

ESSAYS ON PRODUCTIVITY DYNAMICS AND LABOUR
MARKET OUTCOMES

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FOREWORD

The past decades have witnessed a slowdown in aggregate productivity growth (APG) that has been prevalent across OECD countries. The slowdown differed across countries in its outset: in the 1980s for the US (Stiroh, 2002), in the 1990s for Europe (Fernald and Inklaar, 2020), but all countries suffered from a further slowdown in the early 2000s. A widely accepted view holds that this slowdown is not strongly related to changes in the composition of economies in favour of industries with weaker productivity growth, but is instead mostly driven by slowing productivity within industries.

The academic debate frequently places the nature of recent technological change at the heart of this slowdown. Already in 1987, Robert Solow famously remarked that “You can see the computer age everywhere but in the productivity statistics” (Solow, 1987), and this view of an underwhelming productivity impact of the digital transformation has frequently been recited since, also with respect to more recent technologies (e.g. Van Ark, 2016).

A spirited debate has revolved around whether this issue relates to secular stagnation, i.e. a decay of the potential of technological innovation to raise productivity and increasing cost of innovation at the micro level (Bloom et al., 2020), or whether the productivity potential of innovation has remained strong and has continued to augment the productivity of innovative firms, but APG is instead lower due to a slowdown in the rate of technology diffusion to non-innovating firms, for instance due to increasing technological barriers (Andrews, Criscuolo and Gal, 2015) and innovators’ anti-competitive use of intellectual property protection schemes (Akcigit and Ates, 2021). The empirical evidence tends to support the latter view by documenting a sustained rate of productivity growth at the most productive (“frontier”) firms but increasing productivity gaps between the frontier and the rest, even within detailed industries (Andrews, Criscuolo and Gal, 2015; Berlingieri, Blanchenay and Criscuolo, 2017).

Besides the role that the nature of recent technological change may have played in the slowdown of APG, a large body of economic literature studies its impact on the labour market. As discussed in the seminal contribution of Autor, Levy and Murnane (2003), the rapid decline in the price of computing power and associated efficiency gains of automating codifiable, routine tasks depressed the market value and thus both employment and wages of workers initially performing these tasks. This has led to a decline of middle-skill occupations, particularly in

manufacturing, which resulted in the frequently cited phenomenon of “wage polarisation” (e.g. Acemoglu and Autor, 2011) that refers to a shift from a linear widening in the wage distribution towards a U-shaped pattern of employment and wage growth in skill after 1980. As unlike historic productivity growth, this more recent technology trend has impacted the demand for workers in heterogeneous ways according to their skill and task specialisation, the literature has coined the terms of “skill-biased”, and later “task-biased technological change”. Indeed, the task biased view of technological change has been shown to be one of, if not the most important explanation of wage polarization (e.g. Goos, Manning and Salomons, 2014; Michaels, Natraj and van Reenen, 2014; Autor, Dorn and Hanson, 2015).

This view of the digital transformation suggests that technological progress over the past decades has augmented labour market inequality between but also within occupations, depressing the wages of those workers whose more routine-intensive tasks can now easily be replaced by technology, while further augmenting those of highly educated workers with initially higher productivity and wages, whose labour is more complementary to the recent technology trends (e.g. Autor, Levy and Murnane, 2003). Moreover, the reduction of labour demand in the center of the wage distribution may have contributed to the stagnation of median wages and productivity-wage decoupling, with adverse impacts also on the labour share (e.g. Autor and Salomons, 2018; Schwellnus et al., 2018). Furthermore, while existing research does not support the view of digital innovation as a major destructor of jobs in the backward-looking perspective, concerns about looming technological mass unemployment have been raised repeatedly (e.g. Frey and Osborne, 2013; Brynjolfsson and McAfee, 2014).

Therefore, developed economies – and policymakers intending to set a framework for markets that produces socially optimal outcomes – appear to face a complex challenge. The established consensus is that productivity growth is ultimately the main driver of economic well-being; therefore it appears key to accelerate the rate of innovation and technology diffusion to revive productivity growth. However, given the nature of the modern technological environment, doing so may come at the cost of jobs and reduced wages for some workers as well as increased inequality, both within the labour market and between workers and the owners of capital. This may foster social polarisation and a decay of social cohesion, and reallocation from the lower to the upper end of the wage distribution may also reduce social welfare due to decreasing marginal returns to consumption.

The purpose of this thesis is to contribute to the academic knowledge about the structural implications that the kind of technological change that we have observed more recently, and we may continue to observe over the decades to come, may have for aggregate productivity growth and labour market outcomes on the one hand, and to discuss concrete options available

to policymakers to address the key challenges in the nexus of productivity and labour markets on the other.

Chapter 1 investigates the empirical link of APG to productivity divergence, i.e. increasing gaps between the most and least productive firms in an industry, to understand if widening productivity gaps may not only be a by-product of the slowdown of APG but also a dynamic cause of its persistence. It does so by empirically investigating the dynamic link of divergence to APG over the period 2000–2018 for 13 OECD countries, using the OECD MultiProd database. The analysis relies on the estimation of local projections as proposed by Jordá (2005) for different components of APG, derived from the productivity growth decomposition of Melitz and Polanec (2015). The results show that the impact of productivity dispersion shocks on APG is the result of two countervailing forces. On the one hand, such shocks dynamically increase the rate of productivity growth at larger firms, further improving the efficiency of resource allocation. On the other, they persistently reduce the rate of average firm-level productivity growth, consistent with a slowdown of technology diffusion. While divergence in the upper tail of the productivity distribution is related positively to APG due to a strong allocative efficiency component, productivity gaps have been increasing more in the lower tail more recently, and this phenomenon is negatively related to APG due to a stronger negative diffusion component. Therefore, productivity divergence may have indeed contributed to the slowdown of productivity growth, and a simple calculation suggests that around 10% of the total slowdown can be explained by divergence in the lower tail. With respect to sectoral heterogeneity, the analysis finds that divergence is less positively related to APG in intangible-intensive environments, which is however where productivity gaps have been increasing most. Beyond productivity growth, the results link upper tail divergence to skill-biased technological change. Therefore, this phenomenon may be more ambiguous in terms of the social impact as the positive APG link would suggest.

Chapter 2 turns specifically to the productivity-employment nexus, and investigates whether also in light of the more recent direction of technology change, productivity growth and employment have remained complementary rather than alternative policy targets. Whereas existing contributions have focused on productivity mostly indirectly and have considered the employment impact of either specific types of innovations (e.g., process vs. product innovation) or technologies (e.g., artificial intelligence or robot imports) without explicitly accounting for the induced productivity change, the analysis inverts the approach and focuses on productivity growth *directly*, making its heterogeneous sources the indirect dimension. This empirical perspective is brought to the OECD MultiProd database which was also exploited in Chapter 1 to study the productivity-employment nexus from a cross-country perspective and over a

recent time period in which labour-replacing technologies may have played an important role for productivity. Importantly, the analysis is able to derive insights on different levels of aggregation (the firm, the industry and the economy) from the same root dataset, and therefore overcomes limitations related to the limited comparability of heterogeneous data sources employed in by publications that focus only on the firm or the industry level, respectively.

The findings in Chapter 2 highlight that productivity growth boosts employment at the firm level due to a mechanism that relates to the improved competitiveness of the firm: while an increase in productivity implies that the firm uses less labour at constant outputs due to higher efficiency and the possibly task-level labour-replacing nature of productivity growth, firms that improve their productivity performance relative to their competitors in the same country and industry are able to strongly expand their market and increase output, which in turn augments their demand for all factors of production, including labour. While some of this positive micro-level employment effect is offset at the industry level due to competitive externalities that induce overtaken firms to shrink or exit, the results do not provide any evidence of negative own-industry impact of productivity growth on employment on average. However, a negative own-industry relationship may emerge if initially less productive firms are limited in their ability to expand their market in response to a productivity catch-up due to a limited degree of market contestability, as this factor depresses the micro-level impact of productivity growth on employment. In a last step, the analysis turns to spillovers of productivity growth along value chains, and shows that productivity growth in a given industry may boost employment in connected downstream industries, both domestically and abroad. Finally, a similar positive association between productivity growth and wage dynamics is identified across levels of aggregation. Therefore, the chapter concludes that also more recently, productivity growth continues to be positively associated with the growth of employment and wages, both at the firm level and at the more aggregate one. In this view, productivity is not only a standalone economic objective, but well-designed and complementary policies also have the potential to help translate technological and organisational change into higher employment and wages.

While Chapter 2 focuses directly on productivity growth as a key policy objective that encompasses multiple determinants, Chapter 3 zooms in on automation technologies as one specific driver of productivity growth, arguably the most ambiguous one in terms of its impact on employment. The chapter exploits a partial equilibrium model of the labour market to shift the focus from the firm to the worker, and more specifically investigates the role of workers as “task-aggregating institutions”. Existing theories of the labour market impact of automation technologies have thus far simplified the worker’s role as an agent that supplies tasks to the labour market, which are complementary only at high levels of aggregation (i.e., the in-

dustry or country), but not within the firm or the worker itself. Yet in the real world, single workers use multiple skills and perform multiple tasks in their jobs. This can be seen from, among others, the OECD's PIAAC Survey of Adult Skills, which shows that almost all surveyed workers spend a non-negligible proportion of their time outside their two most frequent tasks. Under the assumption of an efficient allocation of workers to tasks, this suggests that the tasks performed by workers must be complementary and the worker's task-level productivities are interdependent – otherwise firms would unbundle tasks from jobs and let single workers perform only single tasks, as assumed in the existing literature.

The model in Chapter 3 structurally accounts for the circumstance that workers operate in an environment where they perform multiple tasks that are complementary to each other, and that automation technology may not target all these tasks at once but only a subset of them. Under this premise, the predictions of the labour market impact of automation are both complementary to and different from existing work. My theoretical investigation suggests that when automation concerns only some of the complementary tasks that a worker performs, the impact of automation on labour demand is not trivially negative, but instead involves a trade-off between the impact of the shift in the mix of production inputs away from labour on the one hand, and increased capital-labour complementarity on the other hand. The more complementary the tasks are, the stronger is the latter force, and the more workers are shielded from adverse impacts of automation. A key role in this context is played by the elasticity of demand. This role arises due to the productivity-enhancing nature of automation, and the link of labour demand to output prices: in an industry that automates, product supply increases and output prices fall, which also reduces the marginal revenue product of labour and therefore labour demand. The negative mechanism is stronger the less elastic product demand is, and the framework therefore predicts that all else equal, automation has a differentially negative impact on labour demand when it occurs on the supply side of more saturated markets.

The importance of this demand mechanism suggests that on the more aggregate level, differently from the perhaps common perspective, automation does not appear to affect only those workers that see some of their tasks replaced by technology, but instead all those that work at firms which operate on the output market in which automation occurs on the supply side. Furthermore, the conclusions of my analysis allow to discuss the thus far insufficiently understood origin of possible positive micro-level relationships between the adoption of automation technology and firm-level employment: at sufficiently competitive firms that take prices as given, demand is sufficiently elastic, and if their workers have one or more complementary non-automatable tasks left after automation occurs, the labour demand impact of automation is mainly determined by the capital-labour channel. I indeed show that in this context, au-

tomation necessarily increases the marginal product of labour, and thus also labour demand.

These findings resonate well with the results in Chapter 2: while at the firm level, even technological progress that replaces employment at the task level boosts a firm's competitiveness and allows it to expand its market, the aggregate implications crucially depend on the demand side of the output market, and the extent to which output is allowed to scale in response to a positive productivity change. The lower the elasticity of demand, the weaker is the indirect mechanism related to sales expansion that compensates for the direct negative employment effect associated with a reduced demand for factors at constant output (and possibly capital-labour substitution).

Furthermore, the degree of task complementarity appears to crucially affect the rate of the reduction in the labour share associated with automation. While at the outset, automation unambiguously reduces the labour share due to its capital-labour substituting nature, the degree to which further improvements in technology at the intensive margin, which occur in environments where workers work complementary to the machines, reduce the labour share is lower the higher the degree of complementarities between tasks, or put differently, the complementarity between workers and machines in the automation state. Finally, the conclusions of the analysis in Chapter 3 also inform about the structural determinants on the supply side of the labour market, and may be relevant to education policy. Different to the common perception that workers are best-advised to specialise in tasks that are as far away as possible from the spectrum of automatable tasks, the trajectory of labour demand may be most stable for workers that have invested in general purpose skills such as e.g. literacy, numeracy, social skills, flexibility and the ability to learn, which make them proficient across a broad range of tasks. According to the model, such workers gravitate more naturally to more complementarity-intensive occupations where the impact of automation is not felt as harshly (or is even welcomed), and broad specialisation may also be a "safe bet" if the trajectory of technology, or respectively the set of tasks that become automatable over a worker's career, are uncertain.

In combination, the insights from the three works in this thesis suggest that recent technological change – with innovation and technology diffusion as two complementary but distinct engines – has wide-reaching and multi-faceted impacts on the economy. Beyond its direct link to APG, Chapter 1 has shown that the associated changes the productivity distribution may have also dynamic impacts on aggregate productivity dynamics. In particular, closing productivity gaps in the lower tail of the productivity distribution appear to have dynamic returns to APG through a further indirect acceleration of diffusion.¹ As such, investments in technology

¹To this end, (Berlingieri et al., 2020) argue that large productivity gaps may be related to low absorptive capacities of less productive firms, among others due to heterogeneous production structures and skill shortages – or

diffusion, including public ones, may have a multiplier that strictly exceeds one, and appear key in reviving APG. Within-firm productivity growth is further shown to be positively linked also to employment growth in Chapter 2, and both the heterogeneous effects analysis in this chapter as well as the theoretical analysis in Chapter 3 suggest that this positive link may persist even when productivity growth is related to technological change that replaces labour at the task level.

Beyond within-firm dynamics, my work highlights the important role of reallocation. In Chapter 1, larger productivity gaps – in particular those at the top – are found to be linked to a further concentration of productivity growth at the top, and a productivity-enhancing shift of value added shares in favour of ex-ante more productive firms. While productivity growth at the top is found to be less related to the reallocation of input factors in Chapters 1 and 2, Chapter 2 shows that initially less productive firms do increase their share in employment if they improve their productivity relative to their competitors, and thereby provides evidence of a productivity-driven reallocation mechanism that both supports the efficient allocation of inputs and employment growth. The evidence in Chapter 2 suggests that the strength of the reallocation mechanism depends on the elasticity of demand, as firms which improve their productivity grow the more in employment the more they can grow their sales. As mentioned above, the analysis of Chapter 3 finds that this demand mechanism may be a key determinant also for the concrete context of labour-replacing technological change.

In terms of the dynamics of aggregate employment, while academics and the public alike have repeatedly worried about a trade-off between productivity and employment in the digital age, and that a revival of productivity growth could come at significant social cost through adverse labour market impacts, the findings of Chapter 2 suggest that these concerns may be unwarranted. Importantly, productivity growth appears to be positively related to labour market outcomes at the micro level through a mechanism tightly linked to competition in the market. If technology diffuses at a faster rate, this may stimulate firms competitiveness and promote an efficiency-enhancing reallocation of markets, also since firms catching up to the frontier may challenge established positions. If competition is sufficiently strong, there is reason to believe that productivity growth will boost employment even within the industry where it originates. Furthermore, connected industries may benefit from higher supply side productivity in the market for intermediates, which by the complementarity of different factors of production also boosts employment and wages.

But will the picture change if in the future, automation becomes even more prevalent and

more simply put, the sophisticated technologies used by large productivity superstars are not immediately useful to the average firm as it lacks the infrastructure in terms of workers and complementary capital assets, importantly intangible ones. Therefore, technology diffusion may directly close productivity gaps, but also foster absorptive capacities and thereby further promote the diffusion of other technologies.

reaches also more occupations in the service sector? From the insights of Chapter 3, the answer appears to be no: in services, tasks are not as easily unbundled as in manufacturing where automation has been more prevalent thus far, and due to higher complementarities between different tasks performed by individual workers in this sector, the impact of automation may even be differentially positive. However, there indeed appears to be a risk of an excessive productivity focus, and sustainable technological change may require that technology does not exclusively target the occupations where it can operate most independently from labour, even if the productivity gains from automation are highest there. To the extent that these occupations represent the lowest-hanging fruits for automation that may already be harvested, there still appears to be reason for optimism about the role automation may play for labour demand in the future.

1. PRODUCTIVITY DIVERGENCE AND PRODUCTIVITY SLOWDOWN: ARE THE TWO LINKED? AND WHAT ARE THE CHANNELS?

based on joint work with:

Chiara Criscuolo, Alexander Himbert and Francesco Manaresi

Abstract. This work exploits harmonised and comparable data for 13 OECD countries to shed new light on the relationship between productivity divergence, i.e. increasing gaps between the most and least productive firms in an industry, and aggregate productivity growth (APG). The results show that the impact of productivity dispersion shocks on APG is the result of two countervailing forces. On the one hand, such shocks dynamically increase the rate of productivity growth at larger firms, further improving the efficiency of resource allocation. On the other, they persistently reduce the rate of average firm-level productivity growth, consistent with a slowdown of technology diffusion. Divergence in the upper tail of the productivity distribution is related positively to APG due to a strong allocative efficiency component. However, productivity gaps have been increasing more in the lower tail more recently, and this phenomenon is negatively related to APG due to a stronger negative diffusion component. Divergence is less positively related to APG in intangible-intensive environments, which is however where productivity gaps have been increasing most. Upper tail divergence is linked to skill-biased technological change and may thus be more ambiguous in terms of the social impact as the positive APG link would suggest.

1.1 INTRODUCTION

The past decades have witnessed a slowdown in aggregate productivity growth (APG) across OECD countries. The slowdown differed across countries in its outset: in the 1980s for the US (Stiroh, 2002), in the 1990s for Europe (Fernald and Inklaar, 2020), but all countries suffered from a further slowdown in the early 2000s, driven by a drop in within-industry APG.¹²

To develop effective policies that support a revival of APG, policy-makers need to evaluate the underlying drivers of within-industry APG. This quantity can be decomposed in three factors: within-firm productivity growth, driven by firm-level innovation in and adoption of new technologies and products; growth in allocative efficiency, i.e. the degree of efficiency in the allocation of resources to firms of heterogeneous productivity; and the contribution of creative destruction to APG, coming from two components that capture market entry and exit, respectively. Each of these components may play a role in explaining the slowdown.³

The observed productivity slowdown has been accompanied by an increasing divide between the most and less productive firms within industries in most OECD countries (Berlingieri, Blanchenay and Criscuolo, 2017; Berlingieri et al., 2020; Gal et al., 2019; Cette, Corde and Lecat, 2017; Andrews, Criscuolo and Gal, 2016). The overall change in productivity dispersion, identified even in narrowly defined industries, is usually conveniently decomposed in two different phenomena (Berlingieri et al., 2020): increasing distance between the most productive firms and the median of the distribution (“change in upper dispersion”), and between this and the least productive firms (“change in lower dispersion”).

Evidence shows that, on average across OECD countries, over the past two decades the increase in lower dispersion has been far stronger than the one in upper dispersion. Indeed, Berlingieri, Blanchenay and Criscuolo (2017) show on a sample of 16 OECD countries that while the distance between the top performing firms (firms belonging to the top decile of the productivity distribution in a country-industry) and the median firm grew by 4% between 2000 and 2012, the distance between the median firm and the group of laggards (i.e., firms belonging to the bottom decile of the productivity distribution⁴) increased by 10%.

While increasing productivity dispersion is not negative per se, the simultaneity of produc-

¹The only exceptions are Finland and Korea, which benefited from booming ICT industries in the early 2000s.

²A decomposition of APG over the 2000-18 period into its within- and between-industry components shows that over 70% of the downward trend can be attributed to the within-industry component.

³The literature has studied the role of slower pace of innovation (Gordon, 2018), slower pace of diffusion (De Ridder, 2019), declining competition (Akcigit and Ates, 2021), frictions to allocative efficiency (Andrews and Cingano, 2012; Hsieh and Klenow, 2009), and the slowdown in creative destruction forces (Calvino, Criscuolo and Verlhac, 2020; Akcigit and Ates, 2021) in explaining the slowdown in APG. Recent contributions in macroeconomics point to a larger role for declining competition and a slower pace of technology diffusion among firms (Akcigit, Hanley and Serrano-Velardo, 2021).

⁴Andrews, Criscuolo and Gal (2016) define global frontier firms as the top 5% in an industry across 26 countries.

tivity divergence and productivity slowdown over the past two decades has sparked a debate about the relationship between these two phenomena. Empirical studies of this relationship have proven difficult, also because of the lack of comparable cross-country data that provide coherent micro-to-macro information on these phenomena.

This paper studies empirically the impact of a shock to the dispersion of the productivity distribution (measured as a one-year increase in productivity dispersion between firms in the 95th and 10th percentile of the distribution) on the dynamics of APG and its components over a 5-year horizon. For this purpose, we estimate impulse-response functions using a dynamic panel model that controls for serial correlation in both the dependent and the independent variables using local projections (Jordá, 2005).

The analysis covers 11 OECD countries, studied over the period 2000-18. It exploits the richness of the OECD MultiProd dataset, which collects harmonised cross-country data on the full population of firms (or a representative sample) of the business sector. Thanks to these features, the MultiProd data are particularly suitable for cross-country analyses that need information for the whole distribution of firms, not only covering larger and more productive firms (such as those whose data are collected in commercial databases).

The analysis distinguishes between shocks to upper and lower dispersion. This distinction is crucial as it allows to unveil the role of several structural mechanisms (which we call “channels”) that may affect the relationship between dispersion and APG dynamics, thus assessing their relative strength. A first key factor that may link a shock to productivity dispersion to the dynamics of APG is the pace of innovation. A stronger rate of innovation may augment productivity dispersion to the extent that innovative technologies, practices and products are more frequently developed by firms with initially high productivity and that diffusion is partial. If the full productivity effect of innovation materialises over several years and productivity growth at innovators is therefore strongly and positively serially correlated, this channel may have significant dynamic implications for APG, and in particular its within-firm component. Changes in (both upper and lower) productivity dispersion may also reflect the pace of diffusion of technological gains across firms. Slower diffusion, possibly driven by the structural features of the ongoing digital transformation, may dynamically dampen productivity growth, also through the within-component. We refer to this mechanism as the *diffusion channel*.

A shock to dispersion (both at the upper and lower part of the distribution) may also be associated with changes in the allocative efficiency of resources in the economy (the *allocative efficiency channel*). In particular, a widening gap between most and least productive firms may be followed by an efficient productivity-enhancing reallocation of resources (Autor et al., 2020). The strength of this mechanism may depend, inter alia, on the strength of frictions in factor

mobility (De Loecker, Eeckhout and Mongey, 2021). Allocative efficiency would also increase if the widening in productivity dispersion is driven by a faster productivity growth among (ex-ante) larger firms (thus inducing a stronger association between productivity and size). This second mechanism behind the allocative efficiency channel could signal increasing difficulties by SMEs to keep up with the pace of productivity growth of larger firms. Finally, increased productivity dispersion might also affect APG through the entry and exit components of the *creative destruction channel*, although the direction of this channel is ex-ante ambiguous.⁵

The results of the empirical analysis suggest that a shock to overall dispersion is tightly linked with two countervailing forces. First, the allocative efficiency channel is present both for shocks to both upper and lower dispersion, but is stronger in the former than in the latter. Second, the change in productivity dispersion is also persistently negatively correlated with average within-firm productivity growth, as a result of the diffusion channel (Andrews, Criscuolo and Gal, 2016). The negative and persistent impact on within-firm productivity growth is driven mainly by a shock to lower dispersion; while shocks to upper dispersion are not correlated with the within-firm component. In this case the positive innovation channel may counter-balance the diffusion channel.

Finally, we find some evidence of a positive creative destruction channel, particularly over the longer horizon. Shocks to upper dispersion, in particular, are correlated with a stronger contribution to productivity growth from the exit of less productive firms, consistent with the idea that higher productivity growth among frontier firms exert more competitive pressures on the rest of the productivity distribution. Conversely, the impact of shocks to dispersion on entry is smaller and generally not significant.⁶

Summing up the various effects, an increase in upper (lower) dispersion appears to have a sustained positive (transient negative) link to APG. According to our estimates, a 1% increase in upper dispersion is associated with a rate of APG higher by roughly 0.1 p.p. in each of the five subsequent years. Conversely, a 1% increase in lower dispersion predicts a more transient reduction of APG, which is lower by 0.2 p.p. in the first year, and by 0.1 p.p. in the second year after the shock. A simple back-of-the-envelope calculation suggests that over the sample period, lower divergence was linked to a yearly APG slowdown of around 0.12 p.p. (i.e., more than 10% of the slowdown experienced over the period).

⁵If increased productivity dispersion reflects lower competition and lower selection at exit, i.e. lower exit probability at the bottom of the productivity distribution, it may be linked to a weakening of the cleansing process and, thus, lower contribution of creative destruction to APG. However, increased dispersion might also reflect lower barriers to entry and higher entry rates of (less productive) new firms. To the extent that young contribute disproportionately to economic growth and innovation, and that more entry induce more competitive pressures on incumbents, this mechanism would lead to higher APG.

⁶It must be noticed, however, that the entry component of APG may underestimate the overall extent of creative destruction forces, as the contribution of new firms to APG may unveil during the years after entry, but this contribution is not measured in the entry component of the productivity growth decomposition we consider.

Beyond the average relationships discussed above, we find that industries which use intangible capital assets more intensively are characterised by both a stronger negative diffusion channel and a weaker positive allocative efficiency channel. This raises concerns about the impact of the growing intangibility of the economy on APG, in particular because intangible-intensive industries have featured stronger increases in dispersion over the past decades (Corrado et al., 2021). Furthermore, we provide evidence that the allocative efficiency channel seems to be largely driven by stronger productivity growth in larger firms, while reallocation of physical factors play a limited role. This points to a large growth potential of supporting resource reallocation, through effective policies in labour and capital markets. Finally, we document that changes in dispersion may also play an important role in explaining employment and wage dynamics. Shocks to upper dispersion are linked to a capital-labour substitution and thus possibly to skill-biased technology among the most productive firms. Lower dispersion shocks are followed by reductions in industry-level employment and a reduction in the relative wage of less productive firms, and may therefore reflect a drop in labour demand that may be linked to the worsening performance of highly labour-intensive laggards.

The remainder of the paper is structured as follows. Section 1.2 discusses more in-depth the theoretical channels that may link changes in productivity dispersion to long-term changes in APG. Section 1.3 presents the data used, the MultiProd database, and discusses the measures of productivity dispersion and growth used throughout the analysis. Section 1.4 provides descriptive evidence of the trends in dispersion and productivity growth. Section 1.5 discusses the empirical model used throughout our analysis, while Section 1.6 presents our main findings on the role of the diffusion and allocative efficiency channels as well the evidence on industrial heterogeneity, according to the intangible intensity of the industry. Section 1.7 analyses the link between upper dispersion shocks and the allocative efficiency channel, and estimates the role of changes in dispersion on the labour market. Section 1.8 concludes.

1.2 THE CHANNELS LINKING CHANGES IN PRODUCTIVITY DISPERSION TO AGGREGATE PRODUCTIVITY GROWTH

In this section, we discuss more in depth some of the main channels that have been debated in the economic literature to link the increase in productivity dispersion and the trends in APG. Several analyses trace back these channels to long-term changes in technology (notably, with the role of the digital transformation) and innovation activity.

Several empirical studies have linked increasing productivity dispersion to the digital transformation and the related rise of the intangible economy. In particular, changes in upper dispersion have been related to investments in intellectual property assets and in software and

databases, while lower dispersion shocks have been linked to investments in organizational capital (Corrado et al., 2021).⁷ These phenomena may be differently linked to APG.

At the one hand, the digital transformation may slowdown technology diffusion across less productive firms, augmenting productivity dispersion and reducing APG. Increasing lower dispersion, in particular, has been empirically linked to the rising importance of intangible organizational capital and skills (Berlingieri et al., 2020; Corrado et al., 2021). These assets are crucial complements of advanced digital technologies and are more difficult to accumulate by laggard firms, because of their sunk nature and scalability (Haskel and Westlake, 2018). Therefore, the increasing prevalence of intangible capital assets may not only augment productivity dispersion itself, but also slow down the rate of technology diffusion dynamically. As the lack of technology diffusion may hamper APG both directly and indirectly (Comin and Mestieri, 2018), rising intangibility may be an important source of the diffusion channel.

The diffusion channel may also matter for changes in upper dispersion. Winner-takes-most dynamics and lower competition against frontier firms, driven by the scalability of intangibles and/or by tight intellectual property (IP) protection regimes, may widen the gap in the upper part of the productivity distribution and also affect APG. Empirically, cross-country evidence has shown that investments in software and databases and IP assets are tightly linked to increases in upper dispersion (Corrado et al., 2021). Models of endogenous growth show that these phenomena may reduce innovation incentives for firms competing with the frontier, and also discourage innovative firms from entering the market, thus lowering long term productivity growth (Akcigit and Ates, 2021; De Ridder, 2019).

Changes in dispersion may impact APG by reshuffling input and output allocation, thus affecting the allocative efficiency of resources. Indeed, in a competitive economy without frictions, resources should flow from less to more productive firms within and across industries until their marginal returns are equalized. Under this assumption, productivity divergence may trigger an efficient productivity-enhancing reallocation of resources (Autor et al., 2020). The strength of this mechanism may depend, inter alia, on the strength of frictions to factor mobility (De Loecker, Eeckhout and Mongey, 2021). These implications for the firm-level correlation between productivity and market size are reflected in the allocative efficiency channel.

Finally, increased productivity dispersion might also affect APG through the creative destruction channel, although the direction of this channel is ex-ante ambiguous. Indeed, if changes in dispersion reflects lower competition, they may be linked to a weakening of the se-

⁷Intellectual property protection may insulate innovators from (neck and neck) competition (Akcigit and Ates, 2021), and the cost structure of developing a software or a database (i.e., high fixed and low marginal costs) may generate scale economies which benefit more productive firms (De Ridder, 2019). Organizational and managerial capital is found to be a key complement of technology adoption. Its sunken nature and related lack of pledgeability make it difficult to finance, particularly for micro and small firms (Haskel and Westlake, 2018).

lection and cleansing process and, thus, possibly lower contribution of entry and exit to APG. However, higher dispersion may also put more pressure on less productive firms, which may strengthen the cleansing and selection processes, particularly over time.

While theoretical these channels may all be at play, the empirical relevance of them may highlight which underlying structural mechanisms are driving the process of increasing productivity dispersion and, thus, where policies can more effectively act to support its positive implications for APG (or conversely limit its negative effects on growth and welfare).

1.3 METHODOLOGY, DATA AND MEASUREMENT

1.3.1 DATA SOURCE: THE OECD MULTIPROD PROJECT

The main data source is the OECD MultiProd project, which relies on the distributed micro-data approach to access confidential firm-level data, in collaboration with experts from National Statistical Offices, government departments, and research organisations in 29 countries. For the purpose of the analysis of the different channels linking productivity dispersion and productivity growth, the MultiProd database offers two key advantages absent in most other datasets: firstly, the data is based on the full population of firms (or a representative sample) in most sectors of the economy for a large number of countries, making it suitable for a cross-country analysis not only of innovative frontier firms, but of the whole distribution of firms, including laggards. Relative to other analysis, based on commercial databases covering stock-quoted companies only or only part of the firm population, this paper is therefore able to offer a deeper and more comprehensive empirical investigation of the links between productivity dispersion and APG in the short and medium run and of the contribution from entry and exit, which, is often neglected in other analysis due to the lack of suitable data on business dynamics. Secondly, the MultiProd database contains computations of different productivity decompositions, which make it possible to disentangle the different channels (reallocation, diffusion, innovation, creative destruction) through which dispersion and productivity growth might be linked both in a static and a dynamic setting over different time horizons h ($h = 1, 3, 5$).

From official confidential firm-level data, the MultiProd routine collects statistical moments of the distribution of firm characteristics at different levels of aggregation.⁸ This paper mainly relies on information on detailed country-industries following the SNA A38 classification, within the macro-sectors “Manufacturing [C]” excluding the two-digit industries Coke and Refined Petroleum, “Construction [E]” and “Non-financial Market Services [G-N]” excluding Real Estate, following the ISIC Rev.4/NACE Rev.2 industrial classification. This macro-sector restriction is motivated by our focus on private business industries.

⁸See Appendix A.1 for a more detailed description of the MultiProd database and information on data collection.

1.3.2 MEASURES OF PRODUCTIVITY

The analysis relies on two specific measures of productivity: the first measure of productivity dispersion used in the econometric analysis is based on one of the multi factor productivity (MFP, henceforth) indicators generated in MultiProd, the MFP measure estimated econometrically at the firm-level using the Wooldridge (2009) control function approach with value added as a measure of output. Firms are assumed to have a Cobb-Douglas production function, but not necessarily constant returns to scale:

$$Y_{it} = A_{it}K^{\sigma_K}L_{it}^{\sigma_L}$$

where A_{it} , firm i 's MFP at time t , is typically unobserved and has to be estimated. The Wooldridge (2009) procedure relies on estimating variable inputs with a polynomial of lagged inputs and a polynomial of intermediates. It allows for the identification of the variable input and yields consistent standard errors.

We test the robustness of our results to the use of a different measure of productivity. Labour productivity captures the amount of output produced by a firm relative to the labour input. It is computed at the firm level as the (real) value-added per worker:

$$LP_VA_{it} = \frac{VA_{it}}{L_{it}}$$

where VA_{it} is value-added of firm i at time t and L_{it} is the measure of its labour input.⁹ The advantages of this measure are that it is widely available, straightforward to interpret and relatively immune to measurement error (that commonly affect capital measures). Moreover, it can be aggregated easily into industry-level or country-level labour productivity using employment weights. Establishing robustness of our findings with respect to labour productivity will allow us to argue for the comparability of our results to a broader body of literature which focuses on this measure due to its simplicity in computation and interpretation.

1.3.3 DECOMPOSITIONS OF AGGREGATE PRODUCTIVITY GROWTH

To disentangle the different channels through which productivity dispersion and productivity growth might be linked, it is essential to decompose APG into its components. Such decompositions make it possible to separate effects of reallocation and creative destruction from channels acting at the firm level, such as the strength of innovation and the diffusion of knowledge to laggard firms. Accordingly, these decompositions are key for obtaining a detailed picture

⁹Value added is deflated at the country-industry level based on the OECD STAN database and expressed in 2005 USD PPP. The preferred labour measure for MultiProd is a headcount measure of persons engaged (that is, including both paid employees and working proprietors).

of the link between productivity dispersion and APG, and may identify countervailing mechanisms, that may cancel out and result to a (close-to) zero effect in the aggregate.

The Melitz and Polanec (2015) decomposition (MP decomposition, henceforth) provides an extension to the (static) decomposition of Olley and Pakes (1996) to (dynamically) account for firm entry and exit in addition to reallocation between surviving incumbent firms. More precisely, the MP decomposition disaggregates APG into the growth in the average within-firm productivity of incumbents (“MP-Within”), changes in the allocative efficiency of resources among incumbents (“MP-Covariance”) and contributions of firm entry (“MP-Entry”) and exit (“MP-Exit”). Therefore, the MP decomposition provides a dynamic view of changes in productivity growth by accounting for business dynamics and changes in the universe of firms.

Following the methodology of Melitz and Polanec (2015), year-on-year productivity growth in each SNA A38-industry j and year t is decomposed as follows:¹⁰

$$\Delta AP_{jt} = \frac{1}{N^C} \sum_{i \in C_{jt}} \Delta P_{it} + \Delta_h Cov_{i \in C_{jt}}(\theta_{it}, P_{it}) + \left(\sum_{i \in E_{jt}} \theta_{it} \right) (P_{jt}^E - P_{jt}^C) + \left(\sum_{i \in X_{jt}} \theta_{it} \right) (P_{jt-1}^C - P_{jt-1}^X) \quad (1.1)$$

The first term is the change in the unweighted productivity average of incumbents, i.e. firms that survive between $t - 1$ and t (C_{jt}). The second term is the change in the Olley and Pakes (1996) covariance term among incumbents, i.e., the change in the covariance between the weight θ_{it} and firms’ productivity.¹¹ The weight θ_{it} reflects a firm’s share in industry-year’s value added.¹² The second term thus measures the APG contribution of resource and market share reallocation between incumbents. Notably, next to productivity-enhancing factor reallocation, an increase in MP-Cov may also reflect stronger productivity growth at larger firms.¹³

Finally, E_{jt} is the set of firms entering the industry in year t , and X_{jt} is the set of firms that exit from the industry between $t - 1$ and t . P_{jt}^E , P_{jt}^C and P_{jt-1}^X are the weighted productivity averages of, respectively, market entrants, incumbents, and market exiting firms computed in the relevant time period and with weights that sum up to one within each group.

The sign and strength of the relationship between the MP-Within component and changes in dispersion allow to estimate the relevance the innovation and diffusion channels, as stronger innovation at the frontier would imply a positive relationship between increases in dispersion and the MP-Within component, while a lack of diffusion would imply a negative relationship. Furthermore, the MP-Covariance component provides a direct means to assess the allocative efficiency channel (for incumbent firms), and the MP-Entry and MP-Exit components of the MP

¹⁰The formula omits the index c as the decomposition is performed for each country individually.

¹¹The Olley and Pakes (1996) covariance term, also called OP gap, has been used as a measure of allocative efficiency. It increases if more productive firms use a higher share of resources in the industry.

¹²For the decomposition of labour productivity, θ_{it} is the firm’s share in industry-year level employment.

¹³Note that without factor reallocation, market shares increase in productivity growth by $\Delta \log VA_i = \Delta \log A_i$.

decomposition directly relate to the creative destruction channel. Hence, the MP decomposition is able to address all channels linking productivity growth and productivity dispersion discussed above. The main empirical analysis will therefore focus on all of the different components of the MP decomposition. Due to our focus on the dynamic response of year-on-year productivity growth rates to initial changes in productivity dispersion, we focus on the MP decomposition over 1 year, which is readily available in the MultiProd database.

1.3.4 MEASURES OF DISPERSION

Making use of the richness of the MultiProd database, several measures of productivity dispersion within 2-digit industries are calculated. By measuring different moments, and specifically percentiles of the productivity distribution, it becomes possible to investigate the implications of productivity dispersion between the most and least productive firms using e.g. the 90-10 ratio. The 90-10 productivity ratio is defined as the ratio between the 90th and the 10th percentile of the productivity distribution. It is used widely in the economic literature to assess the dispersion of the distribution of wages and productivity. The measure has an intuitive interpretation, since a ratio of X informs that firms at the top of the productivity distribution, proxied by firms at the 90th percentile, producing (given the same amount of inputs) X times as much as firms at the bottom of the distribution, proxied by firms at the 10th percentile.

We build on this measure in two ways. First, we proxy the frontier, i.e. firms at the top of the productivity distribution, using the 95th percentile, rather than the 90th percentile. Indeed, as percentiles are generally based on the unweighted distribution of productivity, the 90th percentile may insufficiently proxy the frontier, especially in large industries with many small firms.¹⁴ Second, we define upper and lower dispersion as, respectively, the gap between the 95th percentile and the median firm (95-50 ratio) and the gap between the median firm and the 10th percentile of the distribution (50-10 ratio). This paper shows that the distinction between upper and lower dispersion provides useful information on the interplay between reallocation, innovation, diffusion, and creative destruction channels.

1.4 CHARACTERISATION OF THE PRODUCTIVITY DISTRIBUTION

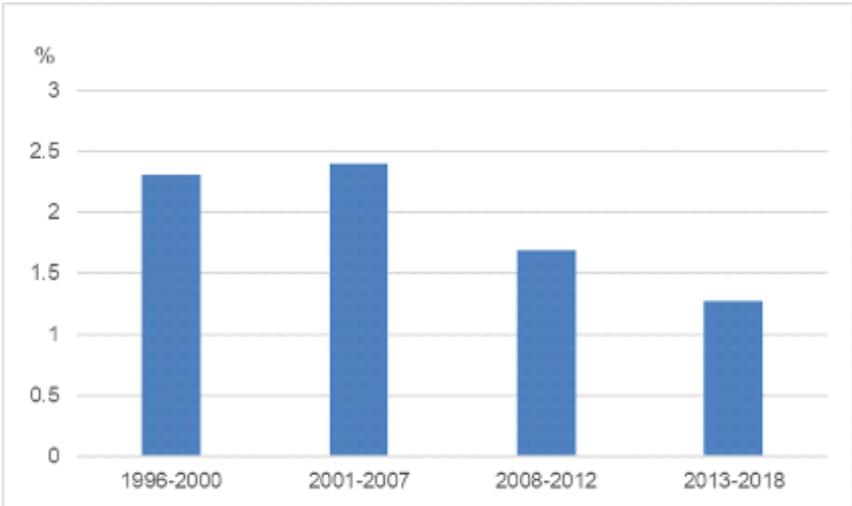
Figure 1.1 plots the average annual MFP growth in 14 countries for three different time periods: 2000-07, 2008-12 and 2013-18. The pattern matches the well-documented slowdown of APG in recent years Andrews, Criscuolo and Gal (2016) with productivity growth in the recovery

¹⁴An alternative measure reflecting more closely the frontier is the average of firm-level productivity within the top decile, i.e. firms above the 90th percentile of the distribution. However, while this measure may cover firms at the very top even better than the 95th percentile (equal to the median within the top decile), this measure may also be more sensitive to outliers. For this reason, we abstain from using it for our analysis.

being unable to match pre-crisis levels. Further analyses show that over 70% of this decline can be related to its within-country-industry component.

Increasing productivity gaps between the most successful firms and those lagging behind raise questions about technology adoption and diffusion, in particular whether the increase is driven by frontier firms pulling away or by firms in the rest of the distribution lagging behind. Indeed, technology and knowledge diffusion might affect firms along the productivity distribution differently. Specifically, the overall increase in productivity dispersion can come from (i) increasingly good performance at the top, (due to, e.g., to augmented knowledge and innovation), or (ii) worsening performance in the rest of the distribution, for the median firms (say, because of the lowering “neck-and-neck” competition with the most productive ones); or for the least productive firms (say, due to a slowdown in technology diffusion). Earlier analysis showed a trend of increasing dispersion at both the top and the bottom of the distribution, with a particularly strong increase at the lower end Berlingieri, Blanchenay and Criscuolo (2017).

Figure 1.1: Aggregate labour productivity growth, cross-country average



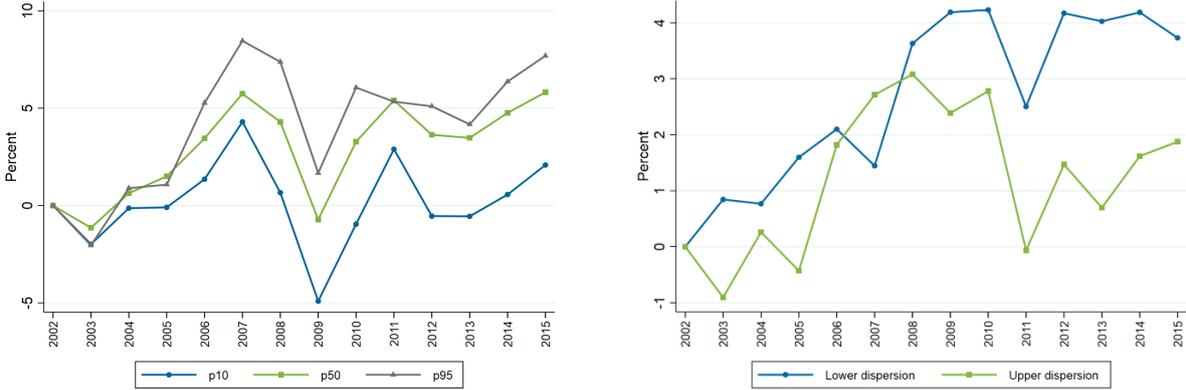
The figure plots the weighted average annual labour productivity growth across countries for different time periods. Country-industry level data (SNA A38) are aggregated to the year level using weighted means (with weights equal to the value added of firms in the country-industry in every year), and bars show the unweighted average across countries and years within periods. Countries included: AUT, BEL, CAN, EST, FIN, FRA, HUN, HRV, ITA, LTU, LVA, PRT, SVN and SWE. Source: Calculations based on the OECD STAN database.

Panel (a) of Figure 1.2 shows the dynamics of MFP for different deciles of the productivity distribution, averaged across countries and industries. Over the period 2002-15, the 95th and the 50th percentile have grown by around 7.5% and 6%, respectively, while the 10th percentile has grown by less than 2% and, by 2015, it had still to reach its pre-crisis levels.¹⁵ Panel (b) shows the direct implications of these trends for productivity gaps in the upper and lower half of the distribution, respectively. As the figure shows, productivity gaps have increased both in

¹⁵We focus on the period 2002-15, omitting later years to ensure that the depicted patterns are not driven by incomplete coverage across countries after 2015.

the upper and the lower tail of the distribution, but the degree of lower divergence we observe is much larger.¹⁶ The weaker divergence at the top has only marginally weakened the right skewness of the productivity distribution: the 95th percentile remains around 3.3 times more productive than the median firm, while the median firm is around 2.45 times more productive than the 10th percentile.

Figure 1.2: Trends in the multifactor productivity distribution, 2002-15



(a) Productivity distribution percentiles.

(b) Productivity dispersion.

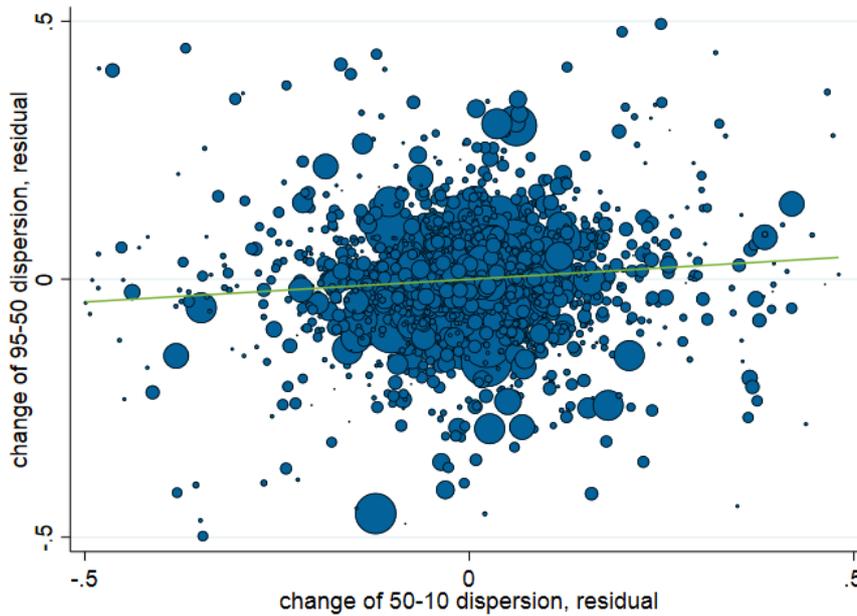
The figure plots average within-country-industry trends of different percentiles of the productivity distribution (Panel (a)) and productivity gaps between different percentiles (Panel (b); lower dispersion is defined as the gap between the 50th and 10th percentile, and upper dispersion as the gap between the 95th and 50th percentile), based on the year coefficients of regressions controlling for country-industry (SNA A38) fixed effects, for the period 2002-15. The regressions weight country-industries (SNA A38) by the value added of firms in the country-industry in the year 2002 (or first year of observation). Each point represents the average cumulative change since 2002. Countries included: BEL, CAN, FRA, HUN, HRV, ITA, LVA, NLD, PRT, SVN and SWE. Source: Calculations based on the OECD MultiProd 2.0 database.

The disconnect between the widening of productivity gaps in the upper and lower tail of the distribution shown in Figure 1.2 suggests that the two phenomena may be only partially correlated. To further corroborate this view, Figure 1.3 shows the relationship between annual changes in, respectively, upper dispersion (the ratio of the 95th and the 50th percentile of the productivity distribution; y-axis) and lower dispersion (the ratio of the 50th and the 10th percentile; x-axis), net of fixed effects accounting for unobserved heterogeneity at the country-industry or country-year levels. The graph shows that the phenomena are positively related, as indicated by the upward-sloping regression line fitted to the observations, but only very weakly so. Indeed, we observe many country-industry-years with a positive deviation of changes in upper dispersion from the absorbed trends, but a negative deviation of changes in lower dispersion and vice versa, and the scatterplot suggests that the two phenomena are largely independent, with possibly different structural sources and implications. The next section discusses how several key macroeconomic phenomena may be highlighted by relating the

¹⁶This pattern is consistent with the descriptions of divergence in Berlingieri, Blanchenay and Criscuolo (2017), who focus on a similar period and sample of countries.

dynamics of changes in upper and lower dispersion to APG and its components.

Figure 1.3: Partial correlation of changes in upper and lower dispersion



The figure plots annual changes in productivity dispersion between the 50th and 10th percentile (x-axis) against those between the 95th and 50th percentile (y-axis). The circles correspond to yearly observations for country-industry level data (SNA A38). The size of circles corresponds to the total value added in the country-industry in the initial year of observation of the country, which is also the weight used in the fitted regression line. Countries included: BEL, CAN, FRA, HUN, HRV, ITA, LVA, NLD, PRT, SVN and SWE. Source: Calculations based on OECD MultiProd 2.0 database.

1.5 EMPIRICAL MODEL

The slowdown in APG and the ongoing productivity divergence between frontier and laggard firms have both characterized the last two decades. This concomitance has brought researchers to conjecture that these two phenomena could be tightly linked. Andrews, Criscuolo and Gal (2016) and Berlingieri et al. (2020) showed that the increase in dispersion was linked with lower technology diffusion among laggard firms, and De Ridder (2019) discussed how this may increase market concentration and reduce the incentive of frontier firms to innovate. Conversely, Autor et al. (2020) emphasize how the rise of superstar firms may positively contribute to the efficiency of resource allocation in the economy, supporting APG. Ultimately, whether the shocks to productivity dispersion are linked to lower or higher APG is an empirical question. To answer such question, it is key to have comparable, representative, cross-country data on productivity distributions, as well as and APG and its components.

Such data is provided by the MultiProd database, which readily measures different moments of the productivity distribution and decompose APG according to established methodologies. Our empirical framework is based on estimating the dynamic impact of changes in up-

per and lower dispersion on the MP components of APG. We argue that the sign and strength of these associations provides prima facie evidence of the allocative efficiency, diffusion, innovation and creative destruction channels.

Estimating the link between shocks to productivity dispersion and APG (and its components) faces several empirical challenges. One important issue emerges if we try to estimate the simultaneous association between these variables, and is a sort of “reflection problem” (Manski, 1993). Indeed, because APG is the result of the growth of each parts of the productivity distribution, regressing APG on simultaneous changes in upper (lower) dispersion ultimately implies regressing the growth of the top (bottom) 10% of the distribution on itself. The more we control for changes in other parts of the productivity distribution, the stronger the reflection problem.

To address this challenge, we estimate a dynamic model that relates changes in productivity dispersion today to APG (components) at horizons $h > 0$ in the future, a relationship that is unaffected by the contemporaneous mechanical component and promises to reveal economically meaningful relationships, once properly estimated.

A second challenge is the serial correlation of the dependent and independent variables overtime, which may induce spurious correlation. The estimation strategy we adopt to overcome this empirical issue is to estimate impulse response functions relying on the projection method of Jordá (2005).¹⁷ We build on this model and adjust it to the needs of our specific application, bearing in mind that both left and right hand side variables are structurally related to the productivity distribution.

In particular, for every horizon $h = 1, 2, \dots, 5$, the following panel fixed effect model is estimated for country c , industry j , and year t :

$$DV_{jct+h} = \beta_U^h \Delta PD_{cjt}^{95-50} + \beta_L^h \Delta PD_{cjt}^{50-10} + \beta_H^h \Delta p50_{cjt} + \gamma X_{jct}^{lag} + \delta X_{jct}^{lead} + \gamma_{jc} + \theta_{ct} + \varepsilon_{jct} \quad (1.2)$$

where DV_{jct} is a dependent variable DV in an industry j , country c and year t . Five dependent variables are considered: the change in aggregate (log) productivity, and the four components of the MP decomposition. All dependent variables refer to year-on-year APG or the composition thereof, i.e. the period $t + h - 1$ to $t + h$. PD_{cjt}^{95-50} is the main measure of productivity dispersion in the upper tail, i.e. the log of the ratio between the 95th and the 50th percentile of the productivity distribution. In analogy, PD_{cjt}^{50-10} measures productivity dispersion in the lower tail as the log of the ratio between the 50th and 10th percentile. $p50_{cjt}$ is the log productivity of the median productivity firm. We further include vectors of time series controls

¹⁷A technical discussion of our preferred method, including a comparison against alternative specifications such as a panel (S)VAR model, is given in Appendix A.2.

to account for serial correlation in the system’s variables and to isolate indirect effects through autoregressive mechanisms. X_{jct}^{lag} is a vector of backward-looking controls that includes two lags of ΔPD_{cjt}^{95-50} , ΔPD_{cjt}^{50-10} and $\Delta p50_{cjt}$, and further two lags of each component of the MP decomposition. X_{jct}^{lead} is a vector of forward-looking controls that includes intermediate divergence, i.e. $\Delta PD_{cjt+k}^{95-50}$ and $\Delta PD_{cjt+k}^{50-10}$ for $k = 1, 2, \dots, h$,¹⁸ and further intermediate changes in the dependent variable, i.e. ΔDV_{jct+k} for $k = 0, 1, \dots, h-1$. Controlling intermediate impulses follows e.g. Autor and Salomons (2018) and ensures estimation of the direct response to an impulse at time t . Controlling intermediate levels of the dependent variable rules out that our estimates are confounded by spurious reverse to the mean patterns in industry-level productivity, and reinforces the interpretation as the direct/additional response of productivity growth (components) at time $t+h$ to an impulse at time t , given the changes up until $t+h-1$. γ_{jc} is a vector of country-industry fixed effects controlling for time-invariant characteristics of industry j in country c , and θ_{ct} is a vector of country-year fixed effects.

At every estimated horizon $h = 1, 2, \dots, 5$, the key coefficients of interest are β_U^h and β_L^h , estimates of the direct response of the dependent variable at time $t+h$ to an idiosyncratic unit change in upper/lower dispersion at time t (“upper/lower dispersion shock”). The coefficient β_H^h captures the response to growth of median productivity. As we hold fixed the changes in upper and lower dispersion in estimating this coefficient, it captures movements in median productivity where the 95th and the 10th percentile move in the same fashion, and thus measures almost a linear shift in the productivity distribution which we refer to as homogeneous productivity growth below.

The sample consists of 1791 observations from 11 countries over the period 2001-18.¹⁹

1.6 MAIN FINDINGS

This section presents our main results. As discussed above, our framework allows us to estimate the response of key variables of interest to idiosyncratic changes in productivity dispersion that are not explained by the past and future levels of the variables we consider. In the following, we refer to these idiosyncratic changes in dispersion as “dispersion shocks”.

1.6.1 GENERAL LINK BETWEEN PRODUCTIVITY DISPERSION AND PRODUCTIVITY GROWTH

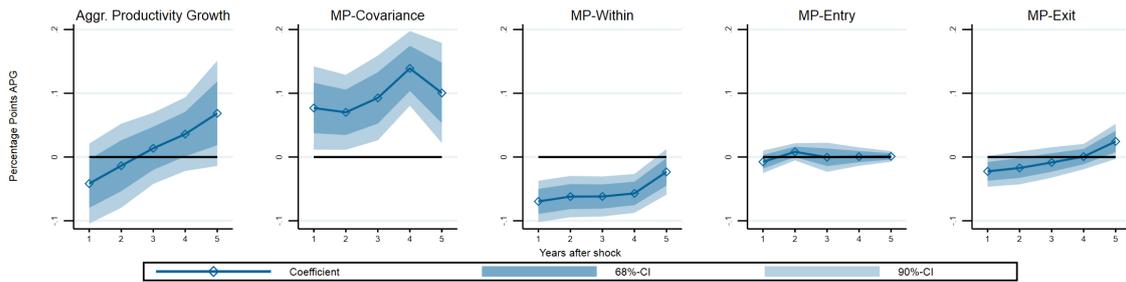
Figure 1.4 presents results from a simplified version of the empirical model presented in Section 1.5 where dispersion is not split into the upper and lower half, but the model considers

¹⁸Crucially, the model does not hold constant the forward-looking change in median productivity $p50_{cjt}$. Doing so would preclude the identification of the diffusion channel, which is likely to shift the productivity distribution to the left by lowering the rate of technology diffusion.

¹⁹Included countries: BEL, CAN, EST, FRA, HRV, HUN, ITA, LVA, NLD, PRT, SVN, SWE.

instead overall dispersion shocks, i.e. shocks to the log-ratio of the 95th and 10th percentile of the productivity distribution. The results show that holding constant the change in median productivity, increases in the distance between frontier and laggard firms are followed by a persistent increase of the MP-Covariance component (second column), but also a persistent reduction of the MP-Within component (third column). Furthermore, the response of MP-Exit tends to be positive at longer horizons (fifth column). Here, the quantitative significance for APG is limited due to the lower overall role of this component in the MP decomposition, but relative to the total variation in the MP-Exit component, this pattern may well be non-negligible. The role of overall dispersion shocks for the entry component appears limited (fourth column). In sum, the estimated total response of APG (first column) is always positive but never statistically significant, and the countervailing impacts of the covariance- and within-components appear to roughly cancel each other on average.

Figure 1.4: Baseline results, 95-10 MFP dispersion shock

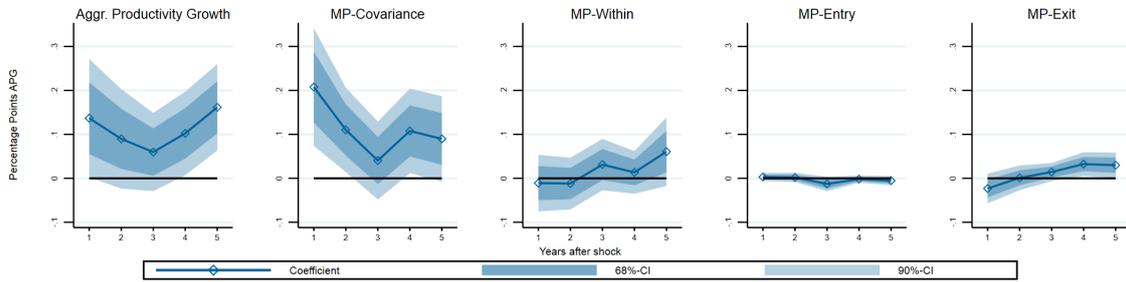


The figures show the regression estimates for the response of productivity growth and its components to a top/bottom dispersion shock, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

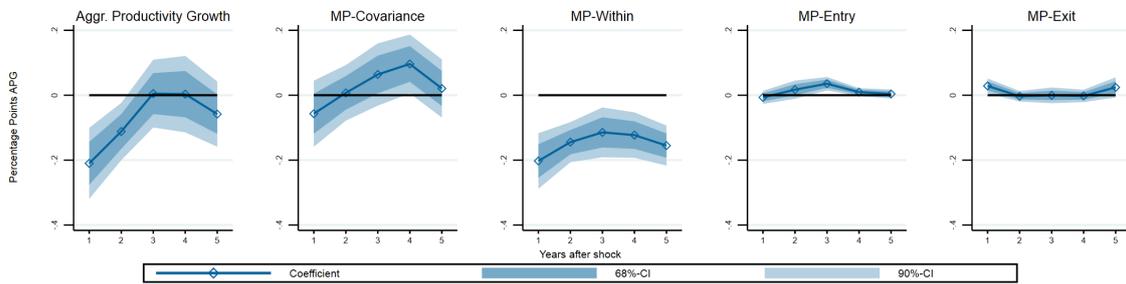
Figure 1.5 shows the results from the main specification. The estimates of β_U^h , the coefficient for the upper dispersion shock, are shown in panel (a), and those for β_L^h , the coefficient for the lower dispersion shock, are shown in panel (b). Comparing the responses of MP-Covariance (second column), the results suggest the positive response to changes in overall dispersion is predominantly driven by upper dispersion shocks, especially at shorter horizons. Conversely, the negative and persistent response of the MP-Within component is entirely driven by lower dispersion shocks (third column). For the MP components related to business dynamics, increases in lower dispersion may lead to an intermediate surge of the entry component (fourth column), whereas the positive response of the exit component appears to be related more strongly to upper dispersion shock (fifth column). Summing up across these components, the response of APG to increases in upper dispersion is positive and pointwise significant at least at the 68%-level at every horizon, while lower dispersion shocks display a

negative (non-positive) elasticity at short (longer) horizons (first column).²⁰

Figure 1.5: Baseline results, 95-50 and 50-10 MFP dispersion shock



(a) Upper dispersion shock.



(b) Lower dispersion shock.

The figures show the regression estimates for the response of productivity growth and its components to productivity dispersion shocks, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Summing up, these results emphasize how the various structural factors that link productivity dispersion to productivity growth are differently associated with the shocks to upper and lower dispersion, and this ultimately shapes the response of APG to these two different shocks.

For changes in upper dispersion, the MP-Covariance component is a key source of their positive correlation with APG. This relationship appears stronger for increasing dispersion in the upper than in the lower tail, especially at shorter horizons. In the next section, we empirically investigate the economic sources and implications of the covariance channel. We find that this link is predominantly driven by shifts in the distribution of productivity growth in favour of larger firms, rather than the reallocation of physical factors to more productive firms, which may signal that frictional factor reallocation prevents the full exploitation of the productivity potential of increasing productivity dispersion.

The link between changes in dispersion and the MP-Within component of APG is affected by the two countervailing forces of the innovation and diffusion channels. The results point

²⁰The results are robust to a series of alternative specifications. Figures A.1 and A.2 in the appendix show that comparable patterns emerge when considering labour productivity as the measure of productivity, and Figures A.3 and A.4 show the same for MFP when normalising weights at the country level, thereby studying the relationship while weighting all countries the same.

to a strong role of the negative diffusion channel for increases in lower-tail productivity gaps, while the zero response of the MP-Within component to upper dispersion shocks may signal that two mechanisms ultimately offset each other when productivity dispersion in the upper part of the productivity distribution increases.

