education spending increases with the child's education stage. Next, following Lee and Seshadri (2019), the size of government transfers T is set to 2% of output to capture welfare programs. Finally, the value of Ω is set to reflect a social security replacement rate of 33% (Abbott et al., 2019).

As for the production sector, the capital share in the aggregate U.S. data results in the choice of $\alpha_K = 0.36$. The five-year capital depreciation rate δ is computed based on 2.5% of the quarterly depreciation rate. These parameter values are within the range commonly used

[13] θ_p^3 is relevant to kindergarten and preschool. The parameter is calibrated internally as described later.

[14] Details are available in Appendix A.2. These values are in line with the estimates in Lee and Seshadri (2019).

TABLE 2
INTERNALLY CALIBRATED PARAMETERS AND TARGET STATISTICS

Parameter		Target Statistics	Data	Model
Preference				
β	0.949	Equilibrium real interest rate (annual)	0.04	0.04
b	6.37	Average hours of work (age 30–64)	0.287	0.313
φ	0.674	Average hours of work (age 30–44)	0.299	0.301
η	0.224	Average inter vivos transfers/annual GDP per capita	0.750	0.938
Childhood human capital production				
θ_3^x	0.827	Average parental time investments in period 3	0.061	0.062
θ_4^x	0.214	Average parental time investments in period 4	0.036	0.036
θ_5^x	0.145	Average parental time investments in period 5	0.020	0.020
ς	0.166	Rank correlation of parental income and child earnings	0.282	0.305
θ_3^p	0.696	Average parental monetary investments in period 3	0.098	0.091
α_I	0.268	Average parental monetary investments in period 4	0.113	0.102
θ_5^I	0.371	Average parental monetary investments in period 5	0.128	0.120
ζ_3	−3.50	Educational gradients in parental time in period 3 (%)	20.9	19.6
ζ_4	0.315	Educational gradients in parental time in period 4 (%)	14.8	13.8
ζ_5	0.266	Educational gradients in parental time in period 5 (%)	20.2	19.7
College				
ν	0.543	Fraction with a college degree (%)	34.2	34.6
ι	0.542	Degree of positive selection	0.50	0.491
μ_ξ	0.334	Average college expenses/GDP per capita	0.140	0.144
δ_ξ	0.524	Observed college wage gap (%)	75.0	65.8
Remaining parameters				
ρ_ϕ	0.042	Rank correlation of parental income and child income	0.341	0.385
σ_ϕ	0.470	Gini wage	0.370	0.337
σ_z	0.148	Slope of variance of log wage from age 25–29 to age 55–59	0.180	0.185
$\underline{a}$	−0.064	Average unsecured debt relative to annual disposable income	0.010	0.010

in the quantitative macroeconomics literature (e.g., Krusell and Smith, 1998). I set $\rho = 1/3$ so that the elasticity of substitution between skilled and unskilled labor is 1.5 (Ciccone and Peri 2005).[15]

3.2. *Parameters Calibrated Internally.* The rest of the parameters are calibrated internally. Table 2 summarizes a set of parameters that are jointly calibrated by simulating the model economy. These parameter values are determined as minimizers of the distance between the relevant statistics from the data and those from the model-generated data. Despite a relatively large number of parameters and targets, there are clear relationships between them, and the model matches the target statistics quite well. I now explain the role of these parameters in the model and illustrate how each parameter is related to its target statistic, as summarized in Table 2. All statistics regarding time-use are obtained from the 2003–2017 waves of the American Time Use Survey (ATUS), combined with the Current Population Survey (CPS). More details on the data are provided in Appendix A.1.

3.2.1. *Preference.* First, β is households' discount factor. The relevant target for this parameter is set as the annual interest rate of 4%, which is standard in the literature. The equilibrium capital–output ratio is 2.93 at an annual frequency, which is in line with U.S. data. The next parameter b determines the disutility constant for hours worked. The relevant target for b is set to be the average weekly hours of work for those whose age is between 30 and 64. This leads to $30.16/105 = 0.287$, provided that the weekly feasible time endowment is $105 (= 15 \times 7)$ hours, excluding time for sleeping and basic personal care. Similarly, the disutility

[15] As this elasticity is important for policy exercises that strongly influence college decisions, I also present the policy exercise results regarding the effects of college subsidies using $\rho = 2/3$, or the elasticity of 3—which is close to the value used in Abbott et al. (2019)—in Table A.7.

parameter φ affects average parental time investments in all periods. Given the calibration strategy using human capital production technology to control average parental time investments in period j as described below, φ is calibrated to match the average hours worked during the periods when time investments are made (age 30–44): 0.299. Finally, η is calibrated to match the average inter vivos transfers. inter vivos transfers in the model provide young households with financial resources that help complete college education and enjoy capital income over the life cycle. The relevant target is the average total parental transfers made to children, which amounts to 75% of the annual income according to the PSID sample (Daruich, 2020).

3.2.2. *Childhood human capital production.* There are 10 parameters—$\{\theta_j^x\}_{j=3}^5$, ς, θ_3^p, α_I, θ_5^I and $\{\zeta_j\}_{j=3}^5$—which are internally calibrated regarding the human capital production technology (16), (18), and (19). To do so, I use the clear linkages between each parameter and its corresponding target moment in the model economy. First, θ_j^x determines the relative importance of time investments (as compared to monetary investments), and it clearly increases the average parental time investment in period j. Therefore, for each j, the target moment for θ_j^x is set to be the mean parental time investments in period j. To compute statistics regarding parental time investment, I focus on parental time spent directly with children that can promote the development of their human capital (see Appendix A.1 for details). A notable feature of these moments is that they are highest in the early years (0.061 in the model or 6.4 hours per week) and decline with children's age. The calibrated θ_j^x decreases with j, meaning that time is more important than money in earlier childhood (Del Boca et al., 2014). Since ς governs the extent to which parental human capital affects the effectiveness of time investments, it shapes the strength of intergenerational persistence of lifetime income through human capital transmission (or labor income). Therefore, the rank correlation of parental income and child earnings—0.282 (Chetty et al., 2014a)—is set as a target moment.

Next, a higher θ_3^p raises parental monetary investments in period 3 and a higher θ_j^I increases them in period j. As in Fuchs-Schündeln et al. (2020), $\{\theta_j^I\}_{j=3}^5$ is assumed to be a function of j: $\theta_j^I = \theta_5^I + \alpha_I \times (5 - j)$. For θ_3^p, α_I, and θ_5^I, I set the mean private education spending in period j as target moments. As discussed above, average private education spending in the data is constructed as the sum of both private spending and local government spending as in Restuccia and Urrutia (2004) and Holter (2015). This leads to the target statistics of 0.098, 0.113, and 0.128 for $j = 3, 4$, and 5, respectively (see Appendix A.2 for details). Unlike parental time inputs, note that monetary inputs increase with children's education stage. The calibrated θ_3^p is 0.696, indicating that the relative importance of parental investments is much higher in $j = 3$ than in $j = 4, 5$.

Finally, ζ_j governs the elasticity of substitution between time and money in period j. In U.S. data, more educated parents spend more time with children (Guryan et al., 2008; Ramey and Ramey, 2010)). I use this elasticity of substitution as a driver to replicate this salient fact.[16] The empirical moments are obtained from the ATUS data. Educational gradients, estimated by controlling for some observable characteristics of parents, are around 20%, meaning that college-educated parents spend 20% more time with their children than parents without a college degree.[17] To match the stage-specific educational gradients, the baseline specification allows ζ_j to differ by j. In the model, increasing inequality in parental income as parents get older implies that the educational gradient in parental time naturally becomes greater

[16] Zhu and Vural (2013) show how the complementarity between time and money in human capital production affects the wage gradient of parental time in an analytically tractable model with two-period-lived overlapping generations and a single parental investment period.

[17] More precisely, the education gradient refers to the percentage difference in mean parental time investments between education groups. See Appendix A.1 for details.

for the older parents. Therefore, the calibration implies that the elasticity of substitution between parental time and monetary inputs is close to 1 (or Cobb–Douglas) in later periods ($j = 4, 5$).[18] Meanwhile, I find that parental monetary investments are a poor substitute for parental time investments for very young children (e.g., preschool aged) with the elasticity of substitution being 0.22.

3.2.3. *College.* The parameter ν in the aggregate production function (25) is calibrated to match the fraction with a college degree (34.2%), as in Lee and Seshadri (2019). In the United States, people with higher precollege human capital are more likely to have a college degree. Specifically, the probability of being a four-year college graduate is about 50 percentage-point higher for the top precollege human capital quintile than for the bottom quintile (Heckman et al., 2006). Recall that the value of ι in the cost of college (5) governs the relative strength of need-based scholarships in determining college costs. As ι increases, more asset-poor households would be able to go to college (holding other things constant), thereby reducing the degree of positive selection. Therefore, I choose this as a target statistic to discipline the degree of positive selection into college in the model.

The target statistic for μ_ξ in the model is set to be the equilibrium ratio of average (tuition and nontuition) expenses after financial aid to per capita GDP. Specifically, I begin by computing the average ratio of annual college tuition and required fees (excluding room and board) for four-year institutions to per capita real GDP for 1990–2011, which is 0.22 according to the Digest of Education Statistics (2011, Table 349) and the Bureau of Economic Analysis. To approximate actual costs faced by students, I also include nontuition expenses such as books, other supplies, commuting costs, and room and board expenses that would not have to be paid by a person who chooses not to go to college, as in Abbott et al. (2019). These nontuition expenses amount to approximately 30% of average tuition and fees. In 2000–2001, the average grants (federal, state/local, and institutional) received by full-time students in four-year colleges weighted by enrollment are approximately 50% of the average tuition and fees. Based on the above information and assuming that college completion takes four years, the equilibrium ratio of average financial college costs to the five-year per capita GDP is 0.14. Finally, as the variability of college costs σ_ξ increases, the observed wage premium tends to decline. The observed college premium, or the ratio between the average wage of those with a college degree and the average wage of those without a college degree ranges from 70% to 80% in the ATUS samples depending on the age bands. Thus, I choose 75% as a target, which is also in the range of recent estimates in Heathcote et al. (2010).

3.2.4. *Remaining parameters.* A higher ρ_ϕ leads to a higher degree of economic associations across generations. I set its relevant target as the rank correlation of family income of 0.341 (Chetty et al., 2014a), which has been relatively stable in the United States (Chetty et al., 2014b). Due to the data limitation, Chetty et al. (2014a) estimate intergenerational persistence using the proxy income variable instead of lifetime income. The rank correlation from the model, which is used as a target statistic, is also obtained based on the proxy incomes as in Chetty et al. (2014a) (see Section 5 for the precise definition of proxy income).

Recall that the idiosyncratic shocks to adult human capital z, following a normal distribution, have a mean of zero with the standard deviation of σ_z. Since both σ_ϕ and σ_z are exogenous sources of the cross-sectional dispersion of wages in the model, I choose the Gini coefficient of wages (0.37) as a target statistic. Note that although the degree of wage inequality

[18] I also consider an alternative calibration strategy where $\zeta_j = \zeta$ for all j, and report the results in Table A.5. When the model is calibrated in this way by matching the overall education gradient (without targeting age-specific gradients), the model implies that educational gradients in parental time investment increase sharply as parents and children get older.

increases with either σ_ϕ or σ_z, their economic mechanism is very different. This is because σ_ϕ affects the variability of the initial condition in human capital, whereas σ_z affects households over an individual's working life. Specifically, holding the overall dispersion of wage constant, in the case when σ_z is relatively larger, households would experience more volatile idiosyncratic shocks to human capital, the effect of which accumulates over the life cycle. As a result, the life-cycle profile of wage inequality would become steeper. Therefore, I choose the difference between the variance of log wages at age 55–59 and that of log wages at age 25–29 as an additional target to pin down the relative contribution of each shock process to overall wage inequality.[19] These statistics on wage inequality in U.S. data for recent periods, obtained from Heathcote et al. (2010), are reported in Table 2.

Finally, the borrowing limit $\underline{a}$ is calibrated so that the average debt in equilibrium amounts to 1% of the five-year GDP per capita. This target moment is in line with Livshits et al. (2010) who find that the average unsecured debt relative to annual disposable income ranged between 5% and 9% in the 1980s and 1990s.

4. ASSESSING THE MODEL AS A QUANTITATIVE THEORY OF INTERGENERATIONAL MOBILITY

Before turning to the quantitative exercises, which will include the counterfactual and policy experiments, this section evaluates the baseline model economy as a quantitative theory of intergenerational mobility. I consider three measures of intergenerational mobility: (i) the intergenerational elasticity of income (IGE); (ii) the rank correlation; and (iii) the quintile transition matrix. The intergenerational mobility estimates reported below are based on family income to be consistent with U.S. data counterparts from Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), family income is the five-year per-parent average of the pretax income defined as either the sum of adjusted gross income, tax-exempt interest income, and the nontaxable portion of social security and disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits (otherwise). In the model, family income is the five-year per-parent sum of labor earnings, interest income, and social security benefits. It is worth noting that family income is preferred as a measure of intergenerational mobility in economic status when samples include both males and females (Chadwick and Solon, 2002), which is the case in Chetty et al. (2014a) as well as in my gender-neutral model.

4.1. *IGE and Rank Correlation.* The first measurement is the IGE, a conventional way to measure the degree of intergenerational persistence. The IGE is the slope coefficient obtained by running the following log-log regression equation:

$$\log \mathcal{Y}_{child} = \rho_0 + \rho_1 \log \mathcal{Y}_{parent} + \varepsilon, \tag{29}$$

where $\mathcal{Y}$ is permanent income. The IGE provides a straightforward interpretation: a 1% increase in parental permanent income is associated with a ρ_1% increase in their children's permanent income. Thus, a high ρ_1 implies low intergenerational mobility. The second way to measure intergenerational mobility is to use a rank-rank specification instead of a log-log specification (Chetty et al., 2014a, 2014b). In other words, I estimate the slope parameter after replacing log income with the percentile rank of income within a single generation in (29). The slope coefficient in a rank-rank specification (or the rank correlation) has a similar interpretation: a 1 percentage-point increase in the parent's percentile rank is associated with a ρ_1 percentage-point increase in their children's percentile rank.[20] Unlike the IGE, the

[19] With the help of this target, the model replicates the life-cycle inequality of wages and earnings over the age quite well, as shown in Figure A.1.

[20] Note that the rank-rank slope estimate is simply equal to the correlation coefficient in percentile rank (or Spearman correlation) since the independent and dependent variables, both of which are normalized by transforming the income level to the percentile ranks, have the same variance.

TABLE 3
INTERGENERATIONAL PERSISTENCE ESTIMATES

	U.S. Data	Model	
	Chetty et al. (2014a)	Proxy Income	Lifetime Income (Discounted)
IGE: log-log slope	0.344	0.341	0.406
Rank corr: rank-rank slope	0.341	0.385	0.387

[20]NOTE: The log-log slope estimate is obtained from a univariate regression equation where the dependent variable is the child's log income and the independent variable is the parent's log income. The rank-rank slope estimate is obtained from an equivalent regression equation replacing log transformation with the percentile rank.

rank correlation is less sensitive to the treatment of zero income observations and is relatively robust to the point of measurement in the income distribution (Chetty et al., 2014a, 2014b).

The biggest challenge in estimations of intergenerational mobility is the data requirement: a data set that contains career-long income histories (or permanent income) for at least two successive generations is needed. In practice, this limitation is overcome by replacing permanent income with proxy income measured at a point in the life cycle. For the purposes of comparison, I present model statistics based on proxy income defined similarly to Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), a child's income is measured when children are around 30 years old, averaged over two years. The parent's income is averaged over five years when parents are roughly 45 years old. Accordingly, in the model, the age at which the parent's income is measured is set to be 45–49 ($j = 6$), and the age at which the child's income is measured is 30–34 ($j = 3$). I also compute the intergenerational persistence measures using present-value lifetime income discounted according to the equilibrium real interest rate (Haider and Solon, 2006).

Table 3 reports these first two measures (i.e., slope estimates) from the model and the data. The first column shows estimates from U.S. data in Chetty et al. (2014a). Recall that the rank-rank slope using proxy income has been used as a calibration target. The estimate of the log-log slope (IGE) using lifetime income is 0.406, which is close to the estimates of around 0.4 in Solon (1999). Moreover, note that this estimate, which uses lifetime income, is considerably larger than the estimate of 0.341 using proxy income. This is in line with empirical-study observations that the short-term income (even multiyear averages) may not represent permanent income, which leads to attenuation bias in estimating the persistence of income across generations. The bias is negligible in the estimate of the rank-rank slope using proxy income instead of lifetime income.

4.2. *Quintile Transition Matrix.* In what follows, I use the quintile transition matrix as a means of evaluating just how successful a candidate model is as a quantitative theory of intergenerational mobility. The income quintile transition matrix is a 5×5 matrix where the (a, b) element gives the conditional probability that a child's lifetime income is in the bth quintile of his generation's distribution, provided that his parent's income is in the ath quintile of her own generation's distribution. This matrix provides a richer description of how economic status is transmitted across generations than the first two measures of correlations. Given that calibration targets do not include any elements in the income quintile transition matrix and that the same correlation of income across generations can be obtained from different disaggregated moments in the quintile transition matrix, comparing the model output to the

TABLE 4
INCOME QUINTILE TRANSITION MATRICES: DATA VERSUS MODEL

Unit: %	U.S. Data					Model									
	Chetty et al. (2014a)					Proxy Income					Lifetime Income				
Parent	Child Quintile					Child Quintile					Child Quintile				
Quint.	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	33.7	28.0	18.4	12.3	7.5	36.5	24.6	18.2	13.6	7.1	36.4	25.0	18.1	13.8	6.7
2nd	24.2	24.2	21.7	17.6	12.3	26.0	23.2	20.9	17.8	12.0	25.9	23.3	21.0	18.1	11.7
3rd	17.8	19.8	22.1	22.0	18.3	19.3	20.9	21.6	21.0	17.2	19.1	20.5	22.2	20.8	17.5
4th	13.4	16.0	20.9	24.4	25.4	13.1	17.9	21.2	23.2	24.7	12.9	17.7	21.0	23.4	25.1
5th	10.9	11.9	17.0	23.6	36.5	5.2	13.3	18.1	24.4	39.0	5.7	13.6	17.7	24.0	39.0

empirical quintile transition matrix would be a straightforward way of evaluating a model as a quantitative theory of intergenerational mobility.[21]

Table 4 compares the transition matrix obtained from U.S. data (Chetty et al., 2014a) to the transition matrices using the model-generated data. Three features in the U.S. data transition matrix are worth noting. First, it shows that the observed positive correlations of income across generations are not simply due to the intergenerational poverty trap but also to the rich families sustaining their economic status over generations. Specifically, the probability of children remaining in the bottom quintile when their parents' income lies in the bottom quintile is 33.7%, whereas the probability of children staying in the top quintile when their parents' income is in the top quintile is even higher: 36.5%. Second, there is quite a bit of mobility in the middle of the income distribution. For instance, children born to parents in the third quintile are almost equally likely to be located in any income quintiles (18–22%). Third, both upward mobility, measured by the probability of moving from the bottom quintile to the top quintile, and downward mobility, measured by the probability of moving from the top quintile to the bottom quintile, are quite low (7.5% and 10.9%, respectively).

The middle panel of Table 4 reveals that the model is able to account strikingly well for these salient features in the U.S. income quintile transition matrix despite the fact that the calibration only targets the overall correlation of income across generations. Specifically, the model generates a high probability of staying in the bottom quintile (36.5%) and an even higher probability of staying in the top quintile (39.0%). The model also predicts a substantial degree of mobility in the middle of the income distribution: children born to third-quintile parents are almost equally likely to end up in any quintile (around 20%). Finally, the upward mobility rate is 7.1% in the model, which is very similar to the data (7.5%).

The right panel of Table 4 reports the quintile transition matrix when lifetime income is used. As shown in Table 3, using lifetime or proxy income barely affects the rank correlation. This is evident from the similar probabilities of remaining in the bottom (36.4%) and in the top (39.0%) income quintiles. The upward mobility rate is slightly lower (6.7%) when lifetime income is used. Given the likely vulnerability of proxy income estimations to attenuation biases (Haider and Solon, 2006), the following sections use lifetime income instead of proxy income to estimate intergenerational mobility measures.

5. HETEROGENEITY IN PARENTAL INVESTMENTS AND INTERGENERATIONAL MOBILITY

In this section, I use the calibrated model to investigate the importance of differences in parental investments across households at different stages of childhood on the intergenerational mobility of lifetime income. In the baseline model, households endogenously choose

[21] Note that this is in the same spirit as the model validation exercises in the quantitative macroeconomics literature on income and wealth inequality. For instance, the same high Gini coefficient can be due to various combinations of sizeable poor households and super rich households.