These results provide evidence of the various channels identified in the previous literature. The positive allocative efficiency channel is mostly associated to changes in upper dispersion in the short-term, and associated to changes in both upper and lower dispersion over time. The sources of this positive association are analysed more in depth in Section 1.7.1.

The persistently negative relationship between lower dispersion shocks and the MP-Within component of APG shows that the diffusion channel represents an important drag to long-term growth. The relationship between changes in upper dispersion and the MP-Within component is affected both by the negative diffusion channel and by the positive association between productivity growth of the top decile and the expansion of the technological frontier (the positive innovation channel). Results show that these two countervailing forces are likely to offset each other, as no significant response to upper dispersion shocks can be identified in any time horizon for the MP-Within component.

Finally, for the creative destruction channel, our estimates show that increasing productivity gaps in the upper tail generate competitive pressure that – after having increased the MP-Covariance component in the short-term – is followed by stronger selection through exit over the longer term. On the MP-Entry component, we find generally smaller responses and that are also less statistically significant, if anything linked with changes in lower dispersion over the medium term.

Weighing the various channels together, increasing upper dispersion is found to be positively and significantly linked to long-term APG. Increasing lower dispersion is instead linked to reduced APG, particularly over the first two years after the initial change in dispersion.

To understand the relative roles of increasing gaps in the upper and lower tail of the productivity distribution, respectively, in explaining the slowdown of APG, it is important to emphasize that, over the last 2 decades, the expansion of gaps in the lower tail has been far more pronounced than the one of gaps in the upper tail, as illustrated also in Figure 1.2.

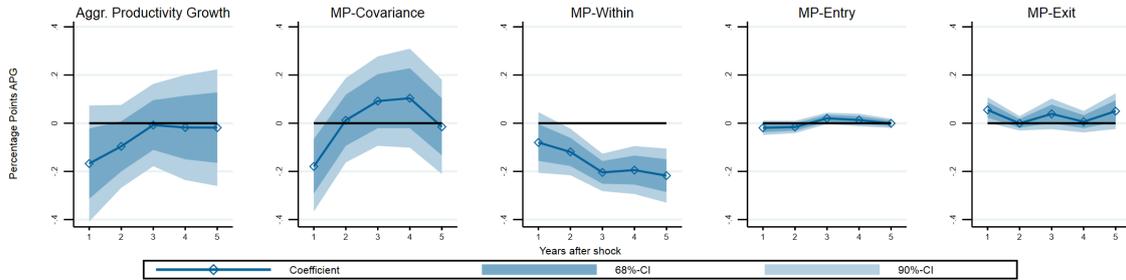
1.6.2 THE ROLE OF INTANGIBLES

A growing empirical evidence suggests that the digital transition and the increased importance of intangible capital as factor of production is an important reason behind the increasing dispersion in firm performance (Akcigit and Ates, 2021; De Ridder, 2019; Corrado et al., 2021).

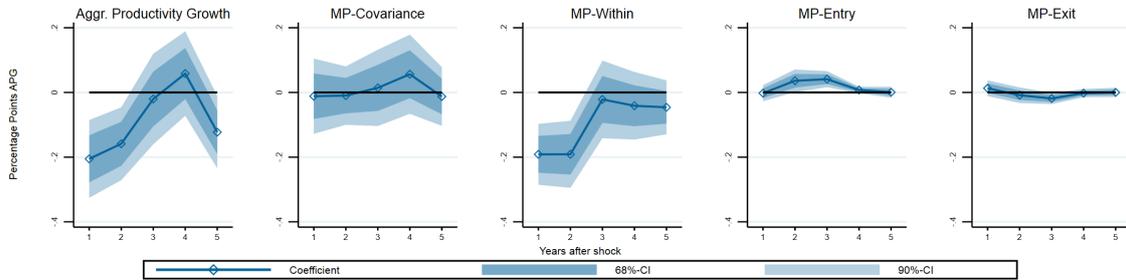
To understand the role that the intangible-intensity of industries plays in our context, we

construct a binary indicator of intangible-intensity at the country-industry level from the MultiProd database, using complementary information from IntanInvest when necessary.²¹ We use this indicator to split our sample into non-intangible-intensive and intangible-intensive country-industries, and estimate our main specification separately for these two samples.

Figure 1.6: Results by intangible intensity, 50-10 MFP dispersion shock



(a) Intangible-intensive.



(b) Non-intangible-intensive.

The figures show the regression estimates for the response of productivity growth and its components to a lower productivity dispersion shock in intangible-intensive and non-intangible intensive industries, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure 1.6 shows the estimates of β_L^h , the coefficient for lower dispersion shocks. Panel (a) shows the results obtained from the subsample of intangible-intensive industries, and (b) those obtained from the subsample of non-intangible-intensive industries. The results suggest that the drag of lower dispersion on MP-Within component (column 3) is driven more strongly by intangible-intensive industries, especially at longer horizons. This is consistent with the possibility that intangibles are key to explain the negative diffusion channel.

However, in opposition to the stronger drag on the MP-Within component, results show that intangible-intensive industries are also driving the positive association between lower dispersion shocks and MP-Covariance (column 2).

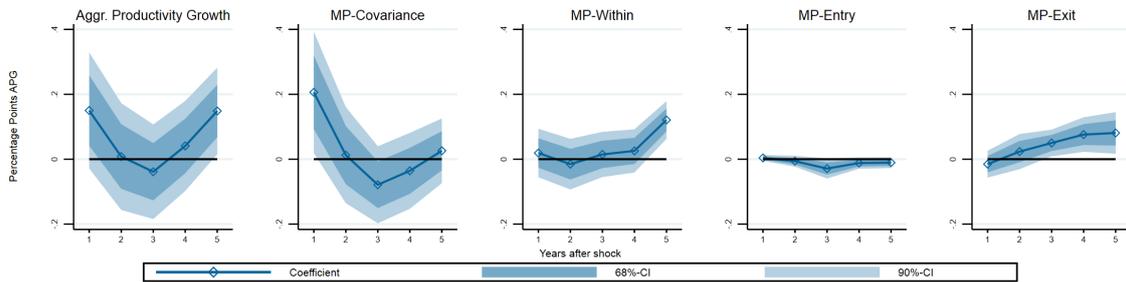
Finally, the result on the entry contribution to APG (column 4) shows a weaker positive response in intangible-intensive industries. If increasing lower dispersion is linked more

²¹A detailed description of this indicator is given in Appendix A.1.2.

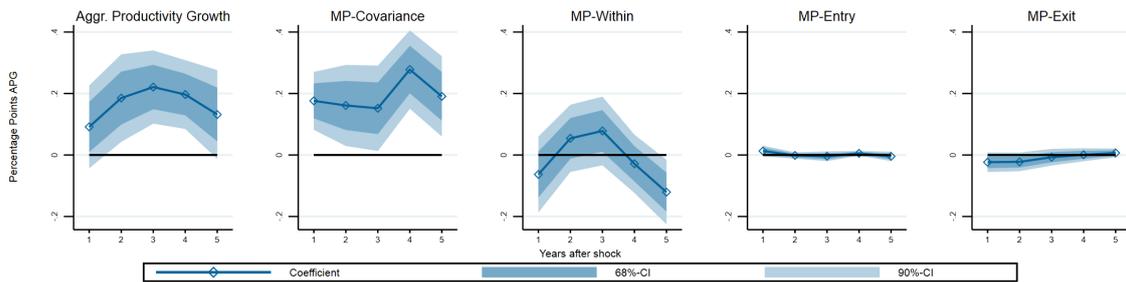
strongly to slower technology diffusion in intangible-intensive industries (as indicated already by the results for the MP-Within component discussed above), then also potential entrants may see stronger limitations to their access to state-of-the-art technology. Therefore, also the result on entry is consistent with a stronger diffusion channel in intangible-intensive industries.

The various differences between intangible-intensive and non-intangible intensive industries counter-balance each other, and we find that changes in lower dispersion predicts a similar response of APG in both groups of industries (column 1).

Figure 1.7: Results by intangible intensity, 95-50 MFP dispersion shock



(a) Intangible-intensive.



(b) Non-intangible-intensive.

The figures show the regression estimates for the response of productivity growth and its components to an upper productivity dispersion shock in intangible-intensive and non-intangible intensive industries, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure 1.7 shows the estimates of β_H^h , the coefficient for upper dispersion shocks. Again, panel (a) shows the results obtained from the subsample of intangible-intensive industries, and (b) those obtained from the subsample of non-intangible-intensive industries. Our results show that the positive response of MP-Covariance to increasing upper dispersion is driven predominantly by non-intangible-intensive industries (column 2), which may seem surprising given the stronger “winner-takes-most” dynamics in intangible-intensive industries that could ex ante generate stronger reallocation to top productivity performers. We return to this issue in the next section when we discuss in more detail the response of the MP-Covariance component, where we argue that this response is driven by shifts in the distribution of productivity

growth rather than the reallocation of physical factors, and this mechanism appears indeed to be stronger in non-intangible-intensive industries.

The positive link between upper dispersion shocks and MP-Exit, particularly over the longer horizon, is entirely driven by intangible-intensive industries, consistent with a “winner-takes-most” dynamics that overtime increases the selection of firms through exit.

Overall, the positive response of APG to changes in upper dispersion seems to be predominantly driven by non-intangible-intensive industries (column 1). This may be concerning, as upper dispersion has increased more in intangible intensive industries (Corrado et al., 2021).

1.7 ADDITIONAL RESULTS ON KEY CHANNELS

The previous section has described our baseline findings on the link of productivity dispersion to APG. In this section, we present further results that shed additional light on the relationships we identified, in particular the responses of the MP-Covariance component to upper dispersion shocks and the allocative efficiency channel. Moreover, we present results related to the labour market impact of changes in dispersion, and we find evidence that changes in upper dispersion are significantly skill-biased, fuelling capital-labour substitution and leading to shifts away from low-skilled workers among top firms.

To investigate the response to changes in dispersion for variables beyond APG and its MP-components, we slightly adapt our empirical strategy when possible. LHS (response) variables that do not measure (changes in) the productivity distribution are not subject to the mechanical relationship discussed before. Accordingly, we are able to recover economically relevant relationships from a model with the LHS in cumulative changes, i.e. with dependent variable $DV_{jct+h} - DV_{jct-1}$ where DV_{jct} is the base dependent variable in (log-) levels. Due to limited concerns about spurious reverse-to-the-mean trends in variables that do not measure aspects of the productivity distribution, we also omit forward-looking controls for the LHS variable.

1.7.1 UPPER DISPERSION AND THE ALLOCATIVE CHANNEL

The previous section has identified a positive and persistent response of the MP-Covariance component to upper dispersion shock, particularly in non-intangible-intensive industries. There are different sources of variation in the covariance between productivity and firms’ share in value added, and understanding which of those are responsible for the relationship we observe is a crucial step towards understanding how policymakers may act to influence this channel.

Any change in the MP-Covariance component, $\Delta_h Cov_{i \in C_{jt}}(\theta_{it}, P_{it})$, can have two sources. On one hand, shares θ_{it} may be reallocated to more productive firms, in particular through the reallocation of physical factors of production (capital, labour). On the other, productivity

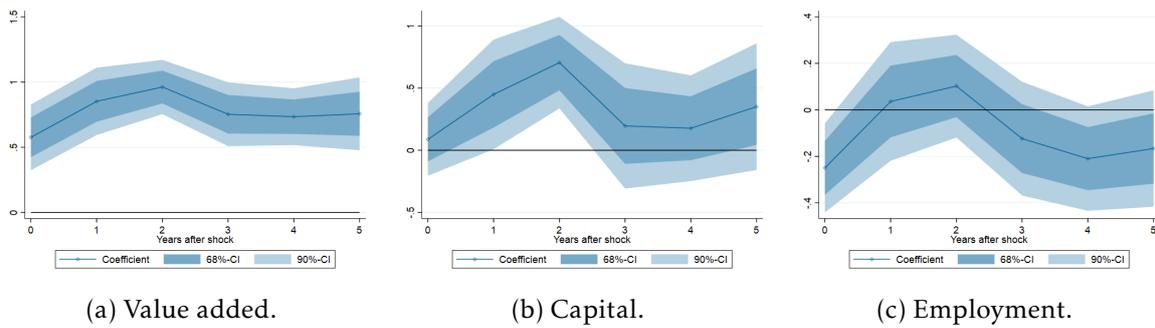
(and productivity growth) may become more concentrated at firms with larger shares, and especially at firms with more employees and a larger capital stock.

We test the first explanation, i.e. the reallocation of physical factors, using as response variable DV_{jct} a log-ratio of averages over different parts of the productivity distribution:

$$DV_{jct} = \log \left(\frac{\bar{x}_{jct}^{p90-p100}}{\bar{x}_{jct}^{p40-p60}} \right)$$

where $\bar{x}_{jct}^{p90-p100}$ is the average of x in the top decile (firms above the 90th percentile) of the MFP distribution in the country-industry-year cjt , and $\bar{x}_{jct}^{p40-p60}$ is the average of x in the two deciles around the median of this distribution (firms between the 40th and the 60th percentile). We study as x the levels of value added, capital, and employment.

Figure 1.8: Reallocation variable responses to upper dispersion shock



The figures show the regression estimates for the responses of key reallocation variables to upper dispersion shock, based on MFP. Every panel shows the response of the log-ratio of the average of the given variable within the top decile and around the median (p40-p60) of MFP, respectively. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure 1.8 shows the cumulative changes of different variables in response to an upper dispersion shock. Panel (a) shows that in response to increased upper dispersion, value added at top firms increases significantly and persistently relative to firms in the middle of the distribution. This appears natural given the increased gap in value-added productivity between the 95th and 50th percentile, respectively. As the responses of capital in Panel (b) and employment in Panel (c) show, this widening of the productivity distribution is associated with shifts in the distribution of physical capital in favour of more productive firms, but the coefficients are modest and not strongly significant at longer horizons. Moreover, average employment at top firms indeed reduces in response to increases in upper dispersion at longer horizons, relative to firms in the middle of the distribution. We return to this aspect in our discussions of the labour market impact of changes in dispersion. Therefore, the reallocation of physi-

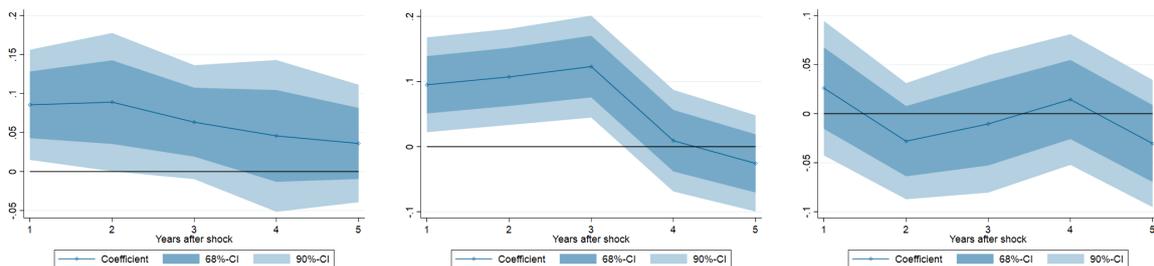
cal resources contributes at best weakly to the observed positive response of the covariance component.²² To ensure that our estimates are not driven by outliers that may drive the average among top productivity performers, we repeat the estimation exercise using the median wage among these firms, and also around the median of the distribution (between the 40th and the 60th percentile). The results are shown in Figure A.5 in the Appendix, and qualitatively confirm the patterns documented here, although the response of capital is somewhat weaker, which further weakens the case for an explanation of the reallocation component based on the reallocation of physical factors. Still, this result may indicate that capital reallocation occurs especially towards the very top of the productivity distribution, consistent with a superstar firm explanation.

If the response of MP-Covariance is not (mainly) due to shifts in the distributions of physical factors in favour of more productive firms, then as initially discussed, it should be associated with a shift of productivity growth in favour of larger firms. To test this mechanism, we consider the dependent variable

$$DV_{jct} = \log \overline{A_{ijct}}^{250+} - \log \overline{A_{ijct}}^{20-49}$$

that measures the difference in relative log productivity growth at large firms with more than 250 employees, $\log \overline{A_{ijct}}^{250+}$, and medium-small firms with 20-49 employees, $\log \overline{A_{ijct}}^{20-49}$. As this variable is again mechanically related to the productivity distribution, we estimate the responses using the baseline model presented in Section 1.5.

Figure 1.9: Responses of the productivity growth ratio of size classes



(a) Upper dispersion shock. (b) Lower dispersion shock. (c) Homog. productivity growth.

The figures show the regression estimates for the responses of the log-ratio of one-year productivity growth within the size classes of large (250+ employees) and medium-small (20-49) firms to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

²²If the reallocation of physical factors explained the *persistent* response of the MP-covariance component, then resources should be reallocated to more productive firms in every period, and the relative stocks of capital and employment at top firms should be steadily upward-sloping. This explanation is clearly inconsistent with the patterns observed especially from horizon 2 onwards.

The response of this variable to changes in upper dispersion is shown in Panel (a) of Figure 1.9. In line with our expectations, the estimated response is positive at all horizons, and significant at least at the 68%-level until $h = 3$. Limited precision prevents us from drawing more definitive conclusions, but together with the absent evidence on the alternative explanation of physical factor reallocation, we are confident that these results signal shifts in productivity growth in favour of large firms that occur after increasing upper dispersion.²³

One concern is that an upper dispersion shock mechanically feeds back into APG over the same period, especially when holding constant the change in median productivity. However, we are confident that our result on productivity growth reallocation between size classes does not reflect a general feature of productivity growth. First, a similar pattern is not observed for homogeneous productivity growth (Panel c), and second, for a lower dispersion shock, a phenomenon with a mechanical negative relationship to APG, we also observe a positive response of the (log) productivity growth-size ratio in the short and medium term.

The response of the productivity growth-size ratio to increasing lower dispersion is also interesting. Recalling the negative response of MP-Within to increasing lower dispersion, the result in Panel (b) of Figure 1.9 suggests that smaller firms are driving more strongly the decline in the growth rate of average productivity. This strengthens our interpretation of the within-result of increasing lower dispersion as a signal of the diffusion channel that hinders the adoption of innovative technologies especially at smaller and lagging firms.

Combining the evidence of Figures 1.8 and 1.9, we conclude that the response of the MP-Covariance component to increasing upper dispersion is predominantly driven by shifts of productivity (growth) in favour of firms that are larger in terms of employment and capital, rather than the reallocation of physical factors. This may have important implications for optimal policy response to ongoing productivity divergence, i.e. the longer-term trend of increasing productivity gaps. Indeed, the absence of systematic evidence for physical factor reallocation may signal sizable frictions in factor markets. In a frictionless environment, factors *should* be reallocated with changes in the productivity distribution, in particular in favour of the firms that grow more in productivity. Our results therefore suggest that the potential of divergence for allocative efficiency may not be fully exploited due to frictional factor reallocation.²⁴

Finally, also the weaker allocative efficiency channel that we measure in the response to

²³The dependent variable we measure using averages within the two size classes 20-49 and 250+ can only give a crude description of the covariance in employment and productivity growth, and we would expect clearer estimates if we could measure this quantity directly.

²⁴As a shift of the productivity growth distribution in favour of larger firms seems to mainly explain the observed response covariance component, it should also explain the differential response across intangible and non-intangible intensive industries shown in Figure 1.7. Figure A.6 in the Appendix shows the estimated responses of the productivity growth-size ratio to increasing upper dispersion which tend to confirm this view, although estimates are somewhat less precise.

the lower dispersion shock does not seem to be driven by the reallocation of physical input factors to more productive firms. This results from the responses of the ratios of average inputs of, respectively, firms in the middle (40th to 60th percentile) and at the bottom (10th to 40th percentile) of the distribution, and also from the ratios of average inputs of, respectively, firms in the top (above the 90th percentile) and at the bottom (10th to 40th percentile) of the distribution. If increasing lower dispersion leads to reallocation of inputs away from less productive firms at the bottom, these ratios should respond positively to the lower dispersion shock. However, as Figure A.7 in the Appendix shows, this does not seem to be the case.

1.7.2 BEYOND AGGREGATE PRODUCTIVITY GROWTH: SKILL-BIASED TECHNOLOGICAL CHANGE AND PRODUCTIVITY DISPERSION

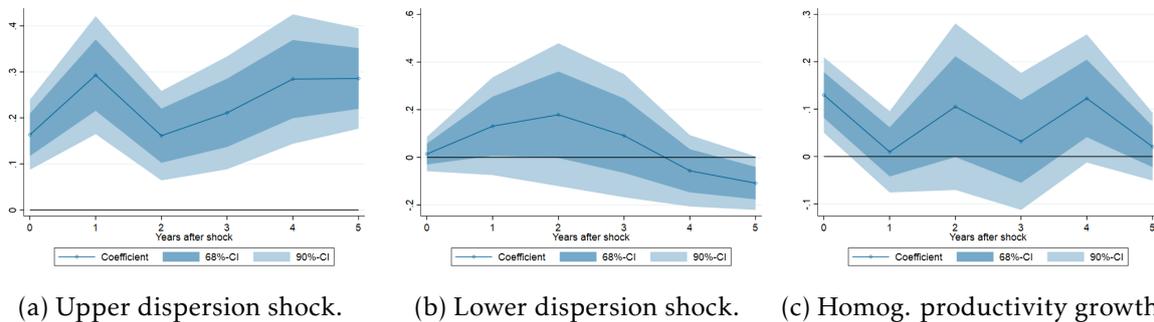
As productivity is tightly linked to wages, there are concerns that persistently increasing productivity gaps may be an impediment towards strong and broad wage growth (Berlingieri, Blanchenay and Criscuolo, 2017; Berlingieri, Calligaris and Criscuolo, 2018). If this productivity divergence has adverse impacts on labour market performance, then its welfare impact cannot be judged only from the productivity implications addressed thus far, and accounting for such labour market impacts is a key step towards a more comprehensive picture. In studying this labour market impact, we focus on patterns within the top decile of productivity to understand how top firms change their labour market behaviour in response to increasing productivity gaps. Moreover, we address the responses of key quantities of the labour market equilibrium at the industry level.

Our first investigation concerns average firm-level wages at top productivity performers. Changes in upper dispersion can be linked to wages of top-firms by two channels. First, a positive shock to the productivity of top firms would likely pass-through on the wage of their workers (Adamopoulou et al., 2021). Second, average wages at top firms could also increase if increasing upper dispersion is linked to the shift to a more skilled workforce. This compositional effect would be consistent with skill-biased technological change and with the potential displacement of low-skilled workers.

Also changes in lower dispersion may be potentially linked to wages at top firms. Indeed, the slowdown of technology diffusion may generate rents among most productive firms, and this may be partly shared with workers. Finally, the growth in median productivity, holding fixed upper and lower dispersion, shows whether a homogeneous increase in productivity across firms passes through on wages at top firms, and in particular whether such pass-through is similar to the one of increasing upper dispersion, where only firms at the top improve their productivity.

From these considerations, the unambiguous expectation is that average wages at top firms increase with increasing upper dispersion, and may also do so for increasing lower dispersion if rent-sharing is relevant. Figure 1.10 shows the estimated responses.²⁵ Panel (a) confirms that average wages of top firms persistently increase in response to a positive upper dispersion shock, with an estimated elasticity of around 0.2 in the short, and 0.25-0.3 in the long term. Therefore, if the 95th percentile increases by 1% relative to the median, we expect a persistent increase of 0.2-0.3% in average wages at top productivity performers. Panel (b) of Figure 1.10 shows the response of average wages at top firms to a lower dispersion shock. The estimates are insignificant at most horizons and even negative at longer horizons. Finally, panel (c) displays the response to homogeneous productivity growth. The impact is somehow positive, though lower than the one of upper dispersion and imprecisely estimated. Therefore, the response to the upper dispersion shock seems to not be fully explained by a productivity pass-through on wages, but must be driven by an additional force. As we argue in the following, this force may be a skill shift in favour of high-skilled workers who earn higher wages, induced by skill-biased technological change.

Figure 1.10: Responses of average firm-level wages within the top decile



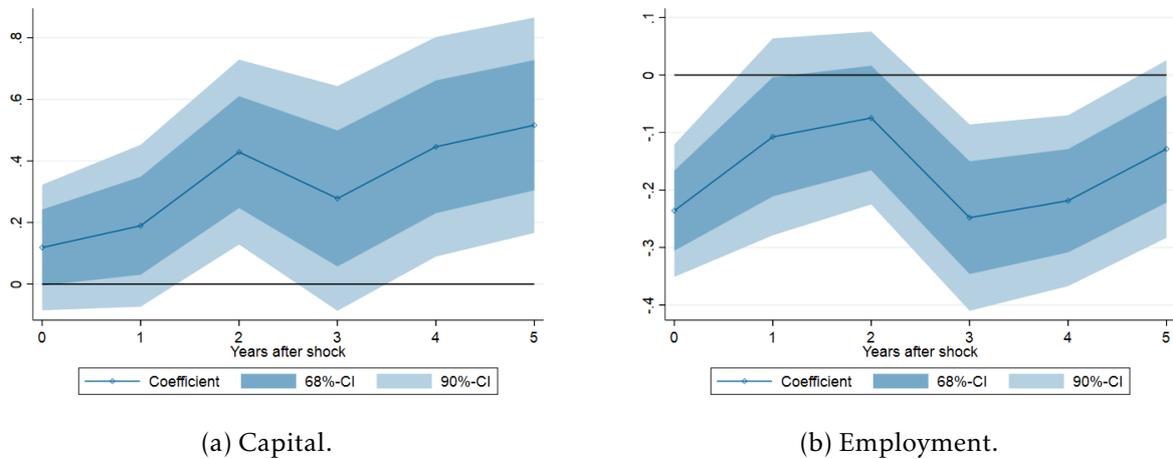
The figures show the regression estimates for the responses of average firm level wages within the top decile of MFP to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

To test whether our estimates are driven by outlier firms that may affect the average wage of most productive firms, we repeat the estimation exercise using the median wage among them. The results are shown in Figure A.9 in the Appendix, and qualitatively confirm the patterns documented here. For the response to increasing upper dispersion, the median is somewhat less responsive, which may indicate that the patterns of the average are driven by firms at the

²⁵We acknowledge that wages may be closely related to the productivity distribution, and the contemporaneous relationship may be simultaneous. To address this concern, we re-estimate the model considering as dependent variable the cumulative change $DV_{jct+h} - DV_{jct}$ (rather than $DV_{jct+h} - DV_{jct-1}$) in response to increasing dispersion from $t-1$ to t , and we additionally control for the change in from $t-1$ to t . The results obtained from this model are shown in Figure A.8 in the Appendix.

very top of the distribution.

Figure 1.11: Responses of key averages within the top decile to upper dispersion shock



The figure plots average within-country-industry trends of different percentiles of the productivity distribution (Panel (a)) and productivity gaps between different percentiles (Panel (b); lower dispersion is defined as the gap between the 50th and 10th percentile, and upper dispersion as the gap between the 95th and 50th percentile), based on the year coefficients of regressions controlling for country-industry (SNA A38) fixed effects, for the period 2002-15. The regressions weight country-industries (SNA A38) by the value added of firms in the country-industry in the year 2002 (or first year of observation). Each point represents the average cumulative change since 2002. Countries included: BEL, CAN, FRA, HUN, HRV, ITA, LVA, NLD, PRT, SVN and SWE. Source: Calculations based on the OECD MultiProd 2.0 database.

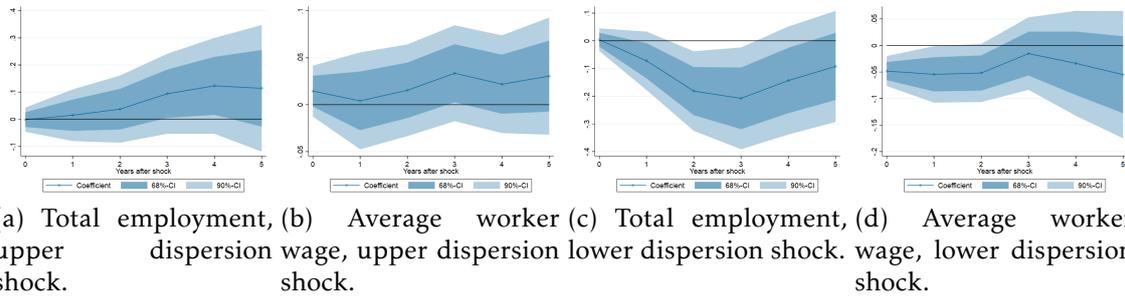
We, then, study whether skill-biased technological change may be behind the response of wages at top firms to increasing upper dispersion. The response of employment at top firms *relative* to firms at the middle, as shown in Panel (c) of Figure 1.8, already indicates that the productivity gains of top performers may come at the expense of employment. To further investigate this aspect, we estimate the responses of input levels at top firms to an upper dispersion shock. The results are shown in Figure 1.11. The estimates suggest that a 1% increase in the top/median productivity ratio leads to an increase of 0.4% in average capital, and a 0.1% decrease in average employment of top firms after 5 years. The initial response of the employment average is even more negative (-0.2%). Therefore, on average, top firms intensify their capital use and decrease employment after increases in upper dispersion, consistent with capital-labour substituting technological change.²⁶

In conclusion, upper dispersion shocks seem to be associated with skill-biased technological change at top firms where the factor mix shifts in favour of capital, and the reduction in the productive importance of labour may be borne especially by less skilled workers who may be displaced from these firms. Besides these firm-level patterns, however, it is crucial to analyse the impact of changes in dispersion on industry-level input accumulation, as micro and macro

²⁶Figure A.10 in the Appendix shows the analogous responses of input levels to lower dispersion shock and homogeneous productivity growth. The responses to lower dispersion shock are never statistically nor economically different from zero, and also those to homogeneous productivity growth are close-to zero.

elasticities may differ markedly (Chetty et al., 2011).

Figure 1.12: Responses of key industry-level quantities



The figures show the regression estimates for the responses of firm level averages within the top decile of MFP for key quantities to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

For this purpose, we estimate the responses to upper and lower dispersion shocks for industry-level aggregates. The responses of industry-level employment and wages are shown in Figure 1.12. Contrary to what observed among top firms, total industry-level employment seems to not decrease with increasing upper dispersion and may even increase in the longer term, although total capital stock displays an even higher elasticity (see Figure A.11 in the Appendix). This may signal that there is a factor composition effect of increasing upper dispersion that shifts the relative mix of factors in favour of capital due to a change in the nature of technology. At the same time, increasing upper dispersion may also entail a factor level effect that augments demand for both factors, as the shock to top-firms' productivity trickles-down overtime to the rest of the economy. The relative strengths of the factor composition and factor level forces ultimately determine the response of total employment.

For the lower dispersion shock, Panels (c) and (d) of Figure 1.12 show the results for employment and wages, respectively. The estimates show a negative response for both variables, consistent with previous findings which show that laggard firms are highly labour intensive (Berlingieri et al., 2020). Indeed, a negative shock to these firms may impact employment and wages and not be fully absorbed by the (less labour-intensive) competitor firms. The negative implications for relative wages at less productive firms can be directly seen from the response of the ratio of average wages at, respectively, the middle of the distribution (40th to 60th percentile) and the bottom (10th to 40th percentile), which is shown in Figure A.12 in the Appendix. The statistically and economically significant and positive response of this ratio implies that average wages at firms at the bottom decline relative to the middle of the distribution, and ongoing dispersion in the lower tail may therefore, next to the adverse consequences for APG, increase labour market inequality across workers at different firms and

threaten inclusive wage growth.

1.8 CONCLUSIONS

This paper provides novel evidence on the link between productivity divergence and productivity growth across OECD countries over the period 2001-18. By exploiting cross-country data that are representative of firms throughout the productivity distribution, and decompositions of aggregate productivity growth, the paper provides evidence of the various channels that link divergence to growth.

Increasing productivity dispersion is associated with higher rate of improvements in allocative efficiency of resources, as productivity growth has become persistently more correlated with firm size overtime. Divergence – particularly the one at the bottom of the distribution – is correlated with a decline in the rate of average within-firm productivity growth, consistent with a slowdown in technology diffusion. Heterogeneity analysis based on country-industry differences in intangible-intensity provide evidence that slow technology diffusion can be linked to the digital transformation and the related rise of the intangible economy. Creative destruction forces are also behind the impact of changes in dispersion on APG, particularly over the longer term. In particular, there is robust evidence of a positive impact of upper dispersion on the contribution of exit to APG, consistent with more productive firms imposing competitive pressure on less productive firms, which then exit the market.

Finally, the richness of the cross-country data allows the analysis to study in-depth the allocative efficiency channel as well as analysing the labour market impacts of upper and lower dispersion shocks. Results point to a labour-substituting effect of upper dispersion among more productive firms, consistent with these firms adopting a more capital-intensive technology. At the same time, at the industry level, changes in upper dispersion has a weakly positive impact on total employment, pointing to a market size effect that overcomes the firm-level labour-substituting effect. Increases in lower dispersion are, instead, linked to drops in total employment and wages.

These results have important policy implications which, crucially, differ markedly between upper and lower dispersion shocks. In the former case, the analysis shows that allocative efficiency does not benefit from input reallocation, but rather from stronger correlation between productivity growth and ex-ante firm size. Considerable productivity gains may thus be obtained by reducing frictions to resource reallocation. Moreover, the analysis also points to the relevance of contribution to exit of APG: to support this channel, policies should focus particularly on easing exit and capital reallocation from exitor firms.

The impact of lower dispersion shock on APG provides compelling evidence of the impor-

tance of the diffusion channel: lack of technology diffusion represents a significant drag to aggregate growth, particularly among intangible-intensive industries. To counter this negative effect, policies should boost technology diffusion by supporting investments in complementary intangible and knowledge-based assets by less productive firms.

The evidence provided in this analysis may be further strengthened and expanded by studying how the various channels have different weights depending on several country-industry features, such as its position in GVCs, or whether they change over the business cycle and in the face of major shocks. These, as well as other avenues of empirical analysis, related to the implications for concentration, mark-ups, or the dynamics of aggregate labour share, are left for future research.

2. THE PRODUCTIVITY-EMPLOYMENT NEXUS: INSIGHTS FROM A MICRO-TO-MACRO STUDY

based on joint work with:

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Abstract. This work uses harmonised and comparable data for 13 countries over the last two decades to comprehensively analyse the productivity-employment nexus at different levels of aggregation. Results highlight that on average productivity growth is positively associated with the growth of employment and wages, both at the firm level and at the more aggregate one. This is the outcome of counteracting mechanisms, related to efficiency and labour-replacement on the one hand and an increase in competitiveness and market shares on the other, and of heterogeneous dynamics across different groups of firms. While an initial positive firm-level impact of productivity growth on employment appears to be partially offset by jobs lost at competitor firms at the industry-level, the aggregate relationship is positive due to value chain spillovers on downstream industries. The contestability of markets is a key factor that determines to which extent productivity growth translates employment growth. The analysis therefore suggests that productivity is not only a standalone economic objective, but that well-designed and complementary policies also have the potential to help translate technological and organisational change into higher employment and wages.

2.1 INTRODUCTION

A vast literature has shown that productivity growth is ultimately the main driver of economic wellbeing. However, the last two decades have been characterised by a slowdown in productivity growth combined with increasing divergences between the most productive firms and the rest, signalling a weakening of productivity growth in some parts of the micro-level distribution and also in the aggregate. These phenomena have ignited a spirited debate among economists and policy makers not only on how to increase productivity growth across countries and over time, but also on how to make it more inclusive.

Even though the importance of a sustained and shared productivity growth is widely acknowledged, the extent to which productivity changes translate into employment growth is still unclear. A long-lasting debate focuses on the role of technological progress – generally viewed as the main driver of productivity in the medium and long run – for employment and labour markets, and the extent to which it may create winners and losers in the economy. However, this debate is still ongoing, and results are mixed.

This debate dates back to discussions by economists such as Keynes and Ricardo and to policy debates at the time of Luddites. Concerns about the possible negative impacts of technological change on employment are further illustrated by recent discussions on the effects of automation. Some studies show, for instance, adverse effects of robotisation on employment and wages, and suggest that the disappearance of routine tasks from the domain of labour may increase inequality. Rapid advances in artificial intelligence and the possibility to automate an increasing set of tasks, including non-routine cognitive ones, have further fuelled public anxiety of a looming technological unemployment. Such views support the idea of a trade-off between employment and productivity growth driven by technological progress, that would compel policy makers to find a complex policy mix to achieve both objectives at the same time.

However, technological progress does not necessarily only destroy jobs, but may also be a powerful driver of employment growth. First, new technologies may favour the emergence of new tasks in which labour can be productively employed, such as tasks related to the design, supervision, maintenance and repair of machines, tasks related to data administrators and analysts (see Acemoglu and Restrepo, 2018b).¹ Second, firm-level adoption of technologies, even those related to automation, can generate employment gains due to increased productivity. Such productivity effects can create jobs at adopting firms that become more competitive and are able to increase sales (e.g., by charging lower prices), but may also induce employment growth at more aggregate levels through changes in aggregate demand and cross-sectoral

¹Acemoglu and Restrepo (2018b) indeed show that over the period 1980-2015, new tasks and new job titles have accounted for a large fraction of US employment growth.

linkages, with gains in downstream consumer industries that may compensate direct negative own-industry effects (Autor and Salomons, 2018). This may support the idea that policy makers could “hit two birds with one stone” and achieve employment growth by addressing the recent productivity growth slowdown.

Understanding the labour market impact of productivity changes is even more relevant today in light of the recent COVID-19 pandemic during which digital technology adoption has been key to support firms’ activities and employment. Firms could focus on investing in product and process innovations and reorganisation of production, as opportunity costs were low during the crisis when demand was low and production halted (Aghion and Saint-Paul, 1998; Barlevy, 2004; Bloom et al., 2021). However, the consequences of the current wave of adoption of ICT-related technologies, especially those that may replace labour from production tasks, are still uncertain, with concerns about their long-term employment impact. Therefore, a deeper understanding of the productivity-employment nexus is of utmost importance.

Taken together, the discussions in the literature are concerned with two key issues: (i) the labour market implications of weakening productivity performance, derived from a fixed, non-zero (and ex-ante ambiguous) impact of a given rate of productivity growth on employment, and/or (ii) changes in this impact due to trends in technology, i.e., how the direction of technological change affects the changes in employment resulting from a given productivity change.

From the lack of a comprehensive analysis of these issues across countries and different levels of aggregation, a key question arises: *Is there a trade-off between employment and productivity growth, or can productivity growth instead contribute to boosting employment?* This work aims to answer this question by investigating how productivity growth relates to the dynamics of employment (and wages) at different levels of aggregation and over different countries, industries, and time horizons. Exploring the productivity-employment nexus can provide useful insights on a relationship that encompasses a broad range of economic mechanisms. This can inform policy makers about the extent to which boosting productivity growth may at the same time help foster employment growth, about which policies may be more likely to help achieve possible double dividends, as well as warn about trade-offs that may arise. Due to the literature’s predominant focus on employment, this work mainly emphasises the productivity-employment link, but further provides some analysis also on the role of productivity growth for wages to address the labour market impact of productivity growth more comprehensively.

Studying the productivity-employment nexus is challenging. First, productivity encompasses many facets, some of which affect employment in different ways and through countervailing mechanisms. Second, the strength and direction of productivity effects may markedly differ according to the level of aggregation considered. For instance, analyses at the firm level

are usually not able to account for competitive externalities of firm-level productivity growth, associated with the exit of less productive firms and reallocation of labour towards more productive ones. Observing such effects requires the study of either more aggregate levels or data representative of the full population of firms within an industry/country. Third, different compensation mechanisms may be at work at different levels of aggregation. For instance, reductions in production costs following a productivity increase may allow lower prices, and in turn increase sales and labour at the firm level; further, direct own-industry negative effects associated with productivity-improving technologies at the sectoral level may be compensated by positive spillovers across industries. Fourth, productivity growth may affect labour demand in a dynamic way. Short-term impacts on employment may be different from longer-term ones, as changes in demand, adjustments to new technologies or market conditions take time and may require complementary investments by firms. Overall, this implies that the effects of productivity on employment and wages may be heterogeneous along different dimensions as well as over time. A lack of comprehensive data at different levels of aggregation, over long-time horizons across countries and industries has so far limited the scope of related analyses.

This paper aims at addressing these challenges taking advantage of the OECD MultiProd data, a unique cross-country infrastructure collecting micro-aggregated representative data based on firm-level information. Previous analyses of the productivity-employment nexus have generally focused on specific facets of productivity, or on single countries/aggregation levels (see for instance Decker et al. (2020) focusing on the United States or Autor and Salomons (2018) using industry-level data, and the related literature section below).

Thanks to the uniqueness of the data used, this paper contributes to the existing literature by investigating the heterogeneity of the link between productivity growth and employment dynamics at different levels of the economy (within-industry vs. cross-industry) over a long-time horizon, and from a cross-country cross-sectoral perspective. This allows to dissect the complexity of the productivity-employment nexus and the potential mechanisms at play. In particular, this analysis exploits statistics collected at different levels of aggregation to investigate the link between productivity and employment growth from two complementary perspectives: at the micro-economic level focusing on within-firm growth and firm survival, and at a more aggregate level focusing on industry-level growth and its economy-wide implications. The dataset used covers 22 SNA A38 industries in manufacturing and non-financial market services for 13 countries, over the period 2000–2018.

More specifically, within individual firms, both the current level of productivity and the rate of productivity growth are positively linked to changes in firm size (i.e., the firm's number of employees) and firm survival. Therefore, firm-level productivity and productivity growth

seem to secure the jobs of the firm's current workforce and create further job opportunities. Here a key mechanism is related to the firm's productivity performance relative to some of its potential competitors, i.e., firms in the same country-industry. The positive relationship between firm-level productivity growth and employment growth is indeed strongly related to the degree to which firms improve their position in the productivity ranking within their industry. In addition, leading firms at the frontier of the productivity distribution experience on average higher employment growth than other firms in the same country and industry.

The correlation between productivity growth and employment growth in principle can be shaped by two opposite forces: a direct negative effect, i.e., a negative labour-saving effect due to higher efficiency and, hence, less inputs being required to produce the same amount of output; a positive indirect effect, i.e., a positive effect on employment growth due to an increase in demand experienced by firms increasing their productivity channeled by a potential decrease in (quality-adjusted) prices and, hence, an increase in sales. In this paper the within-firm positive correlation between productivity and employment growth suggests that the positive indirect employment effect prevails over the direct negative one.

When looking at the link between productivity growth and changes employment and wages at the industry level, the paper finds again a positive correlation, although weaker than at the firm-level. First, this can relate to the fact that at the industry level employment gains of well-performing firms may be in part negatively compensated by losses in firms which are less productive or improve their productivity less. Second, industry-level demand may be less elastic than the firm-level one, implying that industry-level productivity gains translate into employment gains at a slower rate than at the firm-level. To corroborate this finding, additional results show that the industry-level positive link tends to be stronger when productivity growth occurs in relation to increased participation in global value chains, that is when markets expand and industry-level demand may be more elastic.

Employment may also be positively influenced by productivity growth in other domestic and foreign industries. To this end, the results show that especially productivity growth in upstream industries may stimulate labour demand further down the value chain. Stated differently, employment growth in a given industry is positively correlated to productivity growth in supplier industries, further corroborating the idea that productivity gains are on average labour enhancing not only at the firm level, but also at the more aggregate one.

Finally, the contestability of markets appears to significantly shape the strength of the positive link between productivity and employment, and the productivity-employment link is found to be stronger in more contestable environments. A higher dispersion in market power in some industries might prevent firms with initially lower productivity (for which productiv-

ity growth correlates more strongly with employment growth) to increase their employment as much as they would do in more competitive industries. This might be due to the fact that in environments with less contestable markets these firms might be prevented to fully benefit from the indirect benefits associated with productivity growth, notably through sales expansion.

Overall, the evidence shown in the paper suggests that labour demand and productivity represent complementary rather than alternative policy targets, and that well-designed complementary policies enhancing productivity have the potential to help translate the outcomes of technological and organisational change into higher employment and wages.

The rest of the document is organised as follows. Section 2.2 provides an overview of the existing evidence relevant to the productivity-employment nexus. Section 2.3.2 presents a simple framework that conceptualises the key aspects of the empirical analysis. Section 2.3 presents the data used for the analysis. Section 2.4 discusses the methodology adopted to investigate the relationship between productivity growth and employment growth at the level of the firm, and then presents the corresponding results. Section 2.5 presents the methodology and results related to the relationship at the level of detailed industries, as well as related to spillovers between industries along value chains. Section 2.6 discusses key takeaways and the policy implications of the empirical results. Section 2.7 concludes.

2.2 RELATED LITERATURE

The effect of productivity growth – or more specifically of technological progress – on employment growth has been the subject of lively debates in the economic literature. These debates have been long-lasting, dating back to the Ricardian concept of “technological unemployment” moving to the Keynesian predictions of “mankind solving its economic problem” thanks to improved living standards by 2030.

Even recently, the macroeconomic literature has highlighted contradicting effects of (technology-driven) productivity shocks on hours worked at the aggregate level. In this respect, Galí (1999) finds that hours worked fall in response to technology shocks, Christiano, Eichenbaum and Vigfusson (2003) uncover a positive response, while Basu, Fernald and Kimball (2006) highlight the dynamic pattern of the response, showing that hours worked co-move negatively with a contemporaneous productivity shocks, but then rise with a lag. All in all, conclusions from this stream of literature appear sensitive to the modelling approach adopted.²

Lack of consensus at the macroeconomic level motivates the analyses – largely country-specific – focusing on more disaggregated data, which explore more in detail the nature of

²This literature focuses on the relationship between initial productivity growth and subsequent changes in employment over a business cycle horizon (4-8 years). This perspective is similar to the analysis in this work, which focuses on 5-year employment changes for most part of the analysis.

the relationship between productivity growth and employment growth. More granular data provide additional insights on the heterogeneity of the link between productivity and employment, and the extent to which this link can change at different levels of aggregation, as well as over time and across countries. In this respect this paper follows more closely this literature.

Starting from the micro-economic evidence, Baily, Bartelsman and Haltiwanger (1996) focus on US manufacturing establishments over the 1980s and dissect the conventional wisdom that rising productivity is necessarily accompanied by downsizing. They find that productivity and employment can move either in the same direction or in the opposite one, with plants that increased employment and productivity contributing similarly to overall productivity growth as plants that increased productivity and reduced employment. They further discuss how this may be related to several factors, including the elasticity of demand, technological properties of the production function (e.g., returns to scale), or changes in the skill composition, all insights that are relevant for our analysis. More recently, Decker et al. (2020) further focus on US firms using comprehensive longitudinal data between 1981 and 2013. They provide empirical evidence of a significant positive association between productivity and employment growth. Relevantly, they also find that the responsiveness of employment to productivity shocks has declined in recent decades, which is consistent with rising adjustment frictions.

Still at the micro-economic level, a different stream of literature has focused on the labour demand effects of technological change, highlighting significant heterogeneity in the innovation-employment nexus, and emphasising distinct effects of different types of innovation, as well as the role of possible compensation mechanisms (see Calvino et al. (2018) or Vivarelli (2014) for surveys). In particular, this literature documents a positive link between product innovation and employment, while the link with process innovation appears more ambiguous.³

Firm-level effects of technological change on labour demand may significantly differ from more aggregate industry-level and economy-wide effects. In particular, job creation related to sales expansion for innovating firms can occur at the expense of competing firms, through market share reallocation effects (see for instance the discussion by Harrison et al. (2014) about business stealing and market expansion).⁴

In this context, focusing on industry-level data across 19 countries between 1970 and 2007, Autor and Salomons (2018) find that changes in total factor productivity related to cross-country industry trends (that importantly encompass technology) have a direct negative effect

³Productivity-enhancing process innovations, especially those related to automation, decrease employment at constant output. However, higher production efficiency may translate into lower prices when markets are competitive, which may stimulate output and therefore labour demand.

⁴Relatedly, but focusing on labour shares rather than overall employment, Autor et al. (2020) link the decline in labour share to an increase in the market share of productivity leaders, who use capital more intensely and have higher profits (Koch, Manuylov and Smolka, 2021b).

on employment. However, they show that these losses are reverted when accounting for indirect gains in downstream customer industries and increases in aggregate demand induced by industry-level productivity growth, suggesting that aggregate compensation mechanisms may be strong enough to offset potential negative own-industry effects. Such positive compensations across industries are also highlighted by Acemoglu and Restrepo (2018b, 2020) and Dauth et al. (2021), while less so by Dosi et al. (2021).

Recent waves of innovation and robotisation have revived the debate around the effects of technological change on employment. First, the development and diffusion of ICTs may enable firms to “scale without mass”, i.e., to expand sales and market shares without increasing their employment. Moreover, recent availability of detailed data on robot shipments has allowed assessing in more detail the link between automation technologies and employment. To this end, recent contributions tend to highlight a positive employment effect of automation at the firm level due to increased demand (Acemoglu, Lelarge and Restrepo, 2020; Aghion et al., 2020; Koch, Manuylov and Smolka, 2021b; Domini et al., 2021).⁵ Domini et al. (2021) further find that automation spikes are linked to both higher hiring and lower separation, which together explain an increase in contemporaneous net employment growth.

Net employment growth, however, masks composition dynamics (Autor, Levy and Murnane, 2003; Acemoglu and Autor, 2011; Humlum, 2019; Aghion et al., 2020), with overall effects crucially depending on the extent to which direct displacement effects are counterbalanced by positive productivity effects (Acemoglu and Restrepo, 2019).⁶ In this context, Acemoglu and Restrepo (2020) suggest that productivity effects may not always be strong enough. For instance “so-so technologies” and the “wrong kind of AI” tend to focus more on task-level replacement of workers rather than increases in productive efficiency complementing labour.

For 17 OECD countries and industries, Graetz and Michaels (2018) suggest that this concern does not apply to the adoption of industrial robots over the period 1993-2007, and instead link this phenomenon to sizeable gains in productivity and consumer rents (through lower prices) while not providing evidence for induced reductions in aggregate employment. This points, once again, to the relevance of carrying out analyses at different levels of aggregation, and at the existence of countervailing mechanisms acting in different directions.

Indeed, as also emphasised above, technological progress does not only displace labour at the task level, but can also create new tasks in which labour has a comparative advantage, generating a reinstatement effect. This is shown by Acemoglu and Restrepo (2019), who highlight

⁵Acemoglu, Lelarge and Restrepo (2020) however find that expansion of firms adopting automation technologies comes at the expense of their competitors, inducing an overall negative impact of robot adoption on industry employment.

⁶Jaimovitch and Siu (2020) notice that the job losses related to routine-replacing technological change occur during recessions and contribute to job polarization and to jobless recoveries.

that changes in labour demand depend on the balance between forces of task level substitution and reinstatement. Focusing on the US, they argue that this balance has shifted in the 1990s due to the deceleration in the introduction of technologies reinstating labour and an acceleration of displacement technologies. Further, due to capital-labour complementarity, even task-replacing automation need not always have adverse consequences for affected workers if sufficiently many productive tasks remain within the occupation. To this end, in reviewing existing evidence, Lane and Saint-Martin (2021) find that the types of AI adopted over the past decade is, in the aggregate, not strongly linked to adverse impacts on employment or wages.

Beyond technological change, demand plays an important role for productivity growth. A growing literature indeed suggests that firm-level growth (and survival) is also strongly influenced by demand-side factors (Foster, Haltiwanger and Syverson, 2016). Beyond the role of demand for productivity itself, demand may also matter for the response of employment to technology-induced changes in productivity. On this aspect, Bessen (2019) discusses a potential inverse-U shape of the employment effect of technology over time, which may emerge due to the elasticity of demand declining with technological progress over time.⁷

Building upon the analyses discussed above, this work contributes to the academic and policy debate by investigating directly the link between productivity growth and labour demand (focusing mainly on the dynamics of employment and wages) and characterising the heterogeneity of this relationship. It does so exploiting unique micro-aggregated harmonised representative data that allow studying the link between productivity growth and labour demand in detail, focusing on within-firm relationships for a large number of countries, but also inferring aggregate relationship at more aggregate levels with the same data. This allows to investigate with unprecedented detail the role of compensation mechanisms and dynamics occurring at different aggregation levels, also accounting for the dynamic nature of the relationship between productivity growth and changes in labour demand.

2.3 DATA AND MEASUREMENT

2.3.1 MAIN DATA SOURCE: THE OECD MULTIPROD PROJECT

This subsection presents the MultiProd data used for the analysis. It provides information on the data collection and the coverage and further discusses measures of employment and productivity, as well as the granularity of the information available.

The analysis relies on harmonised and highly representative cross-country data on pro-

⁷Specifically, initial technological change may occur in an environment of elastic demand allowing firms to significantly increase output, and therefore labour-demand, in response to increased productivity. Eventually, the opportunity to scale output with productivity may decrease due to saturation of demand, causing further technological progress to have a lower, potentially negative, effect on employment.

ductivity from the OECD MultiProd project, the implementation of which is based on a standardised Stata routine that micro-aggregates confidential firm-level data. The data collection involves running a common code in a decentralised manner by representatives in national statistical agencies or experts in governments or public institutions who have access to the national micro-level data. The centrally designed, but locally executed, statistical routines generate micro-aggregated data which are the basis of this analysis.

The MultiProd program relies on two main data sources for each country. First, it uses administrative data or production surveys (PS), which contain all the variables needed for the analysis of productivity, but which may be limited to a sample of firms. Second, it exploits business registers (BR) that typically cover the entire population of firms, but for a more limited set of variables.⁸ The current version of the project includes 13 countries (Belgium, Canada, Chile, Croatia, Finland, France, Hungary, Italy, Japan, Latvia, the Netherlands, Portugal, Sweden) and focuses on manufacturing and non-financial market services. The (unbalanced) data cover the period 2000-2018, as detailed in Table 2.1.

To proxy for productivity, this analysis relies on a measure of multifactor productivity (MFP) estimated following the Wooldridge (2009) control function approach, with value added as a measure of output and two inputs (capital and labour). This methodology assumes that firms have a Cobb-Douglas production function, flexibly allows for non-constant returns to scale and yields consistent standard errors. MFP measures in MultiProd are based on production functions estimated at the country-industry level, thus taking into account technological differences across countries and industries. By accounting for the levels of both capital and labour used by firms, MFP offers a more precise view of the efficiency with which firms use their inputs of production compared to simpler measures such as labour productivity. This may be key especially for the given context of employment implications of productivity growth if productivity changes are sometimes associated with changes in the capital-labour ratio.⁹

The micro-aggregated moments of MultiProd are available at different levels of granularity. First, this work leverages data on firm transitions between productivity quantile groups (henceforth “transition matrix”) in each country, SNA A38 industry and year.¹⁰ Secondly, the analysis exploits data aggregated at the country, industry and year. Table 2.1 provides addi-

⁸The BR is not needed when administrative data on the full population of firms are available. When data come from a PS, however, the availability of the business register substantially improves the representativeness of results and, thus, their comparability across countries. Additional details on MultiProd can be found in Berlingieri et al. (2017) and on the MultiProd webpage <https://www.oecd.org/sti/ind/multiprod.htm>. See also Desnoyers-James, Calligaris and Calvino (2019) for more information on metadata.

⁹The measure of MFP is based on revenue-productivity (theoretically given by the product of physical productivity and prices). This is a common feature of analyses on productivity, given the lack of availability of firm-level prices in many datasets. This implies that the measure of changes in MFP may also be affected by demand-side factors and shocks affecting prices, on top of technological factors affecting the production function.

¹⁰SNA-A38 is an industry classification based on 2-digits ISIC revision 4 codes, with some 2-digit industries aggregated together. The correspondence between ISIC rev. 4 and SNA A38 is available in Berlingieri et al. (2017).

Table 2.1: Data coverage.

Country	Period covered	Sectors covered	Industry-level file	Transition matrix file
Belgium	2002-2018	Manuf. & services	available	available
Canada	2000-2018	Manuf. & services	available	not available
Chile	2005-2016	Manuf. & services	available	not available
Croatia	2002-2018	Manuf. & services	available	available
Finland	2000-2018	Manuf. & services	available	not available
France	2000-2015	Manuf. & services	available	not available
Hungary	2000-2018	Manuf. & services	available	available
Italy	2001-2015	Manuf. & services	available	available
Japan	2000-2015	Manuf.	available	available
Latvia	2007-2015	Manuf. & services	available	available
Netherlands	2001-2018	Manuf. & services	available	available
Portugal	2004-2017	Manuf. & services	available*	available
Sweden	2007-2018	Manuf. & services	available	available

This table presents the coverage of the MultiProd data used for this analysis. “Manuf.” refers to manufacturing sectors and “services” refer to non-financial market services. Statistics in transition matrix data are available for the following (initial) years: 2001, 2004, 2007, 2008, 2009, 2010, 2012, 2015. *Data on aggregate productivity, used in the main industry-level regressions, are not available for Portugal.

tional details on coverage and availability of these main datasets, which are further presented below.

In the transition matrix data, statistics are computed for cells defined at a highly disaggregated level according to the country, year, SNA A38 industries, firm productivity quantile group in t and $t+h$ ($h = 5, 7, 10$).¹¹ Results based on these data focus on all firms active at time t and $t+h$ with positive value-added. The analysis mainly employs measures of average changes in productivity, employment and wages among firms in the cell.¹² These averages within a transition cell (composed of a country, industry, year and transition group) may be interpreted as reflecting a firm representative of all firms making the given productivity transition in the country-industry-year. Therefore, the population of firms in the country-industry-year can be represented by the collection of firms representing a transition group within it, and the results based on the transition matrix data directly speaks to the within-firm level.

This work also relies on data aggregated at the SNA A38 sector level (henceforth “industry-level data”), in which cells are defined according to the country, year and SNA A38 indus-

¹¹Within each country-industry-year, there are 5 productivity quantile groups, collecting, respectively, firms below the 10th percentile, firms between the percentiles 10 to 40, 40 to 60, and 60 to 90, and firms above the 90th percentile of the productivity distribution of the country-industry-year.

¹²The relevant variables to investigate firm-level outcomes correspond to a weighted average of firm-level log-changes, with weights corresponding to inverse probability weights computed from the MultiProd re-weighting procedure. More formally, these variables x_C are computed as follows:

$$x_C = \frac{1}{W_C} \sum_{i \in C} w_i (\ln X_{i,t} - \ln X_{i,t-h})$$

where w_i are the inverse probability weights of firm i , derived from the re-weighting procedure (see Berlingieri et al., 2017), and $X_{i,t}$ is the respective base quantity at firm i at time t (e.g. MFP or employment). $W_C = \sum_{i \in C} w_i$ is the sum of weights in cell C (i.e., a country, industry, year and transition group).

try. These data collect information on average MFP (either unweighted, or weighted by value-added to reflect aggregate productivity), as well as on average firm size and total employment, on wages and other relevant variables. Total employment is measured as the average number of employees at a firm within the industry, multiplied by the number of firms, and reflects the total level of workers employed in the country-industry in the given year. The industry-level data are based on all active firms with positive value-added, and enable to infer aggregate links between productivity, employment and wages.

In addition, the analysis further relies on other OECD data. It exploits data from the OECD DynEmp project to compute measures of reallocation. It also relies on measures of: productivity growth at the global frontier from the ORBIS database; ICT intensity based on the work by Calvino et al. (2018); AI and ICT patents retrieved from the OECD Patstat database; forward and backward linkages based on the OECD Inter-Country Input-Output database (ICIO).

Table 2.2 and Table 2.3 present summary statistics for the key variables employed in the analyses at, respectively, the within-firm level and the industry level. Employment and wages appear to be on a positive trend on average, both within firms and at the industry level. All variables exhibit significant dispersion relative to a modest mean, as shown by the standard deviation. Notably, from a comparison of standard deviations, it appears that at both the within-firm and the industry level, short-term productivity changes exhibit a similar degree of variation as the longer-term variation in the key variables to be explained.

Table 2.2: Summary statistics of data from transition matrices.

Variable	Horizon h (change t to $t+h$)	N	mean	sd
Change in productivity	1	19 384	0.0275	0.2911
Change in productivity	5	19 900	-0.0235	1.2733
Change in employment	5	19 900	0.0425	0.3261
Change in average wage	5	19 850	0.0299	0.4519

The statistics refer to the distribution of the average (over firms within a given cell) firm-level log-change of multifactor productivity, employment or wages. All statistics are based on firms active at time t and $t+5$. Statistics are computed based on a sample including 9 countries (Belgium, Croatia, Hungary, Italy, Japan, Latvia, the Netherlands, Portugal, Sweden), 22 SNA A38 industries in manufacturing and non-financial market services, and detailed transition between five productivity groups. Source: OECD MultiProd 2.0 database

2.3.2 MODEL ENVIRONMENT: HICKS-NEUTRAL PRODUCTIVITY GROWTH AND LABOUR-REPLACING TECHNOLOGIES

This subsection presents a simple micro-foundation of the empirical investigations in this work. In particular, it addresses how factor-biased phenomena such as labour-replacing technological change may reflect productivity growth in terms of a Hicks-neutral productivity measure such as the one of MFP observed in the data.

Table 2.3: Summary statistics of industry-level data.

Variable	Horizon h (change t to $t + h$)	N	mean	sd
Change in productivity	1	2 715	0.0112	0.1424
Change in productivity	5	2 729	0.0662	0.2509
Change in employment	5	2 965	0.0098	0.2045
Change in average wage	5	2 752	0.0616	0.1600

The statistics refer to the distribution of the industry-level aggregate log-change of multifactor productivity, employment or wages. Statistics are computed based on a sample including 12 countries (Belgium, Canada, Chile, Croatia, Finland, France, Hungary, Italy, Japan, Latvia, the Netherlands, and Sweden), 22 SNA A38 industries in manufacturing and non-financial market services, and detailed transition between five productivity groups. Source: OECD MultiProd 2.0 database

Consider a mass I of homogeneous and perfectly competitive firms. Following the literature on task-replacing technological change (e.g. Autor and Handel, 2013), it is assumed that firms produce output using a Cobb-Douglas technology that uses several labour tasks and physical (i.e., non-task-performing) capital such as land, buildings, vehicles, etc. Specifically, assume that firms $i \in I$ produce output Y_i according to the equation

$$Y_i = \tilde{A}_i T_{iR}^{\alpha_R} T_{iA}^{\alpha_A} K_i^{\alpha_K} \quad (2.1)$$

where \tilde{A}_i is the Hicks-neutral productivity component, T_{iR} and T_{iA} are, respectively, the levels of a routine task R and abstract task A , and K_i is the level of capital used. The abstract task can only be performed by labour: $T_{iA} = L_{iA}$, where L_{iA} is the level of employment used to perform the abstract task. The routine task can be performed by either capital or labour: $T_{iR} = \lambda_L L_{iR} + \lambda_R R_i$, where L_{iR} and R_i are, respectively, the levels of labour and capital (henceforth: robots) used to perform the routine task.