TABLE 5

QUANTITATIVE EFFECTS OF HETEROGENEOUS PARENTAL BEHAVIORS ON INTERGENERATIONAL MOBILITY

	IGE	Rank Corr.	Upward Mobility (%)	Educ. Gradient in x_j (%) $j=3$	$j=4$	$j=5$	$\bar{a}_1/Y$
Baseline	0.406	0.387	6.7	19.6	13.8	19.7	0.188
Counterfactuals shutting down							
(i) Heterogeneity in parental monetary investment							
- $e_3 = \bar{e}_3$	0.317	0.299	9.5	−6.5	14.1	19.7	0.212
- $e_4 = \bar{e}_4$	0.286	0.269	10.2	18.3	3.3	19.7	0.211
- $e_5 = \bar{e}_5$	0.284	0.267	10.4	18.4	14.1	2.5	0.220
(ii) Heterogeneity in parental time investment							
- $x_3 = \bar{x}_3$	0.377	0.357	7.5	0.0	14.0	19.8	0.188
- $x_4 = \bar{x}_4$	0.406	0.387	6.8	19.7	0.0	19.7	0.187
- $x_5 = \bar{x}_5$	0.405	0.386	6.8	19.7	13.8	0.0	0.187
(iii) Heterogeneity in all parental investments							
$x_j = \bar{x}_j, e_j = \bar{e}_j, j = 3, 4, 5$	0.084	0.070	18.3	0.0	0.0	0.0	0.247
(iv) Heterogeneity in inter vivos transfers							
- $a_1 = \bar{a}_1$	0.410	0.394	7.0	20.6	11.5	17.9	0.188

NOTE: In Panel (i), the government provides the average amount of $\bar{e}_j$ in the baseline economy, financed by labor income taxation (via proportional changes in $\{\lambda_j\}_{j=1}^{9}$), whereas parents are forced to spend zero monetary investments in period j. In Panel (ii), parents are forced to make the average level of time investment $\bar{x}_j$ in the baseline economy in period j. In Panel (iii), (i) and (ii) are jointly imposed for all childhood periods. In Panel (iv), the government provides the same initial asset level $\bar{a}_1$—equal to the average inter vivos transfers in the baseline economy—financed by labor income taxation (via proportional changes in $\{\lambda_j\}_{j=1}^{9}$), whereas parents are prohibited from transferring money individually.

to invest different amounts of time and money. To quantify the importance of heterogeneity in parental investments across households, I impose that all parents invest exactly the average amount of money or time as in the baseline model. Note that the thought experiments in this section are not meant to be realistic. Rather, the goal is to evaluate the role of heterogeneity in different parental behaviors through restrictions imposed within the model. This approach will reveal the effects of such restrictions in the presence of other operating channels that could have reinforcing or dampening effects.

First, I explore the role that heterogeneity in parental monetary investments—a channel that has been studied and highlighted extensively in the literature[22]—plays in shaping intergenerational mobility. This thought experiment consists of equalizing monetary investments, which is practically difficult to implement since low-income families might not be able to afford the average monetary investments in the baseline economy. As such, I perform the following exercise: the government provides the average amount of $\bar{e}_j$ in the baseline economy, financed by labor income taxation through proportional changes in $\{\lambda_j\}_{j=1}^{9}$ in (2). Parents are then forced to refrain from making monetary investments in period j.

Panel (i) of Table 5 shows that heterogeneity in monetary investments plays a significant role in shaping intergenerational mobility. This is in line with findings in the literature on the high importance of parental monetary investments, which are profoundly affected by economic resources (e.g., Restuccia and Urrutia, 2004; Holter, 2015). In particular, these results reveal that the impact of such investments on intergenerational mobility is considerably stronger in later childhood, when monetary investments play a larger role in children's human capital development (i.e., low θ_j^x). It is worth noting that this exercise is based on a model with endogenous parental time investments. Table 5 shows that positive educational gradients in parental time investment disappear when monetary investments are equalized. These results

[22] See, for example, Restuccia and Urrutia (2004), Holter (2015), Lee and Seshadri (2019), and Caucutt and Lochner (2020). Note that in the model, e could also capture costs of living in a better neighborhood with better public schools, which has also been shown to be important for mobility (e.g., Herrington, 2015).

suggest that parental time investment acts as a mechanism that amplifies the effects of monetary investments.

I now move on to the key channel of interest in the article: parental time investment. Panel (ii) in Table 5 reports the results when I impose that all parents invest exactly the average amount of time as in the baseline model. This is feasible without adjusting taxation since the time endowment is inherently equal across households.[23] As above, because parental investments are made in multiple periods, I consider three different ways of removing heterogeneity in parental time investment at different stages of childhood by imposing $x_j = \bar{x}_j$ for each period j individually.

The results show that intergenerational mobility measures change quite significantly when $x_3 = \bar{x}_3$ (i.e., preschool-aged children). Both the IGE and the rank correlation fall by around 7–8% and the upward mobility rate go up by 12%. By contrast, Table 5 also reports that when I impose $x_j = \bar{x}_j$ for $j = 4, 5$ (i.e., school-aged children), all three measures indicate that intergenerational mobility would only marginally increase. For example, both the IGE and the rank correlation fall by less than 1% in both cases.

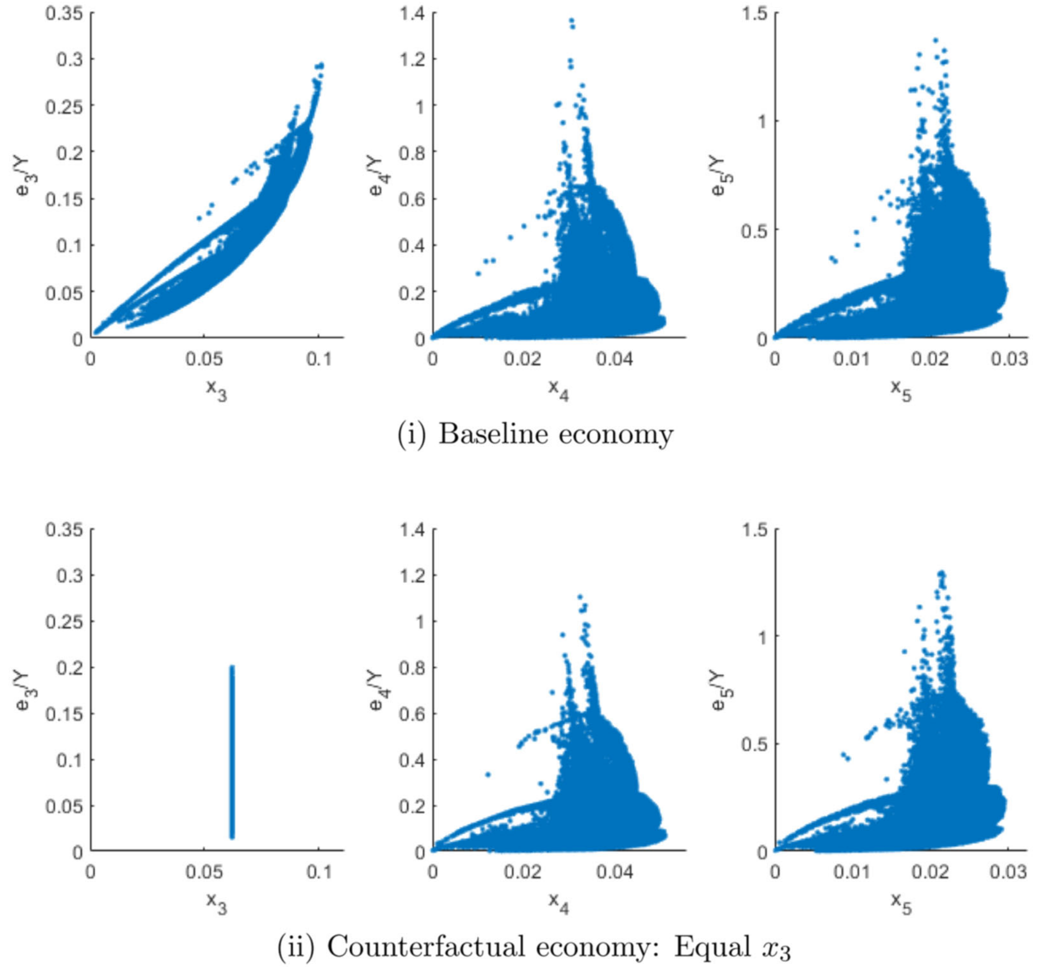
To better understand the mechanism, it is useful to look at the equilibrium relationship between individual time and monetary investment behaviors. The upper three figures of Figure 1 show this relationship in the baseline model economy for each period $j = 3, 4, 5$. In the upper figures, there are clear positive associations between time (x-axis) and money (y-axis) due to complementarities between these two inputs. The bottom three figures show their counterparts when $x_3 = \bar{x}_3$ is imposed. When parents are constrained to invest the fixed mean time $\bar{x}_3$ in the bottom-left figure, the variation in monetary investments also becomes lower. This equalizing force tends to increase intergenerational mobility.

A natural question then is why do the effects of heterogeneity in parental time investments vary by child age? The key to understanding this result is the difference in substitutability between parental time and monetary investments. Specifically, as shown in Table 5, the elasticity of substitution between parental time and money is substantially higher in later childhood (above one in period $j = 4, 5$) than during early childhood (0.2 in period $j = 3$). Since rich parents cannot complement their higher monetary investment demands (due to high available economic resources) with greater time investments when the restriction of $x_j = \bar{x}_j$ is in place, they choose to invest even more money into their children. This substantially increases average monetary investments. For example, with the restriction of $x_4 = \bar{x}_4$, $\bar{e}_4$ increases by 8% (and with $x_5 = \bar{x}_5$, $\bar{e}_5$ increases by 5%). Moreover, as also shown in Table 5, the calibrated θ^x_j increases with j, implying that parental monetary investments are more important in later childhood. Therefore, parental responses—along with the properties of the human capital technology—can substantially mitigate the direct impact of equalizing parental time inputs.

Yet, equalizing parental time investments in period $j = 3$ notably increases intergenerational mobility. Since the elasticity of substitution between parental time and monetary investments is much lower during this period, monetary investments are a poor substitute for parental time investments. Therefore, with the restriction of $x_3 = \bar{x}_3$, average parental monetary investments even decrease slightly (by 1%), whereas large increases in monetary investments are seen for $x_4 = \bar{x}_4$ and $x_5 = \bar{x}_5$. Another technological feature in early childhood is a high value of θ^x_3, which implies that parental time is a much more important input for human capital development. Therefore, despite higher parental demand for monetary investments by rich families, eliminating heterogeneity in parental time investments in early childhood (i.e., $j = 3$) is able to reduce the intergenerational persistence of lifetime income.

Panel (iii) of Table 5 reports the result when I impose that all parents invest the same amount of both monetary and time investments in all periods, unlike the above age-dependent exercises designed for each input. This exercise helps to quantify the effects of

[23] Thus, the government budget is balanced through G without changing taxes. In the Appendix, Table A.8 reports the results when labor income taxes are adjusted to balance the government budget. Since the required tax changes are small, the results remain virtually unchanged.



NOTES: The upper panels are obtained from the baseline model, whereas the bottom panels are the result of a counterfactual exercise where $x_3 = \bar{x}_3$ for all households. Each dot represents the choices of a simulated sample for time investments on the x-axis and monetary investments (relative to baseline output) on the y-axis.

FIGURE 1

TIME AND MONETARY INVESTMENTS IN SIMULATED DATA

both monetary and time investments in all periods while taking into account complementarity across different periods as well. The result shows that although parents raise the amount of inter vivos transfers as an alternative effort to transmit their economic status intergenerationally, the intergenerational persistence of lifetime income decreases substantially: the IGE becomes 0.084 and the rank correlation decreases to 0.070.

Finally, heterogeneity in inter vivos transfers could be an important channel leading to the persistence of lifetime income, since parents can use such transfers to financially support their child's college decision and affect lifetime capital income flows. To explore the role of this channel, in what follows I consider equalizing the inter vivos transfers at its mean value in the baseline economy. This exercise is less obvious to implement in the model, since it may not be feasible for some poor families to transfer the mean value. I accordingly impose that all parents are prohibited from transferring any money individually, but allow the government to provide a constant a_1, financed by higher labor income taxes by proportionally adjusting $\{\lambda_j\}_{j=1}^{9}$ in (2). That way, all agents start their life cycle with an equal amount of assets.

Panel (iv) reports the result. It is striking to note the increases in the intergenerational persistence estimates, both the IGE and rank correlation (indicating lower mobility) with the same initial asset level. In fact, Table 5 reveals that more educated parents, who are not allowed to transfer money to their children, choose to invest more time in their young children instead, especially in period $j = 3$. This substitution toward parental investments in human capital, which are disproportionately made by richer parents, leads to increased educational gradients in parental investments, and thus increases intergenerational income persistence and reduces upward mobility.

Appendix A.4 also reports the effects of altering other model elements on intergenerational mobility of lifetime income. In particular, it includes the exercise of relaxing borrowing constraints substantially (10 times larger than the baseline value), given that Caucutt and Lochner (2020) highlight the importance of borrowing constraints as an underlying factor behind the large gap in parental monetary investments. I find that the borrowing constraints play a limited role in affecting intergenerational mobility in my model. See Table A.4 and its associated discussions in the Appendix for details.

6. POLICY EXPERIMENTS

In this section, the baseline model economy is used to conduct policy experiments. The chosen set of policies is motivated by the literature and the results of the previous section. Specifically, I first consider the effect of subsidies for school or college, which have been referred to as the "Great Equalizer" for their role in promoting intergenerational mobility (Downey et al., 2004; Torche, 2011). Such subsidies seek to equalize opportunities by mitigating parental influence and may also improve human capital in general. Since the calibration in the degree of altruism implies that parents care less about children than themselves, parental investments may be too low from the children's perspective. Moreover, since children cannot choose parents, high-ability children born into poor families who lack resources face a market failure from the children's perspective. Therefore, policies that support human capital formation could both increase mobility and have implications for welfare and aggregate efficiency.

Welfare programs in the United States have been steadily growing in size and variety since the 1970s (Ben-Shalom et al., 2011). In line with this trend, I also consider two sets of policies that are more directly targeted toward poor families whose children might be more likely to suffer from the aforementioned market failures. Since heterogeneity in parental time investment during early childhood was found to be important in shaping mobility, a natural policy to consider would be to encourage parents with low socioeconomic backgrounds to invest more time with their children, assuming that the primary objective of government is to raise intergenerational mobility. In practice, however, it is very difficult for government to directly influence parental time investments because time spent with children is mostly home-based and not observable to the government. I thus first consider means-tested subsidies for monetary investments in children, noting that these can indirectly influence parental time investment behavior. In addition, I also consider a means-tested lump-sum provision of time investments at home (e.g., by nannies) financed by government.

Notably, I ensure that the total costs of each of the above policies are identical. Moreover, in all cases, to balance the government budget constraint, government is assumed to adjust labor income taxation through a proportional change in $\{\lambda_j\}_{j=1}^{9}$ in (2). Finally, I also examine the implications of such policies for aggregate output and welfare. This would allow for more informative evaluations of whether policy changes that raise intergenerational mobility are otherwise desirable for the economy.[24]

[24] Welfare changes are measured by a consumption-equivalent premium, as is standard in the literature. Specifically, I use the utilitarian social welfare function to measure the percentage change in consumption for all agents in the baseline model that makes them indifferent to living in the alternative economy. Although this standard welfare measure reflects changes in cross-sectional inequality, it does not reflect changes in intergenerational mobility.

TABLE 6
EFFECTS OF EXPANDING SCHOOLS AND SUBSIDIZING COLLEGE

		Public Educ.			College Subsidy	
		$\Delta g_j = 0.02/Y_{ss}$			GE	FP
	Baseline	$j = 3$	$j = 4$	$j = 5$	$s_c = .375$	$s_c = .252$
IGE	0.406	0.404	0.399	0.399	0.403	0.405
Rank correlation	0.387	0.385	0.381	0.380	0.383	0.387
Upward mobility (%)	6.7	6.8	6.9	6.9	6.8	6.7
Mean *(% chg.)*						
- $\bar{e}_3$ (relative to Y)	0.091	*−5.2*	*−0.4*	*−0.5*	*−0.6*	*2.5*
− $\bar{e}_4$	0.102	*0.2*	*−7.7*	*3.3*	*−1.8*	*4.4*
- $\bar{e}_5$	0.120	*0.2*	*4.4*	*−7.5*	*−2.0*	*4.4*
- $\bar{x}_3$ (hours per week)	6.5	*−5.0*	*1.8*	*1.4*	*−0.7*	*1.6*
- $\bar{x}_4$	3.8	*−0.4*	*−13.9*	*−1.0*	*−0.7*	*2.3*
- $\bar{x}_5$	2.1	*−0.3*	*−0.3*	*−12.1*	*−0.8*	*2.6*
College fraction (%)	34.6	34.6	34.8	34.7	36.0	49.2
Observed col. premium (%)	64.5	64.4	63.3	63.4	57.0	66.7
$\bar{a}_1/Y$ (inter vivos)	0.188	0.188	0.185	0.186	0.188	0.160
Equilibrium interest rate (%)	0.217	0.217	0.216	0.216	0.217	0.217
Aggregate output *(% chg.)*		*0.8*	*4.7*	*3.9*	*−0.4*	–
Aggregate capital *(% chg.)*		*0.8*	*4.9*	*4.0*	*−0.4*	–
Consumption equiv.*(%)*		*0.8*	*6.1*	*4.9*	*−0.2*	–

NOTE: FP (fixed prices) refers to the case where prices are held constant at the baseline level. Output and welfare changes are not reported in these cases. Welfare gains are the consumption-equivalent premium measured by a percentage change in consumption required for all agents to be indifferent to living in an alternative economy.

6.1. *Subsidizing the Great Equalizers.* First, I consider increasing public investment g_j for each period j to quantify the effect of expanding investments in public schools. The size of the change Δg_j is chosen to be 2% of baseline output per capita, which implies that the total cost amounts to 0.17% of the baseline GDP.

Second, I also consider subsidizing college costs as a way of providing easier access to college. Specifically, the college cost (5) in the budget constraint (7) is replaced by

$$(1 - s_c)\psi(\xi, a), \tag{30}$$

where $s_c \in [0, 1]$ is the subsidy rate. For the purpose of comparability across different policies, I search the size of s_c such that the total costs of the college subsidy are equal to those from the expansion of g_j. I consider long-run effects of such policies by comparing steady states before and after each policy change.[25]

Several interesting results emerge in Table 6. When government directly increases public education spending, there are crowding-out effects. Specifically, parental investments (in both time and money) strongly decline in the period associated with the change in public education investments. Intergenerational mobility would be expected to increase since the relative role of the Great Equalizer increases. Overall, the IGE and the rank correlation do decrease, and the change is more prominent when government spending comes in $j = 4$ or 5. In terms of aggregate efficiency, output and welfare increase most substantially when g_4 increases, whereas an increase in g_3 has a much lower effect. Note that since θ_p^4 and θ_p^5 are greater than θ_p^3, the importance of primary and secondary schools is relatively higher than that of preschools and kindergartens in human capital development. This suggests that the most effective approach would be to spend the same amount of money on improving and expanding primary schools

[25] In Appendix A.6, to gauge the extent to which these results are due to distributional changes over time, I also repeat the exercises for a single generation whose initial state variables are drawn from the baseline steady state while holding prices fixed at the baseline level (Table A.9).

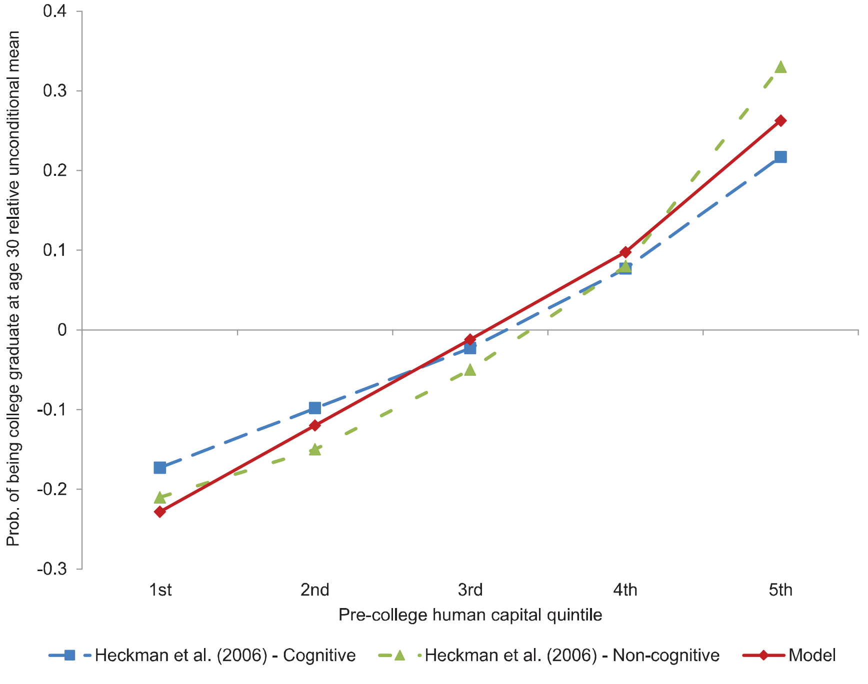


FIGURE 2

PROBABILITIES OF BEING COLLEGE GRADUATES AT AGE 30 RELATIVE TO UNCONDITIONAL MEAN

so as to raise not only intergenerational mobility but also aggregate output and the overall welfare of households.