Firms demand labour and capital from factor markets that elastically supply the factors at unit cost, and sell outputs at price p that they take as given. Therefore, their profit maximisation problem is

$$\max_{L_{iR}, L_{iA}, R_i, K_i \in \mathbb{R}^+} p \tilde{A}_i (\lambda_L L_{iR} + \lambda_R R_i)^{\alpha_R} L_{iA}^{\alpha_A} K_i^{\alpha_K} - (L_{iR} + L_{iA}) - r(R_i + K_i) \quad (2.2)$$

To investigate automation as a productivity-enhancing event, assume that there are two periods. Note that if $\lambda_L \neq \lambda_R$, labour and capital will not be used jointly in the R -task, but firms will exclusively rely on the more efficient factor (i.e., the one with higher task-level productivity λ).¹³ Therefore, to study automation, assume that in the earlier period, $\lambda_{R,0} < \lambda_L$ but $\lambda_{R,1} > \lambda_L$

¹³To see this, note that both factors have the same cost, and the coefficients λ represent per-dollar efficiencies in the R -task. If expenditures on the less productive factor are strictly positive, firms could increase profits by marginally substituting the less efficient factor for the more efficient one in a one-to-one fashion, which maintains cost but increases output. Therefore, strictly positive expenditures on the less efficient factor are not optimal.

so that firms switch from using labour to capital in performing the R-task from $t = 0$ to $t = 1$.

When firms produce the R-task using labour, the optimal assignment of labour to tasks implies¹⁴

$$Y_i^L = \underbrace{\tilde{A}_i \lambda_L^{\alpha_R} \left(\frac{\alpha_R}{\alpha_R + \alpha_A} \right)^{\alpha_R} \left(\frac{\alpha_A}{\alpha_R + \alpha_A} \right)^{\alpha_A}}_{=: A_i^L} L_i^{\alpha_R + \alpha_A} K_i^{\alpha_K}$$

On the other hand, if the R-task is produced using robots, firms produce

$$Y_i^R(\lambda_R) = \underbrace{\tilde{A}_i \lambda_R^{\alpha_R} \left(\frac{\alpha_R}{\alpha_R + \alpha_K} \right)^{\alpha_R} \left(\frac{\alpha_K}{\alpha_R + \alpha_K} \right)^{\alpha_K}}_{=: A_i^R(\lambda_R)} L_i^{\alpha_A} (R_i + K_i)^{\alpha_R + \alpha_K}.$$

In the above equations, A_i^L and A_i^R , respectively, denote the measurable Hicks-neutral productivity component when output is produced using either labour or robots in the routine task.

In the two-period scenario, firm-level productivity grows due to automation if $A_i^L < A_i^R(\lambda_{R,1})$. As a derivation in the Appendix shows, this is necessarily guaranteed irrespective of the level of $\lambda_{R,1}$ (i.e., assuming only $\lambda_{R,1} > \lambda_L$) if $\alpha_A > \alpha_K$, i.e., if the abstract task has a higher production weight than physical capital, or respectively, net of the use of robots, labour still has a higher production weight than physical capital after automation (but not necessarily than automation and other capital combined).¹⁵ Moreover, if the ratio $Y_i^R(\lambda_{R,1})/Y_i^L$ is bounded below by $2^{-(\alpha_K + \alpha_R)}(\lambda_{R,1}/\lambda_L)^{\alpha_K}$ such that if the automation innovation is sufficiently productive, i.e. $\lambda_{R,1}/\lambda_L$ is large, the automation event will necessarily be reflected in a positive change in the measurable Hicks-neutral productivity parameter.

These insights have relevant implications for the empirical investigation to follow, which uses the measure of MFP as described in the previous subsection. In this measure, an increase in the “true” Hicks-neutral component of productivity, \tilde{A}_i , will always be reflected as an increase in the estimated productivity parameter \hat{A}_i . However, also automation events, which always represent actual increases in efficiency in terms of the ratio of output and factor costs, may be reflected in such a positive change in the estimated coefficient \hat{A}_i . As the investigation above has shown, when labour remains sufficiently important after automation (i.e., $\alpha_A \geq \alpha_K$), or otherwise also if the task-level productivity increase associated with automation is sufficiently large, automation events correspond to positive changes in the measurable multiplicative productivity parameter A_i .

When estimating the change in A_i associated with automation using a Cobb-Douglas pro-

¹⁴The derivations are given in Appendix B.1.

¹⁵This circumstance appears reasonable in practice as the relationship between the production weights of labour and physical capital has frequently been estimated as roughly a 2:1 ratio.

duction function in labour and physical capital and a procedure that pools firms across years (as done in the data used), there is an error to be expected that stems from automating firms having non-average factor weights.¹⁶ Indeed, a more capital (labour) intensive production process is associated with an over- (under-)estimation of the change in the measurable parameter A_i that occurs with automation. The error's magnitude increases in the weight α_R of the routine task. At the same time, more capital- (labour-)intensive processes (α_A small (large) relative to α_K) are associated with a more (less) positive change in A_i . Therefore, the issue of automation possibly not reflecting a positive change in A_i in low-labour-intensive environments is, to some extent, mitigated by the associated overestimation of the productivity change. In conclusion, while a Hicks-neutral productivity estimation does not allow to identify the quantitative productivity contribution of automation technologies in terms of the cost-effectiveness of output production in general, automation events may still reflect a positive change also in terms of such measures, which therefore identify these events at least qualitatively.

2.4 FIRM-LEVEL EVIDENCE

2.4.1 METHODOLOGY

This section exploits the MultiProd transition matrices output (further described in the previous section), which provides information at a very granular level on the average characteristics of firms which transition from one group of the productivity distribution to another over various time horizons. Observations therefore correspond to the average firm in a country, industry, year, and detailed productivity transition group.

The main goal of the empirical exercise is to understand how structural, productivity-enhancing developments at a firm are related to its employment dynamics. Because structural change often takes time and the responses of employment to productivity-enhancing events may further not be immediate, the analysis focuses on explaining employment changes over a 5-year horizon. In a first step, a simple model is estimated that correlates the changes of productivity and employment, respectively, that occur over the same five-year horizon.¹⁷ This model has the advantage of being simplistic and easily interpretable, and can further be flexibly extended to perform some relevant heterogeneity analysis of the baseline relationship. To address possible concerns of serial dependence, the analysis also presents results of a dynamic

¹⁶A detailed investigation of this error is given in Appendix B.1. As the Appendix describes, if there is automation in the industry, then at automating firms, the production coefficient of capital (labour) obtained from estimation across all firms understates (overstates) the true firm-level coefficient of capital (labour) after automation, and overstates (understates) it before automation.

¹⁷This choice is guided by the traditional view in the literature which considers productivity as an exogenous process to which inputs and output respond, possibly with some lag if there are relevant adjustment frictions. However, in practice, changes in employment may also affect productivity, e.g. if they induce a change in the average skill level of the workforce. The dynamic IRF model addresses this concern.

impulse response function model following the methodology of Jordá (2005).

The analysis first investigates the link between initial productivity rank at time t and future employment growth (from t to $t + 5$) by reporting the coefficients associated to the initial productivity groups fixed effects (γ_{q_0}), controlling for country-industry-year fixed effects (γ_{cjt}), which are aimed at accounting for confounding factors and common dynamics varying at the country-industry-year level. The model is specified as follows:

$$\Delta_5 y_{it} = \gamma_{q_0} + \theta y_{it} + \gamma_{it} + \varepsilon_{cjt} \quad (2.3)$$

In the equation above, i denotes the panel unit, that is, the country-industry-transition group, and t denotes a year. $\Delta_5 y_{it}$ is the five-year forward looking average within-firm growth rates of outcome y (either employment or wage) from t to $t + 5$. y_{it} denotes the level of the outcome y in the initial year t .

This specification is also the basis for the investigation of the link between productivity growth and employment growth, accounting for the role of initial productivity quantile unveiled with the model corresponding to Equation (2.3). The model is therefore extended to include productivity growth between time t to $t + 5$. It is specified as follows:

$$\Delta_5 y_{it} = \beta \Delta_5 a_{it} + \gamma_g + \theta y_{it} + \gamma_{cjt} + \varepsilon_{it} \quad (2.4)$$

where $\Delta_5 a_{it}$ is the five-year forward looking average within-firm growth rates of productivity from t to $t + 5$ of firms in given country, SNA A38 industry, and productivity transition group (summarised by the panel index i , and a year t).

This equation includes also fixed effects taking into account a wide range of possible unobserved confounding factors at the level of country-industry-year γ_{cjt} , including the dynamics affecting all firms in the same country-industry-year. It therefore exploits productivity and employment variation across firms within the same country, industry, and year. An initial estimation step includes instead a less restrictive set of fixed effects (country-industry and country-year). In this regression the initial productivity group fixed effects γ_{q_0} control for structural differences between low and high productive firms, but also for other characteristics varying systematically across productivity groups.

The main coefficient of interest is β . Given the set of controls, it measures the association between productivity growth between time t and $t + 5$ and the relevant outcome (employment and wage growth between time t and $t + 5$), focusing on average changes occurring within firms. The model is easily extended to analyse the heterogeneity of the relationship (i.e., differences in β) according to the initial position of firms in the productivity distribution, or across sectors or

countries, by interacting productivity growth with relevant continuous or categorical variables.

The regressions are weighted according to the representativeness of an observation within the country, based on inverse probability weights defined by the MultiProd procedure (Berlingieri et al., 2017). All countries are weighted equally.

To investigate the dynamic adjustments of employment and to further account for the role of past productivity and employment growth, the paper also presents results based on impulse response function models (IRF). The modelling strategy follows Autor and Salomons (2018), and implements a local projection estimation approach (Jordá, 2005) to analyse how employment or wages respond at different time horizons to changes in productivity. More specifically, the model is extended to control simultaneously for past productivity growth, as well as for past growth in the outcome variable (i.e., either employment or wage growth), as follows:

$$y_{it+h} - y_{it-1} = \beta^h \Delta a_{it} + \sum_{l=1}^L (\theta_{a,l}^h \Delta a_{it-l} + \theta_{y,l}^h \Delta y_{it-l}) + \gamma_{cjt}^h + \varepsilon_{it}^h \quad (2.5)$$

In the equation above, i again denotes the panel unit, i.e., the country c , industry j , productivity transition group g , and γ_{cjt} denotes country-industry-year fixed effects (or in an initial result alternatively the country-industry and country-year), that absorb specificities of the country-industry-year and the productivity and outcome dynamics common to firms in the same country-industry-year. L denotes the lag order of the model. In the baseline specification, the model considers two-year productivity growth from t to $t+2$ as the impulse Δa_{it} , i.e., the main change of productivity under study, and controls for past productivity and outcome changes from $t-2$ to t , and $t-3$ to $t-2$.¹⁸ The equation is separately estimated for individual horizons $h = 0, 1, \dots, H$ of interest, where H is the maximum horizon considered.

The main coefficient of interest is β_1^h , which measures the response of employment between $t-2$ and $t+h$ to an initial change in productivity between $t-2$ and t , while controlling for both past productivity growth and past employment growth.

The analysis also focuses on whether productivity growth reduces a firm's risk of failure, that is, whether firms that experience growing productivity are less likely to exit the market over the following period. To investigate this, the analysis relies on the estimation of a logistic regression model of the following form:

$$X_{it+5}^2 = g(\beta^X \Delta_5 a_{it} + \theta^X l_{it} + \gamma_{it}^X + \gamma_{q_0}^X) + \varepsilon_{it}^X \quad (2.6)$$

In the equation, X_{it+5}^2 is an indicator that is equal to one if the transition group is one that

¹⁸The choice of the timing is guided by data availability. The estimation of this model indeed relies on transition matrices, which collect information on the dynamics of firms at specific horizons (1, 3, 5, 7, 10) to minimise the burden for participants to the MultiProd project. This imposes some constraints on the structure of the model.

exits over the two subsequent years, i.e., between $t + 5$ and $t + 7$, and equal to zero otherwise. $\Delta_5 a_{it}$ again is the 5-year productivity change from t to $t + 5$, and l_{it} denotes the initial average employment at time t . The tables report average partial effects of productivity growth, the average of the partial derivative of $g(\beta^X \Delta_5 a_{it} + \theta^X l_{it} + \gamma_{cjt}^X + \gamma_{q_0}^X)$ with respect to productivity growth across observations in the data.

2.4.2 RESULTS

This sub-section presents the results of the analysis focusing on firm-level outcomes. It first shows that employment growth is higher for firms at or close to the productivity frontier. It then provides evidence that stronger productivity growth tends to translate into higher employment growth at the firm level, and that this may be particularly related to an increase in sales. It further shows that employment fully adjusts to productivity changes with a lag, and that the link between productivity growth and employment is heterogeneous across different groups of firms, operating in different industries or countries. It uncovers in particular a role of firms' initial position in the within-industry productivity distribution, as well as the industry-level intensity of ICT investments and differences in markups within industries. Finally, it highlights that a positive link emerges also between productivity growth and the growth of wages, and that stronger productivity growth reduces the probability of firm exit.

INITIAL EVIDENCE: MOST PRODUCTIVE FIRMS HAVE STRONGER EMPLOYMENT GROWTH

Columns 1 and 2 of Table 2.4 present the results of the estimation of Equation (2.3) to investigate differences in average employment growth between firms in groups with different initial productivity (i.e., between firms with a different starting point in terms of productivity ranking with respect to firms in the same country-industry-year). The second to fifth rows show the average difference in employment growth for each productivity group relative to the reference group composed of firms in the middle of the productivity distribution, while controlling for the initial size of firms across groups (sixth row).¹⁹

Results show that firms initially at the top of the productivity distribution display on average higher employment growth than other firms in the same country-industry-year. Both frontier firms (labelled "Initial productivity group p90-p100" in the table) and firms closer to the frontier ("Initial productivity group p60-p90") display higher employment growth over the next five years relative to the reference category around the median.²⁰ This indicates

¹⁹The median group, used as the reference group, is composed of firms between the 40th and 60th percentile of the productivity distribution. Note that this set of results accounts for differences in initial size ("initial employment") across groups of firms with different initial productivity.

²⁰At the same time, while the coefficient of the group below the median ("Initial productivity group p10-p40") is still in line with a monotonic relationship between the productivity rank and employment growth, the results also show that future employment growth of firms initially at the very bottom of the productivity distribution

Table 2.4: The firm-level link of employment growth to productivity growth: partial correlations.

	(1)	(2)	(3)	(4)
	5-year change in employment	5-year change in employment	5-year change in employment	5-year change in employment
5-year change in productivity			0.0632*** (0.0104)	0.0648*** (0.0108)
Initial productivity group p0-p10	0.0342** (0.0138)	0.0359** (0.0144)	-0.0579*** (0.0121)	-0.0639*** (0.0130)
Initial productivity group p10-p40	-0.0164*** (0.00599)	-0.0157** (0.00616)	-0.0406*** (0.00485)	-0.0430*** (0.00484)
Initial productivity group p60-p90	0.0429*** (0.00405)	0.0423*** (0.00429)	0.0630*** (0.00463)	0.0653*** (0.00498)
Initial productivity group p90-p100	0.126*** (0.00950)	0.125*** (0.0101)	0.175*** (0.0102)	0.180*** (0.0110)
Initial employment	-0.00885 (0.00901)	-0.00568 (0.0101)	-0.0514*** (0.00812)	-0.0580*** (0.00904)
Observations	19,900	19,875	19,900	19,356
R-squared	0.398	0.457	0.444	0.503
Fixed effects	C-I C-Y	C-I-Y	C-I C-Y	C-I-Y

Estimates obtained from the models in Equation (2.3) (columns 1 and 2) and Equation (2.4) (columns 3 and 4). C-I and C-Y indicate fixed effects for the country-industry and country-year, respectively, and C-I-Y indicate fixed effects for the country-industry-year. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

an increasing relationship between the static level of productivity, and suggests that high-productivity firms contribute importantly to employment dynamics.

PRODUCTIVITY GROWTH IS POSITIVELY ASSOCIATED WITH EMPLOYMENT CHANGES AT THE FIRM LEVEL

While initial productivity is strongly associated with employment growth, the evolution of firms' productivity is also and relevantly related to subsequent employment growth. More specifically, firms that experience stronger productivity growth than other firms in the same country-industry-year also have on average stronger employment growth. This means that all else equal, firms that increase their productivity more also tend to increase more their size relative to competitors.

This is shown in columns 3 and 4 of Table 2.4, which show a positive and statistically significant correlation between productivity growth and employment growth over five years. This result accounts for firms' initial position in the productivity distribution and for their contemporaneous one-year employment growth, together with additional unobserved factors

(“Initial productivity group p0-p10”) is relatively high. This result may be driven by positive selection of firms in this group, as it is populated by a selected group of laggard firms that survive over the next five years. These firms are more likely to be start-ups with a high potential for growth and catch-up (Berlingieri et al., 2020). If start-up environments are characterised by strong “up-or-out” dynamics where young firms either grow rapidly or exit the market, the positive selection of survivors may strongly drive the observed patterns.

affecting employment and productivity in all firms operating in a given country and industry for any given year. This finding is robust to a series of alternative specifications and robustness checks, such as adopting a less restrictive sets of fixed effects, changing the structure of the time lags considered, or taking into account the role of additional confounding factors that are observed only in some countries.²¹

This result echoes the finding of Baily, Bartelsman and Haltiwanger (1996) for US manufacturing, challenging the “myth” that firm-level productivity growth is associated with downsizing. They conclude that productivity growth is also associated with upsizing for a significant share of firms. The results presented in Table 2.4 complement this finding and show that for the period considered (2000-2018) higher productivity growth is on average associated with stronger employment growth within the countries and industries considered.

The results presented in Table 2.4 also suggest that productivity changes relative to firms in the same country-industry-year, and against which they may be competing for market shares, may matter more than the magnitude of the productivity change per se. In other words, the positive link between employment and productivity growth identified in column 4 of Table 2.4 reflects dynamics relative to other firms in the same country, industry, and year. This point already emerges from the comparison of columns 3 and 4 of Table 2.4. The result of column 3, which uses a less restrictive fixed effects structure allowing for other changes in productivity than those relative to competitors, shows an estimate for the coefficient of productivity growth very similar to the one in column 4.²² Additional discussion on the importance of relative improvements in productivity will be carried out in the next subsection.

The faster employment growth associated with an initial change in productivity may be induced by a relative increase in demand experienced by firms increasing their productivity. Firms that increase their efficiency relative to others (regardless of the specific drivers of this change) may be able to charge lower prices, and therefore attract customers and increase sales, which in turn induce a higher demand for factor of productions, including labour.

²¹The estimated coefficient for the association between productivity growth and changes in firm size is similar when not accounting for the initial productivity group. Results are also robust when including average age as a control. Average age is not included in the main model due to lack of data availability for some countries. Results are also confirmed by unreported regressions focusing on the link between contemporaneous employment growth and productivity growth over the same five-years period, as well as regression focusing on five-year employment growth after an initial five-year period of employment and productivity growth (based on transitions of firms across productivity groups over a ten-year horizon). This confirms that results hold when focusing on longer-term changes in productivity which may reflect additional structural changes compared to short-term productivity shocks.

²²A highly similar coefficient estimate is also obtained when using an even less restrictive set of fixed effects absorbing only specificities of countries, years, and industries separately. Table B.1 in the appendix further confirms the importance of relative productivity growth for the total firm-level relationship of productivity growth to employment growth. The results show that a firm’s productivity group at the end of the transition is a strong predictor of its employment growth, and accounting for the productivity group at the end of the transition reduces the estimated relationship between productivity growth and changes in employment significantly (by around one third). Therefore, this result underscores that the extent to which productivity growth helps firms improve their position in the productivity distribution is a key source of its positive link to employment growth.

Table 2.5: The firm-level link of sales growth with productivity growth.

	(1)	(2)
	5-year change in employment	5-year change in sales
5-year change in productivity	-0.126*** (0.0441)	0.582*** (0.0393)
5-year change in sales	0.337*** (0.0629)	
Initial employment	-0.0453*** (0.00791)	
Initial sales		-0.0650*** (0.0160)
Observations	19,873	19,873
R-squared	0.626	0.854
Fixed effects	C-I-Y G	C-I-Y G

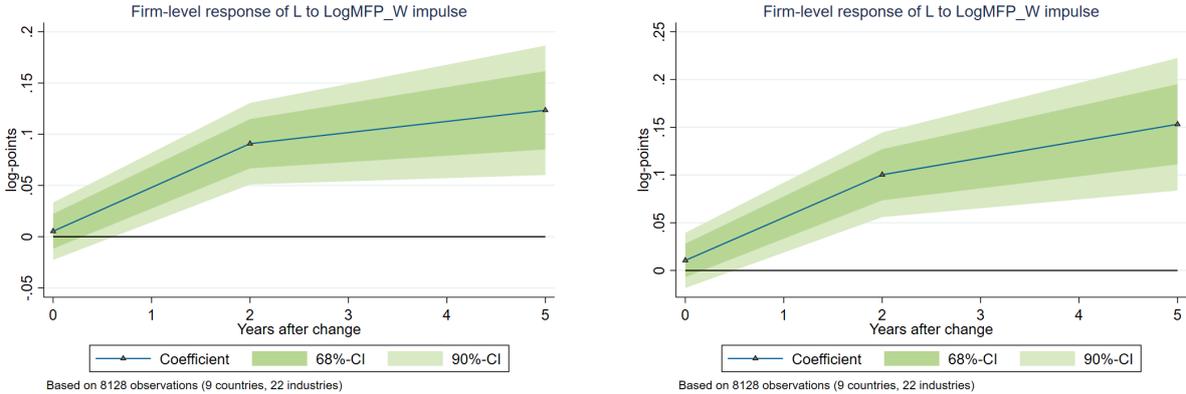
Estimates obtained from modifications of the model in Equation (2.4). C-I-Y indicate fixed effects for the country-industry-year, and G indicate fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

The role of increasing sales in driving the positive link between relative productivity growth and labour demand is confirmed by the results displayed in column 1 of Table 2.5. Once one accounts for the role of contemporaneous changes in gross output over a five-year period ("5-year change in sales" in the table), the estimated link between productivity growth and employment becomes negative and statistically significant. This result may stem from two complementary mechanisms. First, an "efficiency effect", whereby increasing productivity implies that the same output can be produced with less factors of production, including labour. Second, a "displacement effect" linked to capital taking over labour's tasks, which may negatively affect labour demand in instances where productivity is driven by labour-saving technologies (Acemoglu and Restrepo, 2019).

The average positive association between productivity growth and employment growth previously discussed suggests that on average these efficiency and displacement mechanisms seem to be more than compensated by the indirect positive mechanism related to the relative increase in sales. To this end, column 2 of Table 2.5 confirms that firms' relative output markedly increases with an increase in the firm's relative productivity.

Focusing on the results arising from the IRF model estimation (Equation (2.5)), Figure 2.1 confirms the positive response of employment growth to changes in productivity. It further shows that employment may fully respond to initial changes in productivity only with a time lag. The estimates indeed suggest that there is limited contemporaneous adjustment during the same period over which the two-year productivity change occurs ($h = 0$).

Figure 2.1: The firm-level link of employment growth to productivity growth: impulse response



(a) Country-Industry Country-Year Fixed Effects.

(b) Country-Industry-Year Fixed Effects.

This figure illustrates the results of the local projection impulse response regression estimations for the response of employment to a change in productivity using fixed effects for the country-industry and country-year (left) and fixed effects for the country-industry-year (right), based on Equation (2.5). Observations are weighted by the number of firms represented in the full population, normalised at the country level. Confidence bands are based on pointwise estimation of standard errors, clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

Hence, firms may take time to respond to changes in productivity and in their performance relative to competitors.²³ While the bulk of the subsequent adjustment occurs over the first two years after the productivity change, the point estimates suggest that employment may further continue to increase over the whole five-year period considered. The response depicted in Figure 2.1 shows that employment increases over the five-years period and is not reverted at the end of the period (i.e., employment does not revert to the initial level but stays persistently higher over the horizon considered). This suggests that faster productivity gains relative to other firms may induce a persistent change in size.

EXPLORING THE HETEROGENEITY OF THE FIRM-LEVEL LINK

Beyond the average relationship discussed above, the link between productivity growth and employment growth appears heterogeneous across different groups of firms, operating in different industries or countries. First, improvements in relative productivity are more strongly associated with employment growth for non-frontier firms. Second, the link varies according to differences in market power across firms. Third, in an environment characterised by more innovation or the use of technologies with a stronger labour-saving potential the productivity-

²³Comparing panels (a) and (b) of the figure, which rely on two different sets of fixed effects, confirms that also in the impulse response setting, allowing for broader variation of productivity (and employment) growth to dynamics that are not only relative to competitors in the same country-industry-year (as done by the model with country-industry and country-year fixed effects in panel (a)) does not significantly alter the estimated response. This confirms that changes in the position in the productivity distribution (i.e., the change in the ranking in terms of productivity performance) matters more than absolute changes in productivity for employment dynamics.

employment link is still positive, but to a lower extent. This heterogeneity in the dynamics of employment associated to productivity growth is further discussed below.

Firms closer to the frontier display higher employment growth on average (as discussed above and illustrated in Table 2.4), but non-frontier firms are more responsive to relative improvements in their productivity. This is illustrated in Table 2.6 below which shows the strength of the link for the baseline category (p0-p40) in the first row. It also shows the difference, with respect to this baseline, for frontier firms (“Initial productivity group = p90-p100”) which display a positive but lower correlation between productivity and employment growth, and for non-frontier non-laggards firms (“Initial productivity group = p40-p90”) which display a similar correlation as the reference category. Figure B.1. (based on the dynamic IRF model) in the Appendix also confirm a lower responsiveness of frontier firms relative to non-frontier firms.²⁴

Table 2.6: The role of a firm’s initial position for the firm-level link of employment growth to productivity growth.

	(1) 5-year change in employment
5-year change in productivity	0.0838*** (0.00963)
* Initial productivity group = p40-p90	-0.0242*** (0.00793)
* Initial productivity group = p90-p100	-0.0862*** (0.0126)
Initial employment	-0.0426*** (0.00879)
Observations	19,875
R-squared	0.514
Fixed effects	C-I-Y G

Estimates obtained from a heterogeneous effects extension of the model in Equation (2.4). C-I-Y indicate fixed effects for the country-industry-year, and G indicate fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors’ calculations based on the OECD MultiProd 2.0 Database

Efficiency gains in non-frontier firms may allow them to strengthen an initially weaker position on the market, thereby attracting customers and increasing sales. On the contrary, firms at the frontier may have already reached higher levels of efficiency (through the adoption of new technologies, good management and organisational practices, investment in human capi-

²⁴In Table 2.6, the first row shows the link between productivity growth and employment growth for the reference group comprised of the bottom of the productivity distribution (firms below the 40th percentile of the productivity distribution). The second row shows the difference in the relation for firms which are not at the bottom neither the frontier with respect to the reference group. The third row shows the difference between the baseline coefficient and the coefficient for the frontier group. The coefficient for the frontier group is .066 (0.148-0.082) which indicates that employment growth is still positively correlated to productivity growth for frontier firms.

tal, etc.), and may have already built a more stable customer base, allowing them to scale up regardless of further productivity improvements. For such firms, further relative improvements in productivity may therefore be less relevant for future employment. This also suggests that employment in more productive firms may be more resilient to negative productivity shocks (whether they arise from negative technology or revenue shocks), while initially less productive firms may be affected more strongly by such negative shocks.

A second source of heterogeneity in the strength of the productivity-employment growth nexus is related to the use of technologies with a higher potential for labour displacement. Recent waves of innovation in information and communication technologies (ICTs) and their broad diffusion have increased the scope of tasks that can be performed by capital instead of workers, contributing to the displacement of labour from some tasks. This phenomenon may partially offset some of the positive implications of productivity for employment that occur through the expansion of output. When task-replacing technologies are used, any given output target may indeed be achieved with less intense use of labour.

Table 2.7: The role of structural determinants for the firm-level link of changes in employment to productivity growth.

	(1)	(2)	(3)	(4)	(5)
	5-year change in employment				
5-year change in productivity	0.0743*** (0.0101)	0.0590*** (0.0106)	0.0599*** (0.0101)	0.0672*** (0.0104)	0.0736*** (0.00995)
* ICT investment intensity	-0.00755*** (0.00172)				-0.00556*** (0.00163)
* AI patenting intensity		-0.0165*** (0.00560)			
* ICT patenting intensity			-0.352*** (0.0874)		
* Difference p90-p50 of markups				-0.00697*** (0.00211)	-0.00544*** (0.00195)
Initial employment	-0.0583*** (0.00919)	-0.0453*** (0.0104)	-0.0451*** (0.0104)	-0.0556*** (0.00916)	-0.0563*** (0.00920)
Observations	19,875	15,638	15,638	19,832	19,832
R-squared	0.514	0.519	0.520	0.515	0.520
Fixed effects	C-I-Y G				
Countries excluded	–	HRV, JPN	HRV, JPN	–	–

Estimates obtained from heterogeneous effects extensions of the model in Equation (2.4). C-I-Y indicate fixed effects for the country-industry-year, and G indicate fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Column 1 of Table 2.7 shows indeed that employment is less strongly associated to relative changes in productivity in industries that use ICTs (which have the potential to automate some tasks) more intensively.²⁵ Focusing on country-level proxies for innovations, columns 2 and 3

²⁵ICT intensity is measured as investment in ICT equipment as a percentage of total gross fixed capital formation, averaged over the period 2000–2003 and across countries. It varies at the sectoral level only, which helps to address limitations related to data availability. Note that while ICTs were key in the computerisation of routine tasks over the period after 1980 and form the basis of modern automation technologies related to robotics, machine learning and AI, some ICTs also complement labour, e.g., software related to navigation, design, planning and

show that the relation appears weaker in country-years in which ICT and AI patent intensities are higher.²⁶ This suggests that productivity gains associated to a more intensive use of ICTs – or to higher innovation – may be associated to positive employment growth but to a lower extent if compared with productivity improvements related to firm-level changes that have a lower potential for task replacement.²⁷

The competitive environment in which firms operate is another source of heterogeneity for the employment dynamics associated with relative productivity growth. To show this, the analysis investigates how the dispersion in market power (proxied by markups) across firms affects the correlation between productivity and employment growth. This quantity is measured as the difference between high and median markups within a country-industry pair each year, where high markups are proxied by the 90th percentile of the within country-industry-year distribution of markups.²⁸ Markup dispersion is measured in the initial year of the 5-year window over which productivity and employment change to exclude possible concerns of endogeneity.

A high dispersion of markups may indicate a low degree of contestability of markets. When markup dispersion is high, a small number of dominant firms may hold a strong position that allows them to charge high markups while most firms in the industry do not. The latter group of firms may not be able to compete for market shares in the same way they would in environments with less markup dispersion. Relevantly, studies show that markup dispersion has affect welfare through misallocation (see for instance Baqaee and Farhi, 2019; Edmond, Midrigan and Xu, 2018; Peters, 2020). This may further affect the productivity-employment link, as the opportunity of firms catching up in productivity to expand sales – and therefore employment – may be limited by the less contestable position of the market leaders.²⁹ Results in column 4

organisation, surveillance and monitoring, etc., so that the concept is not immediately to be equated with labour-saving technological change.

²⁶The variables on patents are constructed from information from the OECD Patstat database. These data measure the annual number of patents filed related to either technology (ICT, AI) at the country level. The patent count is normalised by the total number of employees in the country-year from the OECD STAN database to measure patenting intensity relative to the size of the economy. Specifically, AI patents are divided by millions, and ICT patents by thousands of employees.

²⁷The estimated coefficients imply that the link between firm-level productivity growth and firm-level employment growth remains positive also in sectors with high ICT intensity. This is also in line with results by Aghion et al. (2020), showing that the firms adopting automation technologies may be able to benefit from productivity growth and increase their market shares.

²⁸The choice of the difference between the 90th and 50th percentile of the markup distribution provides a measure of markup dispersion more specifically at the top. This choice is driven by the aim to have a conservative measure: existing literature shows that differences in markups across firms are driven by the top half of the markups distribution (Calligaris, Criscuolo and Marcolin, 2018; De Loecker, Eeckhout and Unger, 2020), with the bottom part having markups very close to 1. This implies that “few firms have high markups and are large, the majority firms see no increase in markups and lose market share” (De Loecker, Eeckhout and Unger, 2020). Therefore, the measure considered seems to be a more conservative choice when looking at dispersion than, for instance, the difference between 90th and the 10th percentile.

²⁹Low contestability of markets may in particular affect firms with initially lower productivity (which tend to be smaller and younger, with less market power) by reducing their benefits associated with catching up to the frontier. However, according to the results in this analysis, these firms are precisely those with the highest potential for

of Table 2.7 indeed show that the higher the difference in markup across firms, the lower the correlation between relative improvements in productivity and employment growth.³⁰

The lower association between productivity and employment growth related to higher technological intensity on one hand, and to higher markups dispersion on the other seem to capture different mechanisms. Column 5 of Table 2.7 reports the estimated coefficients of a regression including the interaction of productivity growth with both the markup gap measure and the ICT intensity measure, which does not alter strongly the estimated interaction coefficients.

The lower correlation in ICT-intensive sectors suggests that productivity growth related to some specific technologies with a higher scope for capital-labour substitution are positively associated to relative employment growth at the firm-level, but to a lesser extent than productivity growth related to other drivers of efficiency. This is in line with evidence of a displacement effect associated to technologies that automate some tasks (Acemoglu and Restrepo, 2019).

On the contrary the lower association between productivity and employment growth when markup dispersion is higher may reflect barriers related to within-industry differences in market power. Firms with initially low productivity and less market power may indeed face obstacles to increase sales (and therefore employment) when catching up towards the frontier firms. Therefore, while ICTs represent a source of additional firm-level productivity growth that may still allow firms to grow in size, higher markup dispersion may limit the potential of firm-level improvements in productivity to generate employment.

PRODUCTIVITY GROWTH AND OTHER MARGINS: WAGES AND THE RISK OF EXIT

The positive change in labour demand from firms with a relative change in productivity is also confirmed by the comparative evolution of their wages (column 1 and 2 of Table 2.8). Firms with higher productivity growth also increase wages more than other firms. This holds after accounting for a wide range of unobserved confounding factors, similarly to the previous estimations. This result complements findings by Berlingieri, Calligaris and Criscuolo (2018) of a robust productivity-wage premium. This change in wages may be related to firms sharing additional profits with workers, and to firms using wages as a tool to compete for workers on the labour market. Further, it may be linked to a change in the skill composition of workers, through the hiring of high-skill workers to fill new occupations, but also through a skill-shift within occupations (Bessen, Denk and Meng, 2022).

employment growth in response to productivity growth, and any barriers they face in expanding their market may therefore significantly impact the average relationship between productivity growth and changes in employment.

³⁰The results on market power dispersion are also relevant for the role of market power itself. Unreported results show similar patterns when considering the 90th percentile of the within-industry distribution of markups, rather than markup dispersion. Hence, firms with higher markups display a lower link between productivity growth and employment changes, suggesting that they might be less prone to increase their production inputs when productivity increases, but rather retain the benefits of lower marginal costs and higher margins.

Table 2.8: The firm-level link of changes in the average wage to productivity growth.

	(1)	(2)
	5-year change in av. wage	5-year change in av. wage
5-year change in productivity	0.291*** (0.0206)	0.300*** (0.0205)
Initial av. wage	-0.201*** (0.0324)	-0.239*** (0.0377)
Observations	19,845	19,820
R-squared	0.760	0.787
Fixed effects	C-I C-Y G	C-I-Y G

Estimates obtained from the model in Equation (2.4). C-I-Y indicate fixed effects for the country-industry-year, and G indicate fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Like the link with employment growth, the role of relative improvements against the competition within the country-industry seem to fully account for the observed relationship of productivity growth with wage growth. This is indicated by the result in column 2 of Table 2.8, which estimates the relationship using a restrictive set of fixed effects that does not allow for further improvements in productivity beyond those relative to competitors, and shows a similar coefficient for productivity growth to the one in column 1.

Higher productivity growth also affects firms' employment through an extensive margin, as higher level of productivity and productivity growth lower the probability of firm exit. This is shown in Table 2.9 which estimates how the productivity growth over five years is related to the risk of exit over the next two years.³¹

Table 2.9: Firm-level productivity growth and the risk of exit.

	Exit (over 2 years)
5-year change in productivity	-0.189*** (0.0262)
Initial employment	-0.101*** (0.0258)
Observations	13,486
Fixed effects	C-I-Y G

Estimates report average partial effects and are obtained from the logistic regression model in Equation (2.6). C-I-Y indicate fixed effects for the country-industry-year, and G indicates fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

³¹Unreported results displaying the quantile fixed effects in the regression reported in Table 2.9 confirm that the probably of exit is also negatively associated to the initial level of productivity. The lower the initial productivity relative to others, the higher the risk of exit.

This indicates a selection mechanism, whereby less productive firms that fail to improve productivity are exiting the market. This selection of more productive firms, and firms which improve their productivity more, contributes to the reallocation of resources towards better performing firms. However, this also suggests that more rapid productivity improvements in some firms may negatively affect other firms as they may deteriorate their relative performance, and thereby increase their probability of exit. While this mechanism may be beneficial overall for the economy and is tightly linked to the productivity-enhancing reallocation process, this may also contribute to higher gross job destruction at the industry level (some of which may be compensated by gross job creations in expanding and more productive firms). The next section returns to this issue and presents results that suggest the productivity-induced reallocation process to be a net positive force for the productivity-employment link.

Furthermore, this result indicates that it may be key for non-frontier firms to at least keep up with the most productive firms in order to avoid the worker-side cost of job destruction associated with the loss of firm-specific wage premia and temporary unemployment. This suggests that policies aimed at supporting broad technology diffusion may have relevant benefits for employment also through this extensive margin.

To sum up, focusing on within-firm employment growth over five years, results highlight a positive average association of productivity growth with the dynamics of employment. Notably, firms that are initially more productive and non-frontier firms that improve their productivity relative to competitors experience stronger and sustained employment growth. This positive link likely reflects indirect effects on employment channelled through increases in sales, which appear to overcompensate direct negative effects related to efficiency and labour replacement (i.e., respectively the fact that less inputs are required to produce a given level of output, and the fact that productivity growth may reflect labour saving technological change). The stronger correlation for non-frontier firms likely reflects an employment growth potential that materialises when they strengthen their market position relative to competitors. Such correlation is dampened by differences in market power across firms and is also lower when labour-saving technology are more prevalent. Finally, productivity growth is also positively and relevantly associated with wage growth and firm survival.

2.5 INDUSTRY-LEVEL RELATIONSHIP AND AGGREGATE MECHANISMS

After uncovering a positive firm-level relationship between productivity growth and employment growth, the analysis investigates the relationship at a more aggregate level. Certainly, the firm-level responsiveness of employment to productivity changes is a key component of the industry-level relationship; however, aggregate dynamics are also shaped by additional mech-

anisms. Indeed, a positive relation at the firm level may not necessarily mean that productivity growth translates into employment growth at more aggregate levels. When looking beyond the firm-level mechanisms, reallocation mechanisms need to be accounted for: an increase in employment at some firms may indeed come at the expense of a reduction in employment at other firms, such as those losing their competitiveness or exiting the market. Thus, within-industry employment outcomes in response to productivity changes are more ambiguous.

The extent to which productivity growth results in net job creation at the more aggregate level may further depend on changes in demand at the industry level, and the importance of a labour-saving mechanism induced by some technologies. Moreover, industries do not operate in isolation, but are interlinked into global value chains. Productivity changes in one industry are therefore likely to spill over into employment changes also in other industries.

This section investigates the relevance of these mechanisms, focusing on aggregate industry-level changes in employment and productivity. Specifically, beyond studying the average industry-level relationship between these quantities, it analyses the role of different factors possibly shaping the industry-level nexus, namely technology, demand and differences market power. Further, it turns to the spillovers in terms of employment outcomes that productivity growth may have on connected industries.

2.5.1 METHODOLOGY

The analysis now turns to the relationship of productivity growth and employment growth (and related outcomes, especially wages) at the level of a given industry within a country, assessing the extent to which the firm-level mechanisms translate into a similar relationship at the industry level. Accordingly, on the one hand the analysis focuses on industry level “totals”, namely total employment at the industry level, and on the other hand focuses on the wage of the average worker in the industry.

In a first step, the analysis again relies on the estimation of the partial correlation between an initial, short-term, change in productivity and the long-term change in employment or wage:

$$\Delta_5 y_{it} = \beta \Delta_5 a_{it} + \theta z_{it} + \gamma_{ct} + \gamma_j + \varepsilon_{it} \quad (2.7)$$

$\Delta_5 y_{it}$ denotes the change in the outcome y_{it} (i.e., total employment and average wage) of the country-SNA A38 industry i from year t to $t + 5$, and $\Delta_5 a_{it}$ denotes the change in industry-level aggregate productivity in country-industry i from time t to $t + 5$. z_{cjt} are control variables capturing the initial state of the system, including the level of productivity and the outcome at time t . γ_{ct} and γ_j are fixed effects controlling for unobserved heterogeneity at the level of the country-year and the industry, respectively, which control for business cycle effects, coun-

try level shocks, systematic differences across countries (also changing over time), as well as industry-specific trends. The model therefore focuses on deviation of productivity and employment growth from the dynamics common to all industries in a country, and across countries for a given industry.

The analysis further relies again on the IRF model introduced in Equation (2.5), where now, the panel index i refers to a country-industry. Due to the higher frequency of data available compared to the firm-level estimations, and considering that industry-level trends may be more persistent, the model includes four lags (that is $L = 4$).

In order to keep a sufficient number of observations, thereby preserving statistical power, the analysis is limited to a response of employment to a maximum horizon of three years after the initial “shock” (i.e., $H = 3$). This model is more suited to control for the correlation of employment and productivity growth with past growth (i.e., autocorrelation), and therefore may more precisely assess the link between employment (or wage) and productivity growth when autocorrelation may be a more relevant issue. To further understand the role of different sources of productivity growth, an instrumental variable approach is applied to the model

$$\Delta_5 y_{it} = \beta \Delta_5 a_{it} + \theta z_{it} + \gamma_{ct} + \gamma_j + \varepsilon_{it} \quad (2.8)$$

that regresses the five-year change in employment on the contemporaneous change in productivity. By instrumenting productivity growth with specific sources that are exogenous to industry-level employment trends (detailed further below together with the results), this method allows to study directly the contemporaneous association of longer-term changes in productivity and employment in a fashion that further overcomes possible endogeneity concerns arising from the quantities being (possibly) simultaneously determined.

Regressions are weighted by the value-added share of a sector within a country-year, and countries are weighted equally. Standard errors are clustered at the country-industry level.

The analysis of value chain spillovers between industries estimates a distributed lag model of the form:

$$y_{it} = \sum_{s=0}^2 \left(\beta_s^{own} \Delta a_{it-s} + \beta_s^S \Delta a_{it-s}^S + \beta_s^C \Delta a_{it-s}^C + \beta_s^{FS} \Delta a_{it-s}^{FS} + \beta_s^{FC} \Delta a_{it-s}^{FC} \right) + \gamma_j + \gamma_{ct} + \varepsilon_{it} \quad (2.9)$$

The outcome y_{it} of the country-industry i in year t is modelled as a function of up to two lags of several terms capturing different changes in productivity. The first term Δa_{it} , as before, is the change in industry-level aggregate productivity in the country-industry. The terms Δa_{it}^S and Δa_{it}^{FS} are weighted averages of industry-level aggregate productivity growth in, respectively, domestic and foreign supplier industries (with weights based on the Leontief inverse matrix

for value added indicating the strength of industry linkages). Here, $\Delta a_{it}^S = \sum_{k \neq j} w_{ckt}^{S,j} \Delta a_{ckt}$ where $w_{cjt}^{S,j}$ is the share of value added in industry j accounted for by the domestic industry $k \neq j$ of country c , and Δa_{it}^{FS} is defined in analogy summing over non-domestic supplier industries.

Conversely, Δa_{it}^C and Δa_{it}^{FC} are weighted averages of industry-level aggregate productivity growth in, respectively, domestic and foreign customer industries. Here, $\Delta a_{it}^C = \sum_{k \neq j} w_{ckt}^{C,j} \Delta a_{ckt}$ where $w_{cjt}^{C,j}$ is the impact of production in industry j on final demand in the domestic industry $k \neq j$ of country c , and Δa_{it}^{FC} is defined in analogy summing over non-domestic customer industries. All these variables are standardised to allow a more natural interpretation of coefficients. γ_j and γ_{ct} again denote fixed effects for the industry and the country-year, respectively, and absorb the role of industry-specific trends and country-year specificities such as the business cycle.

For each productivity term, the coefficient of interest is the sum of coefficients associated with the terms at different lags. This sum reflects the total impact of the productivity term on the outcome. The presented results directly estimate this sum by using that for each term Δa_{it}^T with coefficients β_s^T , $s = 0, 1, 2$,

$$\beta_0^T \Delta a_{it} + \beta_1^T \Delta a_{it-1} + \beta_2^T \Delta a_{it-2} = \sum_{s=0}^2 \beta_s^T \Delta a_{it} + \beta_1^T (\Delta a_{it-1}^T - \Delta a_{it}^T) + \beta_2^T (\Delta a_{it-2}^T - \Delta a_{it}^T).$$

2.5.2 RESULTS

INDUSTRY-LEVEL EMPLOYMENT GROWTH IS POSITIVELY BUT WEAKLY RELATED TO PRODUCTIVITY GROWTH

Focusing on industry-level quantities, results show that the association between productivity growth and employment growth remains positive, albeit weaker (both in terms of magnitude and significance) than what was found at the firm level. This is evident both from Table 2.10, displaying results from the partial correlation model, and from Figure 2.2 illustrating the results from the dynamic IRF model. The former reports a positive but not statistically significant association between changes in aggregate productivity and consequent changes in industry employment, net of other factors which are accounted for. The latter confirms this result by showing that in the IRF model the link between productivity growth and employment growth is positive and statistically significant, but still quantitatively lower than the coefficient estimated at the firm level (cf. Figure 2.1). Notably, in contrast to the negative own-industry link of productivity growth related to cross-country technology trends (e.g. Autor and Salomons, 2018), these results suggest that the relationship of overall productivity growth to own-industry employment is not negative but, if anything, weakly positive.

This weaker industry-level relationship may emerge as the gains of productivity-improving

Table 2.10: The industry-level link of employment growth to productivity growth: partial correlations.

	(1) 5-year change in employment
5-year change in aggregate productivity	0.0214 (0.0215)
Initial total employment	-0.0971*** (0.0201)
Initial aggregate productivity	0.00465 (0.00718)
Observations	2,713
R-squared	0.542
Fixed effects	I C-Y

Estimates obtained from the model in Equation (2.7). I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

firms in terms of employment and sales may be in part negatively compensated at the industry level by reallocation mechanisms. Increasing employment in some firms – such as those catching up towards the top of the productivity distribution – may indeed come at the expense of a reduction in employment for other firms – such as those losing their competitiveness or exiting the market – making the *net* within-industry employment impacts more ambiguous than the within-firm ones. This mechanism is henceforth referred to as the productivity-induced job reallocation process.³²

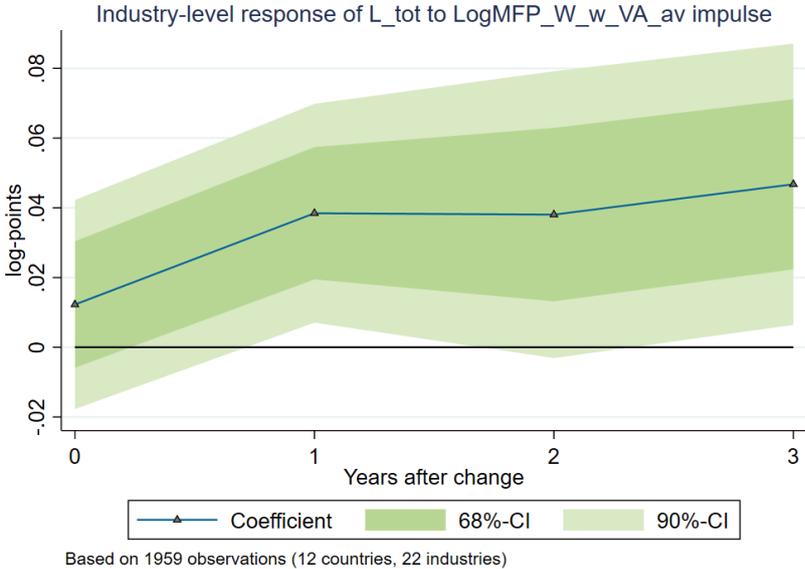
THE PRODUCTIVITY-INDUCED JOB REALLOCATION PROCESS MATTERS FOR INDUSTRY-LEVEL EMPLOYMENT GROWTH

The previous subsection has shown that, also at the industry level, the dynamics of employment are not negatively related to productivity growth. This suggests that – on average – jobs created by productivity-improving firms tend to offer sufficient counterbalance to the negative changes in employment induced by shrinking and exiting firms.

The reallocation process is key for aggregate productivity growth as it enables the reallocation of resources to more productive firms. Results presented below suggest that an efficient reallocation of resources may further be essential for the employment dynamics at the industry level. On the contrary, factors that may slow it down may also imply a less positive (or possibly

³²Table B.5 in the Appendix focuses on a slightly modified firm-level model that regresses long-term job reallocation on an initial one-year change in the productivity distribution to account for the dynamic nature of the nexus in a reduced-form way, shows that a larger degree of dispersion in productivity growth is positively correlated with different measures of job reallocation. These results underscore the relevance of the productivity-induced job reallocation process.

Figure 2.2: The industry-level link of employment growth to productivity growth: impulse response



This figure illustrates the results of the local projection impulse response regression estimations for the industry-level response of total employment to a change in aggregate productivity, based on Equation (2.5). Observations are weighted by the number of firms represented in the full population, normalised at the country level. Confidence bands are based on pointwise estimation of standard errors, clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

negative) response of industry-level employment growth to positive productivity shocks.

The firm-level analysis has shown that firms are limited in their ability to scale their employment when they improve their productivity in environments characterised by a higher degree of markup dispersion. Columns 2 and 3 of Table 2.11 shows that this force also matters at the industry level using two complementary measures based on the difference in market power across firms, proxied by the gap between high and median markups (the measure in column 3 is continuous, and the one in column 2 is a binary variable classifying industries in high or low markup dispersion).³³ Both measures are associated with lower employment growth related to productivity changes, in that it dampens the positive response of employment which may even become negative (although only the binary measure yields a statistically significant coefficient). As highlighted also in the previous section, the contestability of markets may be key in allowing growing firms to benefit from productivity gains by expanding sales, which may, however, be prevented if gaps in market power are large. This, in turn, may contribute to reduced job creation by firms with a high potential for growth.

At the industry level, this may distort the balance between non-frontier firms where productivity growth is net job creating on the one hand, and frontier firms where own-firm em-

³³As in the previous section, the continuous markup dispersion variable measures the difference between high and median markups within a country-industry pair each year, where high markups are proxied by the 90th percentile of the within country-industry-year distribution of markups. See the previous section for further details.

Table 2.11: The role of structural determinants for the industry-level link of employment growth to productivity growth.

	(1)	(2)	(3)
	5-year change in employment	5-year change in employment	5-year change in employment
5-year change in productivity	0.0102 (0.0221)	-0.00557 (0.0247)	-0.00198 (0.0202)
* ICT investment intensity			-0.0107** (0.00531)
* Difference p90-p50 of markups	-0.0147 (0.00983)		-0.00863 (0.00920)
* 1[high difference p90-p50 of markups]		-0.106** (0.0455)	
Observations	1,966	1,998	1,966
R-squared	0.564	0.557	0.570
Fixed effects	I C-Y	I C-Y	I C-Y
Heterogeneity variable(s) controlled	yes	yes	yes
Countries excluded	CAN, CHL, FRA	CAN, CHL, FRA	CAN, CHL, FRA

Estimates obtained from a heterogeneous effects extension of the model in Equation (2.7). I and C-Y indicate fixed effects for the industry and country-year, respectively. In analogy to the baseline model (cf. Table 2.10, Equation (2.7)), the estimated models control the initial levels of aggregate productivity and employment, and the 1-year employment change occurring contemporaneously to the productivity change; coefficients are omitted from the table for brevity. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

ployment implications of productivity growth may be weaker than the induced job destruction at other firms in the same industry on the other hand. This may possibly explain the observed industry-level pattern in case of higher asymmetries in market power.

Column 3 of Table 2.11 confirms that the measures of differences in markups used here capture a distinct association with respect to the one captured by the ICT intensity of industries. The coefficient for the differential relationship of productivity growth with employment growth according to the markup difference in industries (row 3) is virtually unchanged even when including the ICT intensity of the industry as a further heterogeneity variable. This distinction is important since ICTs are also structurally relevant for the productivity-employment nexus as shown before, and existing research shows that markups have been higher in ICT-intensive environments especially more recently (Calligaris, Criscuolo and Marcolin, 2018).

THE TYPE OF TECHNOLOGY AND THE SCOPE FOR INCREASING SALES DETERMINE THE STRENGTH OF DIRECT AND INDIRECT MECHANISMS

The responsiveness of employment to productivity is affected by the balance between a direct negative mechanism related to efficiency and labour replacement, and an indirect positive mechanism related to changes in demand through sales. While the previous section has investigated these mechanisms in detail at the firm level, their relevance at the industry level is now

further investigated focusing on the different responses of employment to productivity when its change is driven by different factors.

Table 2.12: The role of the source of productivity growth for the industry-level link of employment growth to productivity growth: instrumental variable estimates.

	(1) 5-year change in employment	(2) 5-year change in employment
5-year change in productivity	-0.206* (0.106)	0.353*** (0.134)
Initial employment	-0.0953*** (0.0122)	-0.113*** (0.0141)
Initial aggregate productivity	-0.00556 (0.00767)	0.0224*** (0.00829)
Observations	2,310	2,617
R-squared	0.389	0.358
IV	innovation (AI/frontier)	Δ trade exposure
F stage 1	16.7	11.6
Fixed effects	I C-Y	I C-Y
Countries excluded	HRV, JPN	JPN

Estimates obtained from the model in Equation (2.8). I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Taken in isolation, the component of productivity growth more strongly related to automation and frontier technological progress tends to be negatively correlated with employment growth over the studied period, in which technological progress was significantly related to innovations allowing capital to perform workplace tasks. This is shown in column 1 of Table 2.12, in which productivity growth is proxied by productivity changes at the global frontier together with the change in AI patent intensity across countries and years (using the instrumental variable approach of Equation (2.8)).³⁴ This proxy based on the dynamics of the global frontier and of AI patenting aims at capturing the part of productivity growth which is driven to a larger extent by innovation in technologies with a stronger labour-saving component. The negative and significant coefficient of “5-year change in productivity” in column 1 indicates that these sources of productivity growth may be associated with a relatively slower employment growth at the industry level.³⁵ This relation does not however take into consideration other more labour-enhancing sources of productivity growth and the role of inter-industry

³⁴More specifically, the instrument is constructed as the product of five-year productivity growth at the global frontier for each industry (computed from the ORBIS dataset) and the change in annually filed AI patents (computed from the PATSAT dataset) normalised by millions of employees (computed from the STAN dataset), in each country.

³⁵This finding is consistent with the direct negative own-industry effect found by Autor and Salomons (2018).

linkages, which will be discussed in the next subsection.

The differentially negative link between productivity growth and changes in industry-level employment in environments where recent technology trends are more relevant is also evident from the results in Table 2.13, which shows that 5-year productivity changes are more negatively correlated with 5-year employment changes in industries that are investing more in ICTs.

Still, this more labour-saving source of productivity growth does not necessarily generate aggregate net job losses at the industry level if the compensating indirect mechanisms related to sales expansion are sufficiently strong. To this end, evidence suggests that productivity growth driven by factors more related to the possibility of a market expansion is indeed positively and significantly associated with employment growth. This is illustrated in column 2 of Table 2.12, which links employment growth with productivity growth related to a change in trade exposure – measured through the interaction of global value chain linkages and a country-year measure of change in trade exposure.

Table 2.13: The role of recent technology trends for the industry-level link of employment growth to productivity growth.

	(1) 5-year change in employment
5-year change in productivity	0.0110 (0.0188)
* ICT investment intensity	-0.0112** (0.00432)
Initial employment	-0.0958*** (0.0200)
Initial aggregate productivity	0.00750 (0.00722)
Observations	2,713
R-squared	0.548
Fixed effects	I C-Y

Estimates obtained from a heterogeneous effects extension of the model in Equation 2.7. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

The internationalisation of industries through higher exports and connection to global value chains may be associated with stronger productivity growth, through higher market selection and competition. At the same time, such productivity gains occur in a context in which firms may be able to expand sales through international trade, possibly overcoming limitations related to limited domestic market size and business stealing phenomena. Furthermore, as in-

industries are interconnected through global value chains, efficiency gains in one industry may boost sales and employment in other industries. The role of globalisation through connection to value chains is explored further below.

The industry-level baseline relationship and the patterns by markup dispersion and ICT intensity are also found when considering different measures of productivity, as shown in the Appendix in Table B.3 for labour productivity and Table B.4 for gross output multifactor productivity estimated using the approach of Akerberg, Caves and Frazer (2015) that takes intermediates as a production input into account.

INDUSTRY-LEVEL PRODUCTIVITY GROWTH ALSO AUGMENTS WAGE GROWTH

Productivity growth may impact labour markets and thereby welfare, in addition to the quantity of work studied thus far, through its role for the quality of work. To this end, wages and in particular industry-level wage dynamics are an important proxy that are examined below.

Table 2.14: The industry-level link of wage growth to productivity growth: partial correlations.

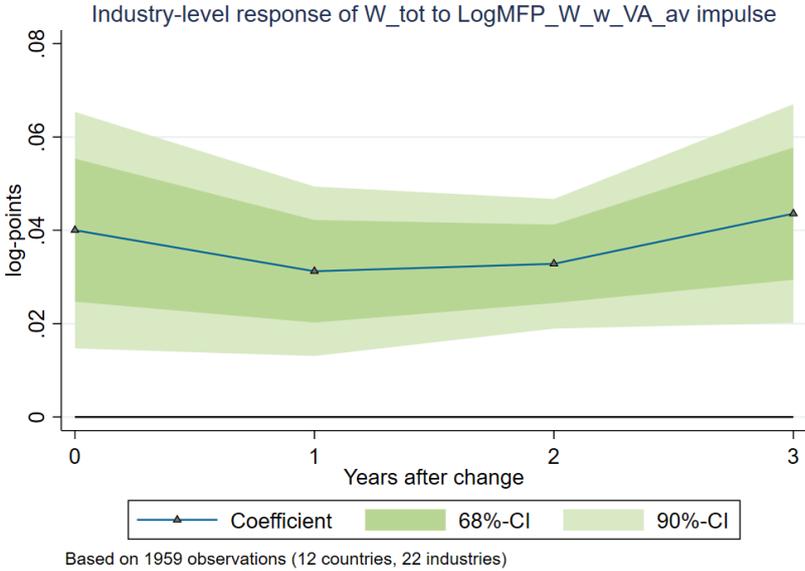
	(1) 5-year change in av. wage
5-year change in aggregate productivity	0.0312*** (0.0100)
Initial av. worker wage	-0.154*** (0.0164)
Initial aggregate productivity	0.00496 (0.00384)
Observations	2,713
R-squared	0.879
Fixed effects	I C-Y

Estimates obtained from the model in Equation 2.7. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors’ calculations based on the OECD MultiProd 2.0 Database

The evidence indicates that productivity growth may lead to differentially positive changes in the wage of the average worker of the industry, and policies addressing the productivity slowdown may therefore also be able to address sluggish wage growth. As Table 2.14 shows, five-year productivity growth is positively associated with wage growth over the same period.

Turning to the dynamic pattern, Figure 2.3 shows the estimation results from the IRF model. The response of wages to productivity growth occurs largely contemporaneously, and wages remain persistently elevated after an initial increase in productivity over the subsequent years.

Figure 2.3: The industry-level link of employment growth to productivity growth: impulse response



This figure illustrates the results of the local projection impulse response regression estimations for the industry-level response of the average worker’s wage to a change in aggregate productivity, based on Equation (2.5). Observations are weighted by the number of firms represented in the full population, normalised at the country level. Confidence bands are based on pointwise estimation of standard errors, clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

The faster adjustment of wages relative to employment may be linked to different factors. First, productivity-improving firms may find it easier to increase wages than to increase employment in the short term, as labour market matching takes time. Second, and relatedly, wages may be used as a competitive tool that productivity-improving firms use to attract workers on the labour market, leading wages to adjust before employment is fully reallocated. Third, industry level productivity growth may, to some extent, reflect the exit of low productivity, low wage firms. Finally, it may also reflect the upskilling of the workforce, which also implies a change in the average wage due to higher wages earned by more qualified workers.³⁶

BETWEEN-INDUSTRY AND BETWEEN-ECONOMY SPILLOVERS

Productivity growth in an industry (relative to other industries) appears to be only moderately related to relative employment growth within the same industry, as discussed in the previous sub-section. However, aggregate outcomes related to industries’ productivity growth are not shaped only by the aggregation of industry outcomes, but also by the linkages of industries through value chains. These supplier-customer linkages may be a source of propagation of productivity spillovers on employment in connected industries.

³⁶However, this last factor is unlikely to account for a large proportion of the observed positive average relationship due to the limited role of the rate of economy-level upskilling for the variation of industry-level productivity.

Therefore, this section focuses on employment dynamics related to productivity growth in supplier and customer industries, i.e., industries for which, respectively, the originating industry is upstream or downstream in the production chain.³⁷ It uses data from the OECD Inter-Country Input-Output tables to measure the linkages among country-industry pairs available in the MultiProd database.

Table 2.15: Employment spillovers of industry-level productivity growth along the value chain.