What if the government spends the same amount of money to promote college education? Table 6 shows that the college fraction increases from 34.6% to 36.0% in GE where prices (w_1, w_2, and r) are allowed to adjust to clear the markets.[26] The IGE and the rank correlation fall slightly, raising mobility. When I fix the price adjustments (FP), the college-educated fraction increases sharply to 49.2%. With this much stronger response, intergenerational mobility actually increases less. Note that these results capture long-run effects.

To better understand this inefficacy of college subsidies in raising intergenerational mobility, it is useful to note that in the model, college decisions do not depend solely on financial conditions, but also on precollege human capital. The discrete decision rule for college education features threshold-based behavior: holding other things constant, an individual chooses to go to college if his or her human capital is above some threshold level. This property of the college decision rule leads to positive selection in equilibrium, meaning that those who have higher precollege human capital are more likely to obtain a college education. Note that selection is not perfect because college costs are stochastic and depend negatively on assets.

To visualize the quantitative importance of precollege human capital in the model, Figure 2 plots the probability of being a college graduate at age 30 for each quintile of precollege human capital. The data counterparts shown are from Heckman et al. (2006) for both cognitive and noncognitive factors.[27] The results clearly show that high precollege human capital raises the probability of becoming a college graduate, indicating positive selection for college

[26] This GE effect hinges on the elasticity of substitution between skilled and unskilled workers, shaped by the parameter ρ. In Table A.7, I consider an alternative calibration where I double ρ. The same policy exercises show that the college fraction increases more in GE, but is still much weaker than when prices are fixed.

[27] The samples considered in Heckman et al. (2006) have a lower unconditional mean probability. To focus on the slope instead of the level, Figure 2 plots probabilities relative to the unconditional mean probability.

in both the model and the data. The strength of selection is in line with the data thanks to the calibration strategy, which broadly targets this overall slope.

Since the college decision rule features positive selection, as shown above, the marginal households affected by the subsidy tend to have lower precollege human capital than those who would already be going to college. On average, such marginal college graduates do benefit from higher lifetime wages but only up to a level beneath that of those who already chose to go to college, leading to few rank reversals.

6.2. *Means-Tested Subsidy for Parental Investments.* First, I consider a subsidy $s_{e,j}$ proportional to private education spending e, which is provided only to those with assets less than the threshold level $\tilde{a}$. In other words, the left-hand side of the resource constraint in period j is replaced by

$$c + a' + (1 - s_{e,j}\mathcal{I}(a < \tilde{a}))e, \tag{31}$$

where $\mathcal{I}(a < \tilde{a})$ is an indicator function. Given that this policy is expected to encourage the affected parents to increase monetary investments, it could boost parental time investments, which are complementary inputs in the skill formation technology. The threshold level $\tilde{a}$ is set to 0.5, which approximately corresponds to the 30th percentile of the baseline wealth distribution among parents with children. To control for the total cost, $s_{e,j}$ is chosen such that the total cost of these subsidies is equal to those from the previous exercises.

Second, I consider the provision of time investments by government $\underline{x}_j$ in period j. I assume that this time input augments parental time inputs: $x+\underline{x}_j$. As shown in (12), these time inputs are then aggregated with parental monetary inputs, which essentially capture the quality of parental time investments. The hourly fiscal cost of this nonparental time provision is assumed to be the parents' wage, thereby leading to the total cost of $w_\kappa h \underline{x}_j$ for an agent with education κ and human capital h. In other words, if the time inputs are provided by the government, they are conducted by a nanny or a school teacher whose wage is equivalent to that of the parents. As above, this is assumed to be means-tested: only families with assets less than the threshold level $\tilde{a}$ are eligible. The goal is straightforward: to directly subsidize parents who invest less time in children. The total costs are controlled by setting $\underline{x}_j$ to achieve cost neutrality.

As above, I focus on long-run equilibrium effects of such policies by comparing stationary equilibrium outcomes (Table 7) and report their short-run effects with a single generation and fixed prices in Appendix A.6 (Table A.10). Table 7 summarizes the results. As expected, this subsidy $s_{e,j}$ increases both monetary investments (by 16–26%) and time investments in the period j targeted by each policy. Most notably, intergenerational persistence measures decrease by 6% if the means-tested subsidies are given to parents of children under the age of five, yet they increase with subsidies in $j = 4, 5$. Furthermore, these subsidies to monetary investments increase output and welfare, especially if provided to parents of young children ($j = 3$).[28]

Why does the same monetary subsidy have such divergent effects on intergenerational mobility depending on when it is provided? Monetary investment subsidies provided in $j = 3$ lead to the greatest percentage increase in average parental time investment relative to the percentage increase in monetary investments. This is due to the high complementarity between parental time and monetary investments during this period. Moreover, the means-tested nature of this policy implies that its impact across the income distribution may differ sharply. Figure 3 plots average parental time investments in three target periods by income quintile, both in the baseline economy and after each policy reform. The disproportionate increases in parental time investments among lower income quintiles are much more pronounced in the case of $s_{e,3}$, whereas parental responses are much more uniform for both $s_{e,4}$

[28] Welfare gains are even larger than output gains since this policy is redistributive that would bring additional welfare benefits under the equally weighted utilitarian welfare function.

TABLE 7
EFFECTS OF MEANS-TESTED SUBSIDIES TO PARENTAL INVESTMENTS

	Baseline	Money Subsidies $s_{e,3} =$ 0.507	Money Subsidies $s_{e,4} =$ 0.245	Money Subsidies $s_{e,5} =$ 0.229	Time provision $\underline{x}_3 =$ 0.019	Time provision $\underline{x}_4 =$ 0.017	Time provision $\underline{x}_5 =$ 0.015
IGE	0.406	0.381	0.422	0.417	0.409	0.406	0.406
Rank correlation	0.387	0.363	0.404	0.399	0.391	0.388	0.388
Upward mobility (%)	6.7	7.4	6.2	6.3	6.7	6.8	6.8
Mean *(% chg.)*							
- $\bar{e}_3$ (relative to Y)	0.091	*15.7*	*0.8*	*1.2*	*−0.5*	*0.7*	*0.4*
- $\bar{e}_4$	0.102	*1.6*	*25.7*	*3.8*	*−1.5*	*−2.3*	*1.0*
- $\bar{e}_5$	0.120	*1.7*	*0.8*	*18.2*	*−1.3*	*−2.1*	*−1.0*
- $\bar{x}_3$ (hours per week)	6.5	*12.8*	*1.6*	*1.8*	*−13.3*	*0.2*	*0.2*
- $\bar{x}_4$	3.8	*−1.3*	*2.5*	*−0.3*	*−0.1*	*−18.8*	*0.0*
- $\bar{x}_5$	2.1	*−0.8*	*−0.1*	*2.9*	*−0.2*	*−0.2*	*−40.9*
College fraction (%)	34.6	36.0	34.5	34.8	34.6	34.9	34.8
Observed col. premium (%)	64.5	56.8	66.6	65.1	64.2	64.5	64.9
$\bar{a}_1/Y$ (inter vivos)	0.188	0.185	0.186	0.184	0.188	0.187	0.186
Equilibrium interest rate	0.217	0.218	0.221	0.222	0.219	0.220	0.220
Aggregate output *(% chg.)*		*2.5*	*2.4*	*2.0*	*−0.4*	*−0.3*	*−0.0*
Aggregate capital *(% chg.)*		*2.4*	*1.7*	*1.3*	*−0.9*	*−0.9*	*−0.6*
Consumption equiv.*(%)*		*2.9*	*2.7*	*2.2*	*−0.6*	*−0.6*	*−0.3*

TABLE 8
EFFECTS OF MEANS-TESTED SUBSIDIES TO PARENTAL MONETARY INVESTMENTS WITHOUT ENDOGENOUS PARENTAL TIME RESPONSES

	Baseline	Money Subsidies with Exogenous x: $s_{e,3} =$ 0.507	$s_{e,4} =$ 0.245	$s_{e,5} =$ 0.229
IGE	0.406	0.388	0.423	0.421
Rank correlation	0.387	0.370	0.405	0.403
Upward mobility (%)	6.7	7.1	6.2	6.1
Mean *(% chg.)*				
- $\bar{e}_3$ (relative to Y)	0.091	*15.8*	*0.9*	*1.3*
- $\bar{e}_4$	0.102	*0.5*	*25.8*	*4.2*
- $\bar{e}_5$	0.120	*0.4*	*0.9*	*18.4*
- $\bar{x}_3$ (hours per week)	6.5	*0.7*	*1.0*	*1.0*
- $\bar{x}_4$	3.8	*−0.1*	*0.8*	*0.7*
- $\bar{x}_5$	2.1	*0.2*	*0.3*	*1.3*
College fraction (%)	34.6	35.9	34.5	34.7
Observed col. premium (%)	64.5	57.6	66.6	65.4
$\bar{a}_1/Y$ (inter vivos)	0.188	0.185	0.186	0.184
Equilibrium interest rate	0.217	0.218	0.221	0.222
Aggregate output *(% chg.)*		*1.5*	*2.4*	*2.1*
Aggregate capital *(% chg.)*		*1.3*	*1.6*	*1.3*
Consumption equiv.*(%)*		*1.6*	*2.7*	*2.4*

NOTE: The reported results are based on a model where parental time investment decisions (x) are restricted to remain the same as in the baseline equilibrium.

and $s_{e,5}$. Since the complementarity between money and time is much lower in $j = 4, 5$, the higher monetary investments by poor parents do not result in higher time investments.

To see the role of endogenous parental time investment behind the effects of $s_{e,j}$, Table 8 reports the results in a model where parental time investment decisions are restricted to remain the same as in the baseline equilibrium when simulating data. It shows that endogenous responses of parental time investment account for a significant portion (around 30–40%) of the total policy effects of $s_{e,3}$ on intergenerational mobility reported in Table 7. These results

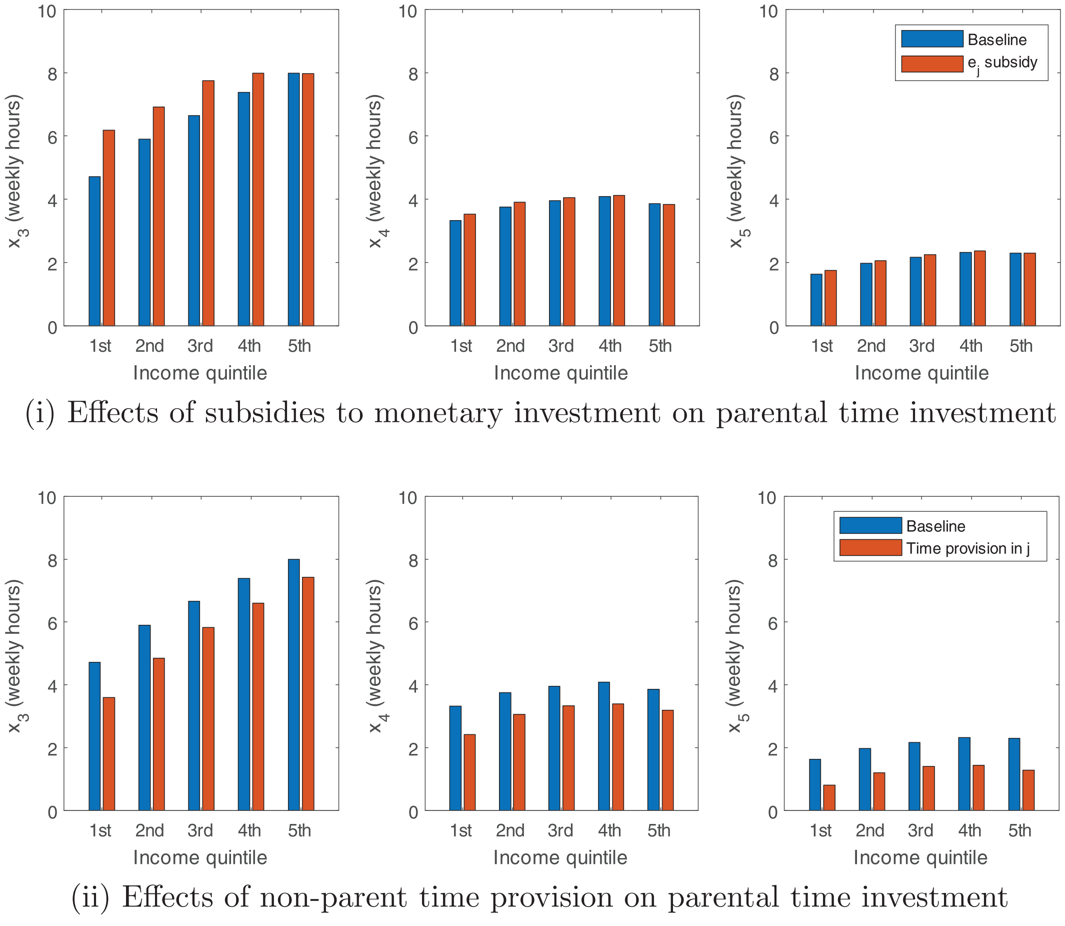


FIGURE 3

EFFECTS OF SUBSIDIZING PARENTAL INVESTMENTS ON PARENTAL TIME INVESTMENT

clearly demonstrate that parental time investments play a role of amplifying the effects of the monetary subsidy in early childhood on intergenerational mobility.

Finally, the last columns of Table 7 show that policies that guarantee a childcare time of $\underline{x}_j$ to parents with low wealth decrease intergenerational mobility. Most notably, parental time investments decline substantially with the additional time provision. Since these policies need to be financed by higher taxes, they lead to negative consequences for aggregate output and welfare. Therefore, these results suggest that attempts by the government to augment parental time investments in a lump-sum fashion are not likely to be effective. Since the (total) parental time investment demand is strictly linked to the amount of monetary investments by the complementarity between the two (i.e., the quality of parental time), the nonparental time provided by the government tends to simply crowd out time spent by parents themselves. In other words, without changes in the quality of parental time, providing disproportionate support to the poor in terms of time spent with children appears ineffective at raising both intergenerational mobility and aggregate efficiency. This is also clearly shown in the bottom panel of Figure 3.

7. CONCLUSION

This article presents a quantitative model of intergenerational mobility that encompasses various standard elements in the quantitative macroeconomics literature. These include GE, incomplete markets, and college decisions, as well as endogenous human capital development as in multiple-period childhood skill formation technology with flexible substitutabilities between parental time and monetary investments. The model successfully accounts for positive educational gradients in parental time investments as well as untargeted distributional aspects of the intergenerational persistence of income, as observed in U.S. data. I find that the intergenerational persistence of lifetime income is reduced by around 7–8% when heterogeneity in parental time investments during early childhood is eliminated, despite the alternative endogenous channels that parents could rely on to strengthen the intergenerational association. In contrast, I find that the intergenerational persistence of lifetime income actually increases when heterogeneity in inter vivos transfers is eliminated.

The policy experiments I examine in this article show that the most cost-effective way to increase intergenerational mobility is to provide a means-tested subsidy to parental monetary investments in early childhood. This intervention also gives rise to substantial output and welfare gains by potentially addressing the market failure noted above. On the other hand, if the government spends the same amount of money to subsidize higher education or provide direct time investments to poor families, intergenerational mobility barely changes and some efficiency losses are incurred due to the distortionary taxes required to finance such policies. An interesting avenue for future work would be to design a more effective and implementable policy scheme that disproportionately encourages poor families to invest more quality time to better address the market failure by facilitating more high-quality time investments in able children born into poor families. In addition, it is important to note that this article abstracts from spillover effects. Consider an example of play centers. If parents can (i) learn parenting skills while watching how other parents spend time with their children in such centers or (ii) share valuable information on parenting while spending time in such centers, they could potentially increase their parenting quality at home as well. These spillover effects could potentially strengthen the effects of the policies discussed in this article on intergenerational mobility.[29]

APPENDIX A

A.1 *Time-Use Data.* Statistics regarding time-use are computed using the 2003–2017 waves of the ATUS, combined with the CPS. The ATUS statistical weights are used for all statistics reported. To compute average hours worked and the fraction that holds a college degree, I consider both men and women and include all individuals whose age is greater than or equal to 30 and less than 65. A person is college-educated if the highest level of completed schooling or highest degree received is a bachelor's degree or above.

To construct the key variable of parental time investment, I focus on interactive activities that require the existence of both a parent and a child in a common space. Such categories include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities. For the time investment variable, I further restrict the sample to households with at least one child and with an age of between 21 and 55 (inclusive), as in Guryan et al. (2008). The statistics for each model period are based on the age of the youngest child: $j = 3, 4, 5$ correspond to ages 0–4, 5–9, and 10–14, respectively. Educational gradients

[29] There can be another type of spillovers from one generation to the next generation directly through children's initial human capital. A previous version of the current article (Yum, 2016) considered this effect. In such a framework, it is very important to consider various fiscal options to finance educational reforms and to distinguish short-run and long-run welfare consequences accordingly (e.g., see Daruich, 2020).

TABLE A.1
EDUCATION GRADIENTS IN PARENTAL TIME INVESTMENTS

	$j=3$	$j=4$	$j=5$
College-educated	1.342	0.561	0.416
	(0.133)	(0.109)	(0.091)
Sex	−2.62	−1.51	−1.20
	(0.123)	(0.101)	(0.083)
Age	−0.041	0.016	0.023
	(0.009)	(0.007)	(0.006)
Married	−0.911	−0.318	−0.102
	(0.085)	(0.064)	(0.053)
R^2	0.023	0.014	0.017
Average x	6.43	3.78	2.06

NOTE: Numbers in parentheses are standard errors. The dependent variable is parental time x (weekly hours).

TABLE A.2
GROSS GROWTH RATES OF HUMAN CAPITAL BY AGE AND EDUCATION

$j=$	1	2	3	4	5	6	7	8
$\gamma_{j,1}$	1.231	1.052	1.017	1.004	0.998	0.995	0.994	0.994
$\gamma_{j,2}$	1.317	1.152	1.101	1.063	1.032	1.004	0.975	0.942

[29]NOTE: The above values are computed based on the estimates from the PSID samples in Rupert and Zanella (2015).

TABLE A.3
PARAMETER VALUES FOR PROGRESSIVE TAXATION AND PUBLIC EDUCATION INVESTMENTS

	τ_j	λ_j		g_j
$j=1,2$	0.1106	0.8177	$j=3$	0.060
$j=3,\ldots,6$	0.1585	0.9408	$j=4$	0.098
$j=7,8,9$	0.1080	0.8740	$j=5$	0.111

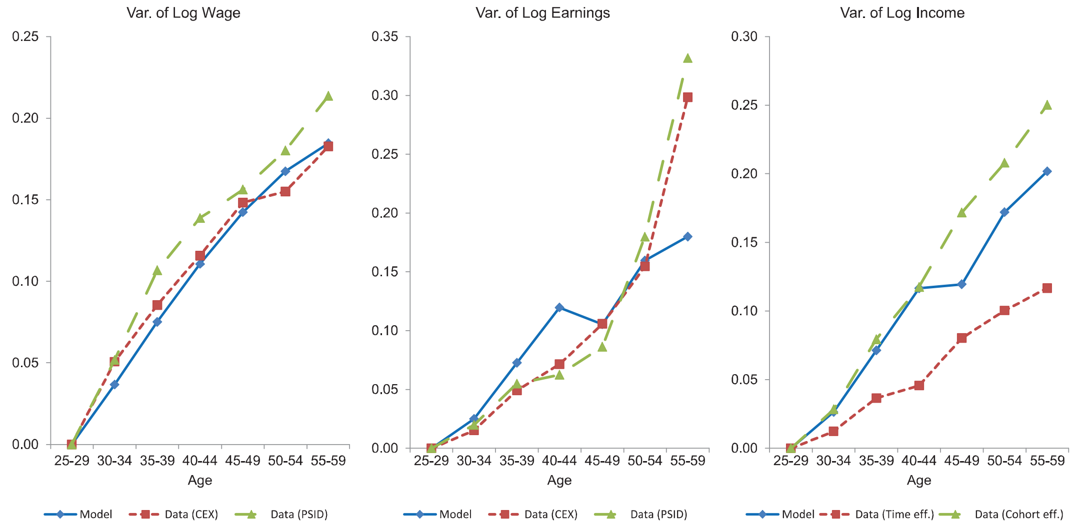
NOTE: τ_j and λ_j are based on the estimates in Holter et al. (2019). Public education investments g_j are based on the 2019 edition of Education at a Glance (OECD).

in parental time investments are obtained by regressing parental time on a college indicator variable while controlling for sex, age, and marital status, as reported in Table A.1. In fact, the coefficients on college are quite stable when control variables are added, in line with Guryan et al. (2008).

The time-diary survey also reports secondary activities, which may also include childcare. However, since the childcare time recorded as secondary activities is expected to be less active and the same hours may not be as effective as an input to skill formation (Del Boca et al., 2014), I focus only on childcare activities reported as a primary activity.