	(1) Change in total employment	(2) Change in average employment
Change in own-industry productivity	0.0208 (0.0261)	0.00532 (0.0170)
Change in domestic supplier productivity	0.00814** (0.00411)	0.00842** (0.00363)
Change in domestic customer productivity	0.00172 (0.00251)	-0.00109 (0.00195)
Change in foreign supplier productivity	0.0114** (0.00566)	0.0107** (0.00420)
Change in foreign customer productivity	0.000393 (0.00297)	0.00124 (0.00226)
Observations	2,821	2,821
R-squared	0.442	0.394
Fixed effects	I C-Y	I C-Y

Estimates obtained from the model in Equation 2.9. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Results of the estimated model, presented in Table 2.15, provide evidence of downstream spillovers, as the positive and significant coefficients reported in the second and fourth row of Table 2.15 suggest. In other words, employment growth in a given industry is positively related to productivity growth in the supplier industries. The positive association is observed for both domestic and foreign suppliers, extending the results from Autor and Salomons (2018) of spillovers arising from domestic suppliers. It is noteworthy that the magnitude of the downstream effect (as measured by the response to a standardized shock) is similar for productivity arising from domestic and foreign suppliers.³⁸ Comparing columns 1 and 2 of Table 2.15, the change in total industry-level employment induced by downstream value chain spillovers appears similar to the one in the average firm's size. This suggests that these spillovers occur by

³⁷An industry k is considered upstream relative to an industry j if k supplies intermediate inputs to j. Industry k is the (upstream) supplier industry and industry j is the customer (downstream) industry.

³⁸Unreported results suggest that omitting the role of foreign suppliers leads to an overestimation of the spillovers arising from domestic suppliers.

facilitating the growth of existing connected firms (rather than through an extensive margin).

This positive link between the productivity growth of supplier industries (domestic and foreign) and the employment growth in customer industries may be related to the change in intermediate prices associated to supplier productivity gains Acemoglu, Akcigit and Kerr (2016). This change in intermediate prices may benefit customer industries, and allow them to raise their sales, in turn inducing this downstream effect of productivity growth.

On the contrary, results suggest that there is no significant association between productivity growth in (both domestic and foreign) customer industries and employment growth in supplier industries. This is also in line with evidence by Autor and Salomons (2018). Increased productivity in customer industries may not necessarily lead to higher demand of intermediates, as increasing sales resulting from lower prices may be partially compensated by higher efficiency, so that less input are required to produce the same level of output. Finally, in line with previous results on the own-industry relationship between productivity growth and changes in employment, the first row of Table 2.15 again identifies a positive but not statistically significant relationship between these two quantities.

Overall, the results suggest that productivity growth in upstream sectors may contribute to higher aggregate employment growth, due to the existing links across value chains. They also suggest that both domestic and foreign linkages may contribute to these spillovers across sectors.

Table 2.16: Wage spillovers of industry-level productivity growth along the value chain.

	(1) Change in average wage	(2) Change in total wage bill
Change in own-industry productivity	0.0560*** (0.0170)	0.0841*** (0.0280)
Change in domestic supplier productivity	0.00539* (0.00322)	0.0106** (0.00474)
Change in domestic customer productivity	0.00301* (0.00165)	0.00379 (0.00280)
Change in foreign supplier productivity	0.00264 (0.00364)	0.0158** (0.00639)
Change in foreign customer productivity	0.00328 (0.00208)	0.00347 (0.00344)
Observations	2,819	2,819
R-squared	0.967	0.912
Fixed effects	I C-Y	I C-Y

Estimates obtained from the model in Equation 2.9. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Productivity growth in connected industries may not only affect employment in a given industry, but also the wages that workers are earning in it. Column 1 of Table 2.16 shows that both upstream and downstream productivity growth in domestic industries appear to have positive value chain spillovers on the average wage in a given industry. For the corresponding terms related to foreign productivity growth, the point estimates are also positive and indicate a similar pattern, but estimation uncertainty disables to draw any definitive conclusion. Furthermore, consistent with previous results (columns 2 and 3 of Table 2.14, Figure 2.3), the first row of this result shows that wage growth of a given industry depends strongly on own-industry productivity growth. Taken together, these results imply a strong and positive role of productivity growth for economy-level wage growth.

This result is also emphasised by column 2 of Table 2.16, which shows the link of productivity growth to the wage bill, i.e., the product of total employment and the average wage at the industry. The total wage bill is strongly and positively related to own-industry productivity growth through the previously discussed link productivity with average wages. The total wage bill is also further related to supplier productivity growth which matter strongly for employment growth, as discussed above. Productivity growth in customer industries also tends to increase the total wage bill, but to a lower extent than productivity growth arising from supplier industries.

Putting together the conclusions of this section, productivity growth does not appear to have a direct negative impact on employment changes at more aggregate levels. Instead, if anything, when considering both within-industry and between-industry patterns, productivity changes appear on average positively related to employment growth at the more aggregate level. This suggests that productivity changes may not only benefit firms that experience those, but they have also positive implications for more aggregate outcomes. The results also highlight that productivity growth benefits workers through higher wages within the same industry, and further through value chain spillovers. Productivity growth has the potential to increase own-industry employment if markets are sufficiently contestable – in which case the amount of job creation associated with the productivity-induced reallocation process may more than offset the negative impacts on shrinking and exiting firms, and the role of productivity growth related to efficiency gains and possibly labour-saving technological change – and if demand is sufficiently elastic, i.e. if industry output can expand strongly in response to productivity growth.

2.6 DISCUSSION

Analysing the link between productivity growth and changes in labour demand is key to better understand the extent to which technological progresses and organisational changes are linked to labour market outcomes. The data collected in the context of the OECD MultiProd project have allowed carrying out a uniquely comprehensive investigation of this relation. Indeed, for the first time this study relies on highly representative official data from several advanced economies that allow focusing on different levels of aggregation with unprecedented detail.

On average, productivity growth is positively associated with employment growth. This net positive relation holds at different levels of aggregation and suggests that boosting productivity is also key for employment. Increasing productivity is also associated with higher wages, further highlighting that productivity-enhancing policies are likely to bring double dividends for other social outcomes.

The analysis has not directly focused on investigating the causality of the productivity-employment link, and the empirical results reflect first and foremost partial correlations. However, the analysis has put forward some hypotheses about causal relationships and mechanisms underlying the estimated correlations, and presented several follow-up analyses that provided further correlation analysis the results of which were consistent with the respective hypothesis of interest. Therefore, while not providing direct causal evidence, the analysis has successfully tested different necessary conditions and relationships that would emerge from the hypothesised phenomena.

The evidence presented suggests that the overall net positive link between productivity and employment is the outcome of counteracting mechanisms. At the firm level, the net relationship depends on direct labour-saving effects – related to efficiency and possibly further to automation and other labour-replacing technologies – and indirect labour-creating effects – related to higher demand and expansion in market shares. The positive link among productivity and employment growth found in the analysis suggests that on average the latter (positive indirect effect) tends to overcompensate the former (direct negative one). Furthermore, the firm-level role of productivity for employment depends on a firm's position in the within-industry productivity distribution, that is, its initial productivity performance relative to other firms in the same industry. While leading firms at the frontier of the productivity distribution experience on average higher employment growth, less productive firms that catch up towards the frontier relevantly experience stronger employment growth than other firms, after accounting for initial differences in productivity. Even when direct negative effects are likely to be stronger, such as in industries that use more intensively ICTs, the link between productivity and employment growth remains positive, although milder. Productivity growth also increases the chances of firm survival, with positive implications for labour demand.

A positive relation at the firm level may not necessarily mean that productivity growth translates into employment growth to the same extent at more aggregate levels. Within the industry in which productivity growth occurs, the induced response of employment is found to be positive but weaker than the one at the firm level. This may be due to the circumstance that increasing employment in some firms may come at the expense of a reduction in employment for other firms, such as those losing their competitiveness or exiting the market.

Less contestable markets, reflected by asymmetries in market power, may limit the ability of firms catching up in productivity – which are those with a high potential for growth – to expand sales and grow in employment. This force is associated with a differentially negative link of productivity growth and changes in employment both at the firm and industry level.

In addition, productivity gains at the industry level contribute to stronger employment growth in other industries through (global) value chains. In particular, productivity growth in upstream industries is positively associated with employment growth in downstream industries. This result is consistent with the idea that upstream productivity growth reflects a positive supply side shock on the market for intermediates, and that downstream industries respond to a decline in the (quality-adjusted) price of one factor of production with an increase in the level of all factors due to input complementarity.

Summing up, the analysis has provided evidence for several key mechanisms that determine the employment impact of a positive shock to productivity growth, which can be summarised in the stylised equation

$$\beta^L = (-D + (1 - r)N + V)s_j. \quad (2.10)$$

Here, s_j is the share of firms in industry j that increase their productivity. D is the direct negative impact of productivity growth on employment at constant output, related to increased efficiency and, depending on the source of productivity growth, also capital-labour substitution at the task level. N is the amount of job creation at productivity-improving firms associated with sales expansion and an increase in the market share.³⁹ $-rN$ is the amount of employment replaced at competitor firms that do not grow in productivity, which is larger if productivity-improving firms create more jobs by increasing their market share (i.e., higher N), and if a given job creation replaces competitor employment at a faster rate r , e.g. if the industry-level elasticity of demand is lower. Finally, V is the amount of job creation through value chains which operates in isolation of the aforementioned within-industry forces,⁴⁰ which is larger the

³⁹Notably, N isolates the (unambiguously positive) output dimension and does not consider job losses related to efficiency and task-level replacement of labour, which is accounted for by (the unambiguously negative component) $-D$. If a firm does not change its employment after an increase in productivity, then $N = D$.

⁴⁰There may also be value chain links within an industry. However, these may be already reflected in the rate r

more upstream the considered industry is in the value chain.

While overlooking relevant heterogeneity of firms and industries, Equation (2.10) may be well-suited to describe the average link of productivity growth and employment. First, it highlights that the employment-reducing force $-D$ operates in isolation of the compensating positive mechanisms. Second, it illustrates that the industry level relationship, $(-D + (1 - r)N)s_j$ should always be unambiguously smaller than the firm-level one, $(N - D)s_j$, emphasising that relative to the firm level, the industry level adds one unambiguously negative additional mechanism, namely the one of externalities of sales growth on competitor firms. In this view, the fact that the industry-level relationship is found to be close to zero but positive on average implies that replacement is only partial, i.e. $r < 1$, such that $-D + (1 - r)N \approx 0$. Third, it highlights the role of the competitive environment and the contestability of markets. If firms can expand sales more after an improvement in their relative productivity performance, they also create more jobs (higher N), and due to incomplete replacement of competitor employment, this also translates into a more positive industry level relationship.

Among others, the firm-level insights of this paper speak to the debate on micro-level effects of automation technology adoption. If adopting firms are broadly spread along the within-industry productivity distribution and operate in a sufficiently competitive environment, the task-level replacement of labour may be more than offset by an increase in sales associated with a productivity improvement over competitors. In this case, the resulting employment effect would be strictly positive, as found for example in Aghion et al. (2020). Conversely, if technology adoption is more concentrated among frontier firms and/or competition is low, the compensating output channel may be weakened. Such environments may generate a negative link of automation and firm-level employment, as found in Bonfiglioli et al. (2020).⁴¹

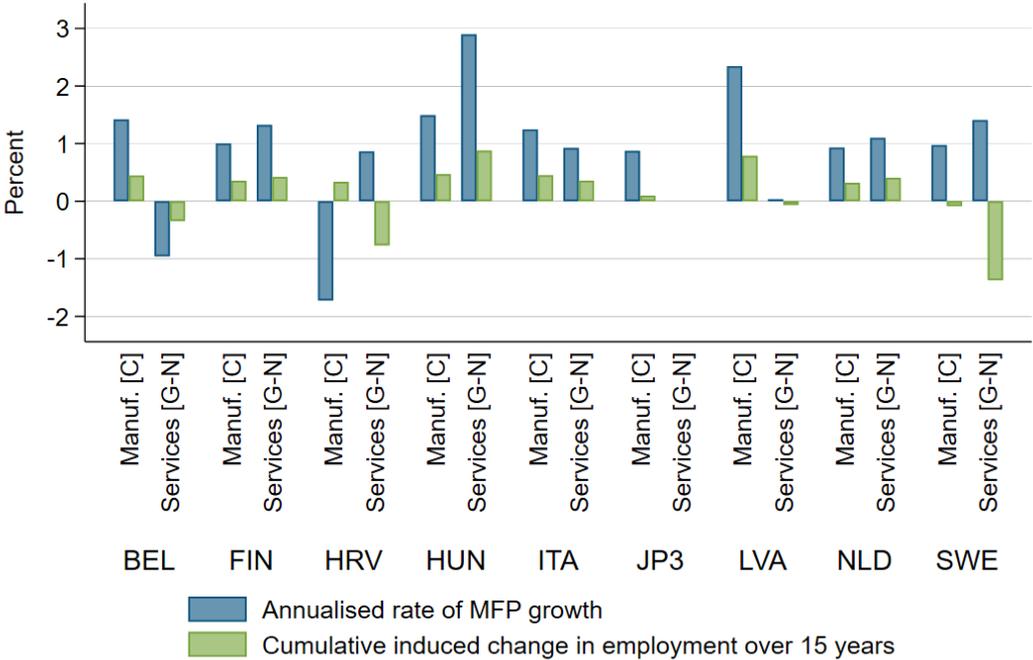
The role productivity growth has played for employment over the sample period has differed across countries. Countries have experienced different rates of productivity growth at the firm and industry level, and a given change in productivity may have translated differently into employment, among others due to differences in the contestability of markets and the structure of value chains across countries. This aspect is illustrated in Figures 2.4 and 2.5.

First, Figure 2.4 calculates the cumulative change in country-level employment over 15 years (corresponding roughly to the sample horizon) associated with productivity growth over the sample period, based on the estimates of column 2 in Table 2.11 that allows the link of

of replacement of competitor employment, which is lower if productivity-improving firms create employment at other firms within the industry.

⁴¹Notably, Bonfiglioli et al. (2020) rely on a measure of robot imports, and describe that firms which import robots are larger and more productive. Moreover, they find robot importing to have a negligible effect on sales at these firms, although they document a positive productivity impact. As such, their evidence may indeed describe technology adoption at dominant productivity leaders with the ability to contract output.

Figure 2.4: Industry-level changes in productivity and employment: the relationship by country

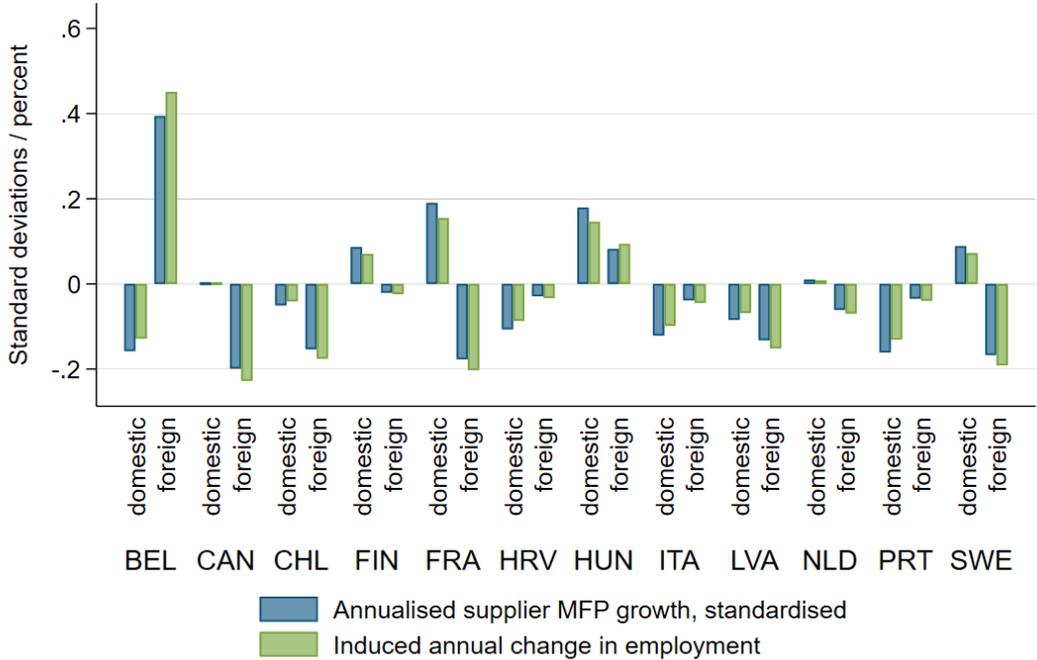


This figure illustrates the included change in employment over the sample period associated with changes in productivity in a given industry, taking into account differences in the contestability of markets across countries, and across industries within countries. The figure augments productivity growth by the coefficients of column 2 of Table 2.11 to assess the associated change in employment within the country-industry-year, and aggregates these contributions to the country-year-one letter sector level using the industry’s share in value added of the country-year as weight. The bars represent unweighted averages across one letter sector-years within the country. Source: Calculations based on OECD MultiProd 2.0 database.

changes in productivity and employment to be heterogeneous according to the degree of the contestability of markets. There are some instances of strong productivity growth that appears to have translated into employment growth even within industries due to a relatively high degree of contestability in originating industries, for instance services in Hungary or manufacturing in Latvia. Conversely, low contestability appears to have prevented productivity growth in Japanese manufacturing to translate into employment growth, and may have even been linked to sizable reductions in employment in Swedish service industries.

Overall, however, the insights from Figure 2.4 illustrate again that the own-industry relationship between productivity growth and changes in employment is quantitatively limited in most cases. To further assess the relevance of productivity growth for employment at the country level, Figure 2.5 therefore calculates the annualised change in employment associated with upstream productivity growth, which was identified as a key potential source of positive employment spillovers across industries in the analysis. Strikingly, supplier productivity appears to have been *declining* or at best stagnant in most countries, with a few exceptions (Finland, France, Hungary and Sweden). In consequence, value chain spillovers seem to have

Figure 2.5: Value chain spillovers of productivity growth on employment: the relationship by country



This figure illustrates the included annual rate of change in employment associated with changes in productivity in upstream industries. The figure augments productivity growth by the coefficients of column 1 of Table 2.15 to assess the associated change in employment within the country-industry-year, and aggregates these contributions to the country-year level using the industry’s share in value added of the country-year as weight. The bars represent unweighted averages across years within the country. Source: Calculations based on OECD MultiProd 2.0 database.

driven employment in connected industries down more frequently than up, both domestically and abroad.⁴² Notably, as can also be seen from Figure 2.4 declines in productivity are not a feature of the average industry, neither in manufacturing nor in services. Therefore, this phenomenon appears to be driven by only a few industries that however have a very high degree of upstreamness. This consideration emphasises that targeted support of upstream industries may possibly be a useful tool in addressing the slowdown in productivity growth that may also have sizeable double dividends in terms of employment.

Well-designed complementary policies have the potential to boost both productivity and employment. To achieve both objectives, a multi-pronged policy approach may aim at enhancing productivity, and promoting the conditions that help translate technological and organisational change into higher employment and wages, also taking into account the fact that gains and losses may contribute differently to welfare and inclusiveness. Policies may thus be articulated around three complementary goals: i) enhancing productivity; ii) providing the condi-

⁴²A notable exception is the positive impact of productivity growth at foreign suppliers on employment in Belgium. This may likely be related to Belgium’s strong reliance on French inputs, where supplier productivity appears to have grown quite steadily. In line with this view, unreported results confirm that the growth of supplier productivity in France is more strongly by the tradeable manufacturing sector.

tions for productivity growth to translate into employment and wage growth; iii) ensuring inclusiveness of productivity growth and the associated change in labour demand. Several policy areas can help achieve these goals: fostering innovation and diffusion; preserving a competitive environment and the reallocation process; consolidating the integration into global value chains; ensuring inclusiveness, as technological progress and organisational changes may result in both gains and losses in terms of employment and wages; supporting demand. An extensive discussion of relevant policy levers is given in Appendix B.2.

2.7 CONCLUSION

This paper investigates how productivity growth relates to the dynamics of employment and wages at different levels of aggregation and over different countries, industries, and time horizons. The micro-aggregated data collected in the context of the OECD MultiProd project, which collects highly representative official data from several advanced economies for the period 2000-2018, enables a uniquely comprehensive investigation of the relation between productivity growth and the dynamics of employment and wages.

These data allow to study the productivity-employment nexus by focusing on different levels of aggregation, looking at *(i)* the link between firm-level productivity growth and firm-level employment (and wage) growth, as well as the link with firm survival: *(ii)* the link between industry-level productivity and employment growth in the same industry and *(iii)* the link between productivity growth in some industries and employment growth in other industries through value chain connections.

Overall, results point to a positive link between productivity growth and both employment and wages growth. This net positive relation holds at different levels of aggregation and suggests that boosting productivity is also key for employment. However, the strength of the link varies according to the level of aggregation considered.

Focusing on within-firm growth, results show a net positive and significant productivity-employment nexus, which is the outcome of mechanisms that act in opposite directions. At the firm level, an indirect labour-creating effect – related to higher demand and expansion in sales – seems to prevail on a direct labour-saving effect – related to efficiency and possibly further to automation and other labour-replacing technologies. Moreover, the extent to which firms' productivity and its growth translate into positive employment changes, although positive on average, is not the same for all firms. While leading firms at the frontier of the productivity distribution experience on average higher employment growth, after accounting for initial productivity, less productive firms that catch up towards the frontier relevantly experience stronger employment growth than other firms.

When looking at the link between productivity growth and changes in employment and wages at the industry level, the analysis finds again a positive correlation, although weaker than at the firm-level. Beyond the direct and indirect mechanisms described before, the industry-level link is relevantly shaped by a reallocation process, which implies that job creation among expanding firms tends to compensate decreasing employment in shrinking or exiting ones. Furthermore, less contestable markets, as signaled by highly asymmetric market power, appear to be a factor that may slow down the positive link between productivity growth and changes employment, as they represent an impediment to the aforementioned reallocation process. Finally, productivity growth in upstream industries is positively associated with employment growth in downstream industries, corroborating the idea that productivity gains are on average labour-enhancing not only at the firm level but also at the more aggregate one.

Overall, this evidence suggests that boosting productivity is not only a key standalone economic objective, but may also contribute to welfare through its role for employment and wages. The fact that productivity growth is on average accompanied with employment and wage growth across different levels of aggregation implies that labour demand and productivity represent complementary rather than alternative policy targets.

This paper has leveraged the MultiProd database to provide an analysis of productivity-employment that shed lights on the relevant mechanisms at play from a complementary micro- and macroeconomic perspective. The analysis could be extended in several ways that focus more in detail on specific mechanisms.

First, as the analysis showed that both at the firm level and at the industry level an indirect labour-creating effect prevails over a direct labour-saving one, it would be interesting to further explore more directly the relative importance of some of the mechanisms at play. Those include variation in prices, variation in sales, and increases in different types of efficiency (technology, managerial efficiency, etc.). This is challenging from an international perspective, but additional insights may arise from the matching of micro-economic databases available in single countries. Second, this paper has highlighted the importance of reallocation and market contestability for the productivity-employment nexus. On the one hand, defining markets is challenging and additional work may refine the analysis providing complementary insights. On the other hand, further empirical analysis of the role of policies in other areas – within or across countries – focusing not only on their impact on employment and productivity separately, but on their role for both outcomes, may provide additional policy-relevant insights.

Third, the role of structural change may be explored more directly, focusing on how long-term shifts in sectoral compositions may affect the productivity-employment nexus in the long run. This would require longer time series than the ones used in this work, which may be

available in the future. Fourth, future work may dissect the weak performance of upstream industries, in particular focused on catalysing factors in the structural and policy environment. Better understanding this aspect promises relevant insights on policymakers' ability to boost job creation through a targeted approach relying on value chain spillovers. Finally, future work may further explore the extent to which productivity changes are associated with changes among specific groups of workers, measuring more directly their skills, or focusing further on workers at disadvantage at labour markets, using linked employer-employee data.

3. HOW DOES AUTOMATION AFFECT LABOUR DEMAND? INSIGHTS FROM A MODEL WITH WORKER-LEVEL TASK AGGREGATION

Abstract. Understanding and directing the impact of automation technologies on labour demand continues to be an issue of great interest to economic discussions. I investigate the previously neglected role of workers as task-aggregating institutions. Whenever there is some complementarity between automatable and non-automatable tasks, automation necessarily increases workers' effective productivity, i.e. productivity net of capital cost. This rationalises the positive micro-level relationship of automation on labour demand that prevails despite the task-replacing nature of automation. At higher levels of aggregation, possible declines of labour demand are driven by output market effects. I identify a close link of the impact of automation on labour demand and on labour shares: the relationship of automation and both quantities is more positive the more complementary tasks are. As workers' propensity to choose complementarity-intensive occupations is inversely related to their degree of specialization in individual tasks, education policy should promote general-purpose skills that support broad worker qualification across tasks. The pace of productivity growth accompanying automation is higher in low-complementarity environments, and there may be a policy trade-off between strong productivity growth and stable labour demand.

3.1 INTRODUCTION

In past decades, the ever-increasing role digital technologies play in our economies, and in society more generally, has attracted broad interest. More recently, with regards to the labour market, great attention has been drawn to the future brought about by a continuation of the digital transformation. Academic work suggests that in scenarios with no or insufficient counterbalancing mechanisms for labour demand, in particular related to the creation of new tasks for labour as existing ones become replaced, automation may indeed be a threat to employment in the long run, but also discuss the possibility that markets may create these counterbalancing mechanisms endogenously (Acemoglu and Restrepo, 2018b). Outside the academic context, the picture painted is often rather dystopian, with fears of mass-unemployment as brilliant technologies render human labour redundant (e.g. Brynjolfsson and McAfee, 2014).

An aspect that economic theory has so far paid little attention to is worker-level complementarities between tasks. However, in real-world labour markets, the multiple tasks workers perform may indeed be very complementary. For instance, if a financial consultant's core tasks are to analyse financial assets and to advise customers on investment strategies, then this job's productivity crucially depends on the combination of both tasks: without knowledge of the quality of assets a consultant will give bad advice, and without advising, the knowledge about the first cannot be put to productive use.

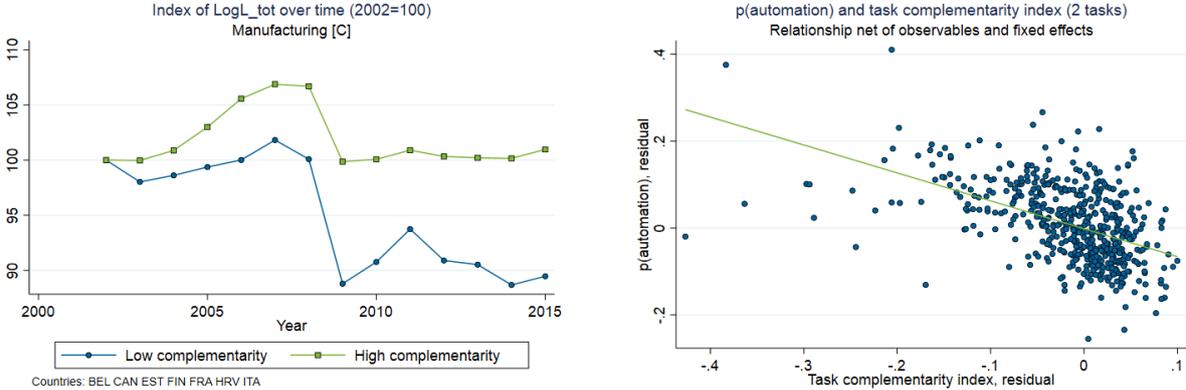
When considering workers as *task-aggregating institutions* who may engage in more than one productive task, the response of labour demand to task-level automation is not trivial, but instead involves a trade-off between task-level replacement of labour on the one hand, and a boost to capital-labour complementarity on the other. Against the popular perception that workers' exposure to task-level automation unavoidably threatens their employment, I show that the latter force usually dominates at the micro level, and may do so even at higher levels of aggregation, including occupations and industries. This result resonates well with the popular example of Bessen (2015) who documents increased bank teller employment in response to the introduction of ATMs, a technology that automated one of the occupation's core tasks, but also freed up the resources of workers to engage in different, and indeed more productive tasks.

This notion is also supported by the patterns shown in Figure 3.1. Panel 3.1a shows trends in manufacturing employment for an average of 7 OECD economies by an index of task complementarity, and demonstrates that low-complementarity industries have been on a differentially lower employment trend in recent years.¹ Panel 3.1b, at the occupation level, shows

¹Within services, the trends of high- and low-complementarity industries are on average similar. However, consistent with the notion of complementarities mitigating adverse impacts of automation on employment, the *between*-sector comparison shows that services occupations are on average much more complementarity-intensive than manufacturing occupations, and have been on a differentially positive employment trend.

the relationship between the task complementarity index and the projected exposure to automation over two decades after 2013. The depicted partial relationship, measured net of key worker characteristics and country-industry specificities, suggests a negative link between complementarity-intensity and automation’s threat to occupations.

Figure 3.1: Task-level complementarities and past and future exposure to automation.



(a) Average employment trends, unweighted across countries. High complementarity = above-median task-complementarity among SNA A38 manufacturing industries.

(b) Controls: education (3 categories), gender (2) and age (5); country times SNA A38 industry fixed effects. Worker-level observations aggregated to 4-digit ISCO-08 occupations. Exposure to (“probability of”) automation based on Arntz, Gregory and Zierahn (2016).

Note: The index of task complementarity is defined as the share of time spent by workers outside the two most frequent tasks, based on a vector of 25 tasks measured in the OECD’s PIAAC survey of adult skills. Panel (a) aggregates this index across employees to the level of the SNA A38-industry, and splits manufacturing industries into groups above and below the median industry. The dynamics of employment are computed using the OECD’s MultiProd v2 database, and reflect a cross-country average across the 7 countries that are available both in MultiProd and PIAAC (Belgium, Canada, Croatia, Estonia, Finland, France, and Italy). SNA A38 industries within manufacturing are aggregated to the country-task complementarity group level using total employment as weight, and the average is unweighted across countries.

Source: Own calculations based on the OECD’s PIAAC and MultiProd v2 databases.

To study the link of worker-level task complementarities to the labour demand impact of task-level automation, I develop a partial-equilibrium labour market model of task-level capital-labour substitution that features workers as task-aggregating institutions. The model abstracts from the relatively well-understood implications of automation at higher levels of aggregation such as between-industry shifts of labour demand and aggregate demand effects, and studies the process of automation at the worker-firm match level within an industry. In this industry, workers differ with respect to their task-skill profile, and firms and occupations differ in the weight of tasks and the degree of complementarity between them. Moreover, my analysis explicitly distinguishes between technology adoption at the extensive and intensive margin, i.e. the introduction of new vs. the improvement of existing automation technologies.

My analysis emphasises the power of complementarities in softening or even reversing the occupation- and industry-level labour demand impact of task-level automation. I identify a shift of *relative* labour demand away from worker types specialised in automatable tasks. However, in environments with perfectly elastic output demand, automation alone does not generate an *absolute* decline in labour demand (for any skill type), as micro-level technology adoption necessarily requires an increase in the marginal product of labour, and therefore labour demand. If product demand is less elastic, output increases less in response to efficiency-enhancing developments such as automation, and the change in the demand for all inputs, including labour, is therefore differentially lower. If product demand is sufficiently inelastic, labour demand may thus decrease with automation. The output market origin of adverse labour demand impacts has two key implications: (i) automation may significantly affect all workers within industries in which automation occurs, not just specialists in the automated tasks, and (ii) increasing demand elasticities, e.g. through competition policy, can soften the adverse employment impact of automation technologies.

At the occupation level, the labour demand impact of automation is proportional to the one on labour shares. Labour shares and labour demand decline more strongly with automation the more substitutive tasks are, especially with technology adoption at the intensive margin, that is, with improvements in automation technologies that are already in use. In these environments, productivity also grows faster with automation. This finding cautions that an excessive policy focus on productivity could jeopardise the long-run stability of labour demand, and could further undermine the labour market's strength as a distributive mechanism, creating a scenario where most workers do not benefit or even suffer from productivity growth.

Moreover, this insight, derived from a rich view on the micro-level impact of automation, invites to revisit the view that employment is especially threatened by “so-so technologies” (those that can replace labour market tasks but offer little to no productivity gain at the task level) rather than “brilliant technologies” (those that significantly raise task-level productivity when replacing labour market tasks) that have been argued to generate sizeable employment-reinstating productivity effects at the economy level (Acemoglu and Restrepo, 2018a,b). In environments with complementary tasks, the own-industry impact of automation is not trivial, and brilliant technologies do not only reduce labour demand more in the same industry, but they may also occur more frequently in environments with lower complementarities where productivity is more responsive to automation. Conversely, a technology that is “so-so” on the task level can be brilliant at the job level if it enables workers to move to more productive tasks (recall Bessen's bank teller example).

The results of the analysis are relevant also to education policy and can guide the optimal

education choice of prospective workers. Previous work suggests or implies that education policy should focus on “bottleneck skills” which are difficult to automate in a foreseeable future (e.g. Nedelkoska and Quintini, 2018; Acemoglu and Restrepo, 2018b). This recommendation suffers from the practical issue of having to predict today which tasks will be non-automatable over a decades-long career – this is a difficult task given the rapid and multi-directional advances in artificial intelligence and machine learning that are still ongoing. My analysis suggests instead that workers may also be well-shielded from negative exposure to automation through proficiency in general purpose skills that imply capabilities across a broad range of tasks, such as e.g. literacy, numeracy, social skills, flexibility and the ability to learn. These educational profiles offer a double dividend: not only do workers with such profiles gravitate naturally to more complementarity-intensive occupations where the impact of automation is not felt as harshly (or is even welcomed), but they may also facilitate transitions between industries, and therefore especially transitions away from heavily automating industries where overall labour demand may decline.

The paper is organised as follows. Section 3.2 reviews the related literature. Section 3.3 introduces the theoretical framework and gives studies the equilibrium that arises in the absence of any automation. Section 3.4 studies the dynamic process of technology penetration when the quality of automation technology increases continuously. Section 3.5 concludes.

3.2 RELATED LITERATURE

Both the theoretical and empirical economic literature has taken great interest in the labour market implications of recent technological progress. Acemoglu and Autor (2011) provide an excellent survey of the earlier literature. In explaining post-1980 trends of wages and employment and, in particular, wage polarisation, i.e., the decline of middle skill wages in co-occurrence with rising low- and especially high-skill wages researchers have argued for a skill-bias (Katz and Murphy, 1992), and later a task-bias of technological change (Autor, Levy and Murnane, 2003; Autor, 2013). The task biased view of technological change rationalises the central features of wage polarization through the emergence of task-replacing technologies that are particularly capable of automating routine-intensive tasks. It is commonly viewed as one of, if not the most important explanation of wage polarization (e.g. Goos, Manning and Salomons, 2014; Michaels, Natraj and van Reenen, 2014; Autor, Dorn and Hanson, 2015).²

²Beyond task-biased technological change, empirical studies of the US economy attribute a significant proportion of the decline in both middle skill wages and the manufacturing sector to international trade, in particular to rising Chinese import competition (Autor, Dorn and Hanson, 2013; Autor et al., 2014). Similar findings exist also for other developed economies (e.g. Dauth, Findeisen and Suedekum, 2018, for Germany). Autor, Dorn and Hanson (2016) includes a comprehensive overview of the literature on labour market effects of Chinese import competition. On the other hand, offshoring has typically been ascribed only a minor role as an independent driver of wage polarization (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Goos, Manning and Salomons, 2014).

Therefore, I consider it essential to perform my analysis through the lens of task-replacing technological change, in line with recent theoretical studies on the link between automation and labour market outcomes (e.g. Acemoglu and Restrepo, 2018b).

More recently, digital technologies, especially robotics and artificial intelligence (AI), are viewed not only as a driver of labour market inequality, but also as a potential threat to employment. As countless examples demonstrate, the range of automatable tasks today, and possibly even more so in the future, extends well into the realm of “non-routine” tasks.³ These trends inspired dreary predictions about labour’s future competitiveness and boosted fear that a wave of technological unemployment may be near (Frey and Osborne, 2013; Brynjolfsson and McAfee, 2014; Autor, 2015; PEW Research Center, 2017).⁴ However, this “alarmist” view of technology typically neglects counterbalancing general equilibrium effects of occupation- and industry-level automation, especially productivity-driven growth of national incomes and an ensuing stimulation of aggregate demand, as well as the direct link of technological progress and the creation of new tasks (Arntz, Gregory and Zierahn, 2016; Acemoglu and Restrepo, 2018a,b).⁵ At the economy level, the cross-industry spillovers appear to have so far dominated the negative own-industry employment impact of routine-replacing technological change, and Gregory, Salomons and Zierhan (2016) attribute almost half of European employment growth over the period 1999–2010 to this phenomenon. For the rising adoption of industrial robots, a narrow-sense task-replacing technology, the evidence is mixed. Investigating local US labour markets, Acemoglu and Restrepo (2020) identify a negative link between robot adoption and employment that persists even after taking into account compensating cross-region effects. On the other hand, such a negative link was found to not reduce aggregate employment in 17 OECD countries, including the US (Graetz and Michaels, 2018).⁶

To gauge the future of work in the context of automation, Acemoglu and Restrepo (2018b) study a long-run equilibrium with task-replacing technological change. In this model, long-run employment is sustained if and only if new tasks emerge at least at the same rate as existing tasks are replaced (i.e., the task domain of labour does not contract over time). While endogenous mechanisms may be able to generate this pattern, the model also implies significant dis-

³As given e.g. in Brynjolfsson and McAfee (2014), these examples include the automation of driving cars, medical diagnosis, warehouse organization and recognition, and translation and playback of text and speech.

⁴Frey and Osborne (2013) put as much as 47% of total US employment at high risk of automation in a foreseeable future of 10-20 years, fueling fear of technological unemployment among the media and economists alike (The Guardian, 2015; Brzeski and Burk, 2015; The Economist, 2018). Improving the empirical model of Frey and Osborne, Arntz, Gregory and Zierahn (2016) arrive at the much lower number of 9%. They further point out that “high-risk” refers to technological possibility rather than likelihood of actual automation, so that the initial label of “probability of automation” may have been additionally misleading.

⁵The emergence of new tasks is directly linked to advances in automation by the need to develop, produce, operate, supervise and maintain these technologies. Indeed, roughly 70% of computer software developers were employed in occupations with new job titles in 2000 (Acemoglu and Restrepo, 2018b).

⁶The result of a negative aggregate effect of robots on employment was also not observed for the case of Germany (Dauth et al., 2021)

ruptions to the efficient allocation of workers to occupations so that medium-term adjustment could, in the frictional real world, come at significant adjustment cost.

In the model in Acemoglu and Restrepo (2018b), workers directly supply tasks to the labour market, and wage-maximizing occupational choice instructs workers to engage only in a single task. This work therefore relies on a one-to-one identification of occupations and tasks, which is a useful abstraction in rationalizing empirical trends at the industry and economy level. In contrast, my model enriches the role of workers by allowing them to engage in multiple tasks that are complementary among each other at a lower level of aggregation, thereby zooming in on structural mechanisms that matter at the micro-level.⁷ As my analysis shows, neglecting these complementarities may give overly dire predictions for automation’s labour demand impact at the level of occupations and worker types.

A recent strand of literature investigates the role of technology in the global decline of labour shares after the early 1980s. Karabarbounis and Neiman (2014) link this phenomenon to a decline in investment goods prices, largely attributable to efficiency gains in information technology, and associated shifts in production from capital to labour. Autor et al. (2020) identify sales concentration at the most productive (“superstar”) firms, commonly characterised as more capital-intensive than their competitors, as an important micro-level driver. A more direct link to technology is established in Autor and Salomons (2018), who empirically establish that a common cross-country trend in total factor productivity explains a significant proportion of the declines in labour shares, and argue for automation as an explanation. I contribute to this literature by showing that the impact of automation on labour shares crucially depends on the degree of complementarity between automated and non-automated tasks, and labour shares fall more if tasks are less complementary. Indeed, in environments with highly complementary tasks, labour shares can even rise as in-use automation technologies improve.

A lively debate in recent empirical literature revolves around the firm-level effects of automation on labour demand. Several contributions point to positive labour demand effects at technology-adopting firms (e.g. Aghion et al., 2020; Koch, Manuylov and Smolka, 2021a) that may, however, lead to expansion predominantly at the expense of their competitors, and ensuing “business stealing” effects could lead to a less positive, or even net negative impact on industry-level labour demand (Aghion et al., 2020; Acemoglu, Lelarge and Restrepo, 2020).⁸ Moreover, Bessen et al. (2019) show that automation has increased separations across Dutch

⁷The model indeed assumes that tasks are complementary at the level of the worker-firm match. This is motivated by the empirical observation that workers distribute time across multiple tasks, which can be rationalised only if tasks complement each other at this level, as wage-maximising workers otherwise only supply the task in which they are most productive. However, as I discuss when introducing the model, the insights derived from framework generalises also to the case where tasks are complementary between workers at the firm level.

⁸Aghion et al. (2020) find a positive industry-level relationship that is however entirely driven by internationally competing industries where (domestic) business stealing effects may be weaker.

industries during 2000–2016, resulting in sizeable and persistent earnings losses for affected workers.⁹ As such, frictional adjustment to automation-induced changes in efficient allocations may come at significant welfare cost at shorter horizons.

On the other hand, Bonfiglioli et al. (2020) find weaker and less persistent firm-level sales effects and indeed negative labour demand effects of automation, even though compared to Aghion et al. (2020), they also focus on the French economy and a similar period. Differences between results appear to be driven, among others, by a differences in the measurement of automation. These differences may point to a role for a sales or demand mechanism that determines the direction and strength of firm-level labour demand effects of technology adoption, as firm-level employment was found to increase only when sales also responded positively.¹⁰

Additional work is required to understand *why* we may sometimes observe positive average relationships between employment and automation at the micro-level, and why these may be different across industries and institutional environments. Only with such understanding one may forecast the extent to which insights from the past continue to be valid in a future shaped by a different composition of automation technologies, and address how policy-makers can act to shape their impact on labour markets. To the best of my knowledge, there is no existing research, theoretical or empirical, that comprehensively studies the role of worker and occupational characteristics for the micro-level effect of automation on labour demand. Existing research has focused on distinctions between skill types (high- vs. low-skill workers), but also the reduced-form insight that high-skilled workers are more complementary to technologies remains to be explained structurally.¹¹

3.3 MODEL ENVIRONMENT

The model presented in this section focuses on a given industry in an economy and considers the partial equilibrium of this industry only. This perspective is chosen as direct effects of automation on labour demand likely occur mostly in the automating industry or industries, whereas other industries are mainly affected indirectly through general equilibrium adjust-

⁹This result concerns not the level of employment or labour demand, but the level of labour reallocation, and is not to be taken as a signal of negative firm-level labour demand effects. Changes in the efficient allocation and automation-induced labour flows are also a key prediction of my model, see Section 3.4.2.

¹⁰This view echoes the key role the elasticity of demand may play for the automation-employment link (e.g. Bessen, 2019), and further resonates well with the discussion of “so-so” vs. “brilliant” technologies: a given productivity improvement may yield a differentially more positive employment change if demand is more elastic, and a larger productivity improvement may have stronger positive competitive effects that allow firms to also grow in employment (see also Chapter 2 of this thesis). The differences between the results in Bonfiglioli et al. (2020) and Aghion et al. (2020) could therefore be, in part, explained by the focus on technologies that are of different quality and adopted in different environments with respect to the elasticity of demand and firms’ ability to control prices.

¹¹The implicit assumption may be that high-skilled workers are not as exposed to routine-task automation and thus see no countervailing force to increased capital-labour complementarity, but if high-skill occupations do not use automatable tasks it is not clear why capital levels would increase, or how high-skilled workers become exposed to automation technologies in the first place.

ments. However, the structure of the model equally allows for an interpretation in terms of economy-level labour demand effects, an aspect that is discussed later in Section 3.4.3.

3.3.1 PRODUCTION AND MARKET STRUCTURE

In what follows, I consider an industry populated by a mass I of workers and $J = 4$ occupational sub-industries (hereafter, for simplicity, “occupation”), each populated by a homogeneous mass F_j of firms, $j \in J$. At a firm f_j or equivalently, in occupation $j \in J$, per unit of labour, a worker $i \in I$ produces output $y(i|j)$ with a standard CES technology:

$$y(i|j) = \left[\lambda_j^{\frac{1}{\sigma_j}} t_{R,ij}^{\frac{\sigma_j-1}{\sigma_j}} + (1 - \lambda_j)^{\frac{1}{\sigma_j}} t_{A,ij}^{\frac{\sigma_j-1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j-1}} \quad (3.1)$$

where $t_{R,ij}$ and $t_{A,ij}$ are, respectively, the levels of a routine task R and an abstract task A that the firm uses in production.

I further assume that firms aggregate output across workers additively, and output is also additively aggregated across firms to the occupation level:

$$y(j) = \int_{i \in I} l_{ij} y(i|j) di \quad (3.2)$$

where $l_{ij} \geq 0$ is the amount of time worker i spends working in occupation j .

This structure introduces the key distinction to other contributions to this literature, namely the assumption of the complementarity of different production tasks at the worker level. So long as firms do not employ workers from different occupations, the analysis would give identical predictions for occupation-level labour demand if complementarities instead occurred between workers but within firms and occupations.¹² I nonetheless assume the worker level as the source of complementarity, because (i) it is the most micro level at which complementarities can occur, and therefore the deepest possible structural source, (ii) this assumption can rationalise the empirical fact that workers distribute time across different tasks within their occupation, and (iii) the conclusions from this framework easily extend to the case where firms can employ workers in more than one occupation.

Finally, the industry’s final output good is aggregated across occupations using Cobb-Douglas technology:

$$Y = J \prod_{j \in J} y(j)^{1/J}. \quad (3.3)$$

This final output good is supplied to a market on which the industry is a price-taker and

¹²In this case, workers would supply tasks to firms with a CES-aggregation structure like the one in Equation (3.1).

faces an exogenous demand function $P(Y)$. The baseline analysis assumes that $P(Y) = P$, i.e. that demand for the industry good is perfectly elastic. The implications of weakening this assumption are then discussed later in Section 3.4.3.

I assume that there exists a technology that can automate the routine task t_R with efficiency α , that is, k units of automation capital can produce αk units of the routine task t_R . For analytical simplicity, I assume that automation capital is elastically supplied to the industry exogenously at a unit price.¹³

On the other hand, workers $i \in I$ can produce either task $k \in \{A, R\}$ with efficiency ϕ_{ik} . This gives $t_{R,ij} = \eta_{ij}\phi_{iR} + \alpha k_{ij}$ and $t_{A,ij} = (1 - \eta_{ij})\phi_{iA}$, where k_{ij} is the amount of automation capital per unit of the labour input used, and $\eta_{ij} \in [0, 1]$ the share of time i allocates to the R -task in the (i, j) -match.

For simplicity, workers are assumed to have a unit time endowment, and to spend all of it working in the occupation(s) in which they obtain the highest wage.¹⁴

Lastly, firms are assumed to be perfectly competitive in the labour and product markets and therefore maximise profits taking prices as given.

3.3.2 WORKER AND FIRM HETEROGENEITY

To assess the importance of different structural worker and firm characteristics, namely task complementarity and further routine task intensity on the firm and task specialisation on the worker side, I study an environment where firms and workers differ along these dimensions.

Specifically, I assume that occupations $j \in J$ differ in the level σ_j of complementarity in task aggregation and in the weight of the routine task λ_j . Each combination of values $0 < \sigma_L < \sigma_H$ and $0 < \lambda_L < \lambda_H < 1$ constitutes one of the $J = 4$ occupations, denoted $j(\sigma, \lambda)$.

Further, workers differ with respect to their skill profile $\Phi_i = (\phi_{iR}, \phi_{iA})$. In each skill $k \in \{A, R\}$, workers $i \in I$ have either a normalised high skill $\phi_{ik} = 1$ or low skill $\phi_{ik} = \phi \in (0, 1)$. The set of considered types is $\Theta = \{A, R, U\}$ with $\Phi_A = (\phi, 1)$ (abstract-specialised), $\Phi_R = (1, \phi)$ (routine-specialised) and $\Phi_U = (1, 1)$ (unspecialised) and captures the *relative* strength profiles of workers in the industry.¹⁵ This implies that U -workers have an *absolute* advantage over the other types. While this is very convenient for keeping the algebra simple, it may blur the focus on the relative advantage associated with task specialisation. Some results therefore also focus on the adjusted skill profile $\tilde{\Phi}_U = (1 - \phi/2, 1 - \phi/2)$ for unspecialised workers.

¹³This implies that the industry is a price-taker on the market for capital, and plausible for smaller industries on domestic markets for automation capital, but more importantly, given the relatively geocentric supply of automation capital, also for relatively small country-industries on global markets for automation capital.

¹⁴This can be rationalised through workers maximising a strictly increasing utility from consumption c_i (but not leisure) subject to a budget constraint $c_i = w_i h_i$, where w_i is the wage and h_i is hours worked.

¹⁵Specialised worker symmetry is imposed for analytical simplicity. A unit mass with skill $c\Phi$, $c > 0$, is equivalent to $1/c$ workers with profile Φ , so that tracing relative strengths is sufficient.

For $\theta \in \Theta$, I denote by $s_\theta \in (0, 1)$ the *exogenously given* mass of type- θ workers in the industry, normalised to $\sum_{\theta \in \Theta} s_\theta = 1$. The assumption of a fixed mass of worker types in the industry may be a counter-intuitive one, as automation may in practice affect the worker mix by differently complementing or substituting different worker types. However, the assumption allows to focus on the role of automation for labour demand, as any change in the equilibrium wage at a fixed amount of labour directly corresponds to the associated shift in the labour demand function. As automation is predominantly a supply-side phenomenon, this simplification appears justified. This means that the analysis is less suited to disentangle price (wage) and quantity (employment) effects in the labour market, but more directly focuses on labour demand impacts of automation by worker type.

For what follows, as workers of a given type are homogeneous, I replace the index i for a worker with the index θ for a worker type whenever appropriate. Accordingly, e.g. $y(\theta|j)$ is the output a worker i of type $\theta \in \Theta$ produces in occupation $j \in J$, and $k_{\theta j}$ is the amount of automation capital used per unit of this worker's labour input in j .

Further, for each occupation j , I denote by $s_\theta(j) = \int_{i \in I} l_{ij} di$ the labour input of workers of type $\theta \in \Theta$ used in occupation j . With this, Equation (3.2) can be simplified to

$$y(j) = \sum_{\theta \in \Theta} s_\theta(j) y(\theta|j). \quad (3.4)$$

3.3.3 MARKET CLEARING

To express the market clearing conditions of the product market, the marginal cost of production need to be computed for each occupation. By the production function's homogeneity of degree one, the output worker i produces by working l_{ij} units of time in occupation j is $l_{ij} y(i|j)$. The cost of producing this output is $l_{ij}(w_{ij} + k_{ij})$, and the marginal cost firms in an occupation $j \in J$ incurs when producing output employing a worker of type $\theta \in \Theta$ is $(w_{\theta j} + k_{\theta j})/y(\theta|j)$.

As firms maximise profits, the marginal cost of production are equated across all worker types which they employ. At the occupation level, this implies that

$$\forall j \in J : \forall \theta_1, \theta_2 \in \Theta : \left(\min\{s_{\theta_1}(j), s_{\theta_2}(j)\} > 0 \Rightarrow \frac{w_{\theta_1 j} + k_{\theta_1 j}}{y(\theta_1|j)} = \frac{w_{\theta_2 j} + k_{\theta_2 j}}{y(\theta_2|j)} = MC(j) \right) \quad (3.5)$$

where $s_\theta(j)$ is the mass of type- θ workers that works in occupation j , and $MC(j)$ denotes the marginal cost of production in j . Because firms are perfectly competitive, the supply function of each occupation $j \in J$ is given by the marginal cost of production $MC(j)$.

The demand for an occupation j 's output is given by its marginal revenue product for the

industry aggregate,

$$MRP(j) = \frac{\partial PY}{\partial y(j)} = \frac{\partial(4\Pi_{j \in J} y(j)^{1/4})}{\partial y(j)} P = \frac{Y/4}{y(j)} P$$

where P is the price of the industry's final output good.

The output prices p_j clear the occupation-level product markets, so that for every $j \in J$, $p_j = MC(j)$ and $p_j = MRP(j)$. From the latter, it follows that

$$\forall j \in J : p_j = \frac{Y/4}{y(j)} P = \frac{\Pi_{j' \in J} y(j')}{y(j)} P. \quad (3.6)$$

The final goods price P satisfies $P = \Pi_{j=1}^4 p_j^{\frac{1}{4}}$.¹⁶

To study the conditions under which the labour market clears, I consider the firm problem. Because firms are price takers, the firm's profit maximisation problem is

$$\max_{\{l_{ij}, k_{ij}, \eta_{ij}\}_{i \in I}} \int_{i \in I} (p_j y(i|j) - (w_{ij} + k_{ij})) l_{ij} \quad s.t. \quad \forall i \in I : l_{ij} \geq 0. \quad (3.7)$$

Verbally, firms choose the profit-maximising level of capital and labour inputs, and further determine the share η_{ij} of time a worker spends working in the routine task. Because profits are multiplicative in the labour input l_{ij} , firms' demand for labour of type $\theta \in \Theta$ is infinitely high (zero) if $w_{\theta j} < p_j y(\theta|j) - k_{\theta j}$ ($w_{\theta j} > p_j y(\theta|j) - k_{\theta j}$). While $y(\theta|j)$ and $k_{\theta j}$ only depend on the parameters that characterise occupation and worker heterogeneity, p_j is a function that depends on the distribution of workers across occupations (cf. Equations (3.2) and (3.6)). Hence, the labour demand function of an occupation j (henceforth: "wage offer")¹⁷ is

$$w_{\theta j} := w_{\theta j}(\{s_{\theta}(j)\}_{\theta \in \Theta} | \{s_{\theta}(j')\}_{\theta \in \Theta, j' \neq j}) = \frac{Y/4}{\sum_{\theta \in \Theta} s_{\theta}(j) y(\theta|j)} y(\theta|j) - k_{\theta j} \quad (3.8)$$

where the final output Y depends on the full distribution of types across occupations (cf. Equation (3.3)).

On the other hand, workers do not value leisure and labour supply of each type θ is perfectly inelastic with quantity s_{θ} .

The wages w_{θ} that clear the labour markets for each type θ are such that $\sum_{j \in J} s_{\theta}(j) = s_{\theta}$, i.e. that the mass of workers of the type employed across occupations corresponds to the overall mass of workers of this type. This idea is illustrated in Figure 3.2, which shows the equilibrium mechanism in a simplified illustration.¹⁸ The market clearing conditions for the labour

¹⁶This follows directly from plugging in the first equality of Equation (3.6) for \bar{p}_j into the product $\Pi_{j=1}^4 p_j^{\frac{1}{4}}$.

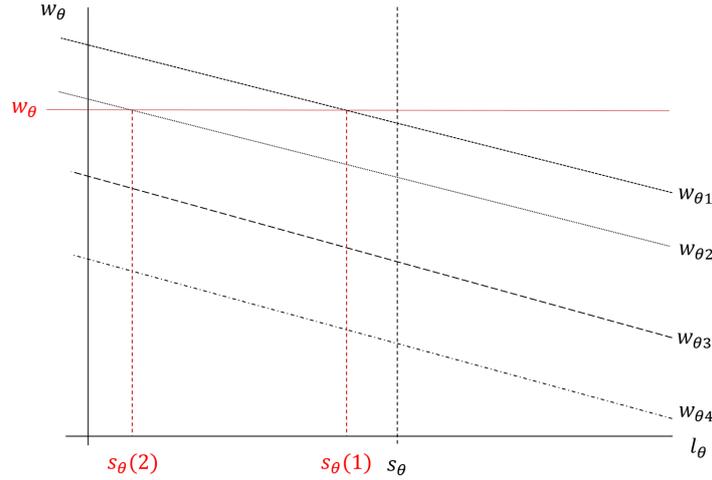
¹⁷The term "wage offer" is used as workers of a given type need not work in all occupations, as is also illustrated by Figure 3.2 below. Throughout the paper, the term "labour demand" refers more to the demand of the industry that is derived from the combination of the wage offers across occupations.

¹⁸In contrast to this visualisation, the labour demand functions $w_{\theta j}$ are generally non-linear and are co-

markets can be derived from this equality by re-arranging Equation (3.8) for the shares $s_\theta(j)$, which gives the **labour market clearing conditions**

$$\forall j \in J \forall \theta \in \Theta : \left(s_\theta(j) = 0 \text{ or } s_\theta(j) = \left(\left(\frac{\prod_{j' \neq j} y(j')^{\frac{1}{4}} y(\theta|j) - k_{\theta j}}{w_\theta} \right)^{\frac{4}{3}} - \sum_{\theta' \neq \theta} s_{\theta'}(j) y(\theta'|j) \right) \cdot y(\theta|j)^{-1} \right).$$

Figure 3.2: Labour market equilibrium for a given worker type.



Note: The figure shows a simplified illustration of the equilibrium in the market for the labour of a given type $\theta \in \Theta$. The downward-sloping functions $w_{\theta j}$ represent the labour demand functions of the different occupations $j \in \{1, 2, 3, 4\}$. w_θ is the equilibrium wage and $s_\theta(j)$ is the mass of workers of type θ employed in occupation $j \in J$, while s_θ denotes the overall mass of workers of this type.

3.3.4 INITIAL EQUILIBRIUM AND EQUILIBRIUM ADJUSTMENT TO AUTOMATION

Defining an equilibrium in a general way is not straightforward as depending on the level of the efficiency α at which automation capital can perform the routine task, automation capital may or may not be used in an occupation, and the assignment of worker types to occupations may also depend on this parameter. Therefore, I pursue a two-step approach to the equilibrium: first, I consider the equilibrium in a state of technology with $\alpha = 0$, i.e. the case where automation capital is unproductive and firms can effectively only produce using labour. Having defined the equilibrium in this state (“no-automation equilibrium”), I then study the change in the equilibrium associated with a continuous change in α . This approach is suited to establish that an equilibrium exists at any value of α , and is further directly suited to study the equilibrium’s adjustment to improvements in automation technology, i.e., increases in the parameter α . Importantly, this analysis can speak to both *extensive margin* effects of automation that occur when occupations start using automation capital instead of labour in the routine task, and *intensive margin* effects of automation that occur when the quality of in-use automation dependent across the labour markets for different worker types θ .

technologies improves and firms adjust their demand for inputs.

The full characterisation of an equilibrium of the introduced economy encompasses the distribution of worker types across occupations. This distribution always depends on the profile $(s_\theta)_{\theta \in \Theta}$ of relative type frequency, and leaving this profile unrestricted would significantly increase the complexity of any analytical exercise. Therefore, I impose Assumption 1 which, as is shown later, ensures that each type is employed in two of the four occupations in the initial no-automation equilibrium. This ensures that either type is relatively common and does not work exclusively in a single occupation, arguably the most neutral assumption that can be imposed. The mechanisms unveiled, however, do not depend on the initial distribution.

Assumption 1 relies on some additional notation that is also useful later in the analysis. I define $Y_\theta := s_\theta y^N(\theta|j(\sigma_H, \lambda_\theta))$ for $\theta \in \{A, R\}$ with $\lambda_\theta = \lambda_H$ for $\theta = R$ and $\lambda_\theta = \lambda_L$ for $\theta = A$ (the specialist type's preferred routine task weight). Then, Y_θ is the maximal occupation-level output that the workers of type $\theta \in \Theta$ can potentially produce, i.e. the output this type would produce if all workers worked in the occupation where their physical productivity is highest, and

$$\rho_\theta := \frac{y^N(\theta|j(\sigma_H, \lambda_\theta))}{y^N(\theta|j(\sigma_L, \lambda_\theta))}$$

denotes the productivity premium of type $\theta \in \{A, R\}$ from working with low complementarity in the occupations specialised in this type. For $\theta = U$, physical productivity is the same in all occupations, and thus $\rho_U := 1$.

Assumption 1 (Relative Frequency of Worker Types). *It holds that*

$$s_U \in \left(\left| \frac{Y_R}{\rho_R} - \frac{Y_A}{\rho_A} \right|, \min \left\{ 3 \frac{Y_R}{\rho_R} - \frac{Y_A}{\rho_A}, 3 \frac{Y_A}{\rho_A} - \frac{Y_R}{\rho_R} \right\} \right).$$

I now turn to characterising the no-automation equilibrium. First, if no capital is used in the match, optimal time allocation of workers implies that for any $i \in I$, $j \in J$, the share η_{ij} of time spent in the routine task is

$$\eta_{ij} = \frac{\lambda_j \phi_{iR}^{\sigma_j-1}}{\lambda_j \phi_{iR}^{\sigma_j-1} + (1 - \lambda_j) \phi_{iA}^{\sigma_j-1}} \quad (3.9)$$

Plugging the implied levels of both tasks ($t_{R,ij} = \eta_{ij} \phi_{iR}$ and $t_{A,ij} = (1 - \eta_{ij}) \phi_{iA}$) into the production function in Equation (3.1), one obtains for the no-automation productivity $y^N(i|j)$ of worker i in occupation j :

$$y^N(i|j) = \left[\lambda_j \phi_{iR}^{\sigma_j-1} + (1 - \lambda_j) \phi_{iA}^{\sigma_j-1} \right]^{\frac{1}{\sigma_j-1}}.$$

Thus, the no-automation wage offers satisfy

$$w_{ij}^N = p_j y^N(i|j) = p_j \left[\lambda_j \phi_{iR}^{\sigma_j-1} + (1 - \lambda_j) \phi_{iA}^{\sigma_j-1} \right]^{\frac{1}{\sigma_j-1}} \quad (3.10)$$

for $i \in I, j \in J$. As no task is automated, both task-based skills matter for the wage offer.