A.2 *More on Parameter Values Calibrated Externally.* Table A.2 reports the gross growth rates of human capital by age and education, computed based on the estimates from the PSID samples in Rupert and Zanella (2015). Table A.3 reports the estimates of two parameters that shape progressive taxation by age, obtained from Holter et al. (2019). Note that estimates for single households are used for $j=1,2$, while estimates for married households are used for the later periods (either with a child for $j=3,\ldots,6$ or without children for $j=7,8,9$).

To compute the public education and private monetary education investments, I use the 2016 information published in the 2019 edition of Education at a Glance by the OECD. In terms of mapping from the model period to education stages, I consider preprimary to correspond to $j=3$, primary to $j=4$, and secondary to $j=5$ in the model. As explained in the main text, I follow the approach of Restuccia and Urrutia (2004) and Holter (2015) by treating state and federal government spending as public investments, whereas local government



NOTES: The unit of the y-axis is the variance of log relative to the initial age group (25–29).

FIGURE A.1

LIFECYCLE INEQUALITY IN THE MODEL AND THE DATA

spending is part of private investment. Note that, in practice, families can choose to live in a richer and more expensive neighborhood with a better public school. By using the local share of public spending as 0.49, I obtain the adjusted shares of private and public investments for each period. Private and public investments are then obtained by multiplying the total education expenditure per child ($j = 3$) or per student ($j = 4, 5$) at each stage of education. Note that mean private and public investments are both approximately in line with the estimates in Lee and Seshadri (2019) based on microlevel data with a relatively small number of samples.

A.3 *Life Cycle Inequality.* Figure A.1 shows life-cycle inequality for wages, earnings, and income in the model and the data (Heathcote et al., 2010). As in Heathcote et al. (2010), the unit of the y-axis is the variance of log relative to the initial age group. The figures show that the model replicates the quantitative patterns of life cycle inequality in that the dispersion of these variables increases with age.

A.4 *Other Channels Shaping Intergenerational Mobility.* In addition to the main exercises regarding heterogeneous parental behaviors reported in the main text, I examine several other channels that shape the intergenerational mobility of lifetime income in the model.

Public education investments $\{g_j\}_{j=3}^{5}$ in the model are provided to every child equally. Therefore, their presence is expected to dampen intergenerational association in the model. To explore the effects of the public education investment channel, the first three rows of Table A.4 report the results when g_j is reduced by 50% for every period $j = 3, 4, 5$. In doing so, the government budget is balanced by adjusting labor income taxation through proportional changes in $\{\lambda_j\}_{j=1}^{9}$ in (2). As expected, intergenerational mobility measures indicate lower mobility in the presence of lower public investment. In particular, the effects are stronger for the periods $j = 4, 5$ where the size and the relative importance of public investments are greater. For instance, the IGE would increase by around 5–6% when public investments are reduced by 50% in either $j = 4$ or $j = 5$.

The next row shows the result when the exogenous source of intergenerational persistence is shut down by setting $\rho_\phi = 0$. Note that the calibrated persistence of ϕ may capture genetic transmission that would tend to increase ρ_ϕ but also any other factors that are not modeled herein that could in principle also reduce the calibrated ρ_ϕ. Given that the calibrated ρ_ϕ was

TABLE A.4
QUANTITATIVE EFFECTS OF OTHER MECHANISMS ON INTERGENERATIONAL MOBILITY

	IGE	Rank Corr.	Upward Mobility (%)	Educ. Gradient in x_j (%) $j=3$	$j=4$	$j=5$	$\bar{a}_1/Y$
Baseline	0.406	0.387	6.7	19.6	13.8	19.7	0.188
Counterfactuals							
- g_3 reduced by 50%	0.410	0.391	6.6	17.9	13.9	19.8	0.187
- g_4 reduced by 50%	0.426	0.408	6.1	22.4	7.2	22.0	0.197
- g_5 reduced by 50%	0.430	0.411	6.0	22.2	15.2	12.2	0.192
- No persistence in ϕ	0.378	0.358	7.6	19.3	14.1	19.8	0.189
- No idiosyncratic shocks	0.406	0.388	6.4	20.2	14.6	20.6	0.168
- Relaxing the borrowing limit ($\times 10$)	0.405	0.387	6.7	18.9	14.5	20.5	0.191

positive, shutting down ability transmission reduces both the IGE and the rank correlation quite considerably (by 7%) and increases the upward mobility rate by 13%. These results show that the external transmission of ability is a quantitatively important source of lifetime income persistence in the model.

Next, I also examine the role of idiosyncratic shocks over the life cycle by setting $\sigma_z = 0$. Note that the most immediate consequence of this restriction is to limit *intragenerational* mobility because idiosyncratic shocks essentially play a role of moving the ranking of adults' wages up or down over the course of their life cycle. When this is shut down, initial conditions at the beginning of adulthood become much more important in determining lifetime income, because the initial gap would be simply amplified through steeper wage growth rates among the college-educated. This implies that parental influence on child's lifetime income could become greater. Interestingly, this change also induces parents to transfer less money, which tends to weaken intergenerational linkage. Overall, the IGE and the rank correlation increase slightly, whereas the upward mobility is significantly reduced by 5%.

Finally, I examine the role of borrowing constraints by relaxing the borrowing limit massively. Specifically, I consider a specification where $\underline{a}$ is 10 times larger than the baseline calibration. As a result, the average debt in equilibrium becomes 6.1% of the five-year GDP per capita, which is substantially larger than the baseline economy (1%). Caucutt and Lochner (2020) find that borrowing constraints are important in determining the large gap in parental monetary investments in their model. My model incorporates additional relevant channels which agents could rely on when it comes to parental monetary investment decisions, as compared to their model, such as endogenous labor supply. Moreover, my model incorporates GE market clearing, unlike Caucutt and Lochner (2020). The final row indeed shows that intergenerational mobility increases only marginally relative to the substantial change in the borrowing limit. The results indicate that in my model, the borrowing constraints have limited effects on intergenerational mobility. This suggests that, in my model, parental monetary investment gaps are not much directly frictional yet are instead largely shaped by the current parent generation's economic status (i.e., human capital and assets), which reflects the dynastic history of luck and choices in the past generations.

A.5 *Sensitivity Analysis.* First, I consider a calibration strategy where I match the overall educational gradient instead of period-specific educational gradients in parental time investments. The overall fit of the model is good except for educational gradients in parental time investment, which increase with age monotonically, as can be seen in Table A.5. The counterfactual exercises that consider the role of heterogeneity in parental time investment show that shutting down heterogeneity in parental time investments in period 3 has weaker effects on intergenerational mobility, as compared to the benchmark model in the main text. This should not be surprising, since the model in this alternative calibration generates a lower

TABLE A.5
COUNTERFACTUAL RESULTS WITH A NONAGE-DEPENDENT ELASTICITY OF SUBSTITUTION BETWEEN PARENTAL INVESTMENTS OF TIME AND MONEY

	IGE	Rank Corr.	Upward Mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	$\bar{a}_1/Y$
Baseline	0.403	0.385	6.8	9.0	31.4	36.4	0.190
Counterfactuals shutting down							
(i) Heterogeneity in parental monetary investment							
- $e_3 = \bar{e}_3$	0.316	0.298	9.3	−2.2	30.6	35.4	0.215
- $e_4 = \bar{e}_4$	0.276	0.259	10.4	7.6	−3.0	34.9	0.226
- $e_5 = \bar{e}_5$	0.270	0.254	10.5	7.7	28.0	−4.0	0.245
(ii) Heterogeneity in parental time investment							
- $x_3 = \bar{x}_3$	0.398	0.380	7.0	0.0	31.5	36.6	0.188
- $x_4 = \bar{x}_4$	0.390	0.372	6.9	9.0	0.0	35.9	0.189
- $x_5 = \bar{x}_5$	0.391	0.373	7.1	9.0	31.3	0.0	0.189
(iii) Heterogeneity in all parental investments							
$x_j = \bar{x}_j, e_j = \bar{e}_j, j = 3, 4, 5$	0.080	0.066	18.5	0.0	0.0	0.0	0.249
(iv) Heterogeneity in inter vivos transfers							
- $a_1 = \bar{a}_1$	0.413	0.397	6.9	8.5	32.1	37.9	0.190
Counterfactuals- g_3 reduced by 50%	0.407	0.389	6.7	7.2	31.7	36.7	0.189
- g_4 reduced by 50%	0.436	0.417	5.6	10.2	25.9	41.0	0.198
- g_5 reduced by 50%	0.438	0.420	5.5	10.0	35.0	29.7	0.193
- No persistence in ϕ	0.380	0.361	7.5	9.1	31.1	35.9	0.191
- No idiosyncratic shocks	0.406	0.388	6.4	9.5	32.6	37.6	0.168
- Relaxing the borrowing limit (×10)	0.403	0.385	6.8	9.0	31.6	37.1	0.192

NOTE: The above results are based on the alternative calibration that imposes $\zeta_j = \zeta$ for $j = 3, 4, 5$.

educational gradient in parental time in the first place (9.0%) and a higher elasticity of substitution in early childhood. On the other hand, the effect of heterogeneity in parental time investments in later childhood becomes much stronger because the education gradients in parental time are higher (counterfactually) and because the elasticity of substitution is lower than the benchmark model that allows for the age dependency ζ_j in the main text.

I also consider another alternative calibration where the persistence of ability is imposed to be higher at $\rho_\phi = 0.15$. I then recalibrate the model with the same set of target statistics, excluding only the intergenerational correlation of income (which is the main target of the parameter ρ_ϕ in the main text). Table A.6 summarizes the quantitative role of various mechanisms in this alternative calibration. Note that the baseline model in this alternative calibration features lower intergenerational mobility as I do not allow ρ_ϕ to be calibrated to match the observed rank correlation. Nevertheless, it is interesting to note that the quantitative role of various channels is very similar to the baseline calibration in the main text. Since the change in mobility measures is similar in magnitude, percentage changes in correlations relative to the baseline are smaller but percentage changes in upward mobility become larger. This is because the baseline model in this alternative calibration features higher IGE and rank correlation but lower upward mobility at the outset.

Table A.7 shows the policy exercises in which college is subsidized with a different elasticity of substitution between skilled and unskilled workers since policy effects may be sensitive to this elasticity. Specifically, I set the value of ρ to 2/3 so that the elasticity becomes 3. This value is quite close to that of 3.3 in Abbott et al. (2019). The results show that the effects of college subsidies on college choices are much stronger with the higher elasticity. However, it is worth noting that the effects on intergenerational mobility are nearly unaffected by this elasticity.