Unspecialised worker productivity is invariant to occupational heterogeneity, i.e. $y(i_U|j) = 1$ for all $j \in J$. Specialised workers are trivially strictly more productive in occupations relying more on their task of specialization, i.e. $y^N(\theta|j(\sigma, \lambda_\theta)) > y^N(\theta|j(\sigma, \lambda_\nu))$ for $\theta, \nu \in \{A, R\}, \theta \neq \nu$. Further, higher σ should intuitively allow specialised workers to focus more on their stronger task and increase their productivity. Proposition 1 verifies that this is indeed true.¹⁹ Assumption 2 ensures the intuitive regularity that workers' physical productivity is always higher in occupations that rely more on the task in which workers are more skilled.²⁰

Proposition 1 (Substitutability and No-Automation Productivity). *Specialised workers are more productive in occupations with lower complementarity across tasks: for $\lambda \in \{\lambda_L, \lambda_H\}$ and $i \in \{A, R\}$,*

$$\frac{\partial y^N(i|j(\sigma, \lambda))}{\partial \sigma} > 0.$$

Assumption 2. *In the no-automation state, specialised workers are strictly more productive in occupations putting higher weight on their task of specialization, i.e.*

$$y^N(i_R|j(\sigma_L, \lambda_H)) > y^N(i_R|j(\sigma_H, \lambda_L)) \quad \text{and} \quad y^N(i_A|j(\sigma_L, \lambda_L)) > y^N(i_A|j(\sigma_H, \lambda_H)).$$

Proposition 2 characterises the unique equilibrium that emerges at the technology level $\alpha = 0$.²¹ This equilibrium trivially emerges also at $\alpha > 0$ so long as α is sufficiently low to prevent any use of automation technology.²²

Proposition 2 (No-Automation Equilibrium). *At technology level $\alpha = 0$, there exists a unique no-automation equilibrium, in which*

- if $s_\theta(j) > 0$ for $j \in J$ and $\theta = A$ ($\theta = R$) [$\theta = U$], then $\lambda_j = \lambda_L$ ($\lambda_j = \lambda_H$) [$\sigma_j = \sigma_L$],
- $p_k = \max_{j \in J} p_j$ for any $k \in J$ with $\sigma_k = \sigma_L$,
- $p_{j(\sigma_H, \lambda_L)} = \rho_A^{-1} \max_{j \in J} p_j$ and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} \max_{j \in J} p_j$.

¹⁹The proof of this proposition is given in Appendix C.1.

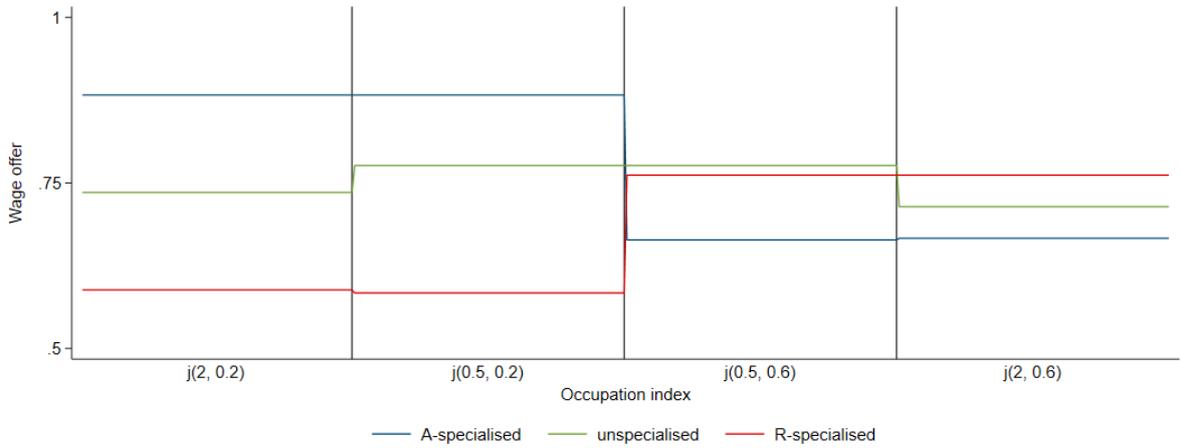
²⁰Assumption 2 is sustained if ϕ is sufficiently bounded away from zero (see Appendix C.1.6). This is reasonable if workers have chosen to work in the considered industry rather than other industries using different sets of tasks.

²¹A detailed investigation of this equilibrium is given in Appendix C.2. Proposition 2 emerges as a corollary of this investigation (Corollary 4 in Appendix C.2).

²²The next section further discusses the conditions under which α is sufficiently low to prevent any adoption.

Figure 3.3 provides a graphical illustration of the unique no-automation equilibrium, based on an exemplary parametrisation of the model that uses the adjusted skill profile of unspecialised workers to focus on the role of workers' relative advantage. In this illustration, $\lambda_L = 0.2 < 0.5 < 0.6 = \lambda_H$, so that for both tasks, there exist occupations in which this task has a larger weight in production. As can be seen, this results in wage offers to be highest for the type specialised in this task in the occupations with a lower degree of task complementarity, i.e. those with $\sigma = \sigma_H = 2$. When tasks are more complementary ($\sigma = \sigma_L = 0.5$), however, occupations may offer higher wages to workers with a more balanced skill profile even if they are less skilled in the task with the highest weight, as can be seen from the higher wage U -workers receive compared to R -specialists in the occupation $j(\sigma_L, \lambda_H) = j(0.5, 0.6)$. On the other hand, if the importance of tasks is sufficiently asymmetric, also more complementarity-intensive occupations may offer higher wages to specialists, as illustrated by the difference in wages of A -specialists and U -type workers in the occupation $j(\sigma_L, \lambda_L) = j(0.5, 0.2)$.

Figure 3.3: Wage offers in the no-automation equilibrium.



Note: The figure shows the profile of equilibrium wage offers across occupations and worker types for an exemplary parametrisation of the model with $\sigma_L = 0.5$, $\sigma_H = 2$, $\lambda_L = 0.2$ and $\lambda_H = 0.6$ for the occupational heterogeneity parameters, and $\phi = 0.5$ for the low-skill penalty of specialised workers. The illustration is based on the adjusted skill profile of unspecialised workers, $\tilde{\Phi}_U = (0.75, 0.75)$.

Closed-form expressions for worker shares may be computed from the restrictions imposed on prices and worker shares by Proposition 2. When $Y^{min} := \sum_{\theta \in \Theta} Y_{\theta} / \rho_{\theta}$ denotes the minimum sum of intermediate outputs produced when workers are employed in occupations specialised in their type, i.e. $\forall \theta \in \Theta : (\forall i_{\theta} \in \theta : \lambda_{j(i_{\theta})} = \lambda_{\theta})$, output is $y^N(j) = \frac{Y^{min}}{4}$ for $j \in J$ with $\sigma_j = \sigma_L$ so that $Y^N = 4 \prod_{j \in J} y(j)^{\frac{1}{4}} = (\rho_A \rho_R)^{\frac{1}{4}} Y^{min}$. Y^{min} may be used to represent types' initial income share

in the industry $Y_0(\theta) := \frac{s_\theta w_\theta}{Y^N}$: for $\theta \in \Theta$,

$$s_\theta w_\theta = \begin{cases} s_\theta y^N(\theta | j(\sigma_L, \lambda_\theta)) p_{j(\sigma_L, \lambda_\theta)} & \text{for } \theta \in \{A, R\}, \\ s_U p_{j(\sigma_L, \lambda_L)} & \text{for } \theta = U \end{cases} = Y_\theta / \rho_\theta \cdot (\rho_A \rho_R)^{\frac{1}{4}}$$

so that $Y_0(\theta) = \frac{Y_\theta / \rho_\theta}{Y^{min}}$. With this result, one obtains

$$\beta^N = \frac{1}{2} \left(1 + \frac{Y_0(A) - Y_0(R)}{Y_0(U)} \right), \quad (3.11)$$

and

$$\frac{s_A^N(j(\sigma_H, \lambda_L))}{s_A} = (4Y_0(A))^{-1}, \quad \frac{s_R^N(j(\sigma_H, \lambda_H))}{s_R} = (4Y_0(R))^{-1}. \quad (3.12)$$

Accordingly, for $\theta \in \{A, R\}$, $Y_0(\theta)$ is positively related to the spread of θ -workers beyond the occupation in which they have the highest physical productivity.²³ Further, the larger the asymmetry in A - and R -income relative to U -income, the more asymmetric is the distribution of U -workers across σ_L -occupations.

3.4 MODEL ANALYSIS: EQUILIBRIUM ADJUSTMENT TO AUTOMATION

Having characterised the unique no-automation equilibrium, I now turn to the core part of the analysis: establishing that a unique equilibrium exists when the parameter of efficiency of the automation capital α increases to level at which automation occurs, and studying the implications of such increases for labour demand, in particular focusing on differences that arise between occupations and worker types according to their heterogeneity. A first set of results characterises key quantities *given that* an occupation uses automation capital. Subsequently, I compare possible firm-level outcomes that would arise, respectively, with and without automation to derive the conditions for α that *trigger* automation.

3.4.1 OUTCOMES WITH AUTOMATION AND AUTOMATION TRIGGER

As capital and labour are perfect substitutes in the R -task, a positive amount of capital will be used if and only if labour is restricted entirely to the A -task. Therefore, For strictly positive levels of automation capital use, $k_i^*(j)$ solves

$$\max_{k_{ij} > 0} p_j \left[\lambda_j^{\frac{1}{\sigma_j}} (\alpha k_{ij})^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - \lambda_j)^{\frac{1}{\sigma_j}} \phi_{iA}^{\frac{\sigma_j - 1}{\sigma_j}} \right]^{\frac{\sigma_j}{\sigma_j - 1}} - k_{ij}$$

²³Initial worker employment in occupations of below-maximum physical productivity is not inconsistent with efficient educational choice, as all workers are employed in occupations that maximise their revenue productivity.

which gives

$$k_i^*(j) = \left(\frac{(1 - \lambda_j)(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{\sigma_j}{\sigma_j - 1}} \frac{\lambda_j}{1 - \lambda_j} \frac{\phi_{iA}}{\alpha} \quad (3.13)$$

for $\lambda_j (p_j \alpha)^{\sigma_j - 1} < 1$, which holds at any level of α at which firms use capital, so that $k_i^*(j) > 0$ is guaranteed.²⁴ Equation (3.13) implies that output in the automation scenario is given by

$$y^A(i|j) = \left(\frac{(1 - \lambda_j)^{\frac{1}{\sigma_j}}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{\sigma_j}{\sigma_j - 1}} \phi_{iA} \quad (3.14)$$

and the automation wage offer, equal to the potential revenue at worker's net-of-capital cost physical productivity (henceforth: effective productivity), is

$$w_{ij}^A = p_j \underbrace{(y^A(i|j) - k_i^*(j)/p_j)}_{\text{effective productivity}} = p_j \left(\frac{1 - \lambda_j}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{1}{\sigma_j - 1}} \phi_{iA} \quad (3.15)$$

for $i \in I, j \in J$. w_{ij}^A increases in α (and p_j), so that net of output market effects, intensive margin technology improvements ($\Delta\alpha > 0$) raise firm- and occupation-level labour demand. As automated workers only perform the A -task, w_{ij}^A is independent of ϕ_{iR} , and A - and U -workers are equivalent conditional on automation. The following denotes by $i_H \in H = A \cup U$ a high automation-productivity worker.

Proposition 3 (Substitutability and Automation Productivity). *For $\sigma \in \{\sigma_L, \sigma_H\}$ and $i \in \Theta \setminus U$,*

$$\frac{\partial \ln w_{ij(\sigma, \lambda)}^A}{\partial \sigma} > 0 \quad \text{for } p_{j(\sigma, \lambda)} \alpha \neq 1.$$

Proposition 4 (Routine Task Weight and Automation Productivity). *For $\sigma \in \{\sigma_L, \sigma_H\}$ and any $i \in I$,*

$$\text{sgn} \left(\frac{\partial \ln w_{ij(\sigma, \lambda)}^A}{\partial \lambda} \right) = \text{sgn}(p_{j(\sigma, \lambda)} \alpha - 1).$$

Propositions 3 and 4 give the key properties of the wage offer equation.²⁵ Post-automation, R -task-intensive occupations use labour in a lower-weight task but also feature higher capital complementarities to labour from capital use. Proposition 4 indicates that if the relative efficiency cost of capital is sufficiently low, i.e. $1/(p_j \alpha)$ is sufficiently small, the latter force

²⁴For $\sigma_j > 1$, $\lambda_j (p_j \alpha)^{\sigma_j - 1} < 1$ by no-arbitrage on the capital market. For $\sigma_j < 1$, if $\lambda_j (p_j \alpha)^{\sigma_j - 1} \geq 1$, per-worker automation output is negative (cf. Eq. (3.14)) and no-automation is chosen by firms within matches.

²⁵These results for partial derivatives, holding prices constant, describe the relative statics of labour demand at the micro-level due to the production set-up, and are not to be interpreted with respect to the heterogeneous impact of automation by model parameters, which is only discussed in the next section. The proofs of these propositions, as well as all remaining propositions in this section, are given in Appendix C.1.

dominates the former.

A key determinant of capital use is the relative cost of the routine task in the automated match (henceforth the “relative efficiency cost of capital”),²⁶

$$c_j^{K,L}(\alpha) = \frac{1/\alpha}{w_{Hj}^A} = \left(\frac{(1 - \lambda_j)(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \right)^{-\frac{1}{\sigma_j - 1}}, \quad (3.16)$$

a strictly monotonically increasing function of the absolute efficiency cost of capital in units of firms’ output, $(p_j \alpha)^{-1}$. The turning point where the efficiency cost of capital fall below those of labour, i.e., where $c_j^{K,L}(\alpha) = 1$, is $\alpha = 1/p_j$. As I show later, this corresponds to the level of α at which firms find it profitable to automate the U -workers that are equally skilled in either task; for these workers, the relative cost of performing the routine tasks is indeed the only aspect firms need to consider when deciding whether or not to automate.²⁷ Together with equation (3.13), capital use can be re-written as follows:²⁸

$$k_i^*(j) = c_j^{K,L}(\alpha)^{-\sigma_j} \cdot \frac{\lambda_j}{1 - \lambda_j} \cdot \frac{\phi_{iA}}{\alpha}. \quad (3.17)$$

Having characterised firm-level revenues net of potential capital expenditures in scenarios with and without automation, respectively, I now turn to comparing them to derive the conditions under which automation is triggered.

As firms are perfectly competitive, automation occurs if and only if it increases profits before wages. As firms compete in the labour market, changes in profits are directly reflected in changes in labour demand, and automation is triggered for worker i in occupation j if $w_{ij}^A \geq w_{ij}^N$, i.e. if

$$\left(\frac{1 - \lambda_j}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{1}{\sigma_j - 1}} \geq \left(\lambda_j(\phi_{iR}/\phi_{iA})^{\sigma_j - 1} + 1 - \lambda_j \right)^{\frac{1}{\sigma_j - 1}}$$

This gives the **automation trigger**

$$\alpha_{ij}^* = \frac{1}{p_j} \frac{\phi_{iR}}{y^N(i|j)} =: \frac{\tau_{ij}}{p_j}. \quad (3.18)$$

τ_{ij} represents a *skill displacement coefficient* of automation relating the (displaced) productivity in automated tasks to i ’s overall no-automation productivity. With Equation (3.18), for all $j \in J$, $\alpha_{iAj}^* < \alpha_{iUj}^* < \alpha_{iRj}^*$ so that any occupation automates A -workers first and R -workers last.

²⁶The expression uses the potential H -wage as it captures the unit cost of the abstract skill in automated matches.

²⁷As I also show later, when workers are instead specialised in the abstract (routine) tasks, firms automate the routine task for this type strictly before (after) cost equality of capital and labour in the routine task, because automation additionally increases (decreases) the workers’ average productivity across performed tasks.

²⁸This equation shows that capital use exhibits some natural properties: it declines in its relative efficiency cost, and more strongly so when inputs are more substitutable, and increases in relative capital productivity α/ϕ_{iA} and the weight of the automated task λ_j .

Thus, within occupations, *assisting* use of technology, i.e. automation of workers' weaker tasks, occurs strictly before substitutive use of technology where workers are displaced from their tasks of specialization. As $\alpha_{ij}^* = \frac{\phi_{iR}}{w_{ij}^N}$ (cf. Equation (3.10)), by wage-maximizing occupational choice, the first triggers hit (simultaneously) are those in a type's occupations of employment.

At the trigger, automation output is

$$y^A(i|j)|_{\alpha p_j = \tau_{ij}} = \left(1 + \frac{\lambda_j}{1 - \lambda_j} \left(\frac{\phi_{iR}}{\phi_{iA}}\right)^{\sigma_j - 1}\right) y^N(i|j) =: m_{ij} y^N(i|j). \quad (3.19)$$

The multiplier $m_{ij} > 1$ satisfies $m_{ij} = (1 - \eta_{ij})^{-1}$ (cf. Eq. (3.9)). The higher η_{ij} , the more resources of the worker are reallocated to the A -task, and the larger the boost in output.

As $m_{ij} > 1$, automation at the trigger has non-zero output effects. As discussed in more detail in Section 3.4, even though it is neutral for labour demand at the firm-level, it is not at the occupation level. If a change in the type-to-occupation matching is induced, the trigger may be represented by a range of values for α , and further triggers for the given type may be hit. Proposition 5 establishes the regularity that triggered occupations triggering automation for the same $\theta \in \Theta$ at some α may only depart from triggers jointly when sustaining positive θ -employment.

Proposition 5 (Joint Departure from Trigger). *Let $j, k \in J$, $\theta \in \Theta$, and suppose that $\alpha = \alpha_{\theta j}^* = \alpha_{\theta k}^*$ and $s_{\theta}(j), s_{\theta}(k) > 0$. If α rises and $s_{\theta}(j), s_{\theta}(k) > 0$ is sustained, then $\alpha \neq \alpha_{\theta j}^*$ if and only if $\alpha \neq \alpha_{\theta k}^*$.*

3.4.2 THE TECHNOLOGY ADOPTION PROCESS

The previous section has laid the foundation for the equilibrium analysis at technology levels α at which automation occurs. This section studies the adjustment of the initial no-automation equilibrium to a continuous rise of α , and shows (i) that an equilibrium exists at any level of α , and (ii) how labour demand and associated quantities change as α increases.

Motivated by the debate in the literature, some of the following results also address how the labour share responds to automation. In the no-automation equilibrium, the labour share is trivially equal to one, which can be thought of as the normalised starting point of the labour share in the state before automation.²⁹ Equations (3.13) and (3.14) can be used to derive the cost share of capital k_j^A ,

$$k_j^A := \frac{k_i^*(j)}{p_j y(i|j)} = \lambda_j (p_j \alpha)^{\sigma_j - 1}, \quad (3.20)$$

²⁹The model analysis generalises in a straightforward way to the case where the occupation-level task aggregate $y(j)$ produced by labour and possibly automation is combined with augmenting capital $K(j)$, e.g. in a common Cobb-Douglas production function $y^{final}(j) = A_j y(j)^{\sigma_L} K(j)^{\sigma_K}$. In this case, the labour share of the occupation is strictly smaller than one also when no automation capital is used.

and by the competitive set-up of the model, the automation labour share in occupation j , l_j^A ,³⁰ is

$$l_j^A = 1 - k_j^A = 1 - \lambda_j(p_j\alpha)^{\sigma_j-1}. \quad (3.21)$$

EQUILIBRIUM ADJUSTMENT BETWEEN TRIGGERS: EXISTENCE, UNIQUENESS, AND CONTINUITY

I first establish a set of results that increase the tractability of the equilibrium adjustment process, and further show that the model indeed obeys to some intuitive expectations.

Proposition 6 (Post-Trigger Automation of Types). *For any $\theta \in \Theta$, if there exists $j \in J$ with $\alpha > \alpha_{\theta j}^*$, then θ -workers are automated in all matches, i.e. $\forall k \in J : w_{\theta k}^A > w_{\theta k}^N$.*

By Proposition 6,³¹ when a U -trigger (R -trigger) is hit in $j \in J$, then $\alpha > \alpha_{i_A j}^*$ ($\alpha > \alpha_{i_U j}^*$) and A -workers (U -workers) do not work at no-automation in any occupation. This directly gives Corollary 1, which establishes that the inverse relationship of routine specialization and automation timing identified above also holds across occupations.

Corollary 1 (Timing of Automation). *When α hits the first U -triggers, all A -workers are automated. Further, when α hits the first R -triggers, all A - and U -workers are automated.*

Corollary 1 establishes that, as one could expect, the industry sequentially automates the routine task for the worker types, so that the degree of workers' specialisation in the routine task is the key determinant for when they become exposed to automation.

Worker indifference conditions (ICs) between occupations of employment are a central tool in the analysis to follow. A j, k -IC for θ -workers, $j, k \in J$ and $\theta \in \Theta$, takes the form $w_\theta = w_{\theta j}^A = w_{\theta k}^A$ if it refers to an automated worker type, and $w_\theta = w_{\theta j}^N = w_{\theta k}^N$ otherwise.³² Worker ICs arise from workers receiving the same wage offer in different occupations, and determine the set of occupations in which a given worker type is employed at a given technology level α . I distinguish between the number of “structural” and “implied” ICs, where elements in the set of structural ICs do not imply each other.³³ Further, a j, k -IC of θ -workers is called “active” when it features positive employment on both sides, i.e. $s_\theta(j), s_\theta(k) > 0$.

As α rises to an automation trigger $\alpha_{\theta j}^*$, $\theta \in \Theta$, $j \in J$ from below at α_0 , at constant prices, adoption of automation technology within matches strictly increases θ -productivity in j (recall:

³⁰The cost share of capital is invariant across worker types in any occupation $j \in J$, so that the worker- and occupation-level labour shares coincide.

³¹All results of this and following subsections are derived in Appendix C.3.

³²“Passive” ICs, i.e. $w_{\theta j}^A = w_{\theta k}^A < w_\theta$ or $w_{\theta j}^N = w_{\theta k}^N < w_\theta$ for occupations of non-employment or under inferior production schemes (e.g. when non-automation always yields a lower wage offer than automation) are not relevant and disregarded in the following.

³³E.g. if θ -ICs hold for any pair of $j, k, l \in J$, then the j, k - and k, l -ICs imply the j, l -IC, and only two of these ICs are structural. Structural ICs impose independent restrictions on $\{p_j\}_{j \in J}$, and more than $J - 1 = 3$ cause over-identification (by $1 = P = \prod_{j \in J} p_j^{1/J}$). Of course, the set of structural ICs is not unique, but only its cardinality.

$m_{\theta j} > 1$). As p_j is bound by $\alpha = \alpha_{\theta j}^* = \frac{\tau_{ij}}{p_j}$ (cf. Eq. (3.18)) and can not change at the trigger, the relative output of j can not increase, so that all θ -workers in j shift to automation at this level of α only if employment adjusts in a way that sustains all worker ICs as only then, prices $\{p_j\}_{j \in J}$ are preserved. Therefore, prices are continuous in α at initial triggers, i.e. α_0 where a θ -trigger in $j \in J$ is first hit.

Otherwise, adoption is gradual and the rise in relative output is bound by $\alpha = \alpha_{\theta j}^*$ until full adoption has occurred. Thus, if j remains at the θ -trigger on an interval $[\alpha_1, \alpha_2]$ for α , then on this interval, $\frac{dp_j \alpha}{d\alpha} = 0$, and for the remaining occupations $k \in J$, if k also employs θ -workers also adopt automation on $[\alpha_1, \alpha_2]$, or p_j/p_k is bound by a non-automated worker IC to some $c \in \mathbb{R}$, then $\frac{dp_k \alpha}{d\alpha} = 0$, and otherwise $\frac{dy(k)}{d\alpha} = 0$ so that $\frac{d \ln p_k \alpha}{d \ln \alpha} > 1$. In conclusion, when triggers are characterised by a range $[\alpha_1, \alpha_2]$ rather than a unique $\alpha \in \mathbb{R}_+$, then prices are differentiable on $[\alpha_1, \alpha_2]$ with $\frac{dp_j \alpha}{d\alpha} \geq 0$.³⁴

In between automation triggers, if the price system is not over-identified (at most $J - 1 = 3$ structural ICs) and all ICs have strictly positive shares of workers on both sides, prices and worker shares are continuously differentiable, as stated in Proposition 7.

Proposition 7. *If at technology level α ,*

(i) *if for $\theta \in \Theta, j \in J, \alpha = \alpha_{\theta j}^*$, then $s_{\theta}(j) = 0$ (no active triggers),*

(ii) *if for $\theta \in \Theta, j, k \in J$, a j, k -IC holds for θ -workers, then it is active, i.e. $s_{\theta}(j), s_{\theta}(k) > 0$, and*

(iii) *any (j, k) -pair features at most one structural IC at α , with at most 3 structural ICs in total,*

then for any $j \in J, p_j$ is continuously differentiable in α with $\frac{dp_j \alpha}{d\alpha} > 0$, and for $\theta \in \Theta$, if θ is not automated, $s_{\theta}(j)$ is continuously differentiable in α . Furthermore, for the set $\Theta_A \subseteq \Theta$ of automated types, $\tilde{s}_E(j) = \sum_{\theta \in \Theta_A} \phi^{-1[\theta=R]} s_{\theta}(j)$ is continuously differentiable.

Proposition 7 makes reference to the effective share of automated workers in a occupation, $\tilde{s}_E(j) = \sum_{\theta \in \Theta_A} \phi^{-1[\theta=R]} s_{\theta}(j)$, $j \in J$, that weighs workers inversely to their abstract skill, which multiplicatively augments their productivity in the automation state (cf. Eq. (3.14)). Defining the effective worker share of all workers in occupation $j \in J$ as

$$s_E(j) = \tilde{s}_E(j) + \sum_{\theta \in \Theta \setminus \Theta_A} \frac{s_{\theta}(j)}{y^A(H|j)} \quad (3.22)$$

³⁴Differentiability for $k \in J$ with $\frac{dy(k)}{d\alpha} = 0$ is by $0 = \Delta \ln P$ for any change $\Delta \ln \alpha$, which gives

$$\sum_{k \in J: \Delta y(k)=0} \Delta \ln p_k = \sum_{j \in J: \Delta \ln p_j = -\Delta \ln \alpha} -\Delta \ln p_j \Rightarrow \frac{\Delta \ln p_l}{\Delta \ln \alpha} = \frac{|\{k \in J : \Delta y(k) = 0\}|}{|\{j \in J : \Delta \ln p_j = -\Delta \ln \alpha\}|}$$

so that $\ln p_l$ is differentiable in $\ln \alpha$ for any $l \in \{k \in J : \Delta y(k) = 0\}$.

allows to compactly represent occupation-level output as $y(j) = s_E(j)y^A(H|j)$.

Verbally, Proposition 7 states that *between triggers and potential transition points for the type-to-occupation matching structure*, prices and worker shares adjust smoothly. Assumption (iii) of Proposition 7 ensures that there are no conflicting ICs and rules out overidentification. This result leaves only transitions between different type-to-occupation allocations between triggers to be characterised. First, if at α_0 , $s_\theta(j)$ declines to zero for some $\theta \in \Theta$, $j \in J$, an active IC is removed. This event is output-neutral, so that for $\alpha > \alpha_0$, the equilibrium obeys Proposition 7 again, and θ -exit from j at most causes a non-differentiable kink in prices and worker shares. Conversely, entry of types to occupations adds new ICs, which may be more disruptive. An extensive study of this issue is given in Appendix C.3.3.

Broadly, when it results from heterogeneous trends in potential wage ratios across types,³⁵ the addition of new ICs at $\alpha = \alpha_0$ may represent a shock to types' relative labour demand. In this case, there are two *simultaneous* ICs that can not be active jointly, so that one IC becomes passive through a discontinuous adjustment in the labour distribution (i.e., abrupt exit of a type from some occupation). If the new IC is over-identifying, that is, there are three active structural ICs in a neighbourhood $(\alpha_0 - \varepsilon, \alpha_0)$, the equilibrium transition is relative-output neutral so that prices are continuous in α at α_0 .³⁶

Corollary 2 (Permanent Triggers and Automation). *Suppose that prices are continuous in α . Then, if at technology level α_0 , for $\theta \in \Theta$, there exists $j \in J$ such that $\alpha_0 > \alpha_{\theta j}^*$, then at any $\alpha \geq \alpha_0$, it holds that $\alpha > \alpha_{\theta j}^*$, and θ -workers do not work at no-automation in any $k \in J$.*

Corollary 3 (Capital Cost Dynamics). *Suppose that prices are continuous in α . Then, for any $j \in J$, the real efficiency cost of capital, $c_j^K(\alpha) = (p_j \alpha)^{-1}$, and the relative efficiency cost of capital, $c_j^{K,L}(\alpha) = \frac{1/\alpha}{w_{Hj}^A}$ decrease in α globally, and strictly so if $\forall k \in J, \theta \in \Theta : \alpha \neq \alpha_{\theta k}^*$.*

Together with this insight, Proposition 7 allows to establish some useful follow-up results. Corollary 2 follows directly from Propositions 6 and 7, and states that automation triggers and automation of θ -workers are not reversed for $\theta \in \Theta$. Further, Corollary 3 states that all relevant cost parameters of capital are globally decreasing with improvements in automation technology, which directly follows from the behaviour of $p_j \alpha$ (cf. Eq. (3.16)).

This concludes the analysis of equilibrium existence and uniqueness between triggers. Proposition 7 has established that except for a (possibly empty) set of singularity points, the equilibrium quantities adjust to automation in a continuous way. At the singularity points, a discontinuous adjustment can occur as a strictly positive mass of workers is reallocated to a

³⁵Note that the trends of both no-automation wage ratios and automation wage ratios are always on the across types, so that heterogeneous trends can occur only for one automated and one non-automated type.

³⁶Otherwise, price continuity is not guaranteed, and equilibrium adjustment may be erratic. It is shown later in this section that this irregularity does not occur for the given model set-up.

different occupation, but also at these points, there is a well-defined way in which the equilibrium adjusts. Having established the key properties of the equilibrium between triggers, I now proceed with the study of labour demand implications of increases in the efficiency of automation capital α between triggers, and subsequently turn to the analysis *at* triggers.

EQUILIBRIUM ADJUSTMENT BETWEEN TRIGGERS: LABOUR DEMAND

In an occupation $j \in J$ that employs some workers with automation technology, an increase in α (“automation deepening”) affects labour demand in two ways. This can be seen from the equilibrium wage offer equation: with Equations (3.14) and (3.15), $w_{ij}^A = [(1 - \lambda_j)y^A(i|j)]^{\frac{1}{\sigma_j}} p_j$ so that

$$\frac{d \ln w_{ij}^A}{d\alpha} = \frac{1}{\sigma_j} \frac{d \ln y^A(i|j)}{d\alpha} + \frac{d \ln p_j}{d\alpha}. \quad (3.23)$$

Accordingly, the impact of automation deepening on wages is driven by of a *product market* effect resulting from higher productivity at the occupation level and the associated impact on occupation’s output prices (second summand). Moreover, wages are determined by a *micro-level productivity* effect that scales with complementarity of tasks ($1/\sigma_j$) (first summand). The first summand is always positive, and highlights that the pass-through of automation-driven productivity on wages crucially depend on workers’ complementarity to automation technologies, as the degree of complementarity determines the impact of technology improvements on workers’ effective productivity, w_{ij}^A/p_j .³⁷ The second summand may be negative, especially in occupations where productivity growth in response to automation is particularly strong. Still, when all occupations have adopted automation technologies, the overall wage effect of automation deepening is strictly positive, a result that, however, crucially depends on elastic industry-level demand ($P = 1$ with $\frac{dP}{d\alpha} = 0$).³⁸

The dynamics of (effective) employment and labour shares offer insights that are complementary to those obtained from the wage equation. In this context, it holds that for any occupations $j, k \in J$ that use automation technology (cf. Eq. (C.4) in Appendix C.3.1 for the first equality, and Eq. (3.21) for the second),

$$\frac{s_E(j)}{s_E(k)} = \frac{1 - \lambda_j(p_j\alpha)^{\sigma_j-1}}{1 - \lambda_k(p_k\alpha)^{\sigma_k-1}} = \frac{l_j^A}{l_k^A} \quad (3.24)$$

so that relative effective employment and relative labour shares coincide, and trends in employment and labour shares are parallel across occupations.

³⁷With $w_{ij}^A = w_{ij}^A/p_j \cdot p_j$, it is easily seen that $\frac{d \ln w_{ij}^A}{d\alpha} = \frac{d \ln w_{ij}^A/p_j}{d\alpha} + \frac{d \ln p_j}{d\alpha}$, so that the first summand in Eq. (3.23) corresponds to the effective productivity effect.

³⁸In this case, this results from $w_{Hj}^A = w_{Hk}^A$ for all $j, k \in J$ and $\max_{j \in J} \frac{d \ln p_j}{d\alpha} \geq 0$.

Proposition 8 (Relative Dynamics of Occupations: Employment and Labour Share). *Let $j, k \in J$ and $\theta \in \Theta$ such that $\alpha \geq \max\{\alpha_{\theta j}^*, \alpha_{\theta k}^*\}$ and $s_{\theta}(j), s_{\theta}(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then,*

1. *if $j = j(\sigma, \lambda_H)$, $k = j(\sigma, \lambda_L)$, $\sigma \in \{\sigma_L, \sigma_H\}$, it globally holds that (i) $s_E(j) < s_E(k)$ and (ii) $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} > 0$ if and only if $\sigma < 1$;*
2. *if $j = j(\sigma_H, \lambda)$, $k = j(\sigma_L, \lambda)$, $\lambda \in \{\lambda_L, \lambda_H\}$, it holds for $\alpha > 1/p_k$ that (i) $s_E(j) < s_E(k)$ and (ii) if $\sigma_H \geq 1$, it furthermore holds that $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} < 0$;*
3. *if $\sigma_j > 1 > \sigma_k$, then $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} < 0$.*

Proposition 9 (Relative Dynamics of Occupations: Productivity and Output). *Let $j, k \in J$ and $\theta \in \Theta$ such that $\alpha \geq \max\{\alpha_{\theta j}^*, \alpha_{\theta k}^*\}$ and $s_{\theta}(j), s_{\theta}(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then, with $\mu_{jk}(\alpha) := \frac{\frac{d}{d \ln \alpha} \ln y^A(H|j)}{\frac{d}{d \ln \alpha} \ln y^A(H|k)}$,*

1. *if $j = j(\sigma, \lambda_H)$, $k = j(\sigma, \lambda_L)$, $\sigma \in \{\sigma_L, \sigma_H\}$, it globally holds that (i) $\mu_{jk}(\alpha) > 1$, (ii) j strictly grows relative to k , i.e. $d \ln \frac{y(j)}{y(k)} / d \ln \alpha > 0$, and (iii) $s_E(j) < s_E(k)$;*
2. *if $j = j(\sigma_H, \lambda)$, $k = j(\sigma_L, \lambda)$, $\lambda \in \{\lambda_L, \lambda_H\}$, it holds for $\alpha > 1/p_k$ that (i) $\mu_{jk}(\alpha) > \sigma_H / \sigma_L$, (ii) j strictly grows relative to k , i.e. $d \ln \frac{y(j)}{y(k)} / d \ln \alpha > 0$ and (iii) $s_E(j) < s_E(k)$.*

For $\alpha > 1/p_k$, the above relationships for worker- and occupation-level growth also hold absolutely.

Propositions 8 and 9 give the results that characterise the relative dynamics of key quantities in equilibrium. The condition $\alpha > 1/p_k$ asserts that the U -trigger is crossed in both occupations (and at most R -workers are not automated), so that the additional results it allows to derive hold for higher-quality technologies with a broader reach across worker types. As wages of automated workers are equated across their occupations of employment by the labour market equilibration process, the relative statics of employment are most informative about changes in the relative labour demand of occupations. As per Proposition 8, after automation, labour demand and labour shares are always lower in more routine-intensive (high λ_j) occupations, and the difference increases with automation deepening in occupations with gross-substitutive tasks. A similar description applies to low-complementarity (high σ_j) occupations, where the same statics hold above a sufficiently high level of technology ($\alpha > 1/p_k$) unless all occupations in the industry are gross-complementary.

To interpret these changes with respect to the relative strengths of the micro-level productivity and product market channels, Proposition 9 can be combined with the insight that for

changes $\Delta \ln \alpha$ (cf. Eqs. (3.14) and (3.15)), it holds that $\Delta \ln y^A(H|j) = \sigma_j \Delta \ln w_{Hj}^A/p_j$, so that

$$\frac{\frac{d}{d \ln \alpha} \ln \frac{w_{Hj}^A}{p_j}}{\frac{d}{d \ln \alpha} \ln \frac{w_{Hk}^A}{p_k}} = \frac{\mu_{jk}(\alpha)}{\sigma_j/\sigma_k}. \quad (3.25)$$

Accordingly, as per Proposition 9, the relative statics of gross micro-level productivity $y^A(H|j)$ apply also to effective productivity w_{Hj}/p_j . This insight allows to conclude that cross-occupation comparisons of the relative strengths of these two effects are *not* informative about differential trends in labour demand, as the rankings of micro-level productivity growth and occupation-level output growth are identical for all scenarios characterised by Proposition 8. Put differently, a stronger micro-level productivity effect will always be accompanied by a weaker/more negative product market effect.

Asymptotic Behaviour. Propositions 8 and 9 give immediate predictions for the model's limit behaviour (i.e., deepening beyond the R-trigger, where all occupations use labour only in the A-task). In industries where tasks are gross-substitutive in at least some occupations, labour demand concentrates in the occupation $j(\sigma_L, \lambda_L)$, whereas productivity and output are highest and grow fastest in occupation $j(\sigma_H, \lambda_H)$, the occupation that also uses the highest level of capital per worker and the lowest amount of labour. Relative to the initial equilibrium, the cumulative change in relative wages after the state of full automation is reached is summarised by

$$\Delta \frac{w_A}{w_U} = 1 - y^N(i_A|j(\sigma_L, \lambda_L)) > 0, \quad \Delta \frac{w_R}{w_U} = \phi - y^N(i_R|j(\sigma_L, \lambda_H)) < 0 \quad (3.26)$$

so that the dynamics of relative wages are inversely related to specialization in the automated task. Further, once all types are automated, all occupations use automation technology, so that the output market effect is weakly positive in at least one $j \in J$, implying strict wage growth.

TRIGGER AUTOMATION, EARLY DEEPENING AND NON-AUTOMATING OCCUPATIONS

The discussion above has presented results for the intensive margin process of technology adoption and for comparisons of occupations that use automated labour, focused at higher levels of α (beyond U -worker triggers). It remains to address the extensive margin, i.e. initial adoption of automation technologies, early-stage intensive margin automation, and the general impacts of technology adoption at both margins on occupations that do not use technology.³⁹

Intuitively, as wage offers are the product of prices and effective (physical) productivity (cf. Eq. (3.15)) and the latter does not change at triggers, the clear expectation is that extensive margin automation should decrease automating occupations' labour demand. Conversely, non-

³⁹Results for automation at the A-trigger and deepening between the A- and U-trigger are derived in Appendix C.3.4, and those for U-trigger automation are derived in Appendix C.3.5.

automating occupations should face a positive “scarcity effect” through the product market, as their outputs become relatively more costly to produce, increasing their equilibrium valuation. Hence, labour demand of these occupations should increase.⁴⁰

If the type-to-occupation matching structure is preserved relative to the initial equilibrium, at the A -trigger, equilibrium prices do not change, and labour demand effects can be inferred from the equilibrium flow of labour. For employment quantities, one obtains

$$\Delta^A \beta := \beta^A - \beta^N = \frac{m_L - m_H}{6m_H + 2m_L} \left(\frac{3Y_0(A) - Y_0(R)}{Y_0(U)} - 1 \right) + \frac{4m_L(m_H - 1)}{6m_H + 2m_L} \frac{Y_0(A)}{Y_0(U)} > 0 \quad (3.27)$$

where $m_k := m_{i_A j(\sigma_k, \lambda_L)}$ is the worker-level output multiplier induced by A -automation in occupation $j(\sigma_k, \lambda_L)$, $k \in \{L, H\}$. $\beta^A - \beta^N > 0$ is implied by Assumption 1 and $m_L > m_H > 1$. Further,

$$\frac{\Delta^A s_R(j(\sigma_H, \lambda_H))/s_R}{\Delta^A \beta} = \frac{Y_0(U)}{2 \cdot Y_0(R)} \in (0, 1) \quad (3.28)$$

and

$$\frac{s_A^A(j(\sigma_H, \lambda_L))/s_A}{s_A^N(j(\sigma_H, \lambda_L))/s_A} - 1 = \frac{4(m_L - 1)}{3m_H + m_L} Y_0(A) - \frac{3(m_H - 1) + (m_L - 1)}{3m_H + m_L}. \quad (3.29)$$

In line with the expectations framed, labour demand increases in both non-automating occupations. The flow of R -workers to $j(\sigma_H, \lambda_H)$ is proportional to the one of U -workers to $j(\sigma_L, \lambda_H)$ but strictly weaker, as R -workers, unlike U -workers, increase their physical productivity when moving to the new occupation. The flow of A -workers between λ_L -occupations is ambiguous, as the increase in gross physical productivity per A -worker is higher $j(\sigma_L, \lambda_L)$, but the occupation employs also U -workers that do not change their productivity.⁴¹ From equation (3.27), it follows that the surge in labour demand in non-automating occupation scales in all parameters of the A -productivity surge, $Y_0(A)$, m_L and m_H . The same applies to the multiplier on industry (and occupation) level outputs.

The type-to-occupation matching structure may change with A -automation. Such “structural breaks” remove or introduce ICs for worker types and may result in occupations remaining at the trigger for an interval of values for α (cf. Subsection 3.4.2). The removal (addition) of an IC means that an increase in labour demand is not (again) met by increased employment, augmenting (dampening) the scarcity effect and resulting in faster (slower) wage growth relative to other occupations.⁴²

Deepening. Denote by $\alpha^A := \max\{\alpha \in \mathbb{R}_+ : (\exists j \in J : \alpha = \alpha_{i_A j}^*)\}$ the level of technology that

⁴⁰This expectation is also in line with Autor and Dorn (2013) who attribute the wage increases in low-skill services occupations, unexposed to automation, to automation-induced productivity gains in middle-skill manufacturing occupations and complementarities in the consumption of output goods.

⁴¹As seen from Eq. (3.29), the flow volume from $j(\sigma_L, \lambda_L)$ to $j(\sigma_H, \lambda_L)$ tends to be positive for large $Y_0(A)$ where β^N is relatively large, and negative when $m_L - m_H$ is modest, i.e. $3(m_H - 1) + (m_L - 1)$ is large.

⁴²The analysis in Appendix C.3.4 studies in detail the individual cases and confirms this general interpretation.

concludes A -adoption. As derived in Appendix C.3.4, the equilibrium at α^A is not characterised by IC-over-identification except for a very specific case in which case three structural ICs hold, so that there is no discontinuous disruption in prices at α^A . Thus, for $\alpha = \alpha^A + \varepsilon$, $\varepsilon > 0$ small, the industry obeys the set-up of Proposition 7. For adjustment in between A - and U -triggers, as established in Proposition 14 in Appendix C.3.4, any structural break occurs at IC over-identification, so that price adjustment is smooth and the monotonicity results for effective capital cost $(p_j\alpha)^{-1}$ apply. While $j(\sigma_H, \lambda_L)$ continues to employ only A -labour prior to the U -trigger (cf. Proposition 13 in Appendix C.3.4), there is little more insight to be obtained without further parameter restrictions. Thus, the following outlines the compound effects of deepening from α^A to $\alpha_0^U = \min\{\alpha \in \mathbb{R}_+ : (\exists j \in J : \alpha = \alpha_{Uj}^*)\}$, the level of α where U -adoption is first triggered. With $p_j\alpha \rightarrow 1$, $w_{Hj}^A \rightarrow p_j$ (cf. Eq. (3.15)) so that around the U -trigger, A -workers become highest-price seeking, and all occupations but $j(\sigma_H, \lambda_H)$ are necessarily at the U -trigger at α_0^U (cf. Proposition 15 in Appendix C.3.4). Proposition 10 gives the key results.

Proposition 10 (A-Deepening: Employment). *In a neighborhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of α_0^U ,*

1. *Both λ_L -occupations employ A -workers, i.e. for any $j \in J$ with $\lambda_j = \lambda_L$, it holds that $s_A(j) > 0$;*
2. *$p_{j(\sigma_L, \lambda_H)} = \max_{j \in J} p_j$;*

and $s_U(j(\sigma_L, \lambda_L))$ is (strictly) smaller than $s_U(j(\sigma_L, \lambda_L))|_{\alpha=\alpha^A}$ (if $s_U(j(\sigma_L, \lambda_L))|_{\alpha=\alpha^A} > 0$).

In a neighbourhood left of the U -trigger, the type-to-occupation matching is similar to the initial equilibrium (cf. Figure 3.3), with both λ_L -occupations employing A - and no R -, $j(\sigma_L, \lambda_H)$ employing U - and $j(\sigma_H, \lambda_H)$ employing R -workers. Similar to the adoption stage, scarcity effects in non-automating occupations promote reallocation of U - and R -workers to non-automating industries. To understand the role of heterogeneous complementarity, unexplained between the A - and U -triggers by Proposition 8, it is useful to note that for $\lambda \in \{\lambda_L, \lambda_H\}$, A -employment in $j(\sigma_H, \lambda)$ and $j(\sigma_L, \lambda)$ at $\alpha_0 < \alpha_0^U$ implies $p_{j(\sigma_H, \lambda)} < p_{j(\sigma_L, \lambda)}$ by Proposition 4. Thus, for $\sigma_H > 1$,

$$1 - \lambda(p_{j(\sigma_H, \lambda)}\alpha)^{\sigma_H-1} \geq 1 - \lambda(p_{j(\sigma_L, \lambda)}\alpha)^{\sigma_H-1} \geq 1 - \lambda(p_{j(\sigma_L, \lambda)}\alpha)^{\sigma_L-1}$$

where the last inequality follows from $p_{j(\sigma_L, \lambda)}\alpha < 1$. This gives $s_E(j(\sigma_H, \lambda)) > s_E(j(\sigma_L, \lambda))$, where at the U -trigger, $s_E(j(\sigma_H, \lambda)) = s_E(j(\sigma_L, \lambda))$ by equality of $y(j)$ and $y^A(H|j)$ for $j \in J$ with $\lambda_j = \lambda$. Thus, for the change from α_0 to α_0^U , one obtains $\Delta[s_E(j(\sigma_H, \lambda))/s_E(j(\sigma_L, \lambda))] < 0$. Accordingly, the insight from Proposition 8 on heterogeneous complementarity at $\sigma_H > 1$ transfers to the compound effect from any α_0 at which both occupations employ automated labour to α_0^U .

Especially, the λ_L -comparison does not depend on $\sigma_H > 1$,⁴³ which emphasises a particularly tight link between complementarities and labour demand at low levels of α . As $s_U(j(\sigma_L, \lambda_L))$ is non-increasing, this also implies that $s_A(j(\sigma_L, \lambda_L))/s_A(j(\sigma_H, \lambda_L))$ increases strictly. In terms of growth, in absence of structural breaks, output increases in all occupations but $j(\sigma_H, \lambda_H)$, and the multiplier on industry output is

$$mult^{U,N}(Y) = \left(\frac{1}{\rho_A}\right)^{\frac{1}{4}} \left(1 + \left(\frac{1/1 - \lambda_L}{y^N(i_A|j(\sigma_L, \lambda_L))} - 1\right) Y_0(A)\right), \quad (3.30)$$

which highlights the two competing forces of A -deepening, A -productivity growth and the relative decline in A -labour demand in the abstract-specialised occupation.⁴⁴

U-trigger Automation. In distinction to the A -trigger, when the U -trigger is hit, automation occurs in a scenario where some types are already automated, but the phenomenon is otherwise very similar to A -trigger automation. Again, the output market effect of adoption shifts non-automated workers away from the automating occupations (see Proposition 16 in Appendix C.3.5). The decline in labour demand due to automation affects all types that “overlap” with U -labour, i.e. types that receive their highest wage offers in occupations that employ U -workers. As A - and U -workers are both highest-price seeking around the U -trigger, A - necessarily overlaps with U -labour, and A -wages may therefore decline if the equilibrium ICs allow $p_{j(\sigma_H, \lambda_H)}$ to increase at the expense of other prices.⁴⁵ If no structural break occurs relative to the initial equilibrium, the multiplier of growth, shared across occupations, is

$$mult^U(Y) = \frac{s_A + s_U + (1 - \lambda_H)Y_R/\rho_R}{(\gamma_A^U - 1)Y_A/\rho_A + Y^{min}} \frac{2}{1 - \lambda_H + 1 - \lambda_L} > 1 \quad (3.31)$$

and unambiguously increases in s_U .

Dynamics after U-automation. The statics of deepening beyond the U -trigger are governed by Propositions 8 and 9 and are discussed in Subsection 3.4.2. Trigger automation of R -workers is trivial, as output market effects are shared across types, and there are no effects on non-automated types. Accordingly, this event is wage-neutral and gross productivity-enhancing, where the productivity surge scales in $Y_0(R)$ and the per-worker multiplier on R -output.

⁴³At the A -trigger,

$$\rho_A = \frac{p_{j(\sigma_L, \lambda_L)}}{p_{j(\sigma_H, \lambda_L)}} = \frac{s_E(j(\sigma_H, \lambda_L))y^A(H|j(\sigma_H, \lambda_L))}{s_E(j(\sigma_L, \lambda_L))y^A(H|j(\sigma_L, \lambda_L))} = \frac{s_E(j(\sigma_H, \lambda_L))}{s_E(j(\sigma_L, \lambda_L))} \frac{m_H}{m_L} \rho_A$$

so that $s_E(j(\sigma_H, \lambda_L))/s_E(j(\sigma_L, \lambda_L)) = (m_H/m_L)^{-1} > 1$ and $\Delta[s_E(j(\sigma_H, \lambda_L))/s_E(j(\sigma_L, \lambda_L))] < 0$.

⁴⁴Appendix C.3.4 formally establishes that this multiplier strictly exceeds one, so that the A -demand effect dampens growth but does not revert it.

⁴⁵Also in a two-task model with more types, all previously automated types would share the output market effects of a new type’s automation at the trigger by trigger indifference between production schemes for this type.

3.4.3 ECONOMY-LEVEL EFFECTS AND INELASTIC DEMAND

Due to the assumption of elastic final demand ($P(Y) = P$), my model generates an increase in labour demand at the industry level. Recalling that labour demand is driven by a productivity and a product market effect (cf. Eq. (3.23)) and the latter effect is shut off when demand is perfectly elastic, this appears natural. Re-interpreting the model as an economy (at which level the numeraire assumption is reasonable) with complementarities between automatable and non-automatable sets of tasks, this result supports an optimistic view of technology in which long-run employment, at the aggregate, is not threatened by advances in automation technologies. While Acemoglu and Restrepo (2018b) condition this result on an endogenous mechanism of task creation outside the automatable domain, my model suggests that the simpler circumstance of task complementarity may also provide strong protection of aggregate labour demand.⁴⁶

For real-world industries, however, especially at higher levels of aggregation (e.g. one-digit industries), elastic demand is hardly a realistic assumption. Without imposing $P(Y) = P$ with $\frac{d \ln P}{d \alpha} = 0$, the equation of the wage impact of automation (cf. Eq. (3.23)) is

$$\frac{d \ln w_{ij}^A}{d \alpha} = \frac{1}{\sigma_j} \frac{d \ln y^A(i|j)}{d \alpha} - \frac{d \ln y(j)/Y}{d \alpha} + \frac{d \ln P}{d \alpha}. \quad (3.32)$$

Here, the third summand captures the industry-level product market effect. The more responsive the industry-level output price is to productivity, the more negative is the industry-level labour demand effect of automation. Especially with very substitutable tasks (high σ_j) where the first summand diminishes, the labour demand effect should be negative.

The conclusion is not as straightforward, since a lower demand elasticity induces stronger price declines at given expansions of output and slows down the use automation capital, especially at the intensive margin, so that the overall role of demand elasticity for the latter two terms in Eq. (3.32) is not trivial. Still, a simple example illustrates that a less than perfectly elastic demand function may generate declining industry level labour demand. Assume that $P = 1/Y$, $\lambda_L = \lambda_H = \lambda \in (0, 1)$ and $\sigma_L = \sigma_H \in \mathbb{R}^+$, and further that $s_R = 0$. Then, $\forall j \in J$, $p_j = 1/y^A(H|j)$ by symmetry of occupations, and in the full-automation stage (cf. Eqs. (3.15))

⁴⁶This reasoning comes with some over-simplification. Contractions of the set of non-automatable tasks would imply increases in λ_L, λ_H that are possibly wage-decreasing at fixed levels of technology (cf. Proposition 4). Still, relative to Acemoglu and Restrepo (2018b), my model additionally studies intensive-margin technology improvements which stimulate labour demand, so that the view on labour demand's trajectory is more optimistic. Further, if a fixed set of tasks remains unautomatable indefinitely (e.g. those requiring human-to-human interaction) there are minimal values $\lambda_L^{min}, \lambda_H^{min}$ that are never crossed, so that the intensive margin dominates asymptotically.

and (3.20) for the first, and Eq. (3.21) for the last equality),

$$w_{Hj}^A = p_j y^A(H|j) (1 - \lambda(p_j \alpha)^{\sigma-1}) = 1 - \lambda(p_j \alpha)^{\sigma-1} = l_j^A \quad (3.33)$$

so that industry-level labour demand, as summarised by the average wage (equal to the shared equilibrium wage across occupations) co-moves with the labour share. By $\Delta \ln p_j = -\Delta \ln y^A(H|j)$, with Eq. (3.14), it results that

$$\Delta \ln p_j = \frac{\sigma}{\sigma-1} \Delta \ln (1 - \lambda(p_j \alpha)^{\sigma-1}) = \frac{\sigma}{\sigma-1} \Delta \ln l_j^A.$$

Therefore, $\Delta \alpha > 0$ that increases gross productivity, i.e. $\Delta \ln y^A(H|j) > 0$, leads to decreased (increased) labour demand in an industry with gross-substitutive, i.e. $\sigma > 1$ (gross-complementary, i.e. $\sigma < 1$) tasks. Hence, in the industry with heterogeneous λ and σ studied thus far, intensive margin automation may reduce labour demand at least if tasks are gross-substitutive on average.

For general interpretation, it is useful to rewrite Eq. (3.32) as

$$\frac{d \ln w_{ij}^A}{d \alpha} = \frac{1}{\sigma_j} \frac{d \ln y^A(i|j)}{d \alpha} - \frac{d \ln y(j)}{d \alpha} + \frac{d \ln PY}{d \alpha}. \quad (3.34)$$

The third summand captures an industry-level revenue effect and can be interpreted as the *factor level effect* that drives demand for both input factors (capital and labour). This force is analogous to the one that applies to firm-level Hicks-neutral productivity growth. The additional within-industry mechanism added by automation is the *factor composition effect* that measures the relative strengths of workers' effective productivity growth (first summand) and occupation level gross output growth that corresponds to gross productivity growth when holding fixed employment. When tasks are gross-substitutive (gross-complementary), effective productivity increases more slowly (faster) than gross productivity with automation, so that the factor composition effect on labour demand is negative (positive). By $(w_{ij}^A/p_j)/y^A(i|j) = w_{ij}^A/p_j y^A(i|j) = l_j^A$, this effect co-moves with the labour share. If effective productivity grows faster (slower) than gross productivity, the productivity contribution of labour increases (decreases), and the factor composition of *productivity* shifts in favour of labour (capital).⁴⁷

3.4.4 DISCUSSION

Having concluded the model analysis, this section discusses first the central assumptions imposed and possible limitations emerging from them, and afterwards additional interpretations

⁴⁷Therefore, labour shares can increase even if the capital-to-labour ratio increases, which occurs for any positive change in α (cf. Eq. (3.17) and the discussion thereafter).

of the model, especially related to labour demand effects at different levels of aggregation, the long-run labour market equilibrium at the economy level and the trajectory of labour shares.

MODEL ASSUMPTIONS

For analytical tractability, I have imposed a number of simplifications. First, the bivariate structure in tasks and heterogeneity eliminates the possibility of sequential automation multiple tasks within the same industry, but also a more general model allowing for this would have the same qualitative implications of trigger automation and subsequent intensive-margin improvements in task-level technologies. Next, specialization in tasks may be only relative but not absolute as I have assumed, e.g. with $\Phi_A = (1, c\phi)$, $\Phi_U = (1, c)$ and $\Phi_R = (\phi, c)$ and $c > 1/\phi$ ($c < \phi$), so that all workers are effectively specialised in the routine (non-routine) task.⁴⁸ Allowing for such a scenario does not affect automation timing (cf. Corollary 1) and the key structural mechanisms, so that the framework’s qualitative predictions are robust to this generalization. Finally, also the share assumption imposed to obtain the initial equilibrium structure does not drive the mechanisms and thus qualitative results.

More crucial assumptions are exogeneity of the types’ shares, i.e. abstraction from a first-stage educational choice problem and possible retraining, and the unit mass of labour in the industry. The latter assumption is unproblematic, as industry-level labour demand effects are readily studied from average wage effects at a constant mass of employment (cf. Section 3.4.3). For the former issue, the only relevant educational decision is (re-)training towards higher abstract skill which augments productivity post-automation. This would accelerate the product market effect, but has otherwise little structural significance for the adjustment mechanism.

In contrast to workers, the structural role of firms is very limited in my model. In particular, firms do not employ workers of different occupations, which also rules out cross-occupational complementarities between workers of different occupations within firms (e.g., boosts of production worker productivity from manager productivity).⁴⁹ However, the outputs of occupations are complementary in the analysis within the industry, and for labour demand effects at any higher level than the firm, it does not matter whether they occur within or between firms.⁵⁰ The discussion of firm-level effects below re-addresses this issue.

Finally, two aspects of the production set-up command discussion. First, the model ab-

⁴⁸This appears plausible for industries that predominantly rely on one task, i.e. both λ_L and λ_H are close to zero/one, where worker selection into the industry will always be associated with a higher skill level in this task.

⁴⁹Within-occupation complementarities between workers are implicitly allowed, cf. the discussion after Eq. (3.1).

⁵⁰One limitation to this view is that even within industries, firms may be heterogeneous in their production structure, and importantly in the degree of task complementarity. Recent work on the German labour market (Freund, 2022) documents that complementarities between co-workers have doubled over the period 1990–2010, which signals that firms increasingly unbundle tasks from the job level and distribute them across different workers of a team within the firm. This may have led to a weakening of within-job complementarities as studied in my model, especially in large firms which may employ a more specialised workforce.

stracts from any capital that does not automate tasks (e.g. real-estate and buildings). So long as capital markets are elastic, however, the model’s mechanism easily generalises to a framework in which intermediates from task aggregation as in Eq. (3.1) are multiplicatively combined with other physical capital assets, e.g. in a Cobb-Douglas function, to produce firm- or industry-level outputs.⁵¹ Second, I neglect that technology may augment the task-level productivity of labour. Most recent technologies complement labour by performing independent tasks (e.g. browsing web databases, running computations and simulations, creating visual models, etc.), add new tasks to the production process (e.g. digital communication, data visualization) or even constitute process innovations (e.g. computers, cloud computing), so that they either fit the framework of the model or represent occupational innovation that reshapes the production structure. Therefore, the structural relevance of task-augmenting technologies appears limited.

The remainder of this section discusses relevant interpretations that emerge from the model analysis. Some of the results derived depend on the condition $\sigma_H > 1$, i.e. that tasks are gross substitutes at least in some occupations. This seems to apply to the majority of real-world occupations, considering that workers tend to spend the majority of time in tasks in which they have been trained/are most skilled (cf. Eq. (3.9)). Therefore, the interpretations to follow rely on the full set of analytical results derived under this additional condition.

FURTHER IMPLICATIONS OF THE ANALYSIS

With respect to the agenda of understanding the effects of automation on labour demand at different levels of aggregation, the model shows that in any environment where workers perform complementary tasks ($\sigma < \infty$), labour demand effects are unambiguously positive at perfectly competitive firms. Since these firms take prices as given, of the forces in Eq. (3.33), the only one relevant at the micro-level is the effective productivity effect, which is zero for adoption and strictly positive for deepening by $\frac{dp_j^\alpha}{d\alpha} > 0$ (cf. Eq. (3.15)). This insight holds also in environments where firms use labour inputs from different occupations, as the effective productivity of non-automated workers remains unchanged.⁵² The crucial assumption, however, is that firms are price takers. Any influence firms exert over prices subjects workers to a negative product market effect, which may reduce or even revert the positive labour demand impact of automation. This insight rationalises well differences in existing empirical research. While Aghion et al. (2020) find positive firm-level effects of automation on sales and employment, Bonfiglioli et al. (2020) show a limited impact on sales and reductions in labour demand, indicating

⁵¹More concretely, the firm-level Cobb-Douglas function is $y(i) = A_i(\gamma_i l(i))^{\varepsilon_L} k(i)^{\varepsilon_K}$ with γ_i as the task-based labour productivity coefficient given by Eq. (3.1) that changes with improvements in automation technology.

⁵²This argument implicitly assumes that workers of different occupations are perfect substitutes at the firm level. If workers are complementary within the firm, the firm-level labour demand effects only become more positive.

that the former work may study automation technologies that are on average prevalent in more competitive environments. This highlights also the potential role of competition policy for the micro-level impact of automation on labour markets.

Notably, at the micro level, beyond the point of capital cost-effectiveness in the R -task ($p_j\alpha > 1$),⁵³ micro-level labour demand effects of intensive margin automation are even stronger in more substitutive environments due to the stronger acceleration of capital use: with Eq. (3.15),

$$\frac{\partial \ln w_{Hj}^A}{\partial \alpha} = \frac{1}{\alpha} \frac{\lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \quad (3.35)$$

and

$$\frac{\partial \ln \lambda_j (p_j \alpha)^{\sigma_j - 1}}{\partial \sigma} = \ln p_j \alpha. \quad (3.36)$$

so that $\frac{\partial^2 \ln w_{Hj}^A}{\partial \alpha \partial \sigma} > 0$ for $p_j \alpha > 1$.

At higher levels of aggregation where the product market effect becomes relevant, the labour share is a key statistic that summarises the dynamics of labour demand, as the change in labour demand is proportional to the change in the labour share.

It can be shown that capital-to-labour ratios are initially higher if capital-labour complementarities are stronger, but eventually it is higher if capital can be used more independently. They are therefore initially larger in high- but eventually larger in low-complementarity occupations with a similar routine task weight.⁵⁴ However, labour shared dynamics depend not only on the capital-to-labour ratio, but also on the wage dynamics.