A.6 *Additional Results.* Table A.8 reports results from the decomposition exercise on the

Table A.6

ALTERNATIVE CALIBRATION 2: QUANTITATIVE EFFECTS ON INTERGENERATIONAL MOBILITY

	IGE	Rank Corr.	Upward Mobility (%)	Educ. gradient in x_j (%) $j=3$	$j=4$	$j=5$	$\bar{a}_1/Y$
Baseline	0.480	0.462	4.7	21.0	13.1	19.4	0.187
Counterfactuals shutting down							
(i) Heterogeneity in parental monetary investment							
- $e_3 = \bar{e}_3$	0.393	0.375	7.0	−6.7	13.4	19.4	0.210
- $e_4 = \bar{e}_4$	0.365	0.347	7.7	19.2	3.8	19.2	0.210
- $e_5 = \bar{e}_5$	0.364	0.347	7.7	19.2	13.4	1.6	0.218
(ii) Heterogeneity in parental time investment							
- $x_3 = \bar{x}_3$	0.451	0.432	5.4	0.0	13.2	19.5	0.186
- $x_4 = \bar{x}_4$	0.480	0.462	4.7	21.1	0.0	19.3	0.186
- $x_5 = \bar{x}_5$	0.479	0.461	4.7	21.1	13.1	0.0	0.186
(iii) Heterogeneity in all parental investments							
$x_j = \bar{x}_j, e_j = \bar{e}_j,\ j = 3, 4, 5$	0.164	0.151	15.0	0.0	0.0	0.0	0.243
(iv) Heterogeneity in inter vivos transfers							
- $a_1 = \bar{a}_1$	0.480	0.465	5.1	21.7	10.9	17.9	0.187
Counterfactuals- g_3 reduced by 50%	0.484	0.465	4.6	19.2	13.1	19.5	0.186
- g_4 reduced by 50%	0.502	0.483	4.0	24.2	6.4	22.1	0.196
- g_5 reduced by 50%	0.504	0.486	4.0	24.0	14.8	11.7	0.191
- No persistence in ϕ	0.380	0.361	7.5	19.4	13.9	19.6	0.191
- No idiosyncratic shocks	0.487	0.471	4.6	21.4	13.7	20.1	0.168
- Relaxing the borrowing limit ($\times 10$)	0.480	0.462	4.6	20.5	13.8	20.3	0.190

Note: The above results are based on the alternative calibration that imposes $\rho_\phi = 0.15$.

Table A.7

GENERAL EQUILIBRIUM EFFECTS OF PROVIDING EASIER ACCESS TO COLLEGE WITH A HIGHER ELASTICITY OF SUBSTITUTION BETWEEN SKILLS

	$\rho = 1/3$ Baseline	College subsidy	$\rho = 2/3$ Baseline	College subsidy
IGE	0.406	0.403	0.406	0.403
Rank correlation	0.387	0.383	0.387	0.383
Upward mobility (%)	6.7	6.8	6.7	6.8
Mean *(% chg.)*				
- $\bar{e}_3$ (relative to Y)	0.091	*−0.6*	0.091	−0.8
- $\bar{e}_4$	0.102	*−1.8*	0.102	−2.1
- $\bar{e}_5$	0.120	*−2.0*	0.120	−2.2
- $\bar{x}_3$ (hours per week)	6.5	*−0.7*	6.5	−0.8
- $\bar{x}_4$	3.8	*−0.7*	3.8	−0.7
- $\bar{x}_5$	2.1	*−0.8*	2.1	−0.8
College fraction (%)	34.6	36.0	34.7	36.7
Observed col. premium (%)	64.5	57.0	64.5	57.5
$\bar{a}_1/Y$ (inter vivos)	0.188	0.188	0.188	0.188
Equilibrium interest rate (%)	0.217	0.217	0.217	0.217
Aggregate output *(% chg.)*		*−0.4*		*−0.3*
Aggregate capital *(% chg.)*		*−0.4*		*−0.3*
Consumption equiv.*(%)*		*−0.2*		*−0.2*

Note: The last columns are based on an alternative calibration where the elasticity of substitution between skilled and unskilled workers is set to 3 (or $\rho = 2/3$). The first two columns are from the baseline economy. The same value of $s_c = 0.4$ is used for both cases.

TABLE A.8

QUANTITATIVE EFFECTS OF HETEROGENEOUS PARENTAL TIME INVESTMENT ON INTERGENERATIONAL MOBILITY, WITH DIFFERENT GOVERNMENT BUDGET BALANCING TOOLS

	IGE	Rank Corr.	Upward Mobility (%)	Educ. Gradient in x_j (%) $j=3$	$j=4$	$j=5$	$\bar{a}_1/Y$
Baseline	0.406	0.387	6.7	19.6	13.8	19.6	0.188
Counterfactuals shutting down							
(ii) Heterogeneity in parental time investment (without tax changes)							
- $x_3 = \bar{x}_3$	0.377	0.357	7.5	0.0	14.0	19.8	0.188
- $x_4 = \bar{x}_4$	0.406	0.387	6.8	19.7	0.0	19.7	0.187
- $x_5 = \bar{x}_5$	0.405	0.386	6.8	19.7	13.8	0.0	0.187
(ii-a) Heterogeneity in parental time investment (with tax changes)							
- $x_3 = \bar{x}_3$	0.376	0.357	7.6	0.0	14.0	19.8	0.188
- $x_4 = \bar{x}_4$	0.406	0.387	6.8	19.6	0.0	19.6	0.186
- $x_5 = \bar{x}_5$	0.405	0.386	6.8	19.6	13.8	0.0	0.187

NOTE: Parents are forced to make the average level of time investment $\bar{x}_j$ in the baseline economy in period j. Panel (ii) reproduces the baseline results reported in Panel (ii) of Table 5 where G is adjusted to balance the government budget without tax changes. In Panel (ii-a), labor income taxation via proportional changes in $\{\lambda_j\}_{j=1}^{9}$ is used to balance the government budget.

TABLE A.9

EFFECTS OF EXPANDING SCHOOLS AND SUBSIDIZING COLLEGE: ONE-GENERATION EFFECTS

	Baseline	Public Educ. $\Delta g_j = 0.02/Y_{ss}$ $j=3$	$j=4$	$j=5$	College Subsidy $s_c = 0.252$
IGE	0.406	0.403	0.395	0.396	0.407
Rank correlation	0.387	0.385	0.379	0.380	0.389
Upward mobility (%)	6.7	6.8	7.0	7.0	6.6
Mean *(% chg.)*					
- $\bar{e}_3$ (relative to Y)	0.091	*−4.7*	*1.2*	*0.9*	*1.8*
- $\bar{e}_4$	0.102	*0.3*	*−8.9*	*2.6*	*3.0*
- $\bar{e}_5$	0.120	*0.4*	*3.4*	*−8.1*	*3.1*
- $\bar{x}_3$ (hours per week)	6.5	*−5.1*	*0.5*	*0.3*	*1.1*
- $\bar{x}_4$	3.8	*−0.3*	*−13.9*	*−0.8*	*1.9*
- $\bar{x}_5$	2.1	*−0.3*	*−0.4*	*−12.3*	*2.2*
College fraction (%)	34.6	34.7	35.7	35.5	48.5
Observed col. premium (%)	64.5	64.5	64.6	64.5	67.4
$\bar{a}_1/Y$ (inter vivos)	0.188	0.186	0.178	0.180	0.164

NOTE: This table reports the results from a single generation whose initial state variables are drawn from the baseline steady state, whereas prices are held fixed at the baseline level. Intergenerational mobility measures are based on the initial parent generation and their subsequent generation.

role of parental time investment heterogeneity considered in Section 5 (Table 5) when G is held fixed yet taxes are adjusted (via $\{\lambda_j\}_{j=1}^{9}$) to balance the government budget. Overall, the baseline results in the main text are barely affected since the required tax changes are very small.

Table A.9 reports the results from a single generation with fixed prices to quantify short-run effects of the above policies. Intergenerational mobility is measured using the initial parent generation and their subsequent generation. It shows that the effects of expanding public education on intergenerational mobility are similar but slightly stronger than those in Table 6, suggesting that the initial impacts tend to be mitigated over time. The effects of college subsidy are also more similar to those with the fixed prices. This suggests that price changes appear more relevant than time itself when it comes to the effects of college subsidy.

Table A.10 reports the results of the same policy changes but with one-generation effects

TABLE A.10

EFFECTS OF SUBSIDIZING PARENTAL INVESTMENTS: ONE-GENERATION EFFECTS

		Money Subsidies			Time Provision		
	Baseline	$s_{e,3} =$ 0.483	$s_{e,4} =$ 0.217	$s_{e,5} =$ 0.196	$\underline{x}_3 =$ 0.017	$\underline{x}_4 =$ 0.015	$\underline{x}_5 =$ 0.014
IGE	0.406	0.383	0.420	0.418	0.408	0.408	0.408
Rank correlation	0.387	0.369	0.402	0.401	0.390	0.391	0.390
Upward mobility (%)	6.7	7.1	6.4	6.4	6.7	6.7	6.7
Mean *(% chg.)*							
- $\bar{e}_3$ (relative to Y)	0.091	*18.2*	*3.0*	*3.2*	*−0.1*	*1.4*	*1.2*
- $\bar{e}_4$	0.102	*2.8*	*26.0*	*6.1*	*−0.4*	*−1.0*	*2.2*
- $\bar{e}_5$	0.120	*2.9*	*2.3*	*19.1*	*−0.4*	*−0.9*	*−0.5*
- $\bar{x}_3$ (hours per week)	6.5	*13.1*	*2.0*	*2.2*	*−11.9*	*0.9*	*0.8*
- $\bar{x}_4$	3.8	*−0.6*	*3.7*	*1.1*	*0.4*	*−17.6*	*0.6*
$-\ \bar{x}_5$	2.1	*−0.2*	*1.3*	*3.9*	*0.3*	*0.4*	*−35.8*
College fraction (%)	34.6	39.3	37.7	37.1	35.7	35.5	35.4
Observed col. premium (%)	64.5	60.6	67.4	66.8	65.1	64.6	64.7
$\bar{a}_1/Y$ (inter vivos)	0.188	0.174	0.158	0.155	0.178	0.175	0.173

NOTE: This table reports the results from a single generation whose initial state variables are drawn from the baseline steady state, whereas prices are held fixed at the baseline level. Intergenerational mobility measures are based on the initial parent generation and their subsequent generation.

with fixed prices as in Table A.9. As above, the size of policies is chosen to be comparable to each other while holding the prices fixed. Qualitatively, the effects of subsidies to monetary investments in each period are consistent with the long-run equilibrium results in Table 7.

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