For technology adoption at the trigger, plugging the automation trigger α_{ij}^* (cf. Eq. (3.18)) into the labour share l_j^A as expressed in Eq. (3.21) gives $l_j^A = 1 - \eta_{ij}$, where η_{ij} is the time spent in the routine task before automation (cf. Eq. (3.9)). Therefore, automation at the trigger reduces the labour share in a fashion proportional to the time use in automated tasks. Moreover, Eq. (3.21), together with the result that $p_j \alpha$ is increasing in α , implies that intensive margin automation decreases (increases) the labour share if $\sigma_j \geq 1$ ($\sigma_j \leq 1$), and therefore further augments (partly offsets) the initial labour share decline associated with adoption.

This insight re-emphasises the close connection of recent trends in labour shares and labour demand, consistent with the within-industry origin of the well-documented global decline of labour shares (Dao et al., 2017) and its possibly tight link to automation (Autor and Salomons,

⁵³The ratio of task-level factor cost, $w_{Hj}^A/(1/\alpha)$ co-moves with $p_j \alpha$ with changes in α (cf. Eq. (3.15)), and is equal to 1 at the U -trigger where $p_j \alpha = 1$. Therefore, task-level cost effectiveness of capital is equivalent to $p_j \alpha > 1$.

⁵⁴Equation (3.17) implies for $j, k \in J$ with $\lambda_j = \lambda_k$ that $\frac{k_{\theta}^*(j)}{k_{\theta}^*(k)} = \left(1/c_j^{K,L}(\alpha)\right)^{\sigma_j - \sigma_k}$ for any $\theta \in \Theta$ employed in both occupations (where $w_{\theta j}^A = w_{\theta k}^A$ and thus $c_j^{K,L}(\alpha) = c_k^{K,L}(\alpha)$). As $c_j^{K,L}(\alpha)$ is strictly decreasing in α with $c_j^{K,L}(1/p_j) = 1$, σ_H -occupations use more (less) capital than σ_L -occupations with equal λ_j if $p_j \alpha \geq 1$ ($p_j \alpha \leq 1$), and $p_j \alpha$ is increasing in α (Corollary 3), so that the σ_H/σ_L -ratio of capital use increases in α .

2018). Further, the model unveils two countervailing channels that may determine the future trajectories of labour shares. Historically, automation has occurred especially in manufacturing where the set of routine tasks was not strongly complemented by abstract (or more generally, non-automatable) tasks. As these “low-hanging fruits” of automation are harvested, new technologies may target also more complex and complementarity-intensive occupations. Therefore, perhaps against the intuitive assumption, more disruptive technological change that extends beyond currently automating industries may indeed slow down the decline of labour shares.

Beyond labour shares, also the long-run trajectory of employment in light of ongoing automation is a highly debated topic, and the task-replacing nature of more recent technological change has raised concerns that the pursuit of productivity growth and strong labour demand is increasingly becoming a trade-off, rather than a synergetic policy objective. To this end, the analysis I present reinforces this view. In the model, for sufficiently capable technologies (i.e., in the state of full diffusion of automation technology), at the occupation level, the degree of task-level complementarities is positively related to labour demand, but negatively related to productivity growth. Moreover, the same degree of industry-level productivity growth is achieved at a lesser reduction in industry-level labour demand if tasks are more complementary.⁵⁵ Therefore, if policy-makers could influence the equilibrium mix of occupations, a stimulation of productivity growth could best be achieved by boosting low-complementarity occupations and therefore accelerating automation, whereas strengthening labour demand would require to support high-complementarity occupations.

Finally, under the premise that task-based skills in non-automatable tasks are more difficult to develop and found more frequently in high-skilled workers,⁵⁶ the model offers a structural explanation for the repeatedly documented empirical complementarity of high-skilled workers to recent technological change. First, these workers are more reliant on non-automatable tasks even within occupations due to their skill profiles, and in this sense, the complementarity is an innate feature of high-skill workers. Furthermore, a significant proportion of this complementarity may also be driven by occupational choice, as high-skilled workers naturally gravitate towards occupations with low routine task weight λ .

3.5 CONCLUSION

I have theoretically investigated the role of worker-level task aggregation for the labour demand effects of task-automating technologies. My framework accounts for occupation-level

⁵⁵This results directly from the relationship between labour shares and labour demand, and the role of complementarity for the trajectory of labour shares.

⁵⁶As a key feature of wage polarization is a shift of relative labour demand in favour of high-skilled workers (e.g. Acemoglu and Autor, 2011), by Eq. (3.26) it is natural to identify these workers as the model’s A -type.

heterogeneity in the production weight of the automatable task, and in the degree of complementarity in task aggregation, and further for worker heterogeneity with respect to task-specific skills. The framework is well-suited to explain empirical trends at all levels of aggregation, i.e. the firm, occupation, industry, and economy. Moreover, it is useful in deriving predictions for the future trajectory of labour demand and labour shares and offers policy-relevant insights.

The key mechanism through which automation technology interacts with workers is described by two countervailing forces. On the one hand, automation increases workers' *effective productivity*, i.e. their productivity net of the additional cost incurred by the use of automation capital, which stimulates labour demand. On the other hand, it also increases workers' gross productivity, and in environments of inelastic product demand, this lowers prices and generates a negative *product market effect* on labour demand. An alternative interpretation characterises this mechanism as a trade-off between a positive *factor level effect* of increased micro-level productivity, i.e. a boost to demand for all inputs associated with increased competitiveness, a force that would be similarly observed for Hicks-neutral productivity growth, and a negative *factor composition effect* that biases the factor mix of production in favour of capital.

Complementarities between job tasks are central for the trajectory of labour demand at all levels of aggregation. Whenever there is at least minimal complementarity between tasks, automation of only one task is unable to depress micro-level labour demand in perfectly competitive environments. Perhaps counter-intuitively, the micro-level effect is even stronger when tasks are less complementary (but not perfect substitutes) when capital is more cost-effective in the automatable task.⁵⁷ When a non-negligible mass of firms automates, the increase in these firms' productivity generates negative spillovers on non-automating competitors ("business-stealing effects") through the product market effect. When tasks are more complementary, capital-labour substitution and gross productivity improvements at automating firms are weaker, and automation generates weaker business stealing effects. Also a given improvement in industry-level productivity through task automation comes at a lower decline in labour demand in more complementarity-intensive environments. Hence, in scenarios with non-zero product market effects, the degree of complementarity is inversely related to the decline of labour demand. Intensive margin automation, i.e., improvements in adopted automation technologies, even stimulates labour demand in environments with gross-complementary tasks.

My analysis is complementary to Acemoglu and Restrepo (2018b) who identify occupations

⁵⁷Task-level cost-effectiveness of capital is not equivalent to its use. If the skill profile of workers is leaning towards the non-automatable (automatable) task, then automation capital is used strictly before (after) the point of equal task-level cost-effectiveness of factors as it increases (decreases) the average task-level productivity of workers.

and tasks a one-to-one fashion and study complementarities between tasks only at higher levels of aggregation, such as the industry or the country.⁵⁸ I do not speak to the disappearance of occupations due automation, a phenomenon that remains to be observed at large scale but may be relevant especially in the longer term. When there are no complementary tasks (remaining) that workers may perform, automation trivially eradicates all occupation-level labour demand,⁵⁹ and the aggregate automation-labour demand link is unambiguously more negative than the one in my analysis. Within industries that do use complementary tasks, I show that automation technologies are intrinsically first labour-friendly before they are “predatory”: the timing of worker automation is inversely related to their specialization in the automatable task. However, any adverse impact on an industry’s labour demand is determined by the product market effect, and is shared by *all* its workers no matter their specialization, which emphasises the role of demand saturation for the labour market effect of automation (cf. Bessen, 2019).

When workers perform multiple complementary tasks pre-automation, occupation- and industry-level trends in labour demand correspond proportionally to those in labour shares. The labour share is shown to generally decrease in response to automation, and indeed more strongly so if the complementarity between the tasks is lower. Therefore, if in the future, automation extends its reach to the higher-complementarity services sector, automation’s role in the global decline of labour shares will be weakened. By the close link of labour shares and labour demand, also the trajectory of employment in the longer term could critically depend on whether future automation technologies are *more-of-the-same* (i.e., manufacturing-focused) or if they are applicable also in the broader economy. In the latter scenario, the higher degree of complementarity in services predicts a less negative own-sector impact of automation, and if the mechanisms of general equilibrium adjustment remain similar to the ones in the past, it predicts also a differentially more positive labour demand effect at the economy level.

This insight has relevant policy implications. The market may have endogenously directed automation towards low-complementarity occupations where productivity gains from automation are higher, and may continue to produce *more-of-the-same* technologies. Seeking higher productivity gains from automation, firms may re-organise occupations to use automatable tasks more independently. Productivity growth that is sustainable in terms of labour demand may therefore require active policy influence, e.g. through R&D support to services automation and by supporting business creation in complementarity-intensive occupations.

Any change in the level of technology induces a (possibly discontinuous) change in the effi-

⁵⁸On the one hand, their model is nested in the one I introduce as the limit case of $\sigma_L, \sigma_H \rightarrow \infty$ that shuts off within-occupation complementarities. On the other, I abstract from several more aggregate factors that are central to their analysis, such as skill heterogeneity of workers across industries and more importantly directed technological change that induces the creation of new tasks in the economy-level equilibrium.

⁵⁹This is easily studied in my framework with the boundary parameter case $\lambda_H = 1$.

cient allocation of labour. This points to a large potential for mis-allocation and inefficiency in economic adjustment to automation. In frictional real-world labour markets, this adjustment may come at significant social cost, consistent with recent empirical evidence (Bessen et al., 2019). Further, for a broad range of levels of technology, the diffusion of automation throughout the industry is partial. The debate on sluggish productivity growth names low diffusion as a key impediment towards strong productivity growth in the digital age (e.g. Van Ark, 2016; Brynjolfsson, Rock and Syverson, 2018), but through the lens of the model, high efficiency of certain technologies in some occupations does not imply that their adoption in other occupations would be efficient, even within industry. Hence, while reducing barriers to technology diffusion certainly has its role in boosting productivity growth especially in environments of frictional financial markets, complementary R&D support to prospective technology innovators could be needed to accelerate intensive margin technology improvements that lead to more mature technologies which boost productivity across a broader range of occupations.

Finally, the significance of complementarities for the response of labour demand at the occupation level has important implications for education policy and workers' optimal educational choice. Changes in educational attainment have played a key role in past adjustments to automation, e.g. in agriculture ("high-school movement", cf. Autor (2015)), yet the efforts to restructure education systems in response to more recent automation and technology trends more generally has been limited. Suggestions by the OECD recommend to emphasise skills in so-called "bottleneck tasks" that are difficult to automate in a foreseeable future (Nedelkoska and Quintini, 2018; Bechichi et al., 2018). However, identifying tasks that are non-automatable over a career of 30-50 years is hardly realistic given the rapid pace of technological change.⁶⁰ As my model shows, specialization is a double-edged sword: on the one hand, specialists in the "right", i.e. automation-complementary tasks benefit most from automation, but specialization in the "wrong" tasks may leave workers worse off, and more importantly, any specialist naturally gravitates towards low-complementarity occupations that see stronger declines in labour demand after automation.

Indeed, workers proficient in general-purpose skills that imply a more balanced skill profile across multiple tasks prefer high-complementarity occupations already pre-automation. Since even voluntary separations may lead to permanent earning losses, the associated incumbency premia offer significant gains to these workers even if the direction of automation is certain. Moreover, when it is not, they are less prone to betting on the "wrong horse" of task specialization. Accordingly, versatility and flexibility may be more important determinants of future

⁶⁰This is indeed underscored by something Nedelkoska and Quintini (2018) state themselves: "[b]y 2013, advances in machine learning (ML) and mobile robotics (MR) extended the list of job tasks that can be performed by machines by a degree that made the question "what is that machines cannot do" easier to answer than [... Autor, Levy and Murnane (2003)]'s question [of what they can do] asked just ten years before."

labour market success than specialization. In this sense, policy should not emphasise the race against the machine by pushing workers as far away as possible from automatable tasks, but should prime them to work *with* the machines in a complementary way by emphasizing e.g. numeracy and logic, social skills and the ability to learn, which promote proficiency across a broad range of tasks and occupations.

The insights of this paper point to two interesting avenues for future research. First, the quantitative importance of this mechanism may be subjected to empirical tests beyond the motivating figures presented in the introduction. Second, the analysis points to a key equilibrium role of occupations in labour markets' adjustment to advances in automation technology. Innovation in occupations, i.e. finding new ways to combine tasks in a productive way or adding previously neglected tasks to occupations when their automation is sufficiently cheap to provide the task in large quantity (as made possible e.g. by the large comparative advantage of computers over labour in performing repetitive computations, and the ever-declining price of computing power), may be central to the long-run fortune of employment and the labour share. On the one hand, this inspires an investigation of the determinants of the pace and direction of investments in such innovations, and the opportunities of policy-makers to take influence on them. On the other, there may be a central role for business dynamism, especially the rate at which new firms emerge and how fast they grow, as in designing the task content of occupations, compared to incumbent firms, new entrants face fewer restrictions imposed by existing infrastructure and operational processes and may thus be more progressive and innovative. This reasoning suggests that local labour markets with a larger role of new firms should differentially adjust better to automation in terms of wages and employment, but also that the recent decline in business dynamism (e.g. Decker et al., 2016; Calvino, Criscuolo and Verlhac, 2020) may have exacerbated adverse impacts of automation on labour demand; verifying these predictions is an interesting empirical exercise.

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Appendices

A. APPENDICES CHAPTER 1

A.1 DATA APPENDIX

A.1.1 MULTIPROD AND THE DISTRIBUTED MICRODATA APPROACH

Researchers and policy analysts increasingly recognise the importance of microdata for understanding how heterogeneity in economic outcomes across firms shapes aggregate performance. Comparing patterns and trends in firm-level developments across countries can provide important insights into the role of policy settings and framework conditions in enhancing productivity growth and convergence. Yet, confidentiality concerns and other administrative issues constitute substantial obstacles to the transnational access to official micro-data.

The “distributed micro-data approach” circumvents these obstacles by collecting statistical moments of the distribution of firm characteristics (employment, output, productivity, wages, age, etc.) by a centrally written, but locally executed routine that is flexible and automated enough to run across different micro-data sources in different countries. This approach to data collection brings three main advantages:

- It puts a lower burden on national statistical agencies and limits running costs for such endeavours while enabling research across a range of different policy areas;
- It overcomes the confidentiality constraints of directly using national micro-level databases while respecting the confidentiality of the underlying data sources;
- It achieves a high degree of harmonisation and comparability across countries, industries, and over time.

The OECD MultiProd project relies on the distributed micro-data approach to access confidential firm-level data, in collaboration with experts from National Statistical Offices, government departments, and research organisations in 29 countries. MultiProd provides a unique comprehensive overview of productivity patterns and other related performance measures over the last two decades. It extends productivity analyses beyond aggregate industry performance and focuses on the underlying dynamics and developments within industries. The resulting micro-aggregated database is harmonised across countries and over time, and hence is suitable for international comparisons – see Berlingieri et al. (2017) for details.

A.1.2 UNDERLYING DATA SOURCES AND COLLECTED DATA

The project typically relies on administrative data that cover the universe of employing firms. When administrative data are not available, it exploits two main other data sources: i) production surveys, which report firm-level usage of factors and intermediate inputs, but may only cover a sample of the population of firms; and ii) business registers, which typically contain less information, but cover the entire population. The programme then re-weights production surveys based on the population structure from business registers to improve representativeness and comparability across countries.

The MultiProd routine collects statistical moments for a set of firm-level performance variables such as labour and multifactor productivity and wages, as well as decompositions of aggregate productivity growth and measures of transition dynamics. It also gathers statistics on the joint distribution of the productivity variables with age, size, ownership (domestic or foreign) and business demographics (entrant, exiting, incumbent).

Data are collected for all industries in the economy, when available. However, analyses generally focus on manufacturing and non-financial market services (“services” for brevity) in order to enhance cross-country comparability. The definition of these two macro-sectors (“sectors”) follows a customised 7-sector aggregation of ISIC Rev.4/NACE Rev.2 industrial classification. Detailed industries within sectors follow the SNA A38 classification. The analysis excludes the Coke and Refined Petroleum industry and the Real Estate industry – see Desnoyers-James, Calligaris and Calvino (2019) for further details.

The data in this paper are generated from the MultiProd 2.0 code, which offers an update of the MultiProd v.1 database and collects several new measures of productivity patterns, including transition dynamics and additional decompositions of productivity growth. The paper focuses on the group of countries for which MultiProd 2.0 data are already available as of March 2021. Table A.1 reports the time coverage and underlying data sources for each country currently in the MultiProd 2.0 database. The data for some countries remain preliminary and are subject to revision. Results may differ from official statistics due to differences in methodology.

Table A.1: MultiProd 2.0 country-time coverage, underlying data source and availability

Country	Time coverage	Data source	Data coverage restrictions
Belgium	2002-2018	New data based on a hierarchy of sources	
Canada	2000-2018	Statistics Canada's National accounts longitudinal micro data file (NALMF)	
Croatia	2002-2018	Annual financial reports of enterprises and Court Register	
France	1995-2018	FICUS/FARE, DADS, LIFI, SIRUS (Contours des entreprises profilées)	
Hungary	1996-2018	Corporate Income Tax data (CIT) of National Tax and Customs Administration	5+ employees
Italy	2001-2015	Bilanci società di capitali con dipendenti, ASIA (Business Register), Indagine sulle grandi imprese (SCI), Database Commercio Estero (COE)	
Latvia	2007-2018	Companies' annual reports to the State Revenue Service and survey "1-annual complex report on activities"	
Netherlands	2001-2018	Productiestatistieken (PS), Algemeen Bedrijvenregister (BR) and Investment survey	10+ employees
Portugal	2004-2017	Integrated Business Accounts System	
Slovenia	2002-2017	Commercial companies' annual reports to AJPES	
Sweden	2003-2018	SBS tax data	

INTANGIBLE INTENSITY INDICATOR

To classify country-industries in our sample with respect to their intangible-intensity, we use an index based on different measures of intangible capital. The indicator variable used is equal to one if the share of intangible capital in all capital assets of the country-industry is above the country median in 2001, or alternatively the first year of the country's observation. This Appendix describes how the share of intangibles in total capital is created.

For a first subset of countries in our sample (BEL, HRV, HUN, ITA, and SVN), we observe both the stock of intangible and tangible capital at the country-industry-year level in the MultiProd data. For these countries, we can compute the share variable of interest directly.

For the next set of countries (FRA and NLD), we do not observe intangible capital in MultiProd, but they are covered by the IntanInvest database. As the stock of physical capital is also covered in IntanInvest, we can compute the share using only information from this database. One complication that needs to be addressed is that the IntanInvest database uses SNA A21, a broader industry classification compared to our main classification SNA A38. SNA A21 does not distinguish between manufacturing sub-industries. To address the arising assignment problem, we assume that the number of manufacturing sub-industries in the high-intangible category is equal to the cross-country average of the countries available in MultiProd when computing the indicator for these countries.

Finally, some countries are covered neither in MultiProd nor in IntanInvest (CAN, LVA, PRT and SWE). For these countries, we assume that in each industry, the share of intangible capital corresponds to the average share across countries observed in the MultiProd database.

A.2 ECONOMETRIC SPECIFICATION: ESTIMATING IMPULSE RESPONSES

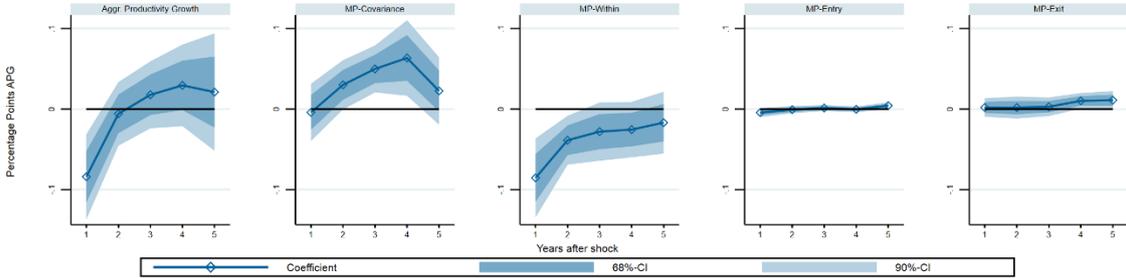
We expect changes in dispersion to have persistent effects on APG, so that current productivity growth is related to the level of current productivity dispersion, even in absence of further divergence. Such a relationship would imply that the data-generating process relates the change in APG to change in dispersion (or alternatively, APG to productivity dispersion, which is practically less useful due to higher persistence and possible issues of non-stationarity of the system variables). We argue that estimating the dynamic effects of shocks to productivity dispersion on APG and its components from such a process is impractical for three reasons.

First, the contemporaneous relationship between left and right hand side variables is confounded by the mechanical component, such that (S)VAR estimation approaches are inappropriate. Second, the change in APG at time t is related to APG at time t and at time $t - 1$. Therefore, even with impulse response estimation, we would have two horizons $h = 0, 1$ with a mechanical relationship between left and right hand side variables, and the first economically meaningful estimates can be obtained at $h = 2$. Such a model misses the share of persistent responses visible already in the first two years, possibly a very large proportion of the total effect, and we are not able to identify the long-term level effect on APG (components) due to our inability of estimating the response at the first two horizons.

The third reason is statistical: computing responses from a DGP in in differences accumulates uncertainty across horizons, so that responses at longer horizons are estimated with very low precision. Therefore, we prefer to estimate directly the response of the level variable. As argued in the seminal contribution on the local projection method (Jordá, 2005), this impulse response model merely estimates the revision in linear predictions for the response variable due to a unit impulse, but remains agnostic about the true DGP relating the system variables to each other. Thus, estimation of level responses is consistent with the assumption of a DGP in differences.

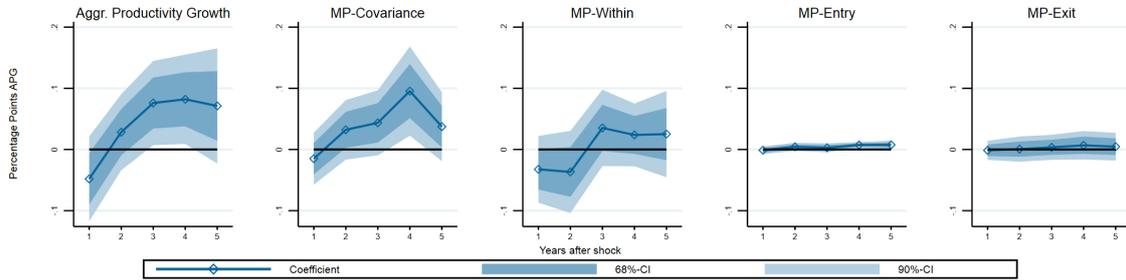
A.3 FIGURES APPENDIX

Figure A.1: Baseline results, 95-10 LP dispersion shock

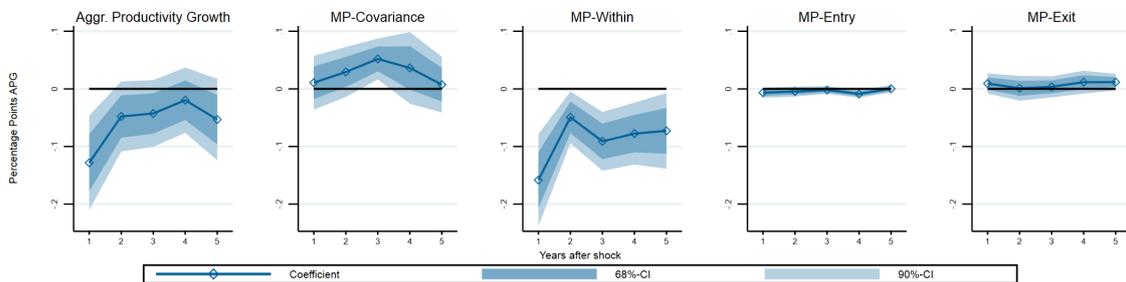


The figures show the regression estimates for the response of productivity growth and its components to a top/bottom dispersion shock, based on labour productivity. Country-industry level data (SNA A38) are weighted by total employment summed over firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

Figure A.2: Baseline results, 95-50 and 50-10 LP dispersion shock



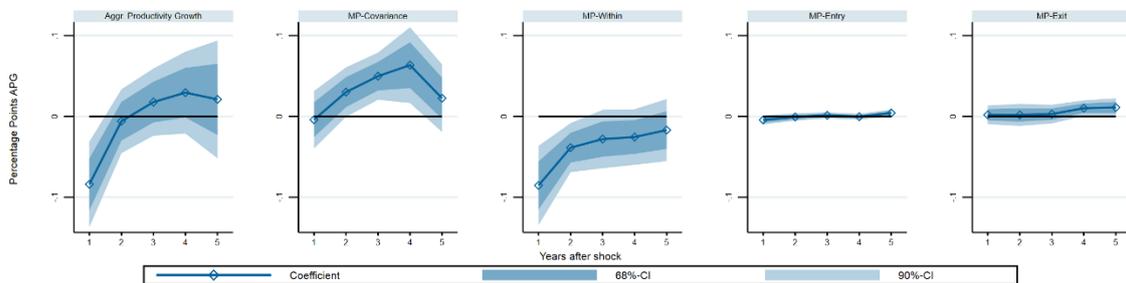
(a) Upper dispersion shock.



(b) Lower dispersion shock.

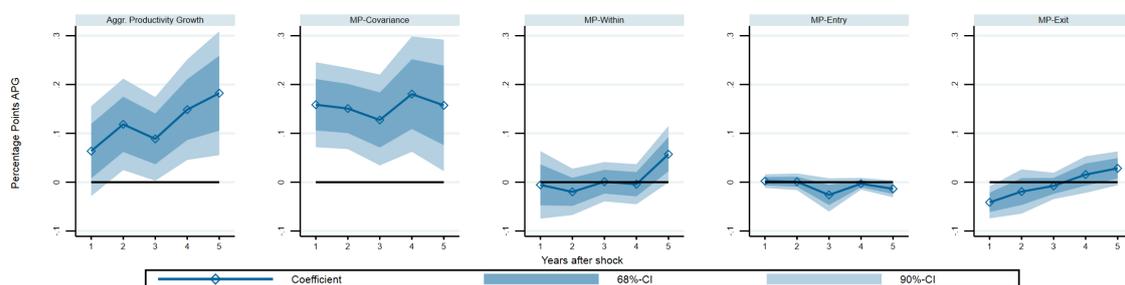
The figures show the regression estimates for the response of productivity growth and its components to productivity dispersion shocks, based on labour productivity. Country-industry level data (SNA A38) are weighted by total employment summed over firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure A.3: Baseline results, 95-10 MGP dispersion shock, equal weight of countries

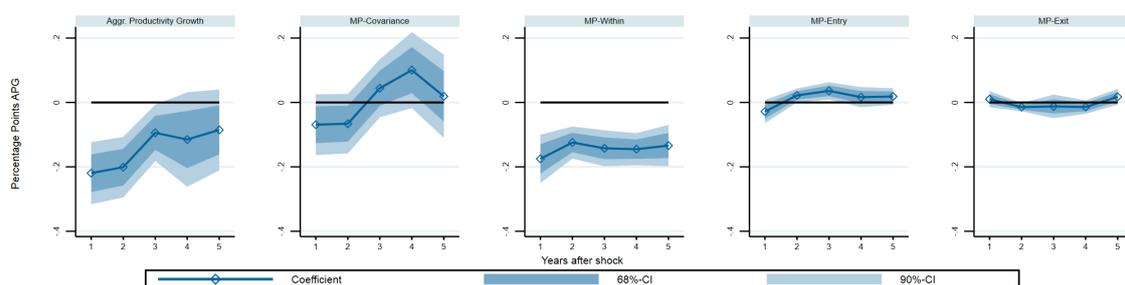


The figures show the regression estimates for the response of productivity growth and its components to a top/bottom dispersion shock, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation, normalised at the country level. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

Figure A.4: Baseline results, 95-50 and 50-10 MFP dispersion shock, equal weight of countries



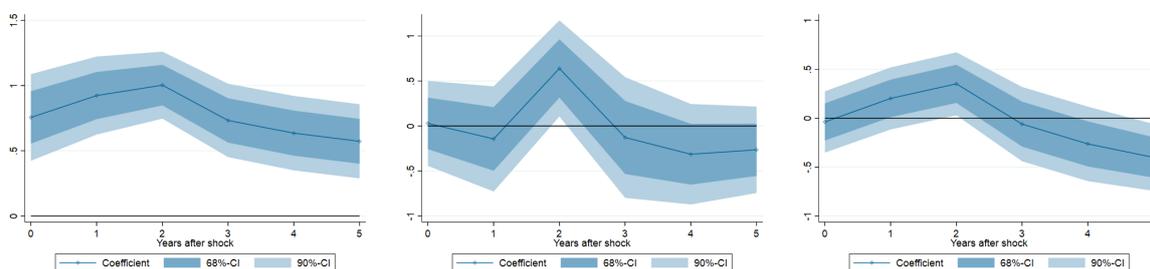
(a) Upper dispersion shock.



(b) Lower dispersion shock.

The figures show the regression estimates for the response of productivity growth and its components to productivity dispersion shocks, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation, normalised at the country level. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure A.5: Reallocation variable responses to upper dispersion shock based on median



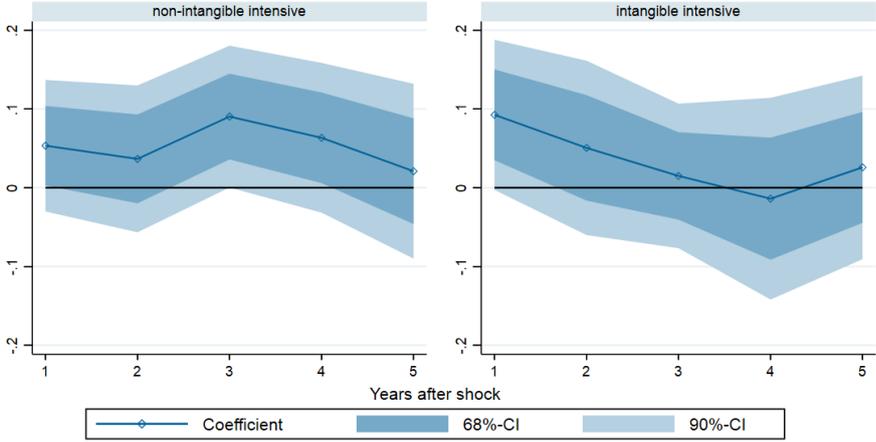
(a) Value added.

(b) Capital.

(c) Employment.

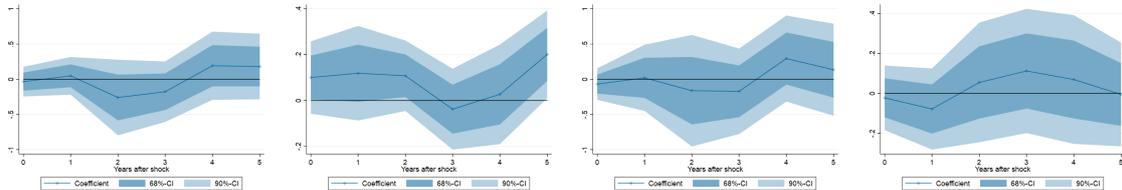
The figures show the regression estimates for the responses of key reallocation variables to upper dispersion shock, based on MFP. Every panel shows the response of the log-ratio of the average of the given variable within the top decile and around the median (firms between the 40th to 60th percentile) of MFP, respectively. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure A.6: Upper dispersion shock: productivity growth ratio of size classes and intangible intensity



The figures show the regression estimates for the responses of the log-ratio of one-year productivity growth within the size classes of large (250+ employees) and medium-small (20-49) firms to an upper dispersion shock, based on MFP. Every panel shows the response of the log-ratio of the average of the given variable within the top decile and around the median (p40-p60) of MFP, respectively. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

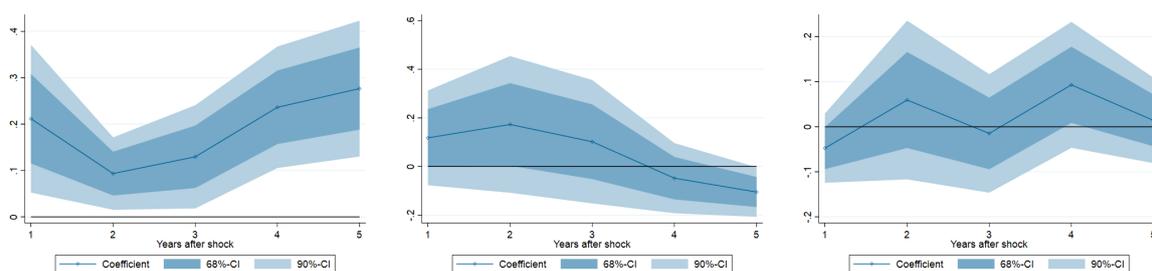
Figure A.7: Lower dispersion shock: responses of input ratios



- (a) Capital, middle-bottom ratio.
- (b) Employment, middle-bottom ratio.
- (c) Capital, top-bottom ratio.
- (d) Employment, top-bottom ratio.

The figures show the regression estimates for the responses of key reallocation variables to a lower dispersion shock, based on MFP. Every panel shows the response of the log-ratio of the average of the given variable within the top decile (top-bottom ratio) or around the median (40th to 60th percentile; middle-bottom ratio) and at the bottom of the distribution (10th to 40th percentile) of MFP, respectively. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

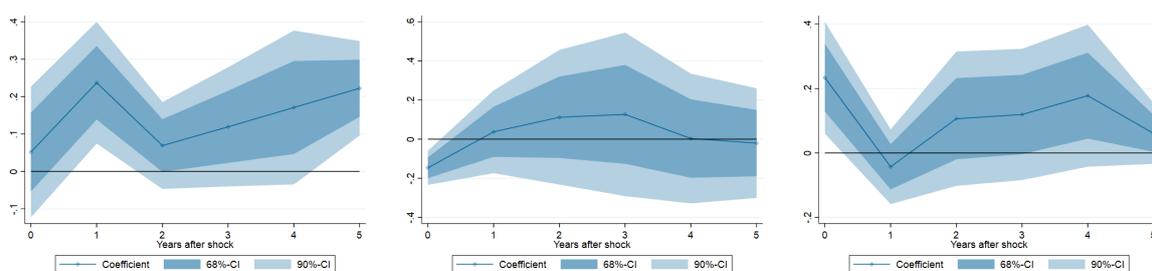
Figure A.8: Responses of average firm-level wages within the top decile: additional control



(a) Upper dispersion shock. (b) Lower dispersion shock. (c) Homog. productivity growth.

The figures show the regression estimates for the responses of average firm level wages within the top decile of MFP to changes in dispersion, based on MFP. The estimates are based on a model that, relative to the baseline specification in Equation (1.2), additionally controls for the growth of the LHS from t to $t + 1$. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

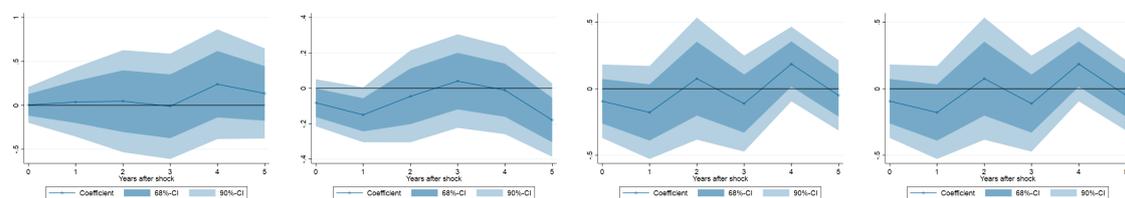
Figure A.9: Responses of the median firm-level wage within the top decile



(a) Upper dispersion shock. (b) Lower dispersion shock. (c) Homog. productivity growth.

The figures show the regression estimates for the responses of the median firm level wages within the top decile of MFP to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

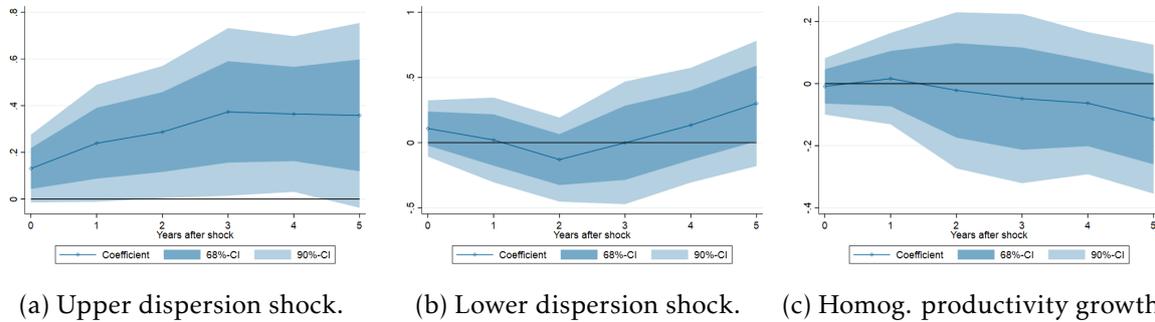
Figure A.10: Responses of averages within the top decile to changes in lower dispersion and homogeneous productivity growth



(a) Capital, lower dispersion shock. (b) Employment, lower dispersion shock. (c) Capital, homog. productivity growth. (d) Employment, homog. productivity growth.

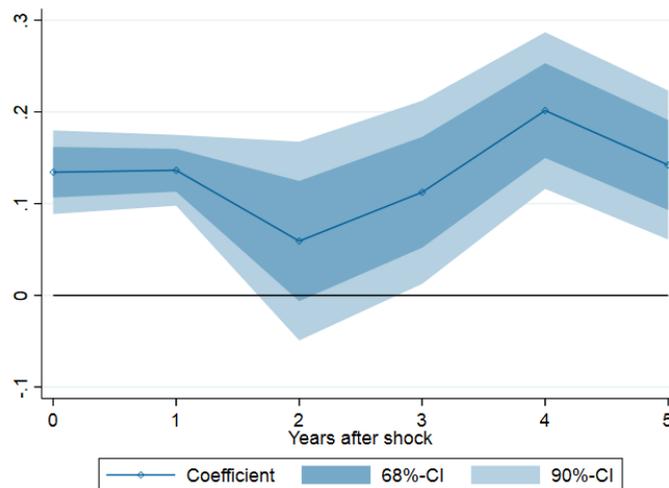
The figures show the regression estimates for the responses of average firm level quantities within the top decile of MFP to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure A.11: Responses of the industry-level capital stock



The figures show the regression estimates for the responses of the industry-level capital stock to changes in dispersion, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on the OECD MultiProd 2.0 database.

Figure A.12: Lower dispersion shock: Ratio of average wages at centre and bottom firms



The figures show the regression estimates for the response of the log-ratio of the average of average wages around the median (40th to 60th percentile) and at the bottom of the distribution (10th to 40th percentile) of MFP, based on MFP. Country-industry level data (SNA A38) are weighted by value added of firms in the country-industry in the first year of observation. Pointwise confidence intervals are obtained from standard errors clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

B. APPENDICES CHAPTER 2

B.1 DERIVATION OF THEORETICAL RESULTS

B.1.1 AUTOMATION AND THE THEORETICAL PRODUCTIVITY COEFFICIENT

Claim 1. For $c > 0$, the function $f_c(x) = \left(\frac{c}{c+x}\right)^c \left(\frac{x}{c+x}\right)^x$ is strictly decreasing in x on $(0, \infty)$.

Proof. The sign of the derivative of $f_c(x)$ can be equivalently studied from $\ln f_c(x)$:

$$\begin{aligned}\ln f_c(x) &= \ln \left[\left(\frac{c}{c+x} \right)^c \left(\frac{x}{c+x} \right)^x \right] \\ &= c(\ln c - \ln(c+x)) + x(\ln x - \ln(c+x))\end{aligned}$$

With this, one obtains

$$\begin{aligned}\frac{d \ln f_c}{dx}(x) &= -\frac{c}{c+x} + \ln \frac{x}{c+x} + x \frac{d}{dx} (\ln x - \ln(c+x)) \\ &= \log \frac{x}{c+x} - \frac{c}{c+x} + x \frac{c+x-x}{x(c+x)} \\ &= \log \frac{x}{c+x} < 0.\end{aligned}$$

As a corollary of Claim 1, one obtains that for $\alpha_K \leq \alpha_A$,

$$\left(\frac{\alpha_R}{\alpha_R + \alpha_K} \right)^{\alpha_R} \left(\frac{\alpha_K}{\alpha_R + \alpha_K} \right)^{\alpha_K} \geq \left(\frac{\alpha_R}{\alpha_R + \alpha_A} \right)^{\alpha_R} \left(\frac{\alpha_A}{\alpha_R + \alpha_A} \right)^{\alpha_A}$$

and due to $\lambda_{R,1} > \lambda_L$, it results that $A_i^R(\lambda_{R,1}) > A_i^L$.

Claim 2. For $c > 0$, the function $g_c(x) = \left(\frac{x}{c}\right)^x \left(\frac{c-x}{c}\right)^{c-x}$ is minimised on $(0, c)$ at $x = c/2$ with $g_c(c/2) = \left(\frac{1}{2}\right)^c$.

Proof. Start again from the derivative of the logarithmic transformation:

$$\begin{aligned}\frac{d \ln g_c}{dx}(x) &= \frac{d}{dx} \left(x \ln \frac{x}{c} + (c-x) \ln \frac{c-x}{c} \right) \\ &= \ln \frac{x}{c} + x \frac{1}{x} - \ln \frac{c-x}{c} - (c-x) \frac{1}{c-x} \\ &= \ln \frac{x}{c-x}\end{aligned}$$

so that $\frac{d \ln g_c}{dx}(x) < 0$ for $x < c/2$ and $\frac{d \ln g_c}{dx}(x) > 0$ for $x > c/2$. The value at the minimum follows

from plugging it into the function. This establishes the claim.

With Claim 2,

$$g_{\alpha_R}(\alpha_K) = \left(\frac{\alpha_R}{\alpha_R + \alpha_K} \right)^{\alpha_R} \left(\frac{\alpha_K}{\alpha_R + \alpha_K} \right)^{\alpha_K} \geq 2^{-(\alpha_K + \alpha_R)}$$

Frurthermore, the corresponding coefficient in A_i^L , $g_{\alpha_R}(\alpha_L)$, is always strictly smaller than one for $\alpha_A, \alpha_R > 0$. Putting this together, it results that

$$\frac{A_i^R(\lambda_{R,1})}{A_i^L} \geq \left(\frac{\lambda_{R,1}}{\lambda_L} \right)^{\alpha_R} \left(\frac{\alpha_R}{\alpha_R + \alpha_K} \right)^{\alpha_R} \left(\frac{\alpha_K}{\alpha_R + \alpha_K} \right)^{\alpha_K} \geq \left(\frac{\lambda_{R,1}}{\lambda_L} \right)^{\alpha_R} \left(\frac{1}{2} \right)^{\alpha_K + \alpha_R}.$$

B.1.2 BIAS IN ESTIMATION OF CHANGES IN THE THEORETICAL PRODUCTIVITY COEFFICIENT

Assume first for simplicity that there are only two periods, and that in the former, no firms automate the R -task. Assume that in both periods there is a sample of N firms, and a share $s \in [0, 1]$ of these firms automates the R -task in period 1. As is the case for the empirical data, assume that productivity estimation is pooled across periods. When $\hat{\alpha}_L$ and $\hat{\alpha}_K$ denote the estimated coefficients for the weight of labour and capital, respectively, one obtains

$$\ln \hat{A}_i^L = \ln A_i^L + (\alpha_R + \alpha_A - \hat{\alpha}_L) \ln L_i + (\alpha_K - \hat{\alpha}_K) \ln K_i$$

for non-automating firms, and

$$\ln \hat{A}_i^R(\lambda_{R,1}) = \ln A_i^R + (\alpha_A - \hat{\alpha}_L) \ln L_i + (\alpha_R + \alpha_K - \hat{\alpha}_K) \ln(K_i + R_i).$$

for automating firms in the second period. With this, when $\Delta \ln A_i := \ln A_i^R(\lambda_{R,1}) - \ln A_i^L$, the estimated productivity change $\Delta \ln \hat{A}_i := \ln \hat{A}_i^R - \ln \hat{A}_i^L$ for automating firms is

$$\Delta \ln \hat{A}_i = \Delta \ln A_i + (\alpha_A - \hat{\alpha}_L) \Delta \ln L_i + (\alpha_K - \hat{\alpha}_K) \Delta \ln(K_i + R_i) + \alpha_R (\ln(K_{i,1} + R_{i,1}) - \ln L_{i,0})$$

As a share s of firms automates, the probability limits of $\hat{\alpha}_L$ and $\hat{\alpha}_R$ are, respectively, $\alpha_A + (1 - 1/2s)\alpha_R$ and $\alpha_K + 1/2s\alpha_R$.¹ Therefore, the error $B_i = \text{plim}_{n \rightarrow \infty} \Delta \ln \hat{A}_i - \Delta \ln A_i$ in estimating the change $\Delta \ln A_i$ at automating firms obeys

$$\begin{aligned} B_i &= \left[-\frac{2-s}{2} \Delta \ln L_i + \frac{s}{2} \Delta \ln(K_i + R_i) + (\ln(K_{i,1} + R_{i,1}) - \ln L_{i,0}) \right] \alpha_R \\ &= \left(\frac{2-s}{2} \ln \frac{K_{i,1} + R_{i,1}}{L_{i,1}} + \frac{s}{2} \ln \frac{K_{i,0}}{L_{i,0}} \right) \alpha_R \end{aligned}$$

¹The factor $\frac{1}{2}$ stems from the fact that automating firms use robots only in the second period, i.e. in one half of all periods. An analogous extension applies to the case of a longer sample.

In environments where the production weight of labour is larger than the one of capital, the second factor can be expected to be negative, yielding $B_i < 0$, i.e. an underestimation of $\Delta \ln A_i$. This distortion is larger if the routine task is more important, i.e. α_R is larger. Furthermore, as the capital-labour ratio is likely to increase with automation, i.e. is larger in period 1, the distortion is smaller the smaller s , i.e. the smaller the share of automating firms. However, for a given weight of the routine task and a share of automating firms, the distortion reduces in the quality of the automating innovation, i.e. in the parameter λ_R , which augments the level R_i of automation capital used.

The investigation thus far has focused on tangible automation capital that is recorded in the capital stock used for estimation. However, some automation capital (e.g. software) may be intangible. Here,

$$Y_i^R(\lambda_R) = \underbrace{\tilde{A}_i(\lambda_R R_i)^{\alpha_R} \left(\frac{\alpha_R}{\alpha_R + \alpha_K}\right)^{\alpha_R} \left(\frac{\alpha_K}{\alpha_R + \alpha_K}\right)^{\alpha_K} L_i^{\alpha_A} K_i^{\alpha_K}}_{=A_i^R(\lambda_{R,1})R_i^{\alpha_R}}$$

and the theoretical productivity coefficient after automation depends additionally on the level R_i of robots used. Considering the same two-period scenario as above, for automating firms in the second period, the productivity estimate is

$$\ln \hat{A}_i^R = \ln A_i^R(\lambda_{R,1}) + \alpha_R \ln R_{i,1} + (\alpha_A - \hat{\alpha}_L) \ln L_i + (\alpha_K - \hat{\alpha}_K) \ln K_i.$$

Here, $\text{plim}_{n \rightarrow \infty} \hat{\alpha}_K = \alpha_K$, as automation capital does not contribute to the physical capital variable. Still, as above, the disappearance of the routine task from the domain of labour at automating firms gives $\text{plim}_{n \rightarrow \infty} \hat{\alpha}_L = \alpha_A + (1-s)/2\alpha_R$. Therefore, at automating firms,

$$\begin{aligned} \Delta \ln \hat{A}_i &= \Delta \ln A_i + \alpha_R \ln R_{i,1} - \left(\frac{2-s}{2} \Delta \ln L_i + \ln L_{i,0}\right) \alpha_R \\ &= \Delta \ln A_i + \underbrace{\left(\ln R_{i,1} - \frac{2-s}{2} \ln L_{i,1} + \frac{s}{2} \ln L_{i,0}\right) \alpha_R}_{\xrightarrow{P} B_i^{intan}} \end{aligned}$$

Whether this distortion is more or less negative than the previous one, i.e. B_i , depends on the relative use of physical capital and robots, respectively. However, if s is small and the distortion is mainly driven by the first period (i.e., $B_i^{intan} \approx \ln R_{i,1} - \ln L_{i,1} < \ln(K_i + R_{i,1}) - \ln L_{i,1} \approx B_i$), the error in estimation of ΔA_i may be even larger in magnitude in this case.²

In conclusion, in both scenarios (tangible and intangible automation capital), there is an

²This comparison relies on more labour being used in period 1, where $B_i < 0$. Otherwise, the errors may be of different sign, and a comparison of magnitude is difficult.

error to be expected when estimating the change in the theoretical productivity coefficient. The error's magnitude increases in the weight α_R of the routine task. Further, the change in theoretical productivity coefficients is underestimated when labour is used more intensively than both capital assets combined in the case of tangible automation capital, and when labour is used more intensively than robots in the case of intangible automation capital. The underestimation is more severe the more labour-reliant the production process is. Conversely, if production is sufficiently capital-dependent, i.e. α_K large and/or α_A small, the estimates may overstate the change in theoretical productivity coefficients.

B.2 DISCUSSION OF POLICY IMPLICATIONS

On the one hand, policy makers can enhance productivity growth by supporting technology diffusion, boosting the capabilities of laggard firms and increasing their potential to catch up, which turns out to have significant positive implications also for job creation and wages.

In this context, policy should boost human capital, strengthening the quality of education systems and allowing the workforce to cope with the changes in skills demand related to technological change. STEM education and training appear particularly relevant in this context. Policy should not only aim at boosting the skills of workers, but also the capabilities of managers, since they play a key role not only in technology adoption decisions, but also in the extent to which new technologies and organisational practices materialise into productivity gains.

To further promote diffusion, policies should aim at alleviating relevant financial barriers, which may be particularly challenging in the intangible economy. Enabling access to ICT and digital infrastructure, supporting research and development, especially for young and small innovative firms, and fostering technology transfer and university-industry collaborations may further help increase absorptive capacity and foster technology diffusion.

On the other hand, policy makers should foster innovation, considering its important role for labour demand as it raises productivity growth. Policy should therefore continue fostering the production and sharing of knowledge. Key policy levers in this area are related to encouraging research – including basic one – and development, supporting the creation of innovation network and innovation ecosystems, and providing incentives or support for R&D.

Ensuring a level playing field and the contestability of markets is equally key in this context. This is not only crucial for productivity but also to allow productivity gains related to innovation and diffusion to generate employment growth. This analysis has shown that asymmetries in market power, signalling the presence of top firms whose strong position on the market cannot be easily challenged, may be associated with less positive employment effects of

productivity-improving technological and organisational changes.

Reducing barriers to entry for new firms, which may drive the introduction of radical innovations, and levelling the playing field, which may also foster post-entry growth, appear particularly important. This is even more relevant considering the recent declines in business dynamism and increases in industry concentration documented by recent OECD analysis. In addition, understanding how competition authorities can develop better tools to limit firms' market power and its adverse consequences on business sector innovation and growth seems also of utmost importance.

The economic environment should relevantly allow leveraging spillovers arising from value chains. Indeed, the analysis highlights that productivity growth in domestic and foreign supplier industries may strengthen employment growth in downstream sectors. Integration to global and domestic value chains and connections to increasingly productive supplier sectors may be therefore particularly beneficial for the economy, and restoring value chain links when these have been disrupted appears therefore relevant. In addition, targeting productivity growth and innovation in upstream industries may contribute to aggregate employment growth indirectly through positive effects on downstream sectors.

Given that both productivity growth and the related job creation and wage increases occur through creative destruction, it is key to maintain an environment in which reallocation of resources occurs, while paying attention to its inclusiveness. Indeed, there is ample evidence that this reallocation process may not benefit all equally, favouring some occupations over others, due to the disappearance of tasks replaced by capital and the emergence of new ones that complement technologies. This results into unequal repartition of gains and losses associated with productivity growth. This in turn advocates for policies supporting both the transition of displaced workers to new occupations, but also for training to allow workers to upskill, enabling those that lose their jobs at shrinking or exiting firms to be then matched with high productivity firms, and supporting this transition.

Finally, policies that support the demand for goods and services are complementary to policies supporting productivity growth through innovation and adoption of new technologies. This underpins policies aimed at supporting aggregate domestic final demand, especially at times of distress, as well as policies supporting internationalisation of firms (e.g., through exports) to allow them expanding on foreign markets.

Summing up, a comprehensive policy mix that leverages synergies across policy areas appears key. One aimed at fostering innovation and boosting technology diffusion, preserving competition and allowing reallocation, while supporting the transition of displaced workers and improving their skills.

Such policies would not only be beneficial for productivity but are likely to have double dividends also for employment and wages, fostering an inclusive economic growth.

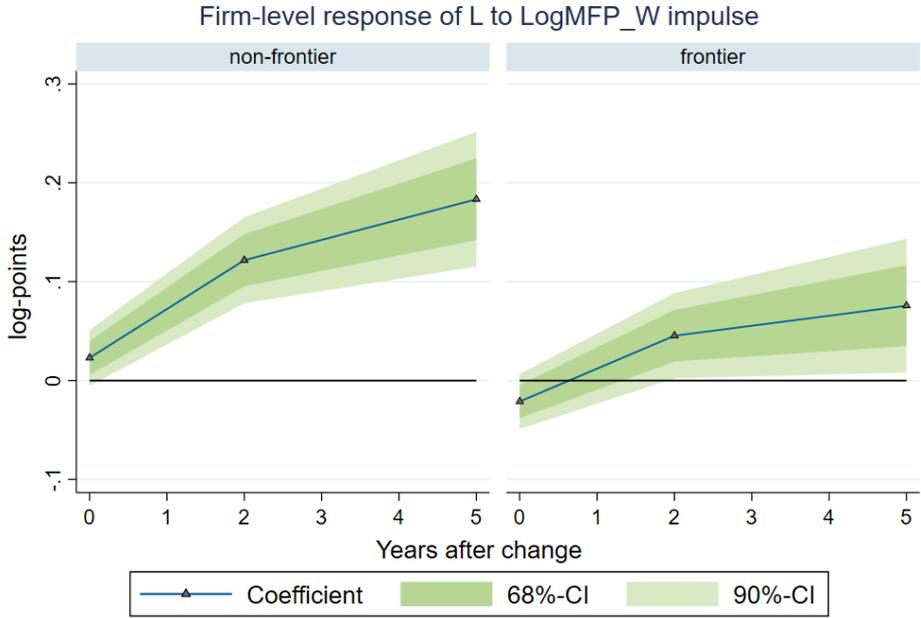
B.3 EMPIRICAL RESULTS

Table B.1: The firm-level link of employment growth to catch-up vs. other aspects of productivity growth: partial correlation model.

	(1)	(2)	(3)
	5-year change in employment	5-year change in employment	5-year change in employment
5-year change in productivity	0.0648*** (0.0108)		0.0427** (0.0178)
Final productivity group p10-p40		0.141*** (0.00819)	0.0905*** (0.0216)
Final productivity group p40-p60		0.202*** (0.0131)	0.131*** (0.0321)
Final productivity group p60-p90		0.216*** (0.0198)	0.126*** (0.0433)
Final productivity group p90-p100		0.144*** (0.0306)	0.0244 (0.0605)
Initial employment	-0.0580*** (0.00904)	-0.0486*** (0.00922)	-0.0491*** (0.00929)
Observations	19,875	19,875	19,875
R-squared	0.503	0.554	0.557
Fixed effects	C-I-Y G	C-I-Y G	C-I-Y G

Estimates obtained from an extension to the model in Equation 2.4. C-I-Y indicates fixed effects for the country-industry-year, and G indicates fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Figure B.1: The role of a firm’s initial position for the firm-level link of employment growth to catch-up: Dynamic model



Based on 8128 observations (9 countries, 22 industries)

This figure illustrates the results of the local projection impulse response regression estimations for the response of employment to a change in productivity, based on a heterogeneous effects extension of the model in (2.5) that further interacts the impulse variable with an indicator that is equal to one if the cell represents firms initially above the 90th percentile of the within-industry productivity distribution (“frontier”) and equal to zero otherwise (“non-frontier”). Observations are weighted by the number of firms represented in the full population, normalised at the country level. Confidence bands are based on pointwise estimation of standard errors, clustered at the country-industry level. Source: Calculations based on OECD MultiProd 2.0 database.

Table B.2: The role of the firm's initial position for the firm-level relationship of productivity growth and the risk of exit.

	(1) Exit
5-year change in productivity	-0.0923*** (0.0192)
* Initial productivity group = p10-p40	-0.211*** (0.0250)
* Initial productivity group = p40-p60	-0.219*** (0.0311)
* Initial productivity group = p60-p90	-0.120*** (0.0440)
* Initial productivity group = p90-p100	0.0221 (0.0338)
Initial Employment	-0.0977*** (0.0230)
Observations	13,486
Fixed effects	C-I-Y G

Estimates obtained from the model in Equation 2.6. C-I-Y indicates fixed effects for the country-industry-year, and G indicates fixed effects for the initial productivity group. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Table B.3: Estimation results based on the labour productivity of value added.

	(1) 5-year change in employment	(2) 5-year change in employment	(3) 5-year change in employment	(4) 5-year change in employment	(5) 5-year change in employment
5-year change in aggregate productivity	-0.0522 (0.0457)	-0.00283 (0.0542)	-0.0431 (0.0433)	-0.0374 (0.0436)	-0.00785 (0.0478)
* 90-50 difference of markups		-0.00330 (0.0203)			0.00271 (0.0169)
* 1[high difference p90-p50 of markups]			-0.0562 (0.0581)		
* ICT investment intensity				-0.0142** (0.00668)	-0.0180** (0.00717)
Initial employment	-0.0908*** (0.0194)	-0.103*** (0.0220)	-0.0882*** (0.0187)	-0.0916*** (0.0195)	-0.103*** (0.0216)
Initial aggregate productivity	0.0254 (0.0242)	0.0834** (0.0348)	0.0269 (0.0240)	0.0236 (0.0258)	0.0713** (0.0346)
90-50 difference of markups		0.000169 (0.00315)			-0.000870 (0.00355)
1[high difference p90-p50 of markups]			-0.0129 (0.0157)		
Observations	2,750	1,988	2,750	2,750	1,988
R-squared	0.552	0.580	0.554	0.559	0.590
Fixed effects	I C-Y				
Countries excluded	-	HRV, JPN	HRV, JPN	-	HRV, JPN

Estimates obtained from a modification of the model in Equation 2.7. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Table B.4: Estimation results based on ACF productivity estimation.

	(1)	(2)	(3)	(4)	(5)
	5-year change in employment				
5-year change in aggregate productivity	0.00168 (0.0451)	0.0452 (0.0504)	0.0325 (0.0489)	0.0166 (0.0504)	0.0435 (0.0536)
tabindent * 90-50 difference of markups		-0.0189** (0.00868)			-0.0195* (0.0105)
tabindent * 1[high difference p90-p50 of markups]			-0.104 (0.0816)		
tabindent * ICT investment intensity				-0.00641 (0.00731)	0.00109 (0.00789)
Initial employment	-0.0982*** (0.0215)	-0.0952*** (0.0219)	-0.0937*** (0.0212)	-0.0976*** (0.0216)	-0.0952*** (0.0219)
Initial aggregate productivity	0.0102* (0.00550)	0.0106* (0.00550)	0.0104* (0.00557)	0.00985* (0.00548)	0.0106* (0.00550)
90-50 difference of markups		7.40e-05 (0.00311)			
1[high difference p90-p50 of markups]			-0.0152 (0.0151)		
Observations	2,056	2,009	2,056	2,056	2,009
R-squared	0.558	0.572	0.561	0.559	0.572
Fixed effects	I C-Y				
Countries excluded	HRV, JPN				

Estimates obtained from a modification of the model in Equation 2.7. The measure of productivity is based on Akerberg, Caves and Frazer (2015) and estimates the Hicks-neutral term in a gross output production function using labour, capital and intermediates. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database

Table B.5: The link between dispersion in productivity changes and job reallocation.

	(1)	(2)	(3)	(4)	(5)	(6)
	St. dev. of 5-year empl. growth	Average excess job reallocation over 5 years	Average excess job reallocation (incum- bents) over 5 years	Average job reallocation rate over 5 years	Average job creation rate over 5 years	Average job destruction rate over 5 years
St. dev. of one-year productivity change	0.0383** (0.019)	3.8359** (1.612)	2.4348** (0.979)	5.1695** (2.006)	2.7007*** (0.946)	2.6704** (1.314)
Initial employment	0.0037* (0.002)	-0.7277* (0.387)	-0.2847 (0.203)	-0.9525** (0.426)	-0.6525** (0.256)	-0.3140 (0.202)
Lagged dependent variable	0.5402*** (0.053)	0.3211*** (0.064)	0.4602*** (0.030)	0.3274*** (0.062)	0.3151*** (0.047)	0.3104*** (0.040)
Observations	2673	1585	1585	1585	1585	1585
Adj. R-Square	0.959	0.841	0.842	0.847	0.866	0.758
Fixed effects	I C-Y	I C-Y	I C-Y	I C-Y	I C-Y	I C-Y
Num countries	12	9	9	9	9	9

Estimates obtained from an extension of the model in from the model in Equation 2.7, focusing on measures of employment growth dispersion of job reallocation as the main regressions. Columns 2 through 6 are unable to use data for Chile, Croatia and France due to the coverage of the dependent variables. I and C-Y indicate fixed effects for the industry and country-year, respectively. Standard errors given in parentheses are clustered at country-industry level, and statistical significance is denoted as follows: *** p<0.01, ** p<0.05, * p<0.1. Source: Authors' calculations based on the OECD MultiProd 2.0 Database and the OECD DynEmp v3 Database

C. APPENDICES CHAPTER 3

C.1 PRODUCTION FUNCTION PROPERTIES

This appendix derives the central results used to characterise the production function and presents a numerical investigation into Assumption 2.

C.1.1 PROPOSITION 1

The derivative of no-automation output with respect to σ is

$$\begin{aligned} \frac{\partial \ln y^N(i|j(\sigma, \lambda))}{\partial \sigma} &= -\left(\frac{1}{\sigma-1}\right)^2 \ln\left(\lambda\phi_{iR}^{\sigma-1} + (1-\lambda)\phi_{iA}^{\sigma-1}\right) \\ &\quad + \frac{1}{\sigma-1} \left(\frac{\lambda\phi_{iR}^{\sigma-1} \ln(\phi_{iR}) + (1-\lambda)\phi_{iA}^{\sigma-1} \ln(\phi_{iA})}{\lambda\phi_{iR}^{\sigma-1} + (1-\lambda)\phi_{iA}^{\sigma-1}} \right) \end{aligned}$$

Multiplying with $[(\lambda\phi_{iR}^{\sigma-1} + (1-\lambda)\phi_{iA}^{\sigma-1})(\sigma-1)^2]^{-1} > 0$, one obtains

$$\begin{aligned} \frac{\partial \ln y^N(i|j(\sigma, \lambda))}{\partial \sigma} &\propto \lambda\phi_{iR}^{\sigma-1} \ln(\phi_{iR}^{\sigma-1}) + (1-\lambda)\phi_{iA}^{\sigma-1} \ln(\phi_{iA}^{\sigma-1}) \\ &\quad - (\lambda\phi_{iR}^{\sigma-1} + (1-\lambda)\phi_{iA}^{\sigma-1}) \ln(\lambda\phi_{iR}^{\sigma-1} + (1-\lambda)\phi_{iA}^{\sigma-1}) \end{aligned}$$

which is strictly positive by strict convexity of $f : \mathbb{R} \mapsto \mathbb{R}, x \mapsto f(x) = x \ln(x)$.

C.1.2 PROPOSITION 3

For a occupation $j(\sigma, \lambda)$ with output price $p_{j(\sigma, \lambda)} = p$,

$$\begin{aligned} \frac{\partial \ln(w_{ij(\sigma, \lambda)}^A/p)}{\partial \sigma} &= \frac{\partial}{\partial \sigma} \ln \left[\left(\frac{1-\lambda}{1-\lambda(p\alpha)^{\sigma-1}} \right)^{\frac{1}{\sigma-1}} \right] \\ &= \left(\frac{1}{\sigma-1} \right)^2 \left[\frac{\lambda(p\alpha)^{\sigma-1} \ln[(p\alpha)^{\sigma-1}]}{1-\lambda(p\alpha)^{\sigma-1}} - \ln \left(\frac{1-\lambda}{1-\lambda(p\alpha)^{\sigma-1}} \right) \right] \end{aligned}$$

Multiplying with $[(1 - \lambda(p\alpha)^{\sigma-1})(\sigma - 1)^2]^{-1} > 0$, one obtains

$$\begin{aligned} \frac{\partial w_{ij(\sigma,\lambda)}^A/p}{\partial \sigma} &\propto \lambda(p\alpha)^{\sigma-1} \ln[(p\alpha)^{\sigma-1}] + (1 - \lambda) \frac{1 - \lambda(p\alpha)^{\sigma-1}}{1 - \lambda} \ln\left(\frac{1 - \lambda(p\alpha)^{\sigma-1}}{1 - \lambda}\right) \\ &\geq \left(\lambda(p\alpha)^{\sigma-1} + (1 - \lambda) \frac{1 - \lambda(p\alpha)^{\sigma-1}}{1 - \lambda}\right) \ln\left((1 - \lambda) \frac{1 - \lambda(p\alpha)^{\sigma-1}}{1 - \lambda} + \lambda(p\alpha)^{\sigma-1}\right) \\ &= 1 \cdot \ln(1) = 0 \end{aligned}$$

where the inequality follows from strict convexity of $f : \mathbb{R} \mapsto \mathbb{R}, x \mapsto f(x) = x \ln(x)$, and is strict for $p\alpha \neq 1$.

C.1.3 PROPOSITION 4

For a function $f(x) = \frac{1-x}{1-cx}$, $c > 0$, at $x > 0$, it holds that $\text{sgn}\left(\frac{\partial f}{\partial x}(x)\right) = \text{sgn}(c - 1)$. Hence,

$$\frac{\partial \ln(w_{ij(\sigma,\lambda)}^A/p)}{\partial \lambda} = \frac{\partial}{\partial \lambda} \ln\left[\left(\frac{1 - \lambda}{1 - \lambda(p\alpha)^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}\right] \propto p\alpha - 1$$

C.1.4 PROPOSITION 5

Without loss of generality, assume that k departs from the trigger while j does not.

If $w_{ik}^A > w_{ik}^N$, with j remaining at the trigger, this gives

$$\frac{1}{p_j} \frac{\phi_{iR}}{y^N(i|j)} = \alpha > \frac{1}{p_k} \frac{\phi_{iR}}{y^N(i|k)}$$

Accordingly, as j remains at the trigger, $w_{ik}^A > w_{ik}^N > w_{ij}^N = w_{ij}^A$, a contradiction to $s_\theta(j) > 0$.

Conversely, if $w_{ik}^A < w_{ik}^N$, j remaining at the trigger, in analogy to before, yields that $w_{ik}^A < w_{ik}^N < w_{ij}^N = w_{ij}^A$, a contradiction to $s_\theta(k) > 0$. \square

C.1.5 PROPOSITION 6

As $\alpha_{\theta j}^* < \alpha$, θ does not work in $k \in J$ at no-automation unless $w_{\theta k}^N \geq w_{\theta j}^A$. This gives

$$\alpha_{\theta k}^* = \frac{\phi_{\theta R}}{w_{\theta k}^N} \leq \frac{\phi_{\theta R}}{w_{\theta j}^N} = \alpha_{\theta j}^* < \alpha,$$

such that $w_{\theta k}^A > w_{\theta k}^N$, and θ does not work in $k \in J$ at no-automation. \square

C.1.6 NUMERICAL INVESTIGATION: ASSUMPTION 2

(λ_L, λ_H)	$(1/6, 2/3)$				$(1/10, 1/2)$
(σ_L, σ_H)	$(2, 5)$	$(1/4, 1/2)$	$(1/2, 2)$	$(1/3, 3)$	$(1/2, 2)$
min. ϕ sustaining Ass. 2	$< 1/40$	$< 1/40$	$< 1/20$	$\in (1/10, 1/5)$	$\in (1/20, 1/10)$

C.2 INITIAL EQUILIBRIUM

C.2.1 EXISTENCE AND UNIQUENESS

Fact 1. *In any no-automation equilibrium, U-workers are highest-price seeking, i.e. if for $j^* \in J$: $s_U(j) > 0$, then $j^* \in \arg \max_{j \in J} p_j$.*

This fact follows directly from $w_{ij}^N = p_j y^N(i|j)$ and $y^N(i_U|j) = 1$ for $j \in J$.

Fact 2. *In any no-automation equilibrium, for $\lambda \in \{\lambda_L, \lambda_H\}$, $p_{j(\sigma_L, \lambda)} \geq p_{j(\sigma_H, \lambda)}$.*

As $y(j) = 0$ can occur for no $j \in J$ in equilibrium, this fact follows from Proposition 1.

Proposition 11. *In any no-automation equilibrium, σ_L -prices are equated, i.e. $p_{j(\sigma_L, \lambda_L)} = p_{j(\sigma_L, \lambda_H)}$.*

Proof. Assume the opposite.

Consider first the case where $p_{j(\sigma_L, \lambda_L)} < p_{j(\sigma_L, \lambda_H)}$. Then, $w_{ij(\sigma_L, \lambda_L)}^N < w_{ij(\sigma_L, \lambda_H)}^N$ for $i \in R \cup U$. Further, by $p_{j(\sigma_L, \lambda)} \geq p_{j(\sigma_H, \lambda)}$ for $\lambda \in \{\lambda_L, \lambda_H\}$, $w_{i_U j(\sigma_H, \lambda_L)}^N < w_{i_U j(\sigma_L, \lambda_H)}^N$, and with Assumption 2 also $w_{i_R j(\sigma_H, \lambda_L)}^N < w_{i_R j(\sigma_L, \lambda_H)}^N$. Thus, only A-workers work in λ_L -occupations, with $\rho_A^{-1} = p_{j(\sigma_H, \lambda_L)} / p_{j(\sigma_L, \lambda_L)}$ (follows from $w_{i_A j(\sigma_H, \lambda_L)}^N = w_{i_A j(\sigma_L, \lambda_L)}^N$). Plugging $y_j = s_A(j) y^N(i_A|j)$ for $j \in J$ with $\lambda_j = \lambda_L$,

$$\rho_A^{-1} = \frac{s_A(j(\sigma_L, \lambda_L)) y^N(i_A|j(\sigma_L, \lambda_L))}{s_A(j(\sigma_H, \lambda_L)) y^N(i_A|j(\sigma_H, \lambda_L))} = \frac{s_A(j(\sigma_L, \lambda_L))}{s_A(j(\sigma_H, \lambda_L))} \rho_A^{-1}$$

and $s_A(j(\sigma_H, \lambda_L)) = s_A(j(\sigma_L, \lambda_L))$. Thus, $p_{j(\sigma_L, \lambda_H)} > p_{j(\sigma_L, \lambda_L)}$ can be sustained only if

$$y(j(\sigma_L, \lambda_H)) < \frac{s_A}{2} y^N(i_A|j(\sigma_L, \lambda_L)) = \frac{1}{2} \frac{Y_A}{\rho_A}. \quad (\text{C.1})$$

where the RHS is the maximum possible output of $j(\sigma_L, \lambda_L)$ consistent with $p_{j(\sigma_L, \lambda_L)} < p_{j(\sigma_L, \lambda_H)}$, obtained when A-workers only work in λ_L -occupations. If $j(\sigma_L, \lambda_H)$ employs some R-workers, solving for R-shares in λ_H -occupations in analogy to above gives $y(j(\sigma_L, \lambda_H)) = \frac{Y_R / \rho_R + s_U}{2}$. Conversely, if $j(\sigma_L, \lambda_H)$ employs only U- and $j(\sigma_H, \lambda_H)$ only R-workers, $p_{j(\sigma_H, \lambda_H)} \geq \rho_R^{-1} p_{j(\sigma_L, \lambda_H)}$, and $s_U \geq Y_R / \rho_R$, which gives $y(j(\sigma_L, \lambda_H)) = s_U \geq \frac{Y_R / \rho_R + s_U}{2}$. Finally, if $j(\sigma_H, \lambda_H)$ employs some U-workers, $y(j(\sigma_L, \lambda_H)) = \frac{Y_R + s_U}{2} > \frac{Y_R / \rho_R + s_U}{2}$. Thus, in any case, $y(j(\sigma_L, \lambda_H)) \geq \frac{Y_R / \rho_R + s_U}{2}$, and equation (C.1) is sustained only if $s_U < Y_A / \rho_A - Y_R / \rho_R$. However, this contradicts Assumption 1.

Now, assume that $p_{j(\sigma_L, \lambda_L)} > p_{j(\sigma_L, \lambda_H)}$. Here, one may proceed in analogy to above:

- Only R-workers work in λ_H -occupations.
- The necessary condition analogous to (C.1) is $y(j(\sigma_L, \lambda_L)) < 1/2 \cdot Y_R / \rho_R$.
- When no R-workers are employed in λ_L -occupations, $y(j(\sigma_L, \lambda_L)) \geq \frac{Y_A / \rho_A + s_U}{2}$, and the necessary condition implies $Y_A / \rho_A + s_U < Y_R / \rho_R$, a contradiction to Assumption 1.

This establishes the proposition. □

With Facts 1 and 2, both σ_L -occupations are highest-price occupations, and by Assumption 2, $j(\sigma_H, \lambda_H)$ employs no A -workers, and $j(\sigma_H, \lambda_L)$ employs no R -workers.

Proposition 12. *In any no-automation equilibrium, σ_H -prices are strictly below the maximum price, with $p_{j(\sigma_H, \lambda_L)} = \rho_A^{-1} \max_{j \in J} p_j$ and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} \max_{j \in J} p_j$.*

Proof. If all prices (and outputs) were equated, it would hold that $Y_\theta \leq s_U/2$ for $\theta \in \{A, R\}$. However, by Assumption 1,

$$2 \frac{Y_A}{\rho_A} = \frac{1}{4} \left[3 \left(\frac{Y_A}{\rho_A} - \frac{Y_R}{\rho_R} \right) + \left(\frac{Y_R}{\rho_R} - \frac{Y_A}{\rho_A} \right) \right] > \frac{3s_U + s_U}{4} = s_U$$

and $2Y_R/\rho_R > s_U$ in analogy. Thus, there exists $j \in J$ with $\sigma_j = \sigma_H$ and $p_j < \max_{k \in J} p_k$.

If $p_{j(\sigma_H, \lambda_H)} < \max_{j \in J} p_j$, then $j(\sigma_H, \lambda_H)$ employs only R -workers. Assume that $p_{j(\sigma_H, \lambda_L)} = \max_{j \in J} p_j$. Then, $j(\sigma_L, \lambda_H)$ does not employ A -workers, so that if $j(\sigma_H, \lambda_H)$ employs all R -workers,

$$y(j(\sigma_L, \lambda_H)) = \frac{s_U}{2} < \frac{Y_R}{\rho_R} = \frac{1}{\rho_R} y(j(\sigma_H, \lambda_H))$$

such that $p_{j(\sigma_H, \lambda_H)} < \rho_R^{-1} p_{j(\sigma_L, \lambda_H)}$ and $w_{i_{Rj}(\sigma_H, \lambda_H)}^N < w_{i_{Rj}(\sigma_L, \lambda_H)}^N$, a contradiction. Thus, $j(\sigma_L, \lambda_H)$ employs some R -workers and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} \max_{j \in J} p_j$. When $\delta \in [0, 1]$ denotes the share of U -workers in λ_H -occupations, $y(j(\sigma_L, \lambda_L)) = (1-\delta)s_U - s_U(j(\sigma_H, \lambda_L))$, $y(j(\sigma_H, \lambda_L)) = Y_A + s_U(j(\sigma_H, \lambda_L))$ and $y(j(\sigma_L, \lambda_H)) = \frac{Y_R/\rho_R + \delta s_U}{2}$ (cf. the proof of Proposition 11). Using that σ_L -prices are equated, one may solve for δ to obtain

$$y(j(\sigma_L, \lambda_H)) = \frac{Y_R/\rho_R + s_U - s_U(j(\sigma_H, \lambda_L))}{3} = y(j(\sigma_L, \lambda_L)).$$

Using Assumption 1 at the second inequality,

$$y(j(\sigma_H, \lambda_L)) = Y_A + s_U(j(\sigma_H, \lambda_L)) > Y_A/\rho_A > \frac{Y_R/\rho_R + s_U}{3} \geq y(j(\sigma_L, \lambda_L)),$$

a contradiction to $p_{j(\sigma_H, \lambda_L)} = p_{j(\sigma_L, \lambda_L)}$.

Conversely, if $p_{j(\sigma_H, \lambda_L)} < \max_{j \in J} p_j$, one may proceed in analogy as Assumption 1 is symmetric with respect to Y_R/ρ_R and Y_A/ρ_A . Specifically, if $p_{j(\sigma_H, \lambda_L)} = \max_{j \in J} p_j$,

- if $j(\sigma_H, \lambda_L)$ employs all A -workers, $y(j(\sigma_L, \lambda_H)) = \frac{s_U}{2} < \frac{Y_A}{\rho_A} = y(j(\sigma_H, \lambda_L))/\rho_A$ so that $p_{j(\sigma_H, \lambda_L)} < \rho_A^{-1} p_{j(\sigma_L, \lambda_L)}$ and $w_{i_{Aj}(\sigma_H, \lambda_L)}^N < w_{i_{Aj}(\sigma_L, \lambda_L)}^N$, a contradiction; thus $p_{j(\sigma_H, \lambda_L)} = \rho_A^{-1} \max_{j \in J} p_j$.
- Solving again for δ gives

$$y(j(\sigma_H, \lambda_H)) = Y_R + s_U(j(\sigma_H, \lambda_H)) > Y_R/\rho_R > \frac{Y_A/\rho_A + s_U}{3} \geq y(j(\sigma_L, \lambda_H)),$$

a contradiction to $p_{j(\sigma_H, \lambda_L)} = \max_{j \in J} p_j$.

In conclusion, $p_k < \max_{j \in J} p_j$ for any $k \in J$ with $\sigma_k = \sigma_H$, which establishes the first part of the proposition.

To prove the second part, assume first that A -workers are isolated in $j(\sigma_H, \lambda_L)$.¹ Then,

$$y(j(\sigma_L, \lambda_H)) = \begin{cases} s_U/2 & \text{if no } R \text{ in } j(\sigma_L, \lambda_H), \\ (Y_R/\rho_R + s_U)/3 & \text{else,} \end{cases}$$

and with $(Y_R/\rho_R + s_U)/3 = s_U/2 + (2Y_R/\rho_R - s_U)/6 \geq s_U/2$ (using Assumption 1), it holds that $(Y_R/\rho_R + s_U)/3 \geq y(j(\sigma_L, \lambda_H))$ and

$$y(j(\sigma_H, \lambda_L))/\rho_A = Y_A/\rho_A > \frac{Y_R/\rho_R + s_U}{3} \geq y(j(\sigma_L, \lambda_H)) = y(j(\sigma_L, \lambda_L))$$

so that $p_{j(\sigma_H, \lambda_L)} < \rho_A p_{j(\sigma_L, \lambda_L)}$ and $w_{i_A j(\sigma_H, \lambda_L)}^N < w_{i_A j(\sigma_L, \lambda_L)}^N$, a contradiction to A -employment in $j(\sigma_H, \lambda_L)$. Thus, $j(\sigma_L, \lambda_L)$ employs some A -workers, and $p_{j(\sigma_H, \lambda_L)} = \rho_A^{-1} \max_{j \in J} p_j$.

Conversely, if R -workers are isolated in $j(\sigma_H, \lambda_H)$, in analogy to above, $y(j(\sigma_L, \lambda_L)) \leq (Y_A/\rho_A + s_U)/3$ and

$$y(j(\sigma_H, \lambda_H))/\rho_R = Y_R/\rho_R > \frac{Y_A/\rho_A + s_U}{3} \geq y(j(\sigma_L, \lambda_L)) = y(j(\sigma_L, \lambda_H))$$

and $w_{i_R j(\sigma_H, \lambda_H)}^N < w_{i_R j(\sigma_L, \lambda_H)}^N$, a contradiction to R -employment in $j(\sigma_H, \lambda_H)$. Thus, $j(\sigma_L, \lambda_H)$ employs some R -workers, and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} \max_{j \in J} p_j$. \square

In conclusion, one obtains the result given as Proposition 2 in Section 3.3.4 as a corollary.

Corollary 4. *There exists a unique no-automation equilibrium, in which*

- if $s_\theta(j) > 0$ for $j \in J$ and $\theta = A$ ($\theta = R$) [$\theta = U$], then $\lambda_j = \lambda_L$ ($\lambda_j = \lambda_H$) [$\sigma_j = \sigma_L$],
- $p_k = \max_{j \in J} p_j$ for any $k \in J$ with $\sigma_k = \sigma_L$,
- $p_{j(\sigma_H, \lambda_L)} = \rho_A^{-1} \max_{j \in J} p_j$ and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} \max_{j \in J} p_j$.

Proof. Immediately implied by Propositions 11 and 12. \square

C.2.2 CENTRAL QUANTITIES

With equation (3.6), using that occupation-level output is additive over workers, closed-form expressions for the assignment of workers to occupations may be obtained. In the unique equilibrium, outputs satisfy (cf. Corollary 4)

$$\frac{y(j(\sigma_H, \lambda_L))}{y(j(\sigma_L, \lambda_L))} = \rho_A \quad \text{and} \quad \frac{y(j(\sigma_H, \lambda_H))}{y(j(\sigma_L, \lambda_H))} = \rho_R. \quad (\text{C.2})$$

¹The label “isolated” refers to a occupation employing only the given type of workers, and the type not working in any other occupation.

Plugging in $y(j) = \sum_{\theta \in \Theta} s_{\theta}(j) y^N(\theta|j)$ and imposing the zero-restrictions on $s_{\theta}(j)$ of Corollary 4, one obtains

$$\frac{s_A(j(\sigma_H, \lambda_L)) y^N(i_A|j(\sigma_H, \lambda_L))}{(s_A - s_A(j(\sigma_H, \lambda_L))) y^N(i_A|j(\sigma_L, \lambda_L)) + (1 - \beta) s_U} = \rho_A$$

and

$$\frac{s_R(j(\sigma_H, \lambda_H)) y^N(i_R|j(\sigma_H, \lambda_H))}{(s_R - s_R(j(\sigma_H, \lambda_H))) y^N(i_R|j(\sigma_L, \lambda_H)) + \beta s_U} = \rho_R.$$

Solving for $s_{\theta}(j(\sigma_H, \lambda_{\theta}))$ in both equations gives

$$\frac{s_A(j(\sigma_H, \lambda_L))}{s_A} = \frac{1}{2} \left(1 + (1 - \beta) \frac{s_U}{Y_A/\rho_A} \right) \quad \text{and} \quad \frac{s_R(j(\sigma_H, \lambda_H))}{s_R} = \frac{1}{2} \left(1 + \beta \frac{s_U}{Y_R/\rho_R} \right)$$

and

$$y(j(\sigma_L, \lambda_L)) = \frac{1}{2} (Y_R/\rho_R + \beta s_U) \quad \text{and} \quad y(j(\sigma_L, \lambda_H)) = \frac{1}{2} (Y_A/\rho_A + (1 - \beta) s_U)$$

Using that $w_{i_U j}^N$ is equated across $j \in J$ with $\sigma_j = \sigma_L$, one may solve $y(j(\sigma_L, \lambda_L)) = y(j(\sigma_L, \lambda_H))$ for β to obtain

$$\beta^N = \frac{1}{2} \left(1 + \frac{Y_A/\rho_A - Y_R/\rho_R}{s_U} \right). \quad (\text{C.3})$$

Plugging this in to previous expressions,

$$\frac{s_A^N(j(\sigma_H, \lambda_L))}{s_A} = \frac{Y^{\min}}{4Y_A/\rho_A} = (4Y_0(A))^{-1}, \quad \frac{s_R^N(j(\sigma_H, \lambda_H))}{s_R} = \frac{Y^{\min}}{4Y_R/\rho_R} = (4Y_0(R))^{-1}$$

and

$$\frac{Y^{\min}}{4} = y^N(j(\sigma_L, \lambda_L)) = y^N(j(\sigma_L, \lambda_H)) = y^N(j(\sigma_H, \lambda_L))/\rho_A = y^N(j(\sigma_H, \lambda_H))/\rho_R.$$

Accordingly, $Y^N = 4 \prod_{j \in J} y^N(j)^{\frac{1}{4}} = (\rho_A \rho_R)^{\frac{1}{4}} Y^{\min}$.

C.3 ADJUSTMENT DYNAMICS

C.3.1 PROPOSITION 7

Lemma 1. *If at technology level α and $j, k \in J$, a change $\Delta\alpha > 0$ induces $\Delta p_j \alpha \leq 0 < \Delta p_k \alpha$, then workers of any type $\theta \in \Theta$ do not move from k to j at α , i.e. $\Delta s_{\theta}(j)/s_{\theta}(k) \leq 0$.*

Proof. For $i \in I$ and $j \in J$, automation wages can be expressed using the relative efficiency cost of capital (cf. Eq. (3.16)) as $w_{\theta j}^A = \phi_{\theta A}/(\alpha c_j^{K,L}(\alpha))$, and $c_j^{K,L}(\alpha)$ moves into the same direction as $p_j \alpha$, i.e. $\text{sgn}(\Delta c_{ij}^{K,L}(\alpha)) = \text{sgn}(\Delta p_j \alpha)$. For $j, k \in J$, the ratios of potential θ -wages, $\theta \in \Theta$,

are

$$\begin{aligned}\frac{w_{\theta k}^A}{w_{\theta l}^A} &= \frac{c_l^{K,L}(\alpha)}{c_k^{K,L}(\alpha)}, \\ \frac{w_{\theta k}^N}{w_{\theta l}^A} &= \frac{1}{\phi_{\theta A}} y^N(\theta|k) \cdot p_k \alpha \cdot c_l^{K,L}(\alpha), \\ \frac{w_{\theta k}^A}{w_{\theta l}^N} &= \phi_{\theta A} \left(y^N(\theta|l) \cdot p_l \alpha \cdot c_k^{K,L}(\alpha) \right)^{-1}, \\ \frac{w_{\theta k}^N}{w_{\theta l}^N} &= \frac{y^N(\theta|k) p_k \alpha}{y^N(\theta|l) p_l \alpha}.\end{aligned}$$

Therefore, with any change $\Delta\alpha > 0$ that induces $\Delta p_j \alpha \leq 0 < \Delta p_k \alpha$, any j, k -ratio of potential θ -wages strictly declines, and $\Delta s_{\theta}(j)/s_{\theta}(k) \leq 0$. \square

Lemma 2. *If at technology level α ,*

(i) *if for $\theta \in \Theta, j \in J, \alpha = \alpha_{\theta j}^*$, then $s_{\theta}(j) = 0$ (no active triggers),*

(ii) *if for $\theta \in \Theta, j, k \in J$, a j, k -IC holds for θ -workers, then it is active, i.e. $s_{\theta}(j), s_{\theta}(k) > 0$, and*

(iii) *if for $\theta_1, \theta_2 \in \Theta, j, k \in J$, a j, k -IC holds for both θ_1 and θ_2 , then $\alpha > \max_{\theta \in \{\theta_1, \theta_2\}} \alpha_{\theta j}^*$ or $\alpha < \min_{\theta \in \{\theta_1, \theta_2\}} \alpha_{\theta j}^*$ (no simultaneous IC for automated and non-automated workers),*

then locally, i.e. for $|\Delta\alpha|$ small, all worker ICs are sustained and new ones are not generated.

Proof. Assume that $j, k \in J$ and $\theta \in \Theta$ are such that a j, k -IC of θ -workers holds at α with $s_{\theta}(j), s_{\theta}(k) > 0$, and that $|\Delta\alpha|$ small breaks the j, k -IC of θ -workers. Without loss of generality, assume that $\Delta\alpha$ induces $w_{\theta j}^P < w_{\theta j}^P$ where $P \in \{N, A\}$ is the production scheme used for θ -workers at α .

Assume first that $\Delta p_k > 0$. If $\Delta p_k > 0$, then by wage-maximizing occupational choice at α , θ -workers join a occupation $l \in J$ only if $\Delta p_l > 0$. By $\Delta y(i_{\bar{\theta}}|l) \geq 0$ for any $\bar{\theta} \in \Theta$,² this is consistent with $\Delta y(l)/Y < 0$ only if there is labour flow from l to $m \in J$. By Lemma 1, this implies $\Delta p_m > 0$. This argument can be applied iteratively to conclude that $\Delta p_s > 0$ for all $s \in J$, a contradiction.

Assume now that $\Delta p_k \leq 0$. Then, $\Delta p_j < 0$, and as labour does not move from k to j by (iii), this is consistent with $\Delta y(j)/Y > 0$ only if there is labour flow from $l \in J$ to $j, l \neq k$. Applying this argument iteratively gives $\Delta p_s < 0$ for all $s \in J$, a contradiction.

Finally, new ICs are generated at $|\Delta \ln \alpha|$ small only if prices change discontinuously, which requires discontinuous movement of labour, i.e. breakdown of existing ICs. \square

By Lemma 2, it may be assumed that ICs are locally sustained when studying continuity and differentiability of central quantities. The following result uses $s_E(j) = \sum_{\theta \in \Theta} s_{\theta}(j) \cdot$

²For $|\Delta\alpha|$ small, per- θ worker productivity in $l, \theta \in \Theta, l \in J$, moves at most with p_l , and Δp_l is positively related to $\Delta y(\theta|l)$.

$y(\theta|j)/y^A(H|j)$ as the mass of effective labour in j in terms of automated labour with unit abstract skill, which satisfies $y(j) = s_E(j)y^A(H|j)$.

Lemma 3. *At technology level α , if for $j, k \in J$ that employ automated labour at α , a change $\Delta \ln \alpha$ small is such that $\text{sgn}(\Delta \ln p_j \alpha) = \text{sgn}(\Delta \ln p_k \alpha) = \text{sgn}(\Delta \ln \alpha)$, and $\text{sgn}(\Delta \ln p_l) \in \{0, -\text{sgn}(\Delta \ln \alpha)\}$ for at least one $s \in J$ that uses automated labour, then $\Delta \ln p_j \alpha$ and $\Delta \ln p_k \alpha$ are $O(\Delta \ln \alpha)$, and*

$$\Delta \ln \frac{s_E(j)}{s_E(k)} = \frac{(\sigma_k - 1)\lambda_k(p_k \alpha)^{\sigma_k - 1}}{1 - \lambda_k(p_k \alpha)^{\sigma_k - 1}} \Delta \ln p_k \alpha - \frac{(\sigma_j - 1)\lambda_j(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \Delta \ln p_j \alpha + o((\Delta \ln \alpha)^2),$$

and if further $\Delta \ln s_E(j)$, $\Delta \ln s_E(k)$ are $O(\Delta \ln \alpha)$, it holds that

$$\begin{aligned} \Delta \ln s_E(k) &= -\frac{s_E(j)}{s_E(k)} \Delta \ln s_E(j) + o(\Delta \ln \alpha), \\ \Delta \ln s_E(j) &= \frac{s_E(k)}{s_E(j)} \frac{(\sigma_k - 1)\lambda_k(p_k \alpha)^{\sigma_k - 1} - (\sigma_j - 1)\lambda_j(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1} + 1 - \lambda_k(p_k \alpha)^{\sigma_k - 1}} \Delta \ln p_j \alpha + o(\Delta \ln \alpha), \\ \Delta \ln p_k \alpha &= \frac{s_E(k)}{s_E(j)} \Delta \ln p_j \alpha + o(\Delta \ln \alpha). \end{aligned}$$

Proof. For $|\Delta \ln \alpha|$ sufficiently small, by $\text{sgn}(\Delta \ln p_s) \in \{0, -\text{sgn}(\Delta \ln \alpha)\}$,

$$|\Delta \ln w_{Hs}^A| \leq \left| \frac{\partial}{\partial \alpha} \ln w_{Hs}^A \right| \cdot |\Delta \ln \alpha|.$$

As automated worker employment is sustained in all occupations, $\Delta \ln w_{Hj}^A = \Delta \ln w_{Hk}^A = \Delta \ln w_{Hs}^A$, so that for $l \in \{j, k\}$, $\lim_{\Delta \ln \alpha \rightarrow 0} \Delta \ln w_{Hl}^A = 0$ and w_{Hl}^A is continuous in α . Because

$$\alpha w_{Hl}^A = \left(\frac{(1 - \lambda_l)(p_l \alpha)^{\sigma_l - 1}}{1 - \lambda_l(p_l \alpha)^{\sigma_l - 1}} \right)^{\frac{1}{\sigma_l - 1}}$$

is an infinitely smooth invertible function of $p_l \alpha$, $p_l \alpha$ (and thus p_l) is continuous in α .

Moreover, by a first order Taylor expansion,

$$\frac{\Delta \ln p_l \alpha}{\Delta \ln \alpha} = \frac{\partial \ln p_l \alpha}{\partial \ln \alpha w_{Hl}^A} \frac{\Delta \ln \alpha w_{Hl}^A}{\Delta \ln \alpha} + \frac{\Delta \ln \alpha w_{Hl}^A}{\Delta \ln \alpha} O(\Delta \ln \alpha w_{Hl}^A).$$

As $\frac{\Delta \ln \alpha w_{Hl}^A}{\Delta \ln \alpha}$ is bounded in absolute value (by $\left| \frac{\partial}{\partial \alpha} \ln w_{Hs}^A \right| + c$ for any $c > 0$),

$$\frac{\Delta \ln p_l \alpha}{\Delta \ln \alpha} = \frac{\partial \ln p_l \alpha}{\partial \ln \alpha w_{Hl}^A} \frac{\Delta \ln \alpha w_{Hl}^A}{\Delta \ln \alpha} + o(1).$$

Thus, also $\frac{\Delta \ln p_l \alpha}{\Delta \ln \alpha}$ is bounded in absolute value, and

$$1. \quad \Delta \ln y^A(H|l) = \frac{\partial y^A(H|l)}{\partial \ln p_l \alpha} \Delta \ln p_l \alpha + o(\Delta \ln \alpha) = \frac{\sigma_l \lambda_l (p_l \alpha)^{\sigma_l - 1}}{1 - \lambda_l (p_l \alpha)^{\sigma_l - 1}} \Delta \ln p_l \alpha + o(\Delta \ln \alpha),$$

$$2. \Delta \ln w_{HI}^A = \frac{\partial \ln w_{HI}^A}{\partial \ln p_l \alpha} \Delta \ln p_l \alpha + o(\Delta \ln \alpha) = \frac{\Delta \ln p_j \alpha}{1 - \lambda_l (p_l \alpha)^{\sigma_l - 1}} + o(\Delta \ln \alpha),$$

By $y(l) = s_E(l) y^A(H|l)$ and $w_{HI}^A/p_l = (y^A(H|l)(1 - \lambda_l))^{1/\sigma_l}$,

$$\begin{aligned} \frac{s_E(j)}{s_E(k)} &= \frac{p_k y^A(H|k)}{p_j y^A(H|j)} = \frac{w_{Hj}/p_j y^A(H|k)}{w_{Hk}^A/p_k y^A(H|j)} \\ &= \frac{(y^A(H|j)(1 - \lambda_j))^{1/\sigma_j} y^A(H|k)}{(y^A(H|k)(1 - \lambda_l))^{1/\sigma_k} y^A(H|j)}. \end{aligned}$$

Plugging in equation (3.14) for $y^A(H|j), y^A(H|k)$ gives

$$\frac{s_E(j)}{s_E(k)} = \frac{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}}. \quad (\text{C.4})$$

By continuity of $p_l \alpha$, this ratio is continuous in α . Moreover, by boundedness of $\Delta \ln p_l \alpha / \Delta \ln \alpha$ in absolute value,

$$\begin{aligned} \Delta \ln \frac{s_E(j)}{s_E(k)} &= \Delta \ln(1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}) - \Delta \ln(1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}) \\ &= \frac{(\sigma_k - 1) \lambda_k (p_k \alpha)^{\sigma_k - 1}}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \Delta \ln p_k \alpha - \frac{(\sigma_j - 1) \lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \Delta \ln p_j \alpha + o(\Delta \ln \alpha), \end{aligned} \quad (\text{C.5})$$

which implies boundedness in absolute value of $(\Delta \ln s_E(j)/s_E(k))/\Delta \ln \alpha$. This establishes the first part of the Lemma.

If furthermore, $\Delta s_E(j), \Delta s_E(k)$ are $O(\Delta \ln \alpha)$, then

$$\Delta \ln s_E(k) = \frac{s_E(j)}{s_E(k)} \frac{\Delta s_E(k)}{s_E(j)} + o(\Delta \ln \alpha) = -\frac{s_E(j)}{s_E(k)} \Delta \ln s_E(j) + o(\Delta \ln \alpha). \quad (\text{C.6})$$

From point 2. above, net of asymptotically dominated terms, equality of $\Delta \ln w_{HI}^A$ across $l \in \{j, k\}$ gives

$$\Delta \ln p_k \alpha = \frac{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \Delta \ln p_j \alpha = \frac{s_E(k)}{s_E(j)} \Delta \ln p_j \alpha.$$

With equations (C.5) and (C.6), this yields

$$\Delta \ln s_E(j) = \frac{s_E(k)}{s_E(j)} \frac{(\sigma_k - 1) \lambda_k (p_k \alpha)^{\sigma_k - 1} - (\sigma_j - 1) \lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1} + 1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \Delta \ln p_j \alpha.$$

This establishes the Lemma. □

For the sake of tractability, Proposition 7 is re-stated here before it is proven.

Proposition 7. *If at technology level α ,*

- (i) *if for $\theta \in \Theta, j \in J, \alpha = \alpha_{\theta j}^*$, then $s_{\theta}(j) = 0$ (no active triggers),*

(ii) if for $\theta \in \Theta, j, k \in J$, a j, k -IC holds, then it is active, i.e. $s_\theta(j), s_\theta(k) > 0$, and

(iii) any (j, k) -pair features at most one structural IC at α , with at most 3 structural ICs in total,

then for any $j \in J$, p_j is continuously differentiable in α with $\frac{dp_j \alpha}{d\alpha} > 0$, and for $\theta \in \Theta$, if θ is not automated, $s_\theta(j)$ is continuously differentiable in α . Furthermore, for the set $\Theta_A \subseteq \Theta$ of automated types, $\tilde{s}_E(j) = \sum_{\theta \in \Theta_A} \phi^{-1[\theta=R]} s_\theta(j)$ is continuously differentiable.

Proof. The proof requires a case distinction of the number of occupations that employ automated worker types.

Case 0: No occupation employing automated labour. Here, α is small enough to not trigger any automation, and the initial no-automation equilibrium is maintained for $|\Delta \ln \alpha|$ small. Thus, for any $j \in J$ and any $\theta \in \Theta$, p_j and $s_\theta(j)$ are continuously differentiable in α with $\frac{dp_j}{d\alpha} = \frac{ds_\theta(j)}{d\alpha} = 0$.

Case 1: All occupations employing automated labour. Consider a change $|\Delta \ln \alpha|$ small that does not affect ICs.

Because all occupations employ automated labour, exactly 3 structural ICs hold, and all for automated worker types. Thus, if still existent, non-automated workers $\theta \in N$, where N is the set of non-automated types, are restricted to single occupations, i.e. $\forall \theta \in N \exists j \in J : \forall i_\theta \in \theta : j(i_\theta) = j$. If $\text{sgn}(\Delta \ln p_j \alpha) = -\text{sgn}(\Delta \ln \alpha)$ for some $j \in J$, then $\text{sgn}(\Delta \ln p_j) = -\text{sgn}(\Delta \ln \alpha)$, and by $\sum_{k \in J} \Delta p_k = 0$, there exists $k \in J$ with $\text{sgn}(\Delta \ln p_k) = \text{sgn}(\Delta \ln \alpha)$, and especially $\text{sgn}(\Delta \ln p_k \alpha) = \text{sgn}(\Delta \ln \alpha)$. Thus, the j, k -IC for automated workers breaks down, a contradiction. Further, $j \in J$ with $\text{sgn}(\Delta \ln p_j) \in \{0, -\text{sgn}(\Delta \ln \alpha)\}$ exists trivially by $\sum_{k \in J} \Delta p_k = 0$.

Thus, the first part of Lemma 3 applies, and $\Delta \ln s_E(j)/s_E(k)$ is $O(\Delta \ln \alpha)$. Further, if $N = \emptyset$, then $\Delta \sum_{j \in J} s_E(j) = 0$, and otherwise

$$\Delta \sum_{j \in J} s_E(j) = \Delta \sum_{\theta \in N} \frac{s_\theta}{y^A(H|j(\theta))} = - \sum_{\theta \in N} s_\theta \cdot \frac{1}{y^A(H|j(\theta))} \frac{\partial \ln y^A(H|j(\theta))}{\partial \ln p_j \alpha} \Delta \ln p_{j(\theta)} \alpha + o(\Delta \ln \alpha)$$

so that $\text{sgn}(\Delta \sum_{j \in J} s_E(j)) = \text{sgn}(\Delta \ln \alpha)$, and

$$\left| \Delta \sum_{j \in J} s_E(j) \right| \leq \max_{\theta \in N} \left| \frac{s_\theta}{y^A(H|j(\theta))} \frac{\partial \ln y^A(H|j(\theta))}{\partial \ln p_j \alpha} \right| \sum_{\theta \in N} |\Delta \ln p_{j(\theta)} \alpha|.$$

In either case, $\Delta \sum_{j \in J} s_E(j)$ is $O(\Delta \ln \alpha)$. By $s_E(j) = \left(1 + \sum_{k \neq j} \frac{s_E(k)}{s_E(j)}\right)^{-1} \sum_{k \in J} s_E(k)$ for $j \in J$,

$$\Delta \ln s_E(j) = - \frac{1}{1 + \sum_{k \neq j} \frac{s_E(k)}{s_E(j)}} \left(\sum_{k \neq j} \frac{s_E(k)}{s_E(j)} \Delta \ln \frac{s_E(k)}{s_E(j)} \right) + \Delta \ln \left(\sum_{j \in J} s_E(j) \right) + o(\Delta \ln \alpha),$$

$\Delta \ln s_E(j)$ is $O(\Delta \ln \alpha)$ for any $j \in J$. Thus, the second part of Lemma 3 applies to any combination of $j, k \in J$. Finally, by $0 = \Delta \ln P = \frac{1}{4} \sum_{k \in J} \Delta \ln p_j$, this gives

$$\Delta \ln p_j \alpha = 4 \Delta \ln \alpha - \sum_{k \neq j} \Delta \ln p_k \alpha = 4 \Delta \ln \alpha - \sum_{k \neq j} \frac{s_E(k)}{s_E(j)} \Delta \ln p_j \alpha$$

so that

$$0 < 4 \left(1 + \sum_{k \neq j} \frac{s_E(k)}{s_E(j)} \right)^{-1} = \lim_{\Delta \ln \alpha \rightarrow 0} \frac{\Delta \ln p_j \alpha}{\Delta \ln \alpha} = \frac{d \ln p_j \alpha}{d \ln \alpha}$$

which yields that $\ln p_j$ is continuously differentiable in $\ln \alpha$ with $\frac{d \ln p_j}{d \ln \alpha} = 4 \left(1 + \sum_{k \neq j} \frac{s_E(k)}{s_E(j)} \right)^{-1} - 1$. Accordingly, also continuously differentiable functions of $p_j \alpha$ like $y^A(H|j)$ and αw_{Hj}^A are continuously differentiable in $\ln \alpha$, and further, by Lemma 3, $\ln s_E(j)$ is differentiable in $\ln \alpha$ with

$$\frac{d \ln s_E(j)}{d \ln \alpha} = 4 \frac{s_E(k)}{s_E(j)} \frac{(\sigma_k - 1) \lambda_k (p_k \alpha)^{\sigma_k - 1} - (\sigma_j - 1) \lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1} + 1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \left(1 + \sum_{l \neq j} \frac{s_E(l)}{s_E(j)} \right)^{-1},$$

where k is any occupation $k \in J$ with $k \neq j$. For any $j \in J$, for $\theta \in N$, $s_\theta(j)$ is unchanged with small changes in α , so that $s_\theta(j)$ is continuously differentiable with $\frac{d s_\theta(j)}{d \alpha} = 0$. Further, as $\tilde{s}_E(j) = s_E(j) - \sum_{\theta \in N: j(\theta)=j} s_\theta / y^A(H|j(\theta))$ and $y^A(H|j(\theta))$ is continuously differentiable in α , $\tilde{s}_E(j)$ is continuously differentiable in α .

Case 2: Three occupations employing automated labour. Consider a change $|\Delta \ln \alpha|$ small that does not affect ICs.

Here, 2 structural ICs hold for automated labour, so that at most one IC may hold for non-automated types. Denote by $J_A \subset J$ the set of occupations using automated labour and by $j_N \in J$ the one that does not. Then, if there exists $j \in J_A$ with $\text{sgn}(\Delta \ln p_j \alpha) = -\text{sgn}(\Delta \ln \alpha)$, by sustained automated worker ICs, $\forall k \in J_A : \text{sgn}(\Delta \ln p_k \alpha) = -\text{sgn}(\Delta \ln \alpha)$. If a non-automated worker IC holds for j_N , $\forall k \in J : \text{sgn}(\Delta \ln p_k) = -\text{sgn}(\Delta \ln \alpha)$, a contradiction to $\Delta P = 0$. If instead no automated worker IC holds for j_N , then $y(j_N)$ is unchanged and $\text{sgn}(\Delta \ln p_{j_N}) = \text{sgn}(\Delta \ln \alpha)$ so that for any $j \in J_A$, $\Delta \ln p_{j_N} / p_j = \Delta \ln y(j) / y(j_N) = \Delta \ln y(j)$ moves in the same direction as α . This contradicts the fact that any $j \in J_A$, $\text{sgn}(\Delta \ln y^A(H|j)) = -\text{sgn}(\Delta \ln \alpha)$.³

Conversely, if for all $j \in J_A$, $\text{sgn}(\Delta \ln p_j) = \text{sgn}(\Delta \ln \alpha)$, then $\text{sgn}(\Delta \ln p_{j_N}) = -\text{sgn}(\Delta \ln \alpha)$, and either, the existing non-automated worker IC breaks, or if there is none, all $y(j)$, $j \in J_A$ move in the opposite direction as α , a contradiction in analogy to above.

Thus, the first part of Lemma 3 applies. Now, if there is no non-automated worker IC, the differentiability argument is analogous to Case 1, using $s_E(j) = \left(1 + \sum_{k \in J_A, k \neq j} \frac{s_E(k)}{s_E(j)} \right)^{-1} \sum_{k \in J_A} s_E(k)$ and an analogous $O(\Delta \ln \alpha)$ -argument for $\sum_{k \in J_A} s_E(k)$. By $\Delta \ln p_{j_N} = \Delta \ln Y = \sum_{j \in J_A} \Delta \ln s_E(j) +$

³If $\Delta \ln \alpha > 0$ (< 0), some $j \in J_A$ weakly reduces (increases) automated labour and $y(j)$ can not increase (decline).

$\Delta \ln y^A(H|j)$ and

$$\Delta \ln p_j \alpha = 4\Delta \ln \alpha - \sum_{k \neq j} \Delta \ln p_k \alpha = 3\Delta \ln \alpha - \sum_{k \in J_A, k \neq j} \Delta \ln p_k \alpha - \Delta \ln Y$$

plugging in the Taylor Approximations for $\Delta \ln s_E(k)$ and $\Delta \ln y^A(H|k)$ and then $\Delta \ln p_k \alpha$ for $k \in J_A, k \neq j$ again gives

$$\Delta \ln p_j \alpha = 3\Delta \ln \alpha - C(\alpha)\Delta \ln p_j \alpha + o(\Delta \ln \alpha)$$

for some parameter $C(\alpha) > 0$ that is continuous in α , so that $\ln p_j \alpha$ is differentiable in $\ln \alpha$ with $\frac{d \ln p_j \alpha}{d \ln \alpha} = \frac{3}{1+C(\alpha)} > 0$. The worker share differentiability argument is then analogous to Case 1.

If instead there is a non-automated worker IC, when $l \in J_A$ is the occupation for which the l, j_N -IC holds for non-automated workers of type $\theta_N \in \Theta$, $\Delta \ln y(l) = \Delta \ln y(j_N)$ (sustained IC) gives

$$\Delta \ln s_E(l) + \Delta \ln y^A(H|l) = \Delta \ln s_{\theta_N}(j_N) \quad \text{or equivalently} \quad \Delta \ln \left(\frac{s_{\theta_N}(j_N)/y^A(H|l)}{s_E(l)} \right) = 0. \quad (\text{C.7})$$

Thus, for $j \in J_A$, with

$$s_E(j) = \left(1 + \sum_{k \in J_A, k \neq j} \frac{s_E(k)}{s_E(j)} + \frac{s_{\theta_N}(j_N)/y^A(H|l)}{s_E(j)} \right)^{-1} \left(\sum_{k \in J_A} s_E(k) + \frac{s_{\theta_N}(j_N)}{y^A(H|l)} \right)$$

and $\Delta \left(\sum_{k \in J_A} s_E(k) + \frac{s_{\theta_N}(j_N)}{y^A(H|l)} \right) = 0$, $\Delta \ln s_E(k)/s_E(j) = O(\Delta \ln \alpha)$ for every $k \in J_A, k \neq j$ and with equation (C.7),⁴ it results that $s_E(j) = O(\Delta \ln \alpha)$ in analogy to previous investigations of $s_E(j)$.

Accordingly, the second part of Lemma 3 applies, and for $j \in J_A$, with $\Delta \ln p_{j_N} = \Delta \ln p_l$, one obtains

$$\Delta \ln p_j \alpha = 4\Delta \ln \alpha - \sum_{k \in J_A, k \neq j} \Delta \ln p_k \alpha - \Delta \ln p_l \alpha.$$

Plugging in the expressions for $\ln p_k \alpha, \ln p_l \alpha$ in terms of $\Delta \ln p_j \alpha$ as obtained from Taylor approximation, for $j \in J_A, j \neq l$, with $l \in J_A \setminus \{j, l\}$, one obtains

$$\Delta \ln p_j \alpha = 4 \left(1 + \frac{s_E(k)}{s_E(j)} + 2 \frac{s_E(l)}{s_E(j)} \right)^{-1} \Delta \ln \alpha + o(\Delta \ln \alpha)$$

⁴This holds as $\Delta \ln \frac{s_{\theta_N}(j_N)/y^A(H|l)}{s_E(j)} = \Delta \ln \frac{s_E(l)}{s_E(j)} + \Delta \ln \frac{s_{\theta_N}(j_N)/y^A(H|l)}{s_E(l)} = \Delta \ln \frac{s_E(l)}{s_E(j)}$ by equation (C.7).

so that $\ln p_j \alpha$ is continuously differentiable with $\frac{d \ln p_j \alpha}{d \ln \alpha} = 4 \left(1 + \frac{s_E(k)}{s_E(j)} + 2 \frac{s_E(l)}{s_E(j)} \right)^{-1} > 0$, and

$$\Delta \ln p_l \alpha = 4 \left(2 + \sum_{k \in J_A, k \neq l} \frac{s_E(k)}{s_E(l)} \right)^{-1} \Delta \ln \alpha + o(\Delta \ln \alpha)$$

so that $\ln p_l \alpha$ is continuously differentiable with $\frac{d \ln p_l \alpha}{d \ln \alpha} = 4 \left(2 + \sum_{k \in J_A, k \neq l} \frac{s_E(k)}{s_E(l)} \right)^{-1} > 0$. By the representation of $\Delta \ln s_E(j)$ for $j \in J_A$ given by Lemma 2, it follows again that also $s_E(j)$ is continuously differentiable in α . Accordingly, for all $j \in J_A$, $y^A(H|j)$ is continuously differentiable in α as a continuously differentiable function of $\ln p_j \alpha$, and by equation (C.7), so is $s_{\theta_N}(j_N)$. Thus, in analogy to Case 1, all $\tilde{s}_E(j)$, $j \in J$ are continuously differentiable.

Cases 3 and 4: One or two occupation(s) employing automated labour. These cases are analytically equivalent to Cases 1 and 2,⁵ however, they require a broader range of sub-case distinctions. For the sake of compactness, they are left out at this point and are available upon request. \square

C.3.2 PROPOSITIONS 8 AND 9

The propositions are shown in inverse order of occurrence for convenience of the formal argument.

Lemma 4. (*Relative Productivity Growth: General Equation*) Let $j, k \in J$ and $\theta \in \Theta$ such that $\alpha \geq \max\{\alpha_{\theta j}^*, \alpha_{\theta k}^*\}$ and $s_{\theta}(j), s_{\theta}(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then, for $\mu_{jk}(\alpha) := \frac{\frac{d}{d \ln \alpha} \ln y^A(H|j)}{\frac{d}{d \ln \alpha} \ln y^A(H|k)}$, it holds that

$$\mu_{jk}(\alpha) = \frac{\sigma_j}{\sigma_k} \frac{\lambda_j (p_j \alpha)^{\sigma_j - 1}}{\lambda_k (p_k \alpha)^{\sigma_k - 1}} = \frac{\sigma_j}{\sigma_k} \frac{\lambda_j / (1 - \lambda_j)}{\lambda_k / (1 - \lambda_k)} \left(\frac{1}{c_k^{K,L}(\alpha)} \right)^{\sigma_j - \sigma_k} \frac{s_E(j)}{s_E(k)}.$$

Proof. With equation (3.14), for any $s \in J$,

$$\frac{d}{d \ln \alpha} \ln y^A(H|s) = -\frac{d}{d \ln \alpha} \left[\frac{\sigma_s}{\sigma_s - 1} \ln(1 - \lambda_s (p_s \alpha)^{\sigma_s - 1}) \right] = \sigma_s \frac{\lambda_s (p_s \alpha)^{\sigma_s - 1}}{1 - \lambda_s (p_s \alpha)^{\sigma_s - 1}} \frac{d \ln p_s \alpha}{d \ln \alpha}.$$

With Lemma 3, $\frac{d \ln p_k \alpha}{d \ln \alpha} = (s_E(k)/s_E(j)) \frac{d \ln p_j \alpha}{d \ln \alpha}$, such that the j, k -ratio of relative per-worker productivity growth is

$$\mu_{jk}(\alpha) = \frac{\sigma_j}{\sigma_k} \frac{\lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} \frac{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}}{\lambda_k (p_k \alpha)^{\sigma_k - 1}} \frac{s_E(j)}{s_E(k)}. \quad (\text{C.8})$$

With equation (C.4), this immediately yields the first equality of the Proposition. Further, as

⁵Case 4 - “one occupation employing automated labour” requires a slight modification of Lemma 3 as there is only one occupation using automated labour. Still, the approach is conceptually identical.

both occupations employ θ -workers, $w_{\theta j}^A = w_{\theta k}^A$, and thus

$$(p_j \alpha)^{\sigma_j - 1} = \frac{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j} \left(\frac{1 - \lambda_k}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \right)^{\frac{\sigma_j - 1}{\sigma_k - 1}} (p_k \alpha)^{\sigma_j - 1}$$

Solving for $(p_j \alpha)^{\sigma_j - 1}$ gives

$$(p_j \alpha)^{\sigma_j - 1} = \frac{\left(\frac{1 - \lambda_k}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \right)^{\frac{\sigma_j - 1}{\sigma_k - 1}} (p_k \alpha)^{\sigma_j - 1}}{1 - \lambda_j + \lambda_j \left(\frac{1 - \lambda_k}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \right)^{\frac{\sigma_j - 1}{\sigma_k - 1}} (p_k \alpha)^{\sigma_j - 1}}$$

and

$$\frac{\lambda_j (p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}} = \frac{\lambda_j}{1 - \lambda_j} \left(\frac{1 - \lambda_k}{1 - \lambda_k (p_k \alpha)^{\sigma_k - 1}} \right)^{\frac{\sigma_j - 1}{\sigma_k - 1}} (p_k \alpha)^{\sigma_j - 1}.$$

Plugging this result into equation (C.8) establishes the proposition. \square

Proposition 9. *Let $j, k \in J$ and $\theta \in \Theta$ such that $\alpha \geq \max\{\alpha_{\theta j}^*, \alpha_{\theta k}^*\}$ and $s_{\theta}(j), s_{\theta}(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then,*

1. *if $j = j(\sigma, \lambda_H)$, $k = j(\sigma, \lambda_L)$, $\sigma \in \{\sigma_L, \sigma_H\}$, it globally holds that (i) $\mu_{jk}(\alpha) > 1$, (ii) j strictly grows relative to k , i.e. $d \ln \frac{y(j)}{y(k)} / d \ln \alpha > 0$, and (iii) $s_E(j) < s_E(k)$;*
2. *if $j = j(\sigma_H, \lambda)$, $k = j(\sigma_L, \lambda)$, $\lambda \in \{\lambda_L, \lambda_H\}$, it holds for $\alpha > 1/p_k$ that (i) $\mu_{jk}(\alpha) > \sigma_H/\sigma_L$, (ii) j strictly grows relative to k , i.e. $d \ln \frac{y(j)}{y(k)} / d \ln \alpha > 0$, and (iii) $s_E(j) < s_E(k)$.*

For $\alpha > 1/p_k$, the above relationships for worker- and occupation-level growth also hold absolutely.

Proof. With Lemma 4,

$$\frac{\mu_{jk}(\alpha)}{\sigma_j/\sigma_k} = \frac{\lambda_j (p_j \alpha)^{\sigma_j - 1}}{\lambda_k (p_k \alpha)^{\sigma_k - 1}},$$

so that with equation (C.4),

$$\text{sgn} \left(\ln \frac{\mu_{jk}(\alpha)}{\sigma_j/\sigma_k} \right) = -\text{sgn} \left(\ln \frac{s_E(j)}{s_E(k)} \right). \quad (\text{C.9})$$

Moreover, with Lemma 3 in analogy to the proof of Lemma 4,

$$\frac{d \ln \frac{y(j)}{y(k)}}{d \ln \alpha} = \frac{d \ln p_k \alpha}{d \ln \alpha} - \frac{d \ln p_j \alpha}{d \ln \alpha} = \left(1 - \frac{s_E(j)}{s_E(k)} \right) \underbrace{\frac{d \ln p_k \alpha}{d \ln \alpha}}_{>0}. \quad (\text{C.10})$$

For case 1., with Lemma 4,

$$\mu_{jk}(\alpha) = \frac{\lambda_H/(1 - \lambda_H)}{\lambda_L/(1 - \lambda_L)} \frac{s_E(j)}{s_E(k)} > \frac{s_E(j)}{s_E(k)},$$

which gives $s_E(j)/s_E(k) < 1 < \mu_{jk}(\alpha)$ by equation (C.9). Further, $s_E(j)/s_E(k) < 1$ directly yields $\frac{d \ln y(j)/y(k)}{d \ln \alpha} > 0$ with equation (C.10). This establishes 1.

For case 2., $\alpha > 1/p_k$ gives $c_k^{K,L}(\alpha) < 1$ (cf. Eq. (3.16); note that $\alpha > 1/p_k$ implies also $\alpha > 1/p_j$ by $c_k^{K,L}(\alpha) = c_j^{K,L}(\alpha)$), and by Lemma 4,

$$\frac{\mu_{jk}(\alpha)}{\sigma_j/\sigma_k} = \left(\frac{1}{c_k^{K,L}(\alpha)} \right)^{\sigma_H - \sigma_L} \frac{s_E(j)}{s_E(k)} > \frac{s_E(j)}{s_E(k)}$$

which gives $s_E(j)/s_E(k) \leq 1 < \mu_{jk}(\alpha)/(\sigma_H/\sigma_L)$ by equation (C.9). Again, $s_E(j)/s_E(k) < 1$ directly yields $\frac{d \ln y(j)/y(k)}{d \ln \alpha} > 0$ with equation (C.10). This establishes 2.

It remains to show the statements' absolute variants for $\alpha > 1/p_k$. If $j = j(\sigma_H, \lambda_H)$ and $p_j = p_k$ at $\alpha = 1/p_k$ or $j \neq j(\sigma_H, \lambda_H)$, at $\alpha = 1/p_k$, $y^A(i|j) = (1 - \lambda_j)^{-1} \geq (1 - \lambda_k)^{-1} = y^A(i|k)$ and $y(j) = y(k)$,⁶ so that (strictly) faster relative growth of j implies (strictly) faster absolute growth of the respective quantity. If instead $j = j(\sigma_H, \lambda_H)$ and $p_j < p_k$ at $\alpha = 1/p_k$, by Propositions 3 and 4, $p_j < p_k$ for $\alpha > 1/p_k$. Thus, with $y^A(H|s) = (w_{Hs}/p_s)^{\sigma_s} (1 - \lambda_s)^{-1}$ for $s \in J$,

$$y^A(H|j) = \frac{1 - \lambda_k}{1 - \lambda_j} (w_H/p_k)^{\sigma_j - \sigma_k} y^A(H|k) > y^A(H|k)$$

by $w_H/p_k > 1$ (cf. Eq. (3.15)). Further, $p_j < p_k$ directly gives $y(j) > y(k)$. \square

By a transitivity argument, Proposition 9 implies an statement analogous to 2. for the comparison $j(\sigma_H, \lambda_H)$ vs. $j(\sigma_L, \lambda_L)$. For $j(\sigma_H, \lambda_L)$ vs. $j(\sigma_L, \lambda_H)$, growth and employment depend on the relative degree of heterogeneity introduced by complementarity and the routine task weight; as $\sigma_H \rightarrow \sigma_L$ ($\lambda_H \rightarrow \lambda_L$), a result in analogy to 1. (2.) holds.

Lemma 5 (*U-Deepening: H-Employment*). *Let $j, k \in J$ with $\alpha \geq \max\{\alpha_{iuj}^*, \alpha_{iuk}^*\}$ and $s_H(j), s_H(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then,*

$$\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} = \frac{\sigma_k - 1}{\sigma_k} \frac{d \ln y^A(H|k)}{d\alpha} - \frac{\sigma_j - 1}{\sigma_j} \frac{d \ln y^A(H|j)}{d\alpha}.$$

Proof. With Equations (3.14) and (3.15), $w_{ij}^A = \left[(1 - \lambda_j) y^A(i|j) \right]^{\frac{1}{\sigma_j}} p_j$ so that

$$\frac{d \ln w_{ij}^A}{d\alpha} = \frac{1}{\sigma_j} \frac{d \ln y^A(i|j)}{d\alpha} + \frac{d \ln p_j}{d\alpha}.$$

As H -employment in j, k is sustained locally around α (by differentiability of $\tilde{s}_E(j)$, cf. Propo-

⁶See Proposition 15.

sition 7), it holds that $\frac{d}{d\alpha} \ln(w_{ij}^A) = \frac{d}{d\alpha} \ln(w_{ik}^A)$, which implies with $p_j/p_k = (y_j/y_k)^{-1}$ that

$$\frac{d}{d\alpha} \ln\left(\frac{y(j)}{y(k)}\right) = \frac{d}{d\alpha} \ln\left(\frac{y^A(i|j)}{y^A(i|k)}\right) + \left(\frac{\sigma_k - 1}{\sigma_k} \frac{d \ln y^A(i|k)}{d\alpha} - \frac{\sigma_j - 1}{\sigma_j} \frac{d \ln y^A(i|j)}{d\alpha}\right).$$

The proposition follows from $y(j)/y(k) = s_E(j)/s_E(k) \cdot y^A(H|j)/y^A(H|k)$. \square

Proposition 8. *Let $j, k \in J$ and $\theta \in \Theta$ such that $\alpha \geq \max\{\alpha_{\theta j}^*, \alpha_{\theta k}^*\}$ and $s_\theta(j), s_\theta(k) > 0$, and assume that the differentiability conditions of Proposition 7 hold at α . Then,*

1. if $j = j(\sigma, \lambda_H)$, $k = j(\sigma, \lambda_L)$, $\sigma \in \{\sigma_L, \sigma_H\}$, it globally holds that (i) $s_E(j) < s_E(k)$ and (ii) $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} > 0$ if and only if $\sigma < 1$;
2. if $j = j(\sigma_H, \lambda)$, $k = j(\sigma_L, \lambda)$, $\lambda \in \{\lambda_L, \lambda_H\}$, it holds for $\alpha > 1/p_k$ that (i) $s_E(j) < s_E(k)$ and (ii) if $\sigma_H \geq 1$, it furthermore holds that $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} < 0$;
3. if $\sigma_j > 1 > \sigma_k$, then $\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} < 0$.

Proof. Part (i) of the proposition is already given in Proposition 9 and just re-stated here to gather the results related to labour.

The equation of Lemma 5 can be re-arranged to

$$\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} = -\frac{\sigma_j - \sigma_k}{\sigma_j \sigma_k} \frac{d \ln y^A(H|j)}{d\alpha} - \frac{\sigma_k - 1}{\sigma_k} \frac{d}{d\alpha} \ln \frac{y^A(H|j)}{y^A(H|k)}. \quad (\text{C.11})$$

For case 1., one obtains

$$\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} = -\frac{\sigma - 1}{\sigma} \frac{d}{d\alpha} \ln \frac{y^A(H|j)}{y^A(H|k)} \propto \frac{1}{\sigma} - 1$$

where proportionality in sign follows from point 1. of Proposition 9.

For case 2., $\frac{d \ln y^A(H|k)}{d\alpha} > 0$ ⁷ gives $\frac{d \ln y^A(H|j)}{d\alpha} > \frac{d}{d\alpha} \ln \frac{y^A(H|j)}{y^A(H|k)}$, and therefore

$$\frac{d}{d\alpha} \ln \frac{s_E(j)}{s_E(k)} < -\frac{\sigma_H - 1}{\sigma_H} \frac{d}{d\alpha} \ln \frac{y^A(H|j)}{y^A(H|k)} \propto \frac{1}{\sigma_H} - 1$$

where proportionality in sign follows from point 2. of Proposition 9.

Finally, case 3. is a corollary of the first equation in Lemma 3 that directly follows from

$$\frac{d \ln p_j \alpha}{d\alpha}, \frac{d \ln p_k \alpha}{d\alpha} > 0. \quad \square$$

⁷Cf. equation (3.14); $\frac{d \ln p_k \alpha}{d \ln \alpha} > 0$ by Proposition 7.

C.3.3 EQUILIBRIUM TRANSITION: NEW ICs

New ICs. Start from a “standard” equilibrium as assumed in the setup of Proposition 7. Here, potential wage ratios $w_{\theta j}^A/w_{\theta k}^A$ and $w_{\theta j}^N/w_{\theta k}^N$ for $\theta \in \Theta$, $j, k \in J$ are infinitely smooth functions of p_j, p_k and α and thus continuously differentiable in α . Suppose that θ -workers are employed in either j but not k at $\alpha = \alpha_0$ and for $P \in \{N, A\}$, $w_{\theta j}^P/w_{\theta k}^P = 1$ at α_0 . Then, α_0 may represent a point of momentary indifference between j and k that does not induce non-zero labour flow, i.e. α_0 is an extremiser of $w_{\theta j}^P/w_{\theta k}^P$ and $\text{sgn}\left(\frac{dw_{\theta j}^P/w_{\theta k}^P}{d\alpha}\right)$ changes around α_0 . In this case, the equilibrium structure is maintained, and prices and worker shares are continuously differentiable at α as the momentary IC does not affect behavior of equilibrium quantities.

Simultaneous and Overidentifying ICs. Assume that at α_0 , a j, k -IC holds for θ_1 -workers on $(\alpha_0 - \varepsilon, \alpha_0)$ for $\alpha_0 \in \mathbb{R}_+$ and $\varepsilon > 0$ small, and that $s_{\theta_2}(j) > 0 = s_{\theta_2}(k)$ for $\alpha \in (\alpha_0 - \varepsilon, \alpha_0)$ for $\theta_2 \in \Theta$ with $\lim_{\alpha \rightarrow \alpha_0^-} w_{i_{\theta_2} j}^{P_2}/w_{i_{\theta_2} k}^{P_2} = 1$ for $P_2 \in \{N, A\}$. It can be shown (see below) that generally,

$$\lim_{\alpha \rightarrow \alpha_0^-} \frac{d \ln w_{i_{\theta_2} j}^{P_2}/w_{i_{\theta_2} k}^{P_2}}{d \ln \alpha} \neq \lim_{\alpha \rightarrow \alpha_0^-} \frac{d \ln w_{i_{\theta_1} j}^{P_1}/w_{i_{\theta_1} k}^{P_1}}{d \ln \alpha}$$

where $P_1 \in \{N, A\}$ is the production method used for θ_1 -workers. Thus, the wage ratios of θ_1 - and θ_2 -workers are on different trends, and there is a labour demand shock at α_0 .

Note that no-automation wage ratios are always on the same trend as $w_{\theta j}^N/w_{\theta l}^N = p_j/p_k \cdot y^N(\theta|j)/y^N(\theta|k)$ where the second factor is invariant to α . Further, automation wage ratios are the same for any type, and thus especially also on the same trend in α . Accordingly, simultaneity can arise only between an automated and a non-automated type. The simultaneity can also be indirect, e.g. when a j, k -IC holds for A -workers, and j, l - and l, k -ICs hold for one or more type(s) of N -workers that bind the relative prices of j, k and l .

This labour demand shock has heterogeneous implications depending on the equilibrium structure on $(\alpha_0 - \varepsilon, \alpha_0)$. If there are three active structural ICs,⁸ the new IC is an *overidentifying* IC that causes indirect simultaneity. Here, labour is mobile between all occupations, and at α_0 , employment adjusts to the j, k -supply shock at sustained ICs (i.e. constant prices and relative outputs) until $s_{\theta}(l)$ reduces to zero for some $\theta \in \Theta$, $l \in J$, i.e. until a θ -IC is deactivated. For $\alpha > \alpha_0$, an overidentifying IC is thus removed, and the equilibrium obeys the setup of Proposition 7 such that adjustment is smooth. As prices are unchanged at α_0 , there is at most a non-differentiable kink in prices, the labour distribution however adjusts discontinuously at α_0 .

If there are strictly less than three active structural ICs, also prices may jump discontinu-

⁸Recall that the j, k -IC for θ -workers is active if $s_{\theta}(j), s_{\theta}(k) > 0$.

ously, as the adjustment to simultaneity in concerned occupations is relative- but not absolute-output neutral, and occupations not bound by an IC generally do not change output. Accordingly, such scenarios may cause discontinuous disruptions of the equilibrium, including price declines that induce sudden reversal of automation triggers. It may be ensured that this issue does not occur in the given model setup, as demonstrated in Section 3.4.

Simultaneity: Sustainability. This investigation shows that at constant relative prices (due to one or multiple non-automated worker ICs), automated worker ICs are generally not sustained.

Constant relative prices as imposed by non-automated worker ICs require

$$\frac{p_j}{p_k} = c \in \mathbb{R}_+, \quad (\text{C.12})$$

and the j, k -IC for automated workers holds if

$$\left(\frac{(1 - \lambda_j)(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{1}{\sigma_j - 1}} = \left(\frac{(1 - \lambda_k)(p_k \alpha)^{\sigma_k - 1}}{1 - \lambda_k(p_k \alpha)^{\sigma_k - 1}} \right)^{\frac{1}{\sigma_k - 1}}. \quad (\text{C.13})$$

Satisfying these equations simultaneously is consistent with smooth, i.e. continuously differentiable equilibrium (especially: price) adjustment only if $\frac{d \ln p_j \alpha}{d \ln \alpha} = \frac{d \ln p_k \alpha}{d \ln \alpha}$ (Eq. (C.12)), and with

$$\frac{d}{d \ln \alpha} \ln \left[\left(\frac{(1 - \lambda_j)(p_j \alpha)^{\sigma_j - 1}}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \right)^{\frac{1}{\sigma_j - 1}} \right] = \frac{1}{1 - \lambda_j(p_j \alpha)^{\sigma_j - 1}} \frac{d \ln p_j \alpha}{d \ln \alpha},$$

this yields $\lambda_j(p_j \alpha)^{\sigma_j - 1} = \lambda_k(p_k \alpha)^{\sigma_k - 1}$ or $\frac{d \ln p_j \alpha}{d \ln \alpha} = \frac{d \ln p_k \alpha}{d \ln \alpha} = 0$. For the latter case, $\frac{d \ln p_j}{d \ln \alpha} = \frac{d \ln p_k}{d \ln \alpha} < 0$, and as $y(\theta|l)$ is unchanged for $l \in \{j, k\}$, $\theta \in \Theta$, either productivity shrinks in, or labour flows to j, k from other occupations $l \in J$, $l \neq j, k$. Thus, either $\frac{d \ln p_l \alpha}{d \ln \alpha} < 0$ at sustained A -employment in $l \in J$, which contradicts $\frac{d \ln p_l \alpha}{d \ln \alpha} = 0$ and sustained j, l -IC for A -workers, or the occupation l from which workers flow to j, k also satisfies $\frac{d \ln p_l \alpha}{d \ln \alpha} = 0$, a contradiction to declined output in l .⁹

On the other hand, for $(p_j \alpha)^{\sigma_j - 1} = \lambda_k / \lambda_j (p_k \alpha)^{\sigma_k - 1}$, if $\sigma_k = \sigma_j = \sigma$, then with equation (C.13)

$$(1 - \lambda_j)(p_j \alpha)^{\sigma - 1} = (1 - \lambda_k) \frac{\lambda_j}{\lambda_k} (p_k \alpha)^{\sigma - 1} \Leftrightarrow \frac{1 - \lambda_j}{\lambda_j} = \frac{1 - \lambda_k}{\lambda_k} \Leftrightarrow \lambda_j = \lambda_k,$$

⁹ $y(l)$ must decline as if it receives labour from $m \in J \setminus \{j, k, l\}$ only if $\frac{d \ln p_m \alpha}{d \ln \alpha} \leq 0$, which contradicts $\sum_{s \in J} \frac{d \ln p_s}{d \ln \alpha} = 0$.

and $j = k$. Therefore, $\sigma_j \neq \sigma_k$, and imposing $(p_j \alpha)^{\sigma_j - 1} = \lambda_k / \lambda_j (p_k \alpha)^{\sigma_k - 1}$ on equation (C.13) gives

$$\begin{aligned} \frac{1 - \lambda_k}{\lambda_k} \lambda_j (p_j \alpha)^{\sigma_j - 1} &= (1 - \lambda_j)^{\frac{\sigma_k - 1}{\sigma_j - 1}} (p_j \alpha)^{\sigma_k - 1} \left(1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}\right)^{\frac{\sigma_j - \sigma_k}{\sigma_j - 1}} \\ \left(\frac{1 - \lambda_k}{\lambda_k} \lambda_j\right)^{\frac{\sigma_j - 1}{\sigma_j - \sigma_k}} (p_j \alpha)^{\sigma_j - 1} &= (1 - \lambda_j)^{\frac{\sigma_k - 1}{\sigma_j - \sigma_k}} \left(1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}\right) \\ \left[\frac{(1 - \lambda_k / \lambda_k)^{\sigma_j - 1}}{(1 - \lambda_j / \lambda_j)^{\sigma_k - 1}}\right]^{\frac{1}{\sigma_j - \sigma_k}} \lambda_j (p_j \alpha)^{\sigma_j - 1} &= 1 - \lambda_j (p_j \alpha)^{\sigma_j - 1}. \end{aligned}$$

Accordingly,

$$\lambda_k (p_k \alpha)^{\sigma_k - 1} = \lambda_j (p_j \alpha)^{\sigma_j - 1} = \left(1 + \left[\frac{(1 - \lambda_k / \lambda_k)^{\sigma_j - 1}}{(1 - \lambda_j / \lambda_j)^{\sigma_k - 1}}\right]^{\frac{1}{\sigma_j - \sigma_k}}\right)^{-1} =: \xi$$

where ξ is invariant to α . Thus, for $l \in \{j, k\}$, $p_l = (\xi / \lambda_l)^{\frac{1}{\sigma_l - 1}} \alpha^{-1}$, and $\frac{d \ln p_j \alpha}{d \ln \alpha} = \frac{d \ln p_k \alpha}{d \ln \alpha} = 1$.

This suggests that a simultaneous j, k -IC for automated workers is sustainable only if $\sigma_j \neq \sigma_k$, the relative outputs of j and k are unchanged, and additionally at the very specific constellation of prices that gives $s_E(j) = s_E(k)$ (cf. equation C.5). Thus, except for singular values of parameters, this adjustment restriction will not be consistent with those imposed by other ICs.

C.3.4 A-AUTOMATION

This appendix derives the analytical results for equilibrium adjustment to A -worker automation at the A -trigger (*adoption*) and between the A - and U -trigger (*deepening*).

ADOPTION: NO BREAKS

Worker Distribution given β , solving for β . A -worker shares are obtained from A -wage equality across λ_L -occupations, i.e. $w_A = w_{i_{Aj}(\sigma_H, \lambda_L)}^A = w_{i_{Aj}(\sigma_L, \lambda_L)}^A$. As at the trigger, $w_{i_{Aj}}^A = w_{i_{Aj}}^N$ for any triggered occupation $j \in J$, it continues to hold that

$$\rho_A^{-1} = \frac{p_{j(\sigma_H, \lambda_L)}}{p_{j(\sigma_L, \lambda_L)}} = \frac{y^A(j(\sigma_L, \lambda_L))}{y^A(j(\sigma_H, \lambda_L))} \quad (\text{C.14})$$

with

$$\begin{aligned} y^A(j(\sigma_H, \lambda_L)) &= s_A(j(\sigma_H, \lambda_L)) m_H y^N(i_A | j(\sigma_H, \lambda_L)), \\ y^A(j(\sigma_L, \lambda_L)) &= (s_A - s_A(j(\sigma_H, \lambda_L))) m_L y^N(i_A | j(\sigma_L, \lambda_L)) + (1 - \beta) s_U. \end{aligned}$$

With equation (C.14), one obtains

$$\begin{aligned} \rho_A^{-1} s_A(j(\sigma_H, \lambda_L)) m_H y^N(i_A | j(\sigma_H, \lambda_L)) &= (s_A - s_A(j(\sigma_H, \lambda_L))) m_L y^N(i_A | j(\sigma_L, \lambda_L)) + (1 - \beta) s_U \\ \Leftrightarrow (m_H + m_L) y^N(i_A | j(\sigma_L, \lambda_L)) s_A(j(\sigma_H, \lambda_L)) &= s_A m_L y^N(i_A | j(\sigma_L, \lambda_L)) + (1 - \beta) s_U \\ \Leftrightarrow s_A(j(\sigma_H, \lambda_L)) &= \frac{s_A}{m_H + m_L} \left(m_L + (1 - \beta) \frac{s_U}{Y_A / \rho_A} \right). \end{aligned}$$

Furthermore,

$$y(j(\sigma_H, \lambda_L)) = s_A(j(\sigma_H, \lambda_L)) m_H y^N(i_A | j(\sigma_H, \lambda_L)) = \frac{m_H}{m_H + m_L} \left(m_L + (1 - \beta) \frac{s_U}{Y_A / \rho_A} \right) Y_A$$

and with equation (C.14), occupation-level output is given by

$$y_{j(\sigma_L, \lambda_L)}^A = m_H \frac{m_L Y_A / \rho_A + (1 - \beta) s_U}{m_H + m_L} \quad \text{and} \quad y_{j(\sigma_H, \lambda_L)}^A = \rho_A y_{j(\sigma_L, \lambda_L)}^A. \quad (\text{C.15})$$

Next, by equality of U -wages (and thus prices) in σ_L -occupations, $y(j(\sigma_L, \lambda_H)) = y(j(\sigma_L, \lambda_L))$, or

$$\beta s_U + s_R(j(\sigma_L, \lambda_H)) y^N(i_R | j(\sigma_L, \lambda_H)) = (1 - \beta) s_U + s_A(j(\sigma_L, \lambda_L)) \underbrace{y^A(i_A | j(\sigma_L, \lambda_L))}_{= m_L y^N(i_A | j(\sigma_L, \lambda_L))}.$$

Plugging $s_R(\cdot)$ and $s_A(\cdot)$ as functions of β gives

$$\beta s_U + \frac{Y_R / \rho_R}{2} \left(1 - \beta \frac{s_U}{Y_R / \rho_R} \right) = (1 - \beta) s_U + \frac{m_L \cdot Y_A / \rho_A}{m_H + m_L} \left(m_H - (1 - \beta) \frac{s_U}{Y_A / \rho_A} \right).$$

Collecting terms,

$$\left(2 - \frac{m_L}{m_H + m_L} - \frac{1}{2} \right) \beta s_U = \left(1 - \frac{m_L}{m_H + m_L} \right) s_U + \frac{m_L m_H}{m_H + m_L} Y_A / \rho_A - \frac{Y_R / \rho_R}{2}$$

and thus

$$\beta^A = \frac{1}{3m_H + m_L} \left(2m_H + 2m_H m_L \frac{Y_A / \rho_A}{s_U} - (m_H + m_L) \frac{Y_R / \rho_R}{s_U} \right). \quad (\text{C.16})$$

This expression can be decomposed into

$$\beta^A = \frac{m_H + m_L}{3m_H + m_L} \left(1 + \frac{Y_A / \rho_A - Y_R / \rho_R}{s_U} \right) + \frac{(m_H(m_L - 1) + m_L(m_H - 1)) \frac{Y_A / \rho_A}{s_U} - (m_L - m_H)}{3m_H + m_L}.$$

With the expression for β^N given in equation (3.11), one obtains

$$\begin{aligned}\beta^A - \beta^N &= \frac{m_L - m_H}{6m_H + 2m_L} \left(1 + \frac{Y_A/\rho_A - Y_R/\rho_R}{s_U} \right) \\ &\quad + \frac{2}{6m_H + 2m_L} \left((m_H(m_L - 1) + m_L(m_H - 1)) \frac{Y_A/\rho_A}{s_U} - (m_L - m_H) \right) \\ &= \frac{m_L - m_H}{6m_H + 2m_L} \left(\frac{3Y_0(A) - Y_0(R)}{Y_0(U)} - 1 \right) + \frac{4m_L(m_H - 1)}{6m_H + 2m_L} \frac{Y_0(A)}{Y_0(U)}.\end{aligned}$$

Worker Distribution: Closed Form Solutions. So long as R -workers are not automated (cf. Appendix C.2),

$$\frac{s_R(j(\sigma_H, \lambda_H))}{s_R} = \frac{1}{2} \left(1 + \beta \frac{s_U}{Y_R/\rho_R} \right)$$

so that

$$\frac{\Delta^A s_R(j(\sigma_H, \lambda_H))}{s_R} = \frac{s_U}{2 \cdot Y_R/\rho_R} \cdot (\beta^A - \beta^N).$$

As Assumption 1 implies $2 \cdot Y_R/\rho_R > s_U$ (cf. Appendix C.2), $0 < \Delta^A s_R(j(\sigma_H, \lambda_H))/s_R < \beta^A - \beta^N$.

For A -worker employment, plugging β^A into the share of A -workers gives

$$\frac{s_A^A(j(\sigma_H, \lambda_L))}{s_A} = \frac{m_L - 1 + Y_0(A)^{-1}}{3m_H + m_L}$$

and thus

$$\frac{s_A^A(j(\sigma_H, \lambda_L))/s_A}{s_A^N(j(\sigma_H, \lambda_L))/s_A} = \frac{4[(m_L - 1)Y_0(A) + 1]}{3m_H + m_L}$$

with

$$\frac{s_A^A(j(\sigma_H, \lambda_L))/s_A}{s_A^N(j(\sigma_H, \lambda_L))/s_A} - 1 = \frac{4(m_L - 1)}{3m_H + m_L} Y_0(A) - \frac{3(m_H - 1) + (m_L - 1)}{3m_H + m_L}.$$

Output. As the relative statics of prices and output are unchanged, real wages do not change.

The multiplier of output, shared across occupations,¹⁰ is

$$\text{mult}^A(Y) = \frac{y^A(j(\sigma_H, \lambda_L))}{y^N(j(\sigma_H, \lambda_L))} = m_H \cdot \frac{s_A^A(j(\sigma_H, \lambda_L))/s_A}{s_A^N(j(\sigma_H, \lambda_L))/s_A} = \frac{4m_H [(m_L - 1)Y_0(A) + 1]}{3m_H + m_L}.$$

Conditions that rule out structural breaks. To sustain the worker-to-occupation matching structure, it is required that the expression obtained for β^A satisfies $\beta^A \leq 1$, and that with $\beta = \beta^A$, $s_R^A(j(\sigma_H, \lambda_H)) \leq s_R$ for the expression $s_R^A(j(\sigma_H, \lambda_H))$ derived under the assumption that the statics are preserved. With equation (C.16), $\beta^A \leq 1$ becomes

$$2m_H m_L \frac{Y_A}{\rho_A} \leq (m_H + m_L) \left(s_U + \frac{Y_R}{\rho_R} \right) \Leftrightarrow Y_0(A) \leq \frac{m_H + m_L}{2m_H m_L + m_H + m_L} \quad (\text{C.17})$$

¹⁰As the worker-to-occupation matching statics are sustained, relative prices and thus relative outputs do not change.

As $2m_H m_L > m_H + m_L$, this requirement is strictly stronger than $Y_0(A) \leq \frac{1}{2}$, the requirement for $\beta^N \leq 1$. Moreover,

$$\frac{s_R^A(j(\sigma_H, \lambda_H))}{s_R} = \frac{1}{2} \left(1 + \beta^A \frac{s_U}{Y_R/\rho_R} \right) \leq 1$$

can be re-arranged to

$$m_H s_U + m_H m_L \frac{Y_A}{\rho_A} \leq (2m_H + m_L) \frac{Y_R}{\rho_R} \Leftrightarrow Y_0(R) \geq \frac{m_H [1 + (m_L - 1)Y_0(A)]}{3m_H + m_L}$$

so that $Y_0(R)/Y_0(A)$ must be sufficiently large.

ADOPTION: POTENTIAL EQUILIBRIA POST-ADOPTION

Proposition 13. *If at most $\theta = A$ is automated, only A -workers are employed in $j(\sigma_H, \lambda_L)$.*

Proof. The proof repeatedly uses contradictions obtained from $Y_A/\rho_A < 1/2 \cdot Y_R/\rho_R$. To see the contradiction, note that by Assumption 1, this relationship gives two contradicting inequalities:

$$s_U > \frac{Y_R}{\rho_R} - \frac{Y_A}{\rho_A} > \frac{1}{2} \frac{Y_R}{\rho_R} \quad \text{and} \quad s_U < 3 \frac{Y_A}{\rho_A} - \frac{Y_R}{\rho_R} < \frac{1}{2} \frac{Y_R}{\rho_R}.$$

If R -workers enter $j(\sigma_H, \lambda_L)$, then $p_{j(\sigma_H, \lambda_L)} > p_j$ for $j \in J$ with $\lambda_j = \lambda_H$. Therefore, A and U are not employed in λ_H -occupations, and¹¹

$$y(j(\sigma_H, \lambda_L)) = \frac{1}{2} \max\{s_A y(i_A | j(\sigma_H, \lambda_L)) + s_U, 2s_A y(i_A | j(\sigma_H, \lambda_L))\} \geq s_A y(i_A | j(\sigma_H, \lambda_L)) > Y_A/\rho_A.$$

As R -labour distributes equally across λ_H -occupations,¹² $y(j(\sigma_L, \lambda_H)) \leq 1/2 \cdot Y_R/\rho_R$. By $p_{j(\sigma_L, \lambda_H)} < p_{j(\sigma_H, \lambda_L)}$, this implies $Y_A/\rho_A < 1/2 \cdot Y_R/\rho_R$, a contradiction. Thus, R -workers do not enter $j(\sigma_H, \lambda_L)$.

Suppose that U -workers enter $j(\sigma_H, \lambda_L)$. Then $\forall j \in J : p_{j(\sigma_H, \lambda_L)} \geq p_j$, and all A -workers are employed in $j(\sigma_H, \lambda_L)$ by Propositions 3 and 4 and $\forall j \in J \setminus \{j(\sigma_H, \lambda_L)\} : y^N(i_A | j(\sigma_H, \lambda_L)) > y^N(i_A | j)$. Therefore, in a scenario where all U - and R -workers do not work in $j(\sigma_H, \lambda_L)$, it holds that $y(j) \geq s_A y(i_A | j(\sigma_H, \lambda_L)) = y(j(\sigma_H, \lambda_L))$ for any $j \in J$, as U -flow to $j(\sigma_H, \lambda_L)$ increases $y(j(\sigma_H, \lambda_L))$ and decreases $y(j)$, $j \neq j(\sigma_H, \lambda_L)$.

For this scenario, U -workers enter $j(\sigma_H, \lambda_L)$ if $2s_U > Y_R$, which contradicts $s_U < 2Y_R/\rho_R$ (implied by Assumption 1). If R -workers enter $j(\sigma_L, \lambda_L)$, then $p_j < p_{j(\sigma_L, \lambda_L)}$ for $j \in J$ with $\lambda_j = \lambda_H$, and U -workers do not work in λ_H -occupations. Accordingly, R -workers distribute equally

¹¹ U -workers enter $j(\sigma_H, \lambda_L)$ if and only if $s_U \geq s_A y(i_A | j(\sigma_H, \lambda_L))$, in which case $y(j(\sigma_H, \lambda_L)) = y(j(\sigma_L, \lambda_L)) = [s_U + s_A y(i_A | j(\sigma_H, \lambda_L))]/2$, and otherwise $y(j(\sigma_H, \lambda_L)) = s_A y(i_A | j(\sigma_H, \lambda_L))$.

¹² By the λ_H -IC for R -labour,

$$\frac{p_{j(\sigma_H, \lambda_H)}}{p_{j(\sigma_L, \lambda_H)}} = \frac{y(j(\sigma_L, \lambda_H))}{y(j(\sigma_H, \lambda_H))} = \frac{s_R(j(\sigma_L, \lambda_H))}{s_R(j(\sigma_H, \lambda_H))} \frac{y^N(i_R | j(\sigma_L, \lambda_H))}{y^N(i_R | j(\sigma_H, \lambda_H))}$$

so that $s_R(j(\sigma_L, \lambda_H))/s_R(j(\sigma_H, \lambda_H)) = w_{i_R j(\sigma_H, \lambda_H)}^N / w_{i_R j(\sigma_L, \lambda_H)}^N = 1$.

across λ_H -occupations,¹³ and $y(j(\sigma_L, \lambda_H)) \leq 1/2 \cdot Y_R/\rho_R$. With $y(j(\sigma_L, \lambda_H)) \geq y(j(\sigma_H, \lambda_L))$, it follows that $1/2 \cdot Y_R/\rho_R \geq Y_A > Y_A/\rho_A$, a contradiction. Similarly, if R -workers are only employed in $j(\sigma_H, \lambda_H)$, then $y(j(\sigma_L, \lambda_H)) = 1/2 \cdot s_U$ which gives $s_U > 2Y_A/\rho_A$, a contradiction to Assumption 1.

Finally, if R -workers work in λ_H - and U -workers in σ_L -occupations, then $s_R(j(\sigma_H, \lambda_L))/s_R = 1/2 \cdot (1 - \beta s_U/(Y_R/\rho_R))$ (cf. Appendix C.2.2), so that equality of σ_L -outputs gives

$$\beta s_U = (1 - \beta)s_U + 1/2 \cdot \left(1 - \beta \frac{s_U}{Y_R/\rho_R}\right) Y_R/\rho_R \Rightarrow \beta s_U = \frac{1}{5} \left(\frac{Y_R}{\rho_R} + 2s_U\right) = y(j(\sigma_L, \lambda_L)). \quad (\text{C.18})$$

Furthermore,

$$\frac{Y_R}{\rho_R} \geq \frac{y(j(\sigma_H, \lambda_H))}{\rho_R} = y(j(\sigma_L, \lambda_H)) \geq y(j(\sigma_H, \lambda_L)) > \frac{Y_A}{\rho_A}.$$

By $y(j(\sigma_L, \lambda_L)) \geq y(j(\sigma_H, \lambda_L))$, it thus follows from equation (C.18) that

$$s_U > \frac{5}{2} \frac{Y_A}{\rho_A} - \frac{1}{2} \frac{Y_R}{\rho_R} = 3 \frac{Y_A}{\rho_A} - \frac{Y_R}{\rho_R} + \frac{1}{2} \left(\frac{Y_R}{\rho_R} - \frac{Y_A}{\rho_A}\right) > 3 \frac{Y_A}{\rho_A} - \frac{Y_R}{\rho_R},$$

which contradicts Assumption 1. Therefore, U -workers do not enter $j(\sigma_H, \lambda_L)$.

Fact 3 (A-Trigger: A-Employment in Abstract-Specialised Sectors). *If there exists $j \in J$ such that $\alpha = \alpha_{i_A j}^*$, then for any $k \in J$ with $\lambda_k = \lambda_L$, it holds that $s_A(k) > 0$.*

Proof. Assume the opposite. By Proposition 13, A -workers are not employed in $j(\sigma_L, \lambda_L)$. For restriction of A -employment to $j(\sigma_H, \lambda_L)$, a contradiction is obtained in analogy to the proof of Proposition 13. When k denotes a λ_H -occupation with $s_A(k) > 0$, by Proposition 5, k is at the A -trigger, and by $w_{i_A j(\sigma_L, \lambda_L)}^N \leq w_{i_A k}^N$,

$$\frac{p_{j(\sigma_L, \lambda_L)}}{p_k} \leq \frac{y^N(i_A|k)}{y^N(i_A|j(\sigma_L, \lambda_L))} < 1$$

so that $p_{j(\sigma_L, \lambda_L)} < \max_{j \in J} p_j$. As $j(\sigma_L, \lambda_L) = \arg \min_{j \in J} y^N(i_R|j)$ and U -workers are highest-price seeking, this contradicts positive employment in $j(\sigma_L, \lambda_L)$. \square

Fact 4 (A-Trigger: Over-identifying ICs). *If there exists $j \in J$ such that $\alpha = \alpha_{i_A j}^*$, if four structural ICs hold at α , three of them apply to A -workers and the fourth to λ_H -occupation indifference for R -workers, and it holds that $y^N(i_A|j(\sigma_H, \lambda_H))/y^N(i_A|j(\sigma_L, \lambda_H)) = \rho_R$.*

Proof. Denote by $\alpha_0^{init} = \min\{\alpha \in \mathbb{R}_+ : (\exists j \in J : \alpha = \alpha_{i_A j}^*)\}$ the level of technology that initially triggers A -adoption. For $\alpha > \alpha_0$ with $\alpha = \alpha_{i_A j(\sigma_H, \lambda_L)}^*$ (ongoing A -adoption), if A -workers do not enter λ_H -occupations, then $\rho_A p_{j(\sigma_H, \lambda_L)} = p_{j(\sigma_L, \lambda_L)} < p_{j(\sigma_L, \lambda_H)}$, and non-automated worker ICs do

¹³See Footnote 12.

not hold for λ_L -occupations. Thus, without A -entry to λ_H -occupations, IC over-identification does not occur.

If a λ_H -occupation A -worker trigger is hit at $\alpha = \alpha_0 > \alpha_0^{init}$, returning to both non-automated worker ICs during A -adoption would restore the no-automation prices, a contradiction to $\alpha = \alpha_{i_A j(\sigma_H, \lambda_L)}^*$ and $\alpha > \alpha_0^{init}$. Moreover, both U - and R -indifference between all occupations $j \neq j(\sigma_H, \lambda_L)$ give $p_{j(\sigma_L, \lambda_L)} = \max_{j \in J} p_j$ and rules out A -employment in λ_H -occupations.

Accordingly, if A -workers enter one λ_H -occupation but not both, IC-over-identification does not occur. If they enter both λ_H -occupations, then $p_j/p_k = (y^N(i_A|j)/y^N(i_A|k))^{-1}$ for any $j, k \in J$. Thus, U -ICs do not hold by $y^N(i_A|j) \neq y^N(i_A|k)$ for $k \neq j$, and the only R -IC potentially consistent with the A -ICs is the λ_H -IC. In this case,

$$\frac{p_{j(\sigma_H, \lambda_H)}}{p_{j(\sigma_L, \lambda_H)}} = \frac{y^N(i_A|j(\sigma_H, \lambda_H))}{y^N(i_A|j(\sigma_L, \lambda_H))} = \frac{y^N(i_R|j(\sigma_H, \lambda_H))}{y^N(i_R|j(\sigma_L, \lambda_H))} = \rho_R. \quad \square$$

DEEPENING

Proposition 14 (Breaks and Over-identification). *If at technology level $\alpha \in (\alpha^A, \alpha_0^U)$, there is IC simultaneity, then it is due to IC over-identification, i.e. there are four structural ICs at α .*

Proof. Any simultaneity is between a non-automated (here: U or R) and an automated (here: A) type (cf. Appendix C.3.3). By the result of Proposition 13, $j(\sigma_H, \lambda_L)$ is exclusively employed by A -workers, so that it is involved in simultaneous ICs at most indirectly. If simultaneity is due to an indirect relationship across occupations $j \neq j(\sigma_H, \lambda_L)$, then there are four structural ICs.

If the simultaneity is direct, there are three cases potentially¹⁴ consistent with the relative productivity rankings of workers (cf. Assumption 2 and Propositions 3 and 4):

1. A - and U -ICs for $j(\sigma_H, \lambda_H)$ and $j(\sigma_L, \lambda_L)$,
2. A - and R -ICs for λ_H -occupations, and
3. A - and R -ICs for $j(\sigma_H, \lambda_H)$ and $j(\sigma_L, \lambda_L)$.

For 1., U -employment in $j(\sigma_H, \lambda_H)$ implies $j(\sigma_H, \lambda_H) = \arg \max_{j \in J} p_j$, so that A - and R -workers do not work in $j(\sigma_L, \lambda_H)$. Accordingly, U -workers are employed in $j(\sigma_L, \lambda_H)$, and at least 2 ICs hold for U - and A -workers each,¹⁵ i.e. there is IC over-identification.

For 2., A -employment in $j(\sigma_L, \lambda_H)$ implies $j(\sigma_L, \lambda_H) = \arg \max_{j \in J} p_j$, so that U - and R -workers do not work in $j(\sigma_L, \lambda_L)$. Accordingly, A -workers are employed in all occupations, and together with the R -IC, there is IC over-identification.

¹⁴All scenarios require very specific parameter constellations, and 1. and 3. are mutually exclusive.

¹⁵ U -workers are employed in all occupations but $j(\sigma_H, \lambda_L)$ and A in all but $j(\sigma_L, \lambda_H)$.

For 3., R -employment in $j(\sigma_L, \lambda_L)$ implies $j(\sigma_L, \lambda_L) = \arg \max_{j \in J} p_j$, so that A - and U -workers do not work in $j(\sigma_L, \lambda_H)$. Accordingly, R -workers are employed in $j(\sigma_L, \lambda_H)$, and at least 2 ICs hold for R - and A -workers each, i.e. there is IC over-identification. \square

Proposition 15. Let $p_j^{U,\lim} := \lim_{\alpha \rightarrow \alpha_0^U} p_j$, and $p_{\max}^{U,\lim} := \max_{j \in J} p_j^{U,\lim}$. Then, for $j \in J$, $j \neq j(\sigma_H, \lambda_H)$ it holds that $p_j^{U,\lim} = p_{\max}^{U,\lim}$.

Proof. Consider a small neighbourhood left of α_0^U , i.e. $N = (\alpha_0^U - \varepsilon, \alpha_0^U)$, $\varepsilon > 0$ small. For what follows, note that $c_j^{K,L}(\alpha) = (\alpha w_{Hj}^A)^{-1}$ implies that if $c_k^{K,L}(\alpha) > c_j^{K,L}(\alpha)$, then automated workers do not work in k by $w_{Hk}^A < w_{Hj}^A$.

Assume first that $p_{j(\sigma_H, \lambda_L)}^{U,\lim} < p_{\max}^{U,\lim}$, i.e. that $p_{j(\sigma_H, \lambda_L)}^{U,\lim} \alpha^A < p_{\max}^{U,\lim} \alpha^A = 1$. Then, (cf. Eq. (3.16))

$$\lim_{\alpha \rightarrow \alpha_0^U} c_{j(\sigma_H, \lambda_L)}^{K,L}(\alpha) > \lim_{\alpha \rightarrow \alpha_0^U} c_j^{K,L}(\alpha) = 1$$

where $j \in J$ is such that $p_j^{U,\lim} = p_{\max}^{U,\lim}$. Thus, A -workers do not work in $j(\sigma_H, \lambda_L)$, a contradiction to Proposition 13 and positive employment in $j(\sigma_H, \lambda_L)$.

Assume now that $p_{j(\sigma_L, \lambda_L)}^{U,\lim} < p_{\max}^{U,\lim}$. Then, U - and R -workers are not employed in $j(\sigma_L, \lambda_L)$, and in analogy to the previous case, this occupation does also not employ A -workers. This contradicts positive employment in $j(\sigma_L, \lambda_L)$.

Assume finally that $p_{j(\sigma_L, \lambda_H)}^{U,\lim} < p_{\max}^{U,\lim}$. Then, U -workers are not employed in $j(\sigma_L, \lambda_H)$, and in analogy to above, also A -workers are not. Accordingly, this occupation exclusively employs R -labour, which gives $\rho_R p_{j(\sigma_H, \lambda_H)}^{U,\lim} \leq p_{j(\sigma_L, \lambda_H)}^{U,\lim}$, so that also $j(\sigma_H, \lambda_H)$ exclusively employs R -labour. By $p_j^{U,\lim} = p_{\max}^{U,\lim}$ for $j \in J$ with $\lambda_j = \lambda_L$, so that for these occupations $\lim_{\alpha \rightarrow \alpha_0^U} y^A(i_A|j) = (1 - \lambda_L)^{-1}$ and $\lim_{\alpha \rightarrow \alpha_0^U} y(j) = \frac{s_A/(1 - \lambda_L) + s_U}{2}$. From $p_{j(\sigma_L, \lambda_H)}^{U,\lim} < p_{\max}^{U,\lim}$, it follows from even R -distribution across λ_H -occupations (cf. Footnote 12) that

$$\frac{1}{2} \frac{Y_R}{\rho_R} = y(j(\sigma_L, \lambda_H)) > y(j(\sigma_L, \lambda_L)) = \frac{s_A/(1 - \lambda_L) + s_U}{2} > \frac{Y_A/\rho_A + s_U}{2}.$$

This gives $\frac{Y_R}{\rho_R} - Y_A/\rho_A > s_U$, a contradiction to Assumption 1. \square

Lemma 6. In a neighborhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of the initial U -trigger α_0^U , A -workers are employed in both λ_L -occupations, i.e. for any $j \in J$ with $\lambda_j = \lambda_L$, it holds that $s_A(j) > 0$.

Proof. By Proposition 13, A -workers are employed in $j(\sigma_H, \lambda_L)$ on $(\alpha_0^U - \varepsilon, \alpha_0^U)$. Suppose that $j(\sigma_L, \lambda_L)$ does not employ A -workers on $(\alpha_0^U - \varepsilon, \alpha_0^U)$. By positive labour in $j(\sigma_L, \lambda_L)$, $p_{j(\sigma_L, \lambda_L)} = \max_{j \in J} p_j$, and $j(\sigma_L, \lambda_H)$ does not employ A -workers (cf. Proposition 3).

If $p_{j(\sigma_H, \lambda_H)}^{U,\lim} < p_{\max}^{U,\lim}$, then $j(\sigma_H, \lambda_H)$ does not employ A -workers on $(\alpha_0^U - \varepsilon, \alpha_0^U)$. If this occupation

employed all R -workers, then $y(j(\sigma_H, \lambda_H)) = Y_R$ and

$$\frac{p_{j(\sigma_H, \lambda_H)}}{p_{j(\sigma_L, \lambda_H)}} = \frac{s_U/2}{Y_R} = \rho_R^{-1} \frac{s_U/2}{Y_R/\rho_R} < \rho_R^{-1}$$

by Assumption 1, a contradiction to R -employment in $j(\sigma_H, \lambda_H)$. Thus, some R are employed in $j(\sigma_L, \lambda_H)$, and solving for β in analogy to the no-automation equilibrium gives

$$y(j(\sigma_L, \lambda_L)) = (1 - \beta)s_U = \frac{s_U + Y_R/\rho_R}{3} < Y_A/\rho_A$$

by Assumption 1. With $y(j(\sigma_L, \lambda_L)) \approx y(j(\sigma_H, \lambda_L))$ on $(\alpha_0^U - \varepsilon, \alpha_0^U)$, this gives

$$y(j(\sigma_H, \lambda_L)) = \frac{s_A}{1 - \lambda_L} = Y_A/\rho_A \frac{y^N(i_A|j(\sigma_H, \lambda_L))}{1 - \lambda_L} < Y_A/\rho_A,$$

which contradicts $y^N(i_A|j(\sigma_H, \lambda_L)) < 1$.

If instead $p_{j(\sigma_H, \lambda_H)}^{U, \lim} = p_{\max}^{U, \lim}$, all R -workers are employed in $j(\sigma_H, \lambda_H)$ on $(\alpha_0^U - \varepsilon, \alpha_0^U)$, and $y(j) \leq s_U/2$ for $j \in J$ with $\sigma_j = \sigma_L$. From $y(j(\sigma_H, \lambda_H)) \geq Y_R$ and $y(j(\sigma_H, \lambda_H)) \approx y(j(\sigma_L, \lambda_H))$, it follows that $Y_R/\rho_R < s_U/2$ or respectively, $s_U > 2Y_R/\rho_R$, a contradiction to Assumption 1 (cf. the proof of Proposition 12 in Appendix C.2). \square

Lemma 7. *In a neighbourhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of the initial U -trigger α_0^U , it holds that $p_{j(\sigma_L, \lambda_H)} = \max_{j \in J} p_j$.*

Proof. Assume the opposite. Then, only R -workers are employed in λ_H -occupations, with $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-1} p_{j(\sigma_L, \lambda_H)} < \max_{j \in J} p_j$ and $y(j(\sigma_L, \lambda_H)) = 1/2 \cdot Y_R/\rho_R$ (cf. Footnote 12). With Proposition 3 and Lemma 6, $j(\sigma_L, \lambda_L) = \arg \max_{j \in J} p_j$. This gives

$$\frac{Y_R}{\rho_R} = 2y(j(\sigma_L, \lambda_H)) > 2y(j(\sigma_L, \lambda_L))$$

and thus

$$\frac{Y_R}{\rho_R} \geq 2 \lim_{\alpha \rightarrow \alpha_0^U} y(j(\sigma_L, \lambda_L)) = s_U + \frac{s_A}{1 - \lambda_L} > s_U + Y_A/\rho_A$$

or respectively, $s_U < Y_R/\rho_R - Y_A/\rho_A$, a contradiction to Assumption 1. \square

Lemma 8 (U -Labour Mobility due to A -Deepening). *For A -deepening from α^A to a neighbourhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of the initial U -trigger α_0^U , if U -workers are still present in $j(\sigma_L, \lambda_L)$ at α^A , $s_U(j(\sigma_L, \lambda_L))$ declines strictly, and if they are not, they do not re-enter this occupation.*

Proof. Assume first that $p_{j(\sigma_H, \lambda_H)}^{U, \lim} < p_{\max}^{U, \lim}$. Here, $j(\sigma_L, \lambda_L)$ employs U -workers only if $p_{j(\sigma_L, \lambda_L)} = p_{j(\sigma_L, \lambda_H)}$ by Lemma 7, and A - (R -) workers are thus employed only in the λ_L - (λ_H -) sub-industry. At the A -trigger, $p_{j(\sigma_L, \lambda_H)} = \max_{j \in J} p_j$ (cf. Section 3.4.2), so that with deepening, $y(j(\sigma_L, \lambda_L))/y(j(\sigma_L, \lambda_H))$

decreases weakly, as does $y(j(\sigma_H, \lambda_L))/y(j(\sigma_L, \lambda_H))$.¹⁶ If $s_U(j(\sigma_L, \lambda_L))$ increases weakly, as A -labour does move towards the λ_H -sub-industry, $y(j(\sigma_L, \lambda_H))$ weakly declines, so that both λ_L -outputs decline weakly. This poses a contradiction, as $y^A(i_A|j)$ increases strictly for any $j \in J$ with $\lambda_j = \lambda_L$, and $s_A(j(\sigma_H, \lambda_L)) + s_A(j(\sigma_L, \lambda_L))$ does not decline.

Assume now that $p_{j(\sigma_H, \lambda_H)}^{U, \lim} = p_{\max}^{U, \lim}$, i.e. $\forall j \in J : p_j|_{\alpha=\alpha_0^U} = 1$. By $p_{j(\sigma_L, \lambda_H)} = \max_{j \in J} p_j$ at the A -trigger, $y(j(\sigma_L, \lambda_H))$ increases relative to any $y(j)$, $j \in J$. As $j(\sigma_H, \lambda_H)$ employs all R -workers on $(\alpha_0^U - \varepsilon, \alpha_0^U)$, there is weak R -flow from $j(\sigma_L, \lambda_H)$ to $j(\sigma_H, \lambda_H)$. If $s_U(j(\sigma_L, \lambda_L))$ increases weakly, then $y(j(\sigma_L, \lambda_H))$ declines. By its relative increase, all $y(j)$, $j \in J$ decline. The decline of $y(j(\sigma_H, \lambda_H))/y(j(\sigma_L, \lambda_H))$ implies strict A -flow away from $j(\sigma_H, \lambda_H)$. However, as $y^A(i_A|j)$ increases strictly for any $j \in J$ with $\lambda_j = \lambda_L$, and $s_A(j(\sigma_H, \lambda_L)) + s_A(j(\sigma_L, \lambda_L))$ does not decline, at least one $y(j)$, $j \in J$ with $\lambda_j = \lambda_L$ increases strictly, a contradiction. \square

Proposition 10. *In a neighbourhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of α_0^U ,*

1. *Both λ_L -occupations employ A -workers, i.e. for any $j \in J$ with $\lambda_j = \lambda_L$, it holds that $s_A(j) > 0$;*
2. *$p_{j(\sigma_L, \lambda_H)} = \max_{j \in J} p_j$;*

and $s_U(j(\sigma_L, \lambda_L))$ is (strictly) smaller than $s_U(j(\sigma_L, \lambda_L))|_{\alpha=\alpha^A}$ (if $s_U(j(\sigma_L, \lambda_L))|_{\alpha=\alpha^A} > 0$).

Proof. Follows directly from Lemmas 6-8. \square

Wage Effects. For wages at the U -trigger, the only relevant compositional component is whether $j(\sigma_L, \lambda_L)$ employs R -labour. If so, $p_j = \rho_R^{\frac{1}{4}}$ for $j \neq j(\sigma_H, \lambda_H)$ and $p_{j(\sigma_H, \lambda_H)} = \rho_R^{-\frac{3}{4}}$. This gives

$$w_A^{U,N} = w_U^{U,N} = \rho_R^{\frac{1}{4}}, \quad w_R^{U,N} = \rho_R^{\frac{1}{4}} y(i_R|j(\sigma_L, \lambda_H))$$

where the index U, N refers to the state at the U -trigger prior to U -adoption. The θ -wage multiplier relative to the initial equilibrium, $m_{w,\theta}^{U,N}$, is thus given by

$$m_{w,A}^{U,N} = \rho_A^{-\frac{1}{4}} \cdot \frac{1}{y^N(i_A|j(\sigma_L, \lambda_L))} \quad m_{w,U}^{U,N} = m_{w,R}^{U,N} = \rho_A^{-\frac{1}{4}}$$

with a relative wage effect of $m_{w,A}^{U,N}/m_{w,\theta}^{U,N} = y^N(i_A|j(\sigma_L, \lambda_L))^{-1} > 1$ for $\theta \in \{U, R\}$. The coefficient $\rho_A^{-1/4}$ captures the output market effect of growth in highest-price occupations, whereas $y^N(i_A|j(\sigma_L, \lambda_L))^{-1}$ refers to A -workers' gain in effective productivity in the highest-price occupation $j(\sigma_L, \lambda_L)$. As output market effects are shared by all workers, the relative $A - \theta$ wage, $\theta \in \{R, U\}$, increases by the effective productivity gain. $m_{w,A}^{U,N} > 1$ is more directly seen from the representation for occupation $j(\sigma_H, \lambda_L)$, $m_{w,A}^{U,N} = \rho_A^{3/4} y^N(i_A|j(\sigma_H, \lambda_L))^{-1}$, where both factors are strictly larger than 1, i.e. both the output market and the effective productivity effect are positive.

¹⁶ $y(j(\sigma_H, \lambda_L))/y(j(\sigma_L, \lambda_H)) = p_{j(\sigma_L, \lambda_L)}/p_{j(\sigma_H, \lambda_L)} \cdot y(j(\sigma_L, \lambda_L))/y(j(\sigma_L, \lambda_H))$, and both factors decrease weakly.

If instead, all R -workers have migrated to the routine-most occupation $j(\sigma_H, \lambda_H)$ at the U -trigger, then $\max_{j \in J} p_j / p_{j(\sigma_H, \lambda_H)} \in [1, \rho_R]$. Relative to the previous scenario, R -labour enjoys a scarcity gain from growth in occupations $j \neq j(\sigma_H, \lambda_H)$, i.e. a positive output market effect, so that R -wages are augmented and other wages depressed. In the most extreme case of $p_{j(\sigma_H, \lambda_H)} = \max_{j \in J} p_j = 1$, the R -wage is higher by the factor $\rho_R^{\frac{3}{4}}$, at the expense of other wages, which are lower by $\rho_R^{-\frac{1}{4}}$, so that A -wages may decrease with deepening if ρ_R is relatively large. The output market effect may dominate only in an industry with strong initial skill mismatch, i.e. ρ_R large, and for relatively ineffective technology.

Industry Growth. Define

$$\gamma_A^U := \frac{y^A(i_A | j(\sigma_L, \lambda_L))|_{\alpha=\alpha_0^U}}{y^N(i_A | j(\sigma_L, \lambda_L))} = [(1 - \lambda_L)y^N(i_A | j(\sigma_L, \lambda_L))]^{-1}$$

as the multiplier on A -output in the initial highest-price occupation of A -employment. Without structural breaks relative to the initial equilibrium, output in occupations $j \neq j(\sigma_H, \lambda_H)$ is

$$y(j) = \frac{1}{2} \left(\frac{s_A}{1 - \lambda_L} + (1 - \beta^{U,N})s_U \right) = \frac{(\gamma_A^U - 1)Y_A/\rho_A + Y^{min}}{4}$$

where $\beta^{U,N}$ solves $y(j(\sigma_L, \lambda_H)) = y(j(\sigma_L, \lambda_L))$ or respectively, $\beta s_U = \frac{1}{2} \left(\frac{s_A}{1 - \lambda_L} + (1 - \beta)s_U \right)$, for β . This gives for $j \in J$ with $\sigma_j = \sigma_L$

$$\frac{y^{U,N}(j)}{y^N(j)} - 1 = (\gamma_A^U - 1)Y_0(A) \quad (C.19)$$

and occupation growth is augmented by the A -worker level productivity surge and their initial income share. As $p_{j(\sigma_L, \lambda_H)} = \rho_R p_{j(\sigma_H, \lambda_H)}$ still holds, equation (C.19) also applies to $j(\sigma_H, \lambda_H)$. Lastly,

$$\frac{y^{U,N}(j(\sigma_H, \lambda_L))}{y^N(j(\sigma_H, \lambda_L))} - 1 = \frac{1}{\rho_A} \left((\gamma_A^U - 1)Y_0(A) - (\rho_A - 1) \right)$$

which may be negative depending on the model parameters, and is in any case strictly smaller than the multiplier of other occupations. The multiplier on industry output is

$$mult(Y) = \left(\frac{1}{\rho_A} \right)^{\frac{1}{4}} \left(1 + (\gamma_A^U - 1)Y_0(A) \right)$$

which highlights the two competing forces of A -deepening, A -productivity growth and the relative decline in A -labour demand in the abstract-specialised occupation. It can be shown

analytically that the former effect always dominates: the multiplier strictly exceeds one if

$$\frac{Y_A/\rho_A}{Y^{min}} > \frac{\rho_A^{\frac{1}{4}} - 1}{\gamma_A^U - 1}$$

for which, by $\frac{Y_A/\rho_A}{Y^{min}} > \frac{1}{4}$,¹⁷ it is sufficient that $(\rho_A^{\frac{1}{4}} - 1)/(\gamma_A^U - 1) \leq 1/4$. As $y(i_A|j) \in [1 - \lambda_j, 1]$ for any $j \in J$, $\rho_A \leq \frac{1}{1 - \lambda_L}$, and $\gamma_A^U = \frac{1}{1 - \lambda_L} \frac{1}{y^N(i_A|j(\sigma_L, \lambda_L))} \geq \frac{1}{1 - \lambda_L}$. Accordingly,

$$\frac{\rho_A^{\frac{1}{4}} - 1}{\gamma_A^U - 1} \leq \frac{(1 - \lambda_L)^{-\frac{1}{4}} - 1}{(1 - \lambda_L)^{-1} - 1} =: f(\lambda_L) \leq \frac{1}{4}$$

where the last inequality is readily seen from closer investigation of the function f .¹⁸ Thus, restructuring, while potentially lowering one occupation level output, it may only dampen but not entirely stop overall output growth.

C.3.5 U-AUTOMATION

ADOPTION

Proposition 16 (*R-workers and U-automation*). *The mass $s_R(j(\sigma_L, \lambda_H))$ of R-workers in $j(\sigma_L, \lambda_H)$ declines with U-adoption, and strictly so if $\lim_{\alpha \rightarrow \alpha_0^U} s_R(j(\sigma_L, \lambda_H)) > 0$.*

Proof. If $\lim_{\alpha \rightarrow \alpha_0^U} s_R(j(\sigma_L, \lambda_H)) > 0$, then in a neighbourhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of the initial U-trigger, $\max_{j \in J} p_j/p_{j(\sigma_H, \lambda_H)} = \rho_R$, and A- and U-workers do not work in $j(\sigma_H, \lambda_H)$. If $s_R(j(\sigma_L, \lambda_H))$ increases, then $\max_{j \in J} p_j/p_{j(\sigma_H, \lambda_H)} = \rho_R$ also at the U-trigger, and only R-workers are employed in $j(\sigma_H, \lambda_H)$. Thus, $y(j(\sigma_H, \lambda_H))$ declines and so does $y(j)/y(j(\sigma_H, \lambda_H))$ for any $j \in J$, such that all occupation-level outputs decline. This may occur only if $\sum_{j \neq j(\sigma_H, \lambda_H)} s_E(j)$ declines, a contradiction.

If instead $\lim_{\alpha \rightarrow \alpha_0^U} s_R(j(\sigma_L, \lambda_H)) = 0$, then in a neighbourhood $(\alpha_0^U - \varepsilon, \alpha_0^U)$ left of the initial U-trigger, $\max_{j \in J} p_j/p_{j(\sigma_H, \lambda_H)} < \rho_R$. R-entry to $j(\sigma_L, \lambda_H)$ at the U-trigger would imply $\max_{j \in J} p_j/p_{j(\sigma_H, \lambda_H)} = \rho_R$, and a contradiction is obtained in analogy to the previous case. \square

Industry Growth. When the initial type-to-occupation matching is preserved, it follows

¹⁷This is an immediate implication of $s_U < 3Y_A/\rho_A - Y_R/\rho_R$.

¹⁸ $f(\lambda)$ is strictly decreasing in λ for $\lambda \in (0, 1)$:

$$\frac{\partial}{\partial \lambda} \ln(f(\lambda)) = -\frac{1}{\lambda} + \frac{1}{1 - \lambda} \left(\frac{1/4 \cdot (1 - \lambda)^{-\frac{1}{4}} - 1}{(1 - \lambda)^{-\frac{1}{4}} - 1} \right) < 0,$$

and by L'Hôpital's rule,

$$\lim_{\lambda \rightarrow 0} f(\lambda) = \lim_{\lambda \rightarrow 0} \frac{(1 - \lambda)^{\frac{3}{4}} - (1 - \lambda)}{\lambda} = \lim_{\lambda \rightarrow 0} \frac{-3/4 \cdot (1 - \lambda)^{-\frac{1}{4}} + 1}{1} = \frac{1}{4}.$$

from the computations in Appendix C.3.4 (section *Industry Growth*) that

$$Y = \frac{\rho_R^{\frac{1}{4}}}{4} \left(1 + (\gamma_A^U - 1)Y_0(A)\right) Y^{min}. \quad (C.20)$$

Denote by $s_H(\lambda_L) := \sum_{j \in J: \lambda_j = \lambda_L} s_H(j)$ the share of homogeneous workers in the λ_L -occupations. Then, by H -worker indifference between σ_L -occupations,

$$\frac{s_H(\lambda_L)}{2} (s_A + s_U) \frac{1}{1 - \lambda_L} = (1 - s_H(\lambda_L)) (s_A + s_U) \frac{1}{1 - \lambda_H} + \left(1 - \frac{s_R(j(\sigma_H, \lambda_H))}{s_R}\right) \frac{Y_R}{\rho_R}$$

which gives

$$s_H(\lambda_L) = \frac{1 - \lambda_L}{1/2(1 - \lambda_H) + 1 - \lambda_L} \left(1 + (1 - \lambda_H) \left(1 - \frac{s_R(j(\sigma_H, \lambda_H))}{s_R}\right) \frac{Y_R/\rho_R}{s_A + s_U}\right)$$

such that for $j \neq j(\sigma_H, \lambda_H)$,

$$y(j) = \frac{s_H(\lambda_L)(s_A + s_U)}{2(1 - \lambda_L)} = \frac{1}{1 - \lambda_H + 2(1 - \lambda_L)} \left(s_A + s_U + (1 - \lambda_H) \left(1 - \frac{s_R(j(\sigma_H, \lambda_H))}{s_R}\right) \frac{Y_R}{\rho_R}\right).$$

Using $y(j(\sigma_H, \lambda_H)) = \rho_R y(j)$ for $j \neq j(\sigma_H, \lambda_H)$,

$$\frac{s_R(j(\sigma_H, \lambda_H))}{s_R} \frac{Y_R}{\rho_R} = \frac{1}{1 - \lambda_H + 2(1 - \lambda_L)} \left(s_A + s_U + (1 - \lambda_H) \left(1 - \frac{s_R(j(\sigma_H, \lambda_H))}{s_R}\right) \frac{Y_R}{\rho_R}\right)$$

and solving for $\frac{s_R(j(\sigma_H, \lambda_H))}{s_R}$ gives¹⁹

$$\frac{s_R(j(\sigma_H, \lambda_H))}{s_R} = \frac{1}{2(1 - \lambda_H + 1 - \lambda_L)} \left(1 - \lambda_H + \frac{s_A + s_U}{Y_R/\rho_R}\right).$$

For $j \neq j(\sigma_H, \lambda_H)$, one obtains

$$\begin{aligned} y(j) &= \frac{1}{\rho_R} y(j(\sigma_H, \lambda_H)) = \frac{1}{\rho_R} \frac{1}{2(1 - \lambda_H + 1 - \lambda_L)} \left(1 - \lambda_H + \frac{s_A + s_U}{Y_R/\rho_R}\right) Y_R \\ &= \frac{(1 - \lambda_H) Y_R/\rho_R + s_A + s_U}{2(1 - \lambda_H + 1 - \lambda_L)} \end{aligned}$$

and thus

$$Y = \rho_R^{\frac{1}{4}} \frac{(1 - \lambda_H) Y_R/\rho_R + s_A + s_U}{2(1 - \lambda_H + 1 - \lambda_L)}.$$

¹⁹With the following result, one may derive the no-break condition

$$\frac{1}{2(1 - \lambda_H + 1 - \lambda_L)} \left(1 - \lambda_H + \frac{s_A + s_U}{Y_R/\rho_R}\right) \leq 1 \Leftrightarrow \frac{Y_R}{\rho_R} \geq \frac{s_A + s_U}{1 - \lambda_H + 2(1 - \lambda_L)}.$$

With equation (C.20), the growth multiplier of U -automation at the trigger is

$$mult^U(Y) = \frac{s_A + s_U + (1 - \lambda_H)Y_R/\rho_R}{s_A/(1 - \lambda_L) + s_U + Y_R/\rho_R} \frac{2}{1 - \lambda_H + 1 - \lambda_L}. \quad (C.21)$$

The multiplier strictly exceeds one, as it applies to all occupations uniformly, and $\psi(j(\sigma_H, \lambda_H))$ strictly increases by Proposition 16. For its comparative statics, it trivially holds that $\frac{\partial mult^U(Y)}{\partial s_U} > 0$ and $\frac{\partial mult^U(Y)}{\partial Y_R/\rho_R} > 0$. Further,

$$\frac{\partial mult^U(Y)}{\partial \lambda_H} = \frac{2}{s_A/(1 - \lambda_L) + s_U + Y_R/\rho_R} \frac{s_A + s_U + (1 - \lambda_H)Y_R/\rho_R - (1 - \lambda_H + 1 - \lambda_L)Y_R/\rho_R}{(1 - \lambda_H + 1 - \lambda_L)^2}$$

which is strictly positive if and only if $s_A + s_U > (1 - \lambda_L)Y_R/\rho_R$. By Assumption 1, $s_U + Y_A/\rho_A > Y_R/\rho_R$, and by $s_A > Y_A/\rho_A$, this equivalent condition is satisfied. Thus, $\frac{\partial mult^U(Y)}{\partial \lambda_H} > 0$. Conversely, $\frac{\partial mult^U(Y)}{\partial \lambda_L} \propto Y_0(U) + Y_0(R) - \frac{1 - \lambda_H}{1 - \lambda_L} \gamma_A^U Y_0(A)$ and $\frac{\partial mult^U(Y)}{\partial s_A} \propto (\lambda_H - \lambda_L)Y_0(R) - \lambda_L Y_0(U)$, and both expressions are ambiguous in sign without further parameter restrictions.

D. EIDESSTADTLICHE VERSICHERUNG

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und ohne fremde Hilfe verfasst und keine anderen Hilfsmittel als die angegebenen verwendet habe.

Insbesondere versichere ich, dass ich alle wörtlichen und sinngemäßen Übernahmen aus anderen Werken als solche kenntlich gemacht habe.

Weiterhin erkläre ich mich damit einverstanden, dass die Universität meine Dissertation zum Zwecke des Plagiatsabgleich in elektronischer Form speichert, an Dritte versendet, und Dritte die Dissertation zu diesem Zwecke verarbeiten